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ABSTRACT

Research indicates that students have great difficulty solving certain algebra word problems. These solutions, moreover, appear to be due to some situational factors characteristic of algebra problems, e.g., presentation and response format. This study investigates students' preconceptions, post facto alternative concepts based on key features of the presentation, and response modes in algebra problems. Sixteen problems were constructed and administered among college and secondary school students. Based on the reading of the literature and analysis of responses, two types of errors were identified: reversal and qualitative errors. By cross-checking these errors with the presentation and response format, and by integrating the performance results of the algebra problems, the causes and types of errors were identified as being either pedagogical (formal) or intuitive. (Contains 23 references.) (Author/NB)

The Role of Presentation and Response Format in Understanding, Preconceptions and Alternative Concepts in Algebra Problems

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Abstract

Research indicates that students have great difficulty solving certain algebra word problems. These solutions moreover, appear to be due to some situational factors characteristic of algebra problems e.g., presentation and response format. This study investigates students' preconceptions, *post facto* alternative concepts based on key features of the presentation, and response modes in algebra problems. Sixteen problems were constructed and administered among college and secondary school students. Based on the reading of the literature and analysis of responses, two types of errors were identified, reversal and qualitative errors. By cross-checking these errors with the presentation and response format, and by integrating the performance results of the algebra problems, the causes and types of errors were identified as being either pedagogical (formal) or intuitive.

INTRODUCTION

In mathematics education, two important aspects of misconceptions, alternative conceptions or preconceptions are suggested. On the one hand, preconceptions may be guided by experiential reality in which students confuse real experiences with formal theories. On the other hand, misconceptions may be caused by some formal instruction where a new conception conflicts with an existing one. In both situations presentation in mathematics plays an important role in students understanding and processing of information.

During the last decade, there has been growing interest among mathematics educators and psychologists in students' misconceptions, intuitive ideas, alternative conceptions, and/or preconceptions in the field of mathematics education. The arousal of interest has come out of a shift of an emphasis from a procedural to a conceptual knowledge of understanding. Misconceptions are no more considered errors but as learned concepts with equal efforts invested in learning concepts in the wrong way.

Two important frames guide my work in this paper. First, there is a view that students' intuitive ideas override a conception or conflict with a theoretical conception. This view is based on theories of information processing which suggest features of imagined appearances that are concrete in character, that is to say the closer these appearances are to objects found in the environment, the less hard it is to construct an image and a functional relation of the quantities in the algebra problem. Secondly, the view that errors due to some pedagogical factor could be related to the cognitive domain and knowledge sequence. Accordingly, in constructing a problem solution, it is found that preconceptions and alternative concepts are guided by prior knowledge of simpler, exemplary or regular "constructs," that are at odds with mathematical theories. These schemes are confusions resulting from instructional re-aligning and shift in modes of instruction or content matter variation. The degree to which the presentational features of the algebra problem model the experiential phenomena and influence students' erroneous responses is questionable. If features of the problem presentation will elicit a response that is in one form of representation or another, it is equally possible that the form of the response which is elicited from students will likewise influence students' approaches or answers.

Generally speaking, preconceptions in mathematics, particularly in algebra, graphs and functions, are primarily pedagogical in character, that is, they are caused by some formal instruction, resulting from "reasonable although unsuccessful attempts to adapt previously acquired knowledge to a new situation" (Matz, 1980, p.95). As evaluated by Tirosh (1990), these conceptions are represented as inconsistencies that arise between students' structure of mathematics and conventional mathematical theory. In general, preconceptions of this sort may become embedded in students' schemas and lead to serious errors in computation. Their reoccurrence is "chronic": they become part of a framework where wrong predictions are made about problem solutions, and they are very hard to remedy. In this study, we add to the knowledge of preconceptions by recognizing that they could be intuitive or cumulatively formal (pedagogical), and in being learned, could be mis-learned.

In this research, I have attempted to study student responses on algebra problems of the propositional relation kind and to draw some conclusions concerning the type and nature of the error. Research on problem solving has provided some insight on student problem-solving approaches and errors in solving a propositional relation algebra problem. The translation of nouns in word problems of the propositional relation kind to algebraic equations, suggests a syntactic approach from words transformed to symbols (Clement, 1982; Sims-Knight & Kaput, 1983a; Gerlach, 1986). From a representational perspective, this error can be explained from a dual operational approach. On one hand, students syntactically observe and automatically copy nouns into symbols in the word problem; on the other hand, they quantitatively relate the objects since there are more of object 'X' and less of object 'Y,' although they attach the noun referents to the symbol as it appears in the syntax of the word problem. In both cases, the errors are so pervasive that this type of error is generally considered to be instructional, either formal or informal. In this paper, I have tried to identify some of the domain-specific errors (either intuitive or formal) in algebra related to the propositional relation algebra problem, through the manipulation and control of the presentation and response format. The analysis of these two salient features of algebra problem, should reveal the phenomena associated with the error. This in turn could help to answer the main question: To what degree do the presentational features of the algebra problem model the experiential phenomena and influence student problem solving, and to what extent do the problem presentations and responses determine the pedagogical or intuitive nature of the error produced? If features of the problem presentation and translation could influence a response, then it is equally possible that the form of the response, which is provided as stimulus and elicited, could indirectly produce the wrong response.

The first step of the present study, identified and operationally defined the type of error produced on the problems of the propositional relation algebra type (Rosnick and Clement, 1980; Clement, 1982; Lochhead and Mestre, 1988; Mestre and Gerace, 1986; Niaz, 1989) as a means of establishing a relation between the errors and type of features of the algebra word problem. In this way, the experiential phenomena, which may be attributed to the algebra problem, can be related to the error. Consequently, certain errors that are projected from student's conceived world will be identified with the real world. Thus, one may to some degree derive or refute the influences of the real world phenomena on students solving the algebra word problem.

Several researchers have viewed students difficulties in solving algebra word problems, especially those of the propositional kind, primarily as a problem in students handling of the verbal structure of the word problem (e.g., Mestre, Gerace and Lochhead, 1982; Mestre and Gerace, 1986). This framework, however, represents a very limited view of algebra word problems and problem-solving behavior. No researcher in this area has approached the problem in terms of the various modes in which the problem may be presented (i.e., pictorially, verbally or symbolically) or the mode of response of the answer, nor in terms of the translation of the relations in these various modes of presentations from one mode to another as a dyad from translation to response. Furthermore, no research has categorized or studied the errors in terms of presentation, response, and cross-translation formats. These very basic limitations in the research literature constitute a principal driving force behind this study.

Algebra Word Problems

Sixteen word problems were constructed for the study. These problems had three different presentation formats: the pictorial, verbal, and symbolic form, with their respective response formats. Accordingly, students had to process and translate each problem from its presentation mode into a particular response format, giving rise to the possibility of six modes or types of cross translations from one mode to another. For example, while the relationship in the problem is stated in verbal terms, the answer must be expressed in symbolic or pictorial modes. In effect, six cross translations from and to a mode of response were generated.

Problem Presentation and Response Format. Problem presentations format (pictorial (P), verbal (V), and symbolic (S)) are linked to problem response formats or modes (P,V and S) and cross translated with the six modes of cross-translations: being verbal-pictorial (vp), verbal-symbolic (vs), pictorial-verbal (pv), pictorial-symbolic (ps), symbolic-verbal (sv) and symbolic-pictorial (sp) (see Table 1).

Response Mode. There are two response types, the generative and passive modes. The generative translation suggests that when given a problem in one representative form, students are asked to give and generate an answer in alternative form. The passive translation explains that when given a problem with two representative forms, students determine the equivalence of the two given, the question and the response. The generative response mode category contains the generative translation which are the ps, vs and vp problems and the passive response mode category contains the pv, sp and sv problems. Our categories are logically interpretable. For example, the translation of a word problem from pictorial form to the verbal form may suggest a large solution pool that could generate a large pool of errors. In this situation, a passive response may be more appropriate where students select among a number of responses suggested. However, the question may arise as to why the symbolic to pictorial (sp) and verbal to pictorial (vp) were placed in two different categories i.e., the sp in the passive and the vp in the generative. Logical analyses may show us that symbolic presentations, e.g., "6f=2G," when translated to the pictorial form, have a large pool of answers with a variety of errors that are exhaustively inconclusive; a passive translation type, however, would limit the responses to the taxonomy of errors conceptualized for this study (i.e., reversal error, qualitative response, or incongruent response- see types of errors section) and hence facilitate an easy measure of the errors.

(Tables 1 and 2 about here)

The two types of response modes (See Table 2 for the generative and passive) require separate but related cognitive operations. In the passive mode, the act of recognition is required, while in the generative translation, the production of the response is entailed (Ashcroft, 1989).

Types of Errors

The errors examined in this study were derived from the literature, see for instance Clement (1982); Clement, Lochhead & Monk (1981); Wollman (1983); Rosnick and Clement (1980); Gerlach (1986); Niaz (1989); Mestre (1985). These studies attempted to explore errors related to the linguistic structure of the verbal problem. Almost all the researchers listed found that students (for the most part first-year engineering students who have taken two semesters of calculus) committed a "variable reversal error." For example, when given the following propositional relation problem:

> "write an equation, using the variables S and P to represent the following statement: There are six times as many students as professors at this university. Use S for the number of students and P for the number of Professors."

students reversed the coefficients in the equations by writing, "6P=S." A number of studies viewed the reversal error in terms of a cognitive operational model in which students were consciously aware of the quantities, but in their attempts to devise a solution, they make a translation error (see Rosnick and Clement, 1980; Clement, 1982). Several other studies have associated the reversal error with linguistic factors, e.g., Clement Lochead and Monk (1981); Mestre; Gerace (1986), suggested that students associated nouns with quantity and directly translated them into symbols.

Gestalt ideas of the error indicate that students may have forged ideas intuitively; generalizing from one situation to another generated either conflicts or wrong responses. In a preliminary study, I found that students would change the discrete quantity to the continuous quantity in response to the question relating the number of dimes to the number of nickels (10 and 5 currency units respectively), i.e., two quarters may qualify as two objects or the value for currency. My view is that students may have forged their response in accordance with their day-to-day familiarity with these quantities and approached the problem in a lay and naive approach. This schema is not that different in the field of science. Physics students, for instance, intuitively predict a projectile of a falling body having a straight vertical drop. Similarly, they believe that only springs have reaction forces that push back objects whereas solid objects have none. The "very lay" approach encouraged by a data-logical frame, creaks a conflict between concepts and generalizations arising from the concepts and applied to new situations.

Some theorists in the field might argue that errors arising from intuitive ideas and pedagogical practice might not really define these as criteria for responses. The approach is to use a nomothetic model of explanation where considerations that are important in explaining classes of action or events are suggested to support the hypothesis. The key to this approach is the problem design into a problem presentation with the type of problem responses desired. In this way, student responses are associated with the question (stimuli) and format of responses. The analyses will reflect not only the answer as it is associated with the question, but in a functional sense from question to response that what effects are between may be considered as the underlying cognitive activities which underline the question-response format, i.e., the type of error found in the responses linked to the question.

Two errors, the reversal error and qualitative error were analyzed and classified through the reading of the literature. In the reversal error, students switches over the quantities on the referents. Qualitative responses are answers given in qualitative terms giving the greater or less than relation is but not specifically the correct relation. Incongruent responses were responses that did not reflect a coherent or task specific response. The errors suggest a taxonomy in which one error may provide some saliency to the correct response where the other may not be as obvious. The scoring scheme was established for the translation of the propositional relation problem, and since dichotomous scores of right and wrong would have severely restricted the variance of the scores, a scoring scheme was devised in which the scores would fall into one of five categories. This scheme of the five categories are taxonomical. The score given on the categories ranged from -2 to 2 based on the analysis of errors found in the literature pertaining to the propositional relation problem. By means of a more focused analysis of the hypothetical but possible responses of the symbolic, verbal, and pictorial responses (a taxonomy and definition of errors is suggested in Table 3), the analyses of the errors can be performed analytically, thereby obviating the need for further qualitative analyses.

(Table 3 about here)

FINDINGS

Sample

A convenience sample of 80 students from a large comprehensive American University was used. Only 37 college students completed the two testing periods. All college students were 19 years of age and above.

A second convenience sample of 193 secondary school students were obtained from two large high schools which serve two cities in the Eastern part of the United States. The age of the composite sample ranged from 11 to 40. The mean age was at 17.25 and median age at 17.00. All the high school and college students had successfully passed their first and second algebra courses. Two periods were allocated for the administration of the 16 algebra word problems.

Validation of the 16 problems was achieved through a consensual validity phase whereby six judges were asked to assess the adequacy, quality and appropriateness of the problems. Several raters were then asked to judge these problems based on the logical analysis and description of each problem. Interclass correlation coefficients indicated that both correctness and agreement levels were extremely high.

The first statistical analysis presented the performance on the generative and passive subset scores followed by the presentation and response format scores. The scores on the problems were aggregated for the presentation and response mode. A verbal-symbolic problem was added to the set of 16 problems to verify a verbal to symbolic problem.

Table 4 presents the means (proportion correct) for each algebra problem. The most challenging item was the passive translation problem with symbolic to verbal (sv) translation (item, SV1). Only one of the college students obtained the correct answer, whereas twelve of the secondary students solved this problem correctly. On the generative response mode problems, a verbal to symbolic item (VS2) seemed to be the most difficult problem for secondary students with a mean proportion of .041. Students had difficulties with problems that needed to be translated from and to a symbolic mode response. Overall, the generative response modes were the most difficult problems, specifically the verbal to symbolic (VS) type, for both college and secondary school students.

A MANOVA was performed to study performance differences between college and secondary students. The first analysis begins with the highest node of the problem feature, i.e., generative and passive modes.

Table 5 presents the means (proportion correct levels) for the passive and generative response modes and the F-ratios for one-way MANOVA (educational level). The main significant effect was found on the generative response mode problems, between high and low educational level.

As discussed earlier, the passive response items required the identification of an answer to the question. In the generative response modes, students had to construct a unique answer to the problem in an active translation activity. The means proportions correct of the passive response items were significantly higher than that of the generative response modes.

(Tables 4 and 5 about here)

One explanation for these results come directly from the passive translation requirements, which merely ask for recognition of the equivalence of two differently, represented forms. The generative translation elicits more elaborate processes demanding broad and extensive cognitive activities and an active abstraction in which students attend to a response that has to be generated.

(Tables 6 and 7 about here)

Tables 6 and 7 present the means (proportion correct levels) on the subset scores for the three types of problem presentation and the means (proportion correct levels) for the cross translations respectively. Included in Tables 6 and 7 respectively, are the one-way (educational level) on the presentation mode and one-way (educational level) on the cross translations MANOVA results. Also included in the tables are the *post hoc* Scheffe' contrasts for the total sample.

A significant difference is shown on the pictorial presented problems by educational level. This significance was not apparent on the symbolic and verbal problems. The pictorial problems were the least difficult problems for college students and secondary students. The cross translation to the verbal format was the least difficult problems for college students and secondary students. The most difficult problems were those cross-translated to symbolic format. Significant main effect of educational level was found on cross translations to the symbolic mode between high and low educational level, with higher scores for the college students.

The *post hoc* tests showed significant differences between all contrasts of problem presentations. Similarly, main significant differences are reported on the cross translation to symbolic form between the two educational levels. College students performed higher than the secondary students did. The *post hoc* tests showed significant differences between the translation to pictorial and symbolic types and between the cross translations to symbolic and verbal formats.

Table 8 presents the analysis done for the verbal to symbolic (vs) and verbal to pictorial (vp) problems by combining the vp problems with the vs problems. The data in the Table indicates the high number of reversal errors presented in the verbal form and translated to the pictorial or verbal form. A large number of students produced the incongruent response and the no answer response, which suggests that students were apparently misled or completely misunderstood the depth of the problem structure.

(Tables 8 and 9 about here)

Tables 8 and 9 present an average count of errors for the three presentation modes and their translations respectively. The average was used because there was an unequal number of problems for the presentation and cross translation modes. The findings in Tables 8 and 9 reveal that the reversal errors were persistent among the presentation to and from the symbolic form of presentation. The results showed a high average of 86 reversal errors on the translation from a symbolic mode compared with an average of 75.17 on a translation to a symbolic mode. For the verbal formats a high reversal error was found on the translations from a verbal format, with a comparable average on the translations to a symbolic form.

The average number of qualitative response errors was relatively comparable among the three translations from the pictorial, symbolic and verbal formats. The highest average count of qualitative responses was found on the pictorially presented problems. In contrast, the cross translations had a higher number of qualitative response errors. Translations to a verbal format showed a high number of qualitative response errors to the verbal mode since these translations were made from the pictorial format. In summary, the above results indicate that those presentation formats that are qualitative (and thus generative) in character have the highest number of qualitative response errors.

Highest count of unanswered problems was on the translation from the verbal format, with an average count difference between verbal and symbolic presentations of 35.11 and a difference of 21.36 between the verbal and pictorial represented formats. The verbal presentation problems as well as their translations to the symbolic format and to the pictorial format have the highest number of unanswered problems. The average frequency data suggests that the larger number of unanswered problems were mostly of the generative problem type, particularly those problems that are perceptually complex.

In summary, the highest average number of reversal errors was found on the symbolically presented problems and the symbolic cross translation. These averages showed those symbolic presentations and cross translations may have encouraged the solver to translate these problems in a syntactical manner. Clement (1982) called this finding the syntactic word order match operation or the "adjacency effect," where students operationally match the quantity to the object as it syntactically appears in the form of presentation; i.e., the naive processor approach.

DISCUSSION AND REVIEW

The main purpose of this study was to understand how the algebra problem presentations of the propositional relation kind are related to errors students make in their response format. The findings in this study suggest that preconceptions or alternative conceptions may arise out of the problems due to conflict with the theoretical-formal rules of a mathematical theory.

The 16 algebra problems presented have two responding formats that required a generative or passive translation of the problem. In the generative mode, the student has to construct the correct answer, whereas in the passive format, the student had only to select the correct answer. These types of response formats or "translations" were in accordance with Clarkson's (1978) view of the cross translation from a presentation to a response mode. The logical analysis revealed that in some definite cases translations are convergent (vs, ps and vp generative translations) and in others, divergent (pv, sp and sv).

Six modes or types of cross translations are possible in the translation from one mode to another. For example, the relationship in the problem is stated in verbal form, but the answer must be expressed in symbolic or pictorial modes. In effect, six cross translations from and to a mode of response are produced; this resulted in 15 combinations of comparisons. A breakdown analysis suggested the categorization of the problems with respect to presentation and response formats. The possibility of significant differences on the presentation and/or response would indicate a connection between the presentation and response chain, and verify a refined and precise understanding of the function of translation. The idea of representation and represented distinguish several facets of the concept function, i.e., translation or transformation (Janvier, 1987).

The most difficult problem was of the passive translation type from symbolic to verbal form, followed by the symbolic to pictorial form. The difficulty found in the latter problems i.e., the translations from a symbolic format or "abstract" presentations can be explained from the theories of Skemp (1982), and more recently Bernardo and Okagaki (1992). According to the latter, symbolic knowledge, which involves tacit knowledge, i.e., knowledge underlying the conventional mathematical theory, is completely hindered when symbolic or "abstract" presentations and the cross translation to an "abstract" format are presented to the student. Students arrive at the problem situation with different backgrounds and with a variety of procedural skills, which become cognitively anchored for specialized tasks. Consequently, when students are given a situation requiring symbolic maneuvering (i.e., use of schemata and strategies), they fail to apply their integral knowledge to that new situation. This is particularly underscored by the low performance on problems, which have an abstract presentation namely, the symbolic formatted problems and translations to a symbolic form. One could conclude with some reservation, that mathematics educators should at least appropriate a substantial part of the curriculum for the planning, provision of lesson plans in problem solving. These problems should include a variety of complex presentations, especially of the generative type.

The findings of this study on the difficulties faced by students on the generative type are so marked as to warrant the recommendation for both college and secondary students, of the inducement of various types of word problems that require some sort of generative response. As Aschcroft (1989) established, generative translations demand data organization and strategies that are cognitively more elaborate, complex, painstaking, specific, and may be important for better storing and retrieving mechanisms in the process of problem solving. The most significant aspect of this translation, however, lies in its facilitation of better learning. As Wittrock (1974) indicated, the reading of an organized textual system of a problem for the appropriate generation of a solution is more conducive to learning than unrelated and unorganized information. These generative processes involve the construction of an organizational structure for storing, and retrieving information, and for relating new to stored information.

Pertinent to symbolic reasoning is the information processing and cognitive development model. A symbolic knowledge development progress in tandem with ontogenetic development, that is the child develops in a continuum from a concrete mode to one of greater abstractness. At the end of this continuum, performance differential tends to diminish among problems of the symbolic and verbal form when compared to more concrete problems of the pictorial mode. The results showed that both groups secondary and university level groups performed poorly underlined by the MANOVA results. Significant differences found on the translation from the pictorial to the symbolic form suggest that functionality as opposed to macro-features, viz., presentation effect, is conduit to the missed concept. Preconceptions, may not be wholly intuitive after all and can therefore no longer have the status of "pre-concepts," but rather alternative concepts. This argument is persuasive in that those errors on the translations to the symbolic were of the reversal type. Given the error taxonomy the reversal is at the end-scale of the taxonomy; in-depth processing by default is a cognitive operational function, which constitutes a pedagogic as opposed to an intuitive alternative conception.

No doubt these models of explanation support our understanding of preconceptions as being of either the intuitive or the formal type. Kintsch and Greeno's (1985) problem-solving model lend support to the view that preconceptions may not be easily identified with one cause. In fact, interaction or a combination of interactions may be more viable for an explanation. For instance, the number of errors found on one of the problems of the passive type compared to others of the same type viz., symbolic to verbal translations, and compared across types viz., pictorial to verbal and symbolic to pictorial may be a better aggregate of analysis. Kintsch and Greeno suggest that students when approaching mathematics problems in their memory make a representation as a result of reading a text which consists of two types—a propositional representation of the information that is in the text and a model of the situation described by the text. The situation model or context is derived from both informations in the text and inferences that can be made from this information by using one's prior knowledge and broad knowledge of the topic area of the text. In fact, the model reflects the complexity and interaction of knowledge-based schemata. Underlying this model are contextual features that are represented by the text and which need to be identified along a continuum of abstract to concrete. Contextual features or situations were not studied in consortium with the presentation. However, identification of key context features along a continuum from concrete to abstract presentation should characterize more comprehensive macro-problem features, and suggest a comprehension-performance model of problem context feature. Hence, these problems should be viewed from within the theoretical framework of Brunner's original idea that cognitive development is related to information presentation, and that responding modes are equally important and in need of investigation, particularly in relation to algebra problems with the responding mode of generative translation.

Error analysis was performed on the algebra problems by isolating the presentation and cross-translation crossed with the type of error. By this method, the highest occurring error is identified with the presentation type and cross-translated format. These presentations and cross-translations were identified with the type of error through an average frequency tables (reported on 8 and 9). For example, students may exhibit a clear understanding of the notational system of an algebra problem and be able to use and manipulate the symbols required, however, they may have little awareness or understanding of the "deep" dimension of the problem structure and/or underlying tacit knowledge needed to solve the problem. The analysis of the errors, with respect to the taxonomy, reflects what may be "error hierarchies" when related to the feature or presentation mode of the algebra problem. At the lowest level are the incongruent responses, followed in ascending order by the reversal error, no response, and the qualitative response.

To illustrate this taxonomy, when students are given a problem whose presentation is symbolic, this type of problem would logically have an abstract and irregular feature relative to the experiential world, which was on the taxonomy scale, an incongruent response followed by the reversal error and lastly the qualitative response (see Table 3). The more abstract the problem presentation, i.e., the symbolic type, the more salient the response or the lower the error index on the taxonomy scale. The results showed a high number of reversals, followed by incongruent responses for the abstract presentation of the symbolic type in the order of incongruent responses, followed by the reversals, and lastly the qualitative. The difference between reversal errors and qualitative responses on abstract presentations of the symbolic showed a higher difference than the more concrete presentation of the pictorial type. These results, which indicate that problem features with valence to reality appeared in their quality and translated into the form which reflects this reality (i.e., pictorial), would argue for the intuitive idea as the cause of preconceptions. A cautious interpretation of these results, however, shows that qualitative and reversal errors occur late and "deep" in the information processing cycle. A qualitative error that is processed from attention, perception and working memory, is retrieved from previously stored and assimilated pieces of information (schema), and "paged" from long term memory to working memory, but cannot be appropriately converted to a full and correct solution. Furthermore, because qualitative errors are partially, but not fully, correct, they are due to something other than the objective character appearance of the problem or student real world experiences. More interesting, though, is the reversal error with a high occurrence on the translation to a symbolic problem. This result may be pertinent to the Kintsch and Greeno problem-solving model, namely that presentation may be at the fore-front of processing, and could no doubt alter the conception in the representation or, even further, in the function of translation. The notion that concrete features complement the order and data-logical frames in word problems, supports the idea that the intuitive spurs a conflict with the formal and theoretical. Like a double-edged sword, the intuitive and pedagogic both "tear-up" concepts in the process of problem solving.

Content analysis of the problems reveals other aspects that may have worked in the problem-solving procedure. For instance, a verbal presented problem that relates quantities asks the student to translate the number of nickels relating to the number of dimes, from the verbal mode to the symbolic mode (i.e., equation). The aim is to have students relate the number of coins. Instead however, they relate the value of the coin's currency, i.e., a dime is 10 cents and a nickel is 5 cents; two nickels make one dime. Hashweh (1986) found this same type of erroneous approach in children's learning of specific scientific theories (i.e., scientific knowledge). "Children learn [that] the quantity of liquid in a glass is affected by the height of the glass" (Hashweh, 1986, p.234), but later they discover that the area of the container may influence the volume. So a situation may play an equally important role in instigating students to activate problem-solving procedures that are not easily encouraged by context or presentation. After all, problem features or the intuitive may not spur at all times pedagogic alternative conceptions. Those alternative conceptions that are essentially pedagogic are due to the fact they build on types of knowledge that bridge their linguistic roots to their mathematical form.

The naive approach or simply the "lay" approach to solving algebra problems may be similar to certain instances found in scientific preconceptions. Many naive approaches to solving algebra problems can be mapped isomorphically to aspects of features of the experiential world, and the stronger the relation between these aspects the more pervasive the preconceptions. The high response errors on student problems involving the translation of the verbal and symbolic presentation to the symbolic, suggest two important points in students' problem-solving approaches. One is that problem features that override the relational aspects of the problem may perceptually guide students; in other words, those problems that have salient features may not be so easily decoded or translated to an abstract form. Second, the operational approach suggests a formal type of training that may lead students to overlook some fundamental procedures of checking their responses. This in itself posits a deep-seated issue in mathematics education and reflects the absolutist views of mathematics education philosophy, namely that mathematical procedures be relegated to the authority of the text or instructor. It is within this framework that the student operates and experiences difficulties, not only in re-evaluating the multiplicity of procedures and approaches, but even in the rechecking of solutions. The need to refocus the goals of mathematics education and help avoid excesses in its procedures is paramount to mathematics educational change for openness to new views and approaches.

In conclusion, alternative concepts in mathematics are content and context based and drawn from the experiential reality of each child. When mathematical content is involved, the majority of alternative concepts tend to be related to interaction between the instructor and the learner; that is to say that errors in mathematics are intrinsic and pedagogically driven. Thus the structure of the algebra problem is a perception, conception and higher order cognitive act and phenomenon (Nasser and Carifio, 1993). Metacognitive preconceptions, therefore, need to be more fully investigated if we are truly to understand, remedy, and alleviate problems of alternative concepts in mathematics.

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Table 1
A Descriptive and Conceptual Characterization of the Algebra Word Problem

Response mode / Presentation mode	Verbal	Pictorial	symbolic
Verbal		4 problems	4 problems
Pictorial	2 problems		2 problems
Symbolic	2 problems	2 problems	

Table 2
Nested Features of the Algebra Problem

Translation Type:	Generative Type			Passive Type		
Cross Translation:	vp	ps	vs	sv	pv	sp

Table 3
Scoring Code for the Passive and Generative Translations

Code	Error	Correct Response
-2	Incongruent Response	
-1	Equivalence Error	
0	No Response	
1	Qualitative Response	
1.5		Static Correct Response
2		Correct Response

1. A "-2" is given for an incongruent response which has no reference to the question.
2. A "-1" is given for the reversal error.
3. A "0" is given for no solution or an unanswered item.
4. A "1" is given for a solution in which the respondent presents the answer in qualitative terms giving the greater or less than relation but not specifically the correct answer.
5. a "1.5" is given for the formulation of a response, that does not express a proportion but a static correspondence to the quantities represented. This score was formulated after piloting the test.
6. a score of "2" is given to a student who had formulates an answer.

Table 4
Means (Proportion Correct Levels) of all the Items Classified Under Passive and Generative Response Modes for Secondary and College Students

Generative Items	Secondary (N=193)	College (N=37)	Passive Items	Secondary (N=193)	College (N=37)
VS1	.187	.243	PV1	.772	.892
VS2	.078	.081	PV2	.632	.865
VS3	.269	.568	SP1	.290	.297
VS4	.041	.054	SP2	.606	.676
VS5	.114	.216	SV1	.062	.027
VP1	.554	.649	SV2	.606	.459
VP2	.746	.757			
VP3	.285	.270			
VP4	.236	.270			
PS1	.150	.216			
PS2	.166	.378			

Table 5
Raw Score Means (Proportion Correct) on the Subset of items of Generative and Passive Macro Feature by Educational Level MANOVA

Educational Level	Generative	Passive	Mult. F-Ratio	p
Secondary (n=193)	.257	.520	2.61	>.05
College (n=37)	.337	.572		

	Educational Level F-ratio df(1,228)
Generative	5.01*
Passive	1.69

*=Significant at the .05 level

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Table 6
Raw Score Means (Proportion Correct) on the Subset of Problem Presentations Mode by Educational Level MANOVA and Post Hoc Tests.

Educational Level	From Pictorial	From Symbolic	From Verbal	Mult. F-Ratio	p
Secondary (n=193)	.43	.39	.30	4.91**	<.01
College (n=37)	.59	.37	.36		

	Educational Level F-Ratio, df(1,228)
From Pictorial	11.98**
From Symbolic	2.77
From Verbal	.49

Scheffe's Post Hoc Test (N=230)

	From Symbolic	From Verbal
From Pictorial	14.09**	18.40**
From Symbolic		64.69**

*=Significant at the .05 level, **=Significant at the .01 level

Table 7
Raw Score Means (Proportion Correct) on the Subset of Items of the Cross Translation by Educational Level MANOVA and Post Hoc Tests

Educational Level	To Pictorial	To Symbolic	To Verbal	Mult. F-Ratio	p
Secondary (n=193)	.45	.15	.52	2.76*	<.05
College (n=37)	.49	.27	.56		

	Educational Level (N=230) F-Ratio, df(1,228)
To Pictorial	.62
To Symbolic	7.36**
To Verbal	1.08

Scheffe's Post Hoc Test (N=230)	To Symbolic	To Verbal
To Pictorial	241.79**	14.09**
To Symbolic		372.60**

*=Significant at the .05 level, **=Significant at the .01 level

Table 8
Average Number of Errors from a Problem Presentation

Problem Presentation	Incongruent Response	Reversal Error	No Response	Qualitative Response
From Pictorial	34.5	42	22.5	26.3
From Symbolic	27	86	8.75	19.25
From Verbal	37.63	56.75	43.86	20.5

Table 9
Average Number of Errors from a Cross Translation to a Response Mode

Translation Presentation	Incongruent Response	Reversal Error	No Response	Qualitative Response
To Pictorial	30	5	29.33	17.67
To Symbolic	20.17	75.17	24	9.67
To Verbal	13.5	59	4.5	32.5

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