Beginning in 1987, a reform movement was heavily funded by the National Science Foundation (NSF) to change the teaching of calculus. Borough of Manhattan Community College (BMCC) received seven NSF grants over an eight-year period, allowing the college to: establish a state-of-the-art calculus computer lab; purchase calculators, graphing calculators, and laptops; send selected faculty to workshops and run workshops for faculty; and hire half-time college lab technicians and support staff. This paper presents results of an assessment of how BMCC's students feel about themselves in pre-calculus and calculus classes and, in particular, how they feel about the use of computers, graphing calculators, working in groups, and creating written research projects and portfolios of their work. Students were asked, through a questionnaire and interviews, on a scale of one to five, how strongly they agreed with 15 statements; comments were also solicited. Data were analyzed for the whole cohort and then by gender to see if women responded differently than men. Results are discussed for each of the 15 statements, and student comments are reproduced. One implication of the study is that there needs to be increased sensitivity in order to make women equal partners in the classroom and laboratory. (AEF)
Research Paper: Curriculum and Instructional Strategies

Equity: Ownership by Minorities and Women of Research Projects

Patricia Wilkinson
Lawrence Sher
Borough of Manhattan Community College
199 Chambers Street
New York, NY 10007
212.346.8530
pbwilk@aol.com
lawsher@yahoo.com

Key Words: women, minorities, presentations, research, students, calculus

Clearly, the last decade has seen a tremendous change nationwide in the teaching of calculus. Starting in 1987, a reform movement got underway. It was a response to high failure rates (as many as 40 percent of undergraduates were failing introductory calculus) and the perception that the students were merely learning by rote, unable to apply mathematics to real world or complex problems. The reform movement was heavily funded by the National Science Foundation which spent 35 million dollars from 1987 to 1995 on dozens of projects to change the teaching of calculus. A survey by the Mathematical Association of America (MAA) found that by 1994 a majority of institutions had made modest or major changes in how calculus was being taught and that a third of the students taking calculus in the United States were enrolled in a reformed course.

The hallmarks of a reformed calculus approach include:

- Emphasis on students working in groups (collaborative learning)
- Working on long-term challenging real world problems (based on Uri Treisman's Berkeley California model)
- Writing and speaking about their work
- Using computers and graphing calculators to visualize and manipulate functions and families of functions
- Using computer algebra systems such as Mathematica and Maple to differentiate, integrate, and "crunch" data
- Encouraging students to do research
Borough of Manhattan Community College has been in the forefront of this movement. The college was already committed to students writing and speaking about their work. We had already learned that collaborative learning and the use of appropriate technology helped developmental mathematics students. We were concerned about low success and retention rates in calculus. More importantly, we were deeply troubled by the fact that those students in calculus were almost exclusively White and Asian American males. The woeful underrepresentation of women in calculus in a college that is two-thirds female was also of great concern. We were already looking for solutions for improving and making inclusive our calculus instruction. Active in our professional organizations, we responded quickly to calls for change by Ron Douglas, Ed Dubinsky, Deborah Hughes Hallet, Tom Tucker, David Smith, and others.

Borough of Manhattan Community College was the first two-year college to receive National Science Foundation funding for calculus reform and for the early years we were the only two-year college in the program. We received a total of seven NSF grants over an eight-year period. Funding allowed us to:

- Establish a state-of-the-art calculus computer laboratory.
- Purchase calculators, graphing calculators, and laptops.
- Send senior faculty to a week-long workshop with Uri Treisman on collaborative learning.
- Send Drs. Sher and Wilkinson to a two-week long workshop at Dartmouth with John Kemenny. The workshop focused on integrating computers into calculus, differential equations, and numerical analysis. We were the only two-year faculty invited. The invitation was based on prototypical calculus projects we had already created.
- Run workshops for City University of New York (CUNY) faculty on the use of MAPLE (a computer algebra system), graphing calculators, and collaborative learning techniques.
- Run two weeks of summer workshops at BMCC for seven to eight years (1992–1999) for hundreds faculty from all over the country.
- Run half-day workshops on computer use for adjunct faculty.
- Hire half-time College Lab Technicians and support staff.

Our program avoided the pitfalls of some reform projects that are currently receiving backlash from very traditional faculty members who long for a return to the good old days of straight lecturing and textbooks that do not emphasize the use of technology. The BMCC program adds labs, supervised by a laboratory technician, to the traditional classroom experience. Faculty are therefore free to use as much or as little changed pedagogy as they feel comfortable learning. Some faculty are totally committed to collaborative learning, even allowing some collaborative testing. Some faculty insist on teaching classes in a lab (or with classroom size amounts of graphing calculators) so the students can actually see graphs, rotate functions, etc.
as the need arises in class discussion. Others have not changed their teaching style, and rely on the lab to add the reform component. In the lab, students work together in groups on challenging problems. They learn to use a wide variety of software (Maple, Mathematica, Derive, True BASIC), and to write up and speak about their projects.

The BMCC program has been successful by a variety of measures. Grades and retention have improved. We have increased by six fold the number of students persevering through Calculus III. The success and increased numbers of calculus students encouraged us to go forward with a mathematics program. The program already has 50 “math majors,” a larger number than some of our four-year sister CUNY colleges. Students are going on and succeeding as math majors at four year institutions. One student showing her portfolio of projects won a scholarship to Columbia. Each semester about 10 of our students are Alliance for Minority Program Students (AMPS) fellows receiving $2,000 stipends from NSF to pursue research on projects originally started in the calculus lab. One of our students won first place presenting “Animations of Taylor Series of Trigonometric Functions” at the fourth annual NSF-AMPS Students Research Conference in Tallahassee, Florida, in June 1996. More importantly, the ethnic make-up of students in the program is now representative of the college as a whole. We have had some, but not as much success, in breaking down the gender barriers. For example, the Fall 1996 evening Calculus III course had only one woman in the room and that woman was the instructor. The program has received national recognition. It tied for first place in the “Student Success Strategies in Applying Technology to Teaching and Learning” competition at the National Council of Instructional Administrators Meeting, Minneapolis April, 1995. The portfolio aspect of the program was the focus of an article “The Student Portfolio: A Powerful Assessment Tool” in ASEE Prism, A Journal of the American Society for Engineering Education, March 1996. Professors Sher and Wilkinson received NISOD awards for their work. They have presented workshop and papers at more than 20 conferences, including the MAA, AMATYC, NYSMATYC, and NCTM. They have spoken and had papers published in the proceedings of international conferences in England, France, and Germany.

What had not been done (the focus of this paper) was a project to elicit the students’ perspectives on these reforms, particularly on the use of technology. Through a questionnaire and student interviews, it was attempted to assess how BMCC’s students feel about themselves in pre-calculus and calculus classes, and in particular, how they feel about the use of computers, graphing calculators, working in groups, and creating written research projects and portfolios of their work. Students were asked, on a scale of the 1 to 5, how strongly they agreed with 15 statements (N = 300). Comments were also solicited. Data was analyzed for the whole cohort and then by gender to see if women responded differently than men. The results follow:

1. I think of myself as a good math student.

The majority of BMCC’s pre-calculus and calculus students agree or strongly agree with this statement, indicating confidence in their ability. However, when data is analyzed by gender or by whether students are advanced or intermediate, there are significant differences. Men rate themselves significantly higher ($\alpha = .05$) than
women and the longer students are in the program, the more highly they rank their ability.

This finding is hardly surprising. In fact, it simply supports hundreds of studies that indicate men are much more confident than women in the mathematics arena. This is often independent of success and/or grades. Successful reform strategies should be pushed back to earlier courses to give women more time to build their confidence. More attention should be paid to the affective aspects of instructing women.

2. I like to solve math problems.

Again, the majority of students support this statement. There were no significant differences by gender. This was a little surprising since several studies on the underrepresentation of women in mathematics hypothesize that women are not “risk takers” and therefore like to avoid math problems. There was, however, a significant difference (α = .01) by course level. The longer students are in the calculus program, the more strongly they support the statement. The program succeeds in encouraging students’ problem solving interest, an important aspect of mathematics.

3. I am nervous when I need to solve math problems.

Students indicate that many are indeed nervous when approaching problems. There is a vast literature on mathematics anxiety and it suggests that even very good students can be somewhat anxious. Like the seasoned actor who still gets opening night jitters, some good math students approach problem solving with their adrenaline pumping. What was surprising was that there were no significant differences, either by gender or advanced standing. So much of the literature characterizes women as math anxious or math phobic but at BMCC there was no real difference. Men as well as women, and advanced students as well as intermediate ones, are nervous about solving problems.

4. I can usually picture in mind what the graph of an equation or function will look like.

The majority of students select neutral, disagree, or strongly disagree, i.e., they do not easily visualize abstractions. However, there are very significant (α = .01) differences by gender and class level. Men report significantly better ability than women and students in higher level classes are more confident of their ability to visualize. Numerous studies have reported that women come to mathematics handicapped by poor spatial relations and visualization skills. Starting in adolescence, women lag behind men in tests involving spatial abilities. Beginning with childhood toys and games, girls have limited practice in spatial visualization, an important tool in mathematics. But it has been shown that even in adulthood, intervention programs can equalize women’s spatial task performance with that of their male peers. Women should be exposed earlier in their mathematics careers to the use of graphing calculations and to computers, to make abstractions “concrete.”
5. Math is something you can learn only by yourself.

Students for the most part reject this statement, i.e., the majority realize group work can produce learning. The higher level students reject the statement more than the intermediate level, although the difference is not statistically significant. However, when means are compared by gender, males are significantly more likely to support the statement (i.e., males think of mathematics as a solitary activity more than females). Collaborative learning apparently taps the verbal and social skills of women. Students, but women in particular, reject the contention that mathematics can only be learned by yourself.

6. I like to work in small groups to solve problems.

This statement closely relates to the previous one and has a similar response. Although not statistically significant, the more advanced students are, the more they prefer group work. Women (α = .05) prefer small group work more them men. Students and women in particular like small group work.

7. I have used computers in math.

Most of our students have used computers in mathematics. Men have more computer experience than women. Advanced level students overwhelmingly report greater use of computers. The difference between advanced level and intermediate level students is significant. We need to find out if lower level courses are not using technology because of faculty disinterest or lack of availability. If the latter, our new ILI Grant for laptops and graphing calculators will help.

8. Using computers helps me to learn math.

Almost half the men and two thirds of the advanced students support this statement. Few students disagree with the statement; although many are neutral. There are statistically significantly more men than women who believe computers are useful, and advanced students, again at statistically significant levels, support computer use over lower level students. We need to be sure women are getting equitable access to computers and introduce them earlier in their mathematics lives.

9. I have used a graphing calculator.

It is interesting to contrast the responses to this statement with those to statement 7 (I have used computers). Here, the responses are absolutely antithetical. Significantly more women than men have been exposed to calculators and significantly more intermediate students have used calculators. Women who enter the program at lower levels (Intermediate Algebra) than men do not get to use computers until later. Again, our ILI Grant for laptops may allow us to bring computers to earlier courses.

10. Graphing calculators help me to solve math problems.

The responses here were similar to those of statement 8. Almost half the men and over half the advanced students support this statement and few students disagree but a large number of students are neutral. Significantly more men than women, and

"Spotlight on the Future"
significantly more advanced students, think calculators are helpful. Calculating a correlation coefficient for response to statement 3 (I am nervous.) found an extremely small negative correlation.

11. Computers and/or calculators help me to visualize graphs better.

A majority of students support this statement. Again, significantly more males than females support technology for visualization and significantly more advanced students (81%) find technology helpful for visualization. Calculating a correlation coefficient between responses to statement 4 (I can usually visualize) and statement 11 found small but positive correlation's for all groups, the largest correlation being for women.

12. Computers help me solve math problems better.

There is strong support for this statement by all students with no difference by gender (i.e., women as well as men find computers helpful to solve problems). There is, however, an extremely large difference by class level; the more advanced students strongly support this statement. Students, particularly students in advanced courses, support the use of technology to help solve problems.

13. I have created written projects in math.

Significantly more men than women, and overwhelming more advanced students, have created written projects.

14. I have created written projects in portfolios.

The use of portfolios is prevalent in advanced courses with 40% of the students supporting this statement, while only 5% of intermediate level students have created portfolios. There is a significant difference by gender; more men have been involved in portfolio creation.

15. I believe projects and portfolios help my understanding of math.

While many students rate this statement as neutral, many (see above) have not worked with portfolios. Advanced students strongly support this statement (only 7% disagree). Men find projects and portfolio more helpful than do women.

The questionnaire had three open ended statements:

1. Computers in math courses are ... 

2. Graphing calculators in math courses are ... 

3. Working with other students on math problems or projects is ...

Students were also invited to make any other comment they wished.

Only a third of respondents chose to add comments. Those that did were almost unanimous in their enthusiasm for the use of computers, calculators, and group
work. For example, only one student said he preferred to work alone. I do not,
however, think the sample of comments represents the population as a whole.
Students who took the time to add comments probably represent those most
enthusiastic about reform activities.

1. The open ended statement “Computers in math courses are” elicited such
typical comments as:

“A whole new approach. I feel I am part of the future.”

“Essential. I am going to live in a high-tech world.”

“Good for seeing what an equation really looks like.”

“Very helpful in solving long, complex problems.”

“Powerful. Maple helps me a lot.”

“Great. I finally understand what polar coordinates are about.”

“At first I was scared, but computers are easy to use and now I like
going to the lab.”

2. The statement “Graphing calculators in math courses are ...” brought
responses such as:

“A good idea, the calculator has many useful modes and options, i.e.,
you can find intercepts and exact points.”

“Useful; easy to use and handy to do long math problems”

“Very valuable. It puts into perspective what is really happening to a
graph.”

“Very helpful in understanding graphs—and fun!” “Makes problems
easier to understand.”

“Exciting”

“Helpful because they help me see things clearer.”

“Very helpful for checking algebraic answers graphically. Gives you an
idea what results really mean.”

3. Working with other students ... elicited the following typical remarks:

“Great, you can discuss different methods with your partners.”

“Working with other people is very interesting. Sometimes we, as math
students, get confused but another person helps us really understand.”

“It is important to see there are different approaches.”

“Spotlight on the future”
“Very helpful. There is always something that somebody understands better than someone else.”

“When I am stranded it helps to be able to ask for some ideas.”

Interviewing students produced very similar results. The students I talked with are not a random sample of students involved in the program. They represent, instead, those who were willing to spend some time talking to me. They were much more likely to be students who “hang out” in the Calculus lab during its open hours, i.e., students heavily involved with team or in the math club. I was not surprised therefore at their universal support for the use of technology and group learning. What was surprising was how articulate, dedicated, and serious they are.

While the outside evaluator of the nondevelopmental math courses was on campus this Fall, he asked to visit the Calculus lab. It was during an open hour (i.e., no class was using the facilities) that I spotted one of the students I had talked with. She was working on an animation project in polar coordinates. By using the software MAPLE, a computer algebra system, she was creating moving images on the screen that showed how the graph changed as she changed variables in her equations. She is an Alliance for Minority Participation (AMPS) fellow and this was her culminating project. The evaluator asked her to explain what she was doing and was amazed by her response. She said: “What level of explanation do you want? Should I start by explaining what polar coordinates are? Do you want to know about the software that allows me to graph my equations? Would you like to know about the program I wrote, that allows me to animate my equations? Do you want to hear what I am concluding based on my animations?” Her grasp of the various levels of knowledge and skills she was using and her articulateness in explaining what she was doing made it clear she was functioning as a working mathematician. She is typical of many students in the program; they are not just “learning” mathematics, they are “doing” mathematics, which is what a working mathematician does.

This study suggested strong gender differences in the calculus classroom at BMCC. Women are less likely to see themselves as good math students (regardless of their grades), less able to picture equations or function in their minds, report less computer use, and are less likely to find calculators and projects helpful. However, they prefer small group work more than men and are less likely to believe math is something you can learn only by yourself. Because the typical BMCC woman enters the college with less academic mathematics preparation than her male cohort, she starts her mathematics career in lower level courses. These courses are much more likely to be taught without technology and in a traditional lecture format. The course is also more likely to be taught by an adjunct than by a full-time faculty member. There is therefore a need to increase our efforts to make adjuncts comfortable using graphing calculators and computers, and to push successful strategies to earlier courses. We need to increase sensitivity to make women equal partners in the classroom and laboratory.

As a result of this study, faculty at BMCC applied for and received an NSF Grant (1997–1999), Portfolios to Increase the Number of Women in Mathematics, NSF-HRD 9710273, aimed at changing the culture and climate of mathematics instruction to improve the representation of women in mathematics.
The grant is providing resources to move the use of technology, and portfolios of work, back to pre-calculus and statistics courses in order to involve women earlier in their academic careers. Students are being encouraged to work in collaborative groups in these earlier courses. Emphasis is being put on visualization of abstract ideas. Students are encouraged to use graphing calculators and computer algebra systems to change, manipulate, and play with functions allowing them to "see" more clearly.

However, what is proving to be one of the most successful aspects of this new program is getting women students involved in research activities. As much of the literature suggests, getting women involved in real research, particularly if they work in groups or under a motivating mentor or role-model, has helped to retain them in science, engineering, and mathematics majors. This may be because some women prefer learning under teachers who do not act as authority figures but as "midwives" to student thinking, as a good research guide does. It also meets the often reported need of some women to tie what they are learning to real and relevant situations.

An example of this is SUNY at Potsdam College's mathematics majors. Investigators expected to cite the college for special sensitivity to gender or additional support for female students. Instead, it was found that teaching techniques were at the core of the success of the program. Teachers worked with students such that they recreated mathematics together. Students played the role of expert and found confidence in their own ability to create mathematical ideas. All students, female and male, were supported through a teaching style "true to the nature of mathematical inquiry." This teaching style appears to be preferred by many women who were interviewed for Women's Ways of Knowing. Research projects favor this way of learning.

Adopting this "midwife" approach to learning, in January 1998, nine women students worked under the direction of their individual mentor on a research project. Topics ranged from papers on women mathematicians and The Mathematical Underpinnings of Childrens' Games in Africa to a sophisticated computer animated analysis of Taylor Series as Symmetry in Three Dimensional Spaces. Most projects utilized graphing calculators or computers. During the summer of 1998, an additional 12 women students completed a variety of research projects. Other recent student activities include presentations:


David Cervetti’s talk on his animated research project was one of the highlights of the student presentations on February 20, 1998, at The Urban University: A CUNY Conference in Science and Engineering.


The Mathematical Association of America (MAA)/Tensor Foundation endorsed our student research by awarding the college a $5,000 grant (matched by the college) to provide stipends for women involved in the program in 1998–99. This endorsement was reaffirmed by a renewed $5,000 grant for women’s research fellowships for 1999–2000.

While the program is still in its initial stage, we have already seen results. Several women who expected to make Calculus I their terminal mathematics course have opted to continue in the calculus sequence, giving themselves more options in the SEM areas. Several students have extended their initial research projects to earn honors designations for their subsequent calculus course. Students have used their research projects in interviews for admission to private four year colleges, and in several instances have earned scholarships.

The Mathematics faculty at Borough of Manhattan Community College are committed to finding the most effective ways to use technology to encourage women and minorities to feel comfortable in a mathematics environment. Research projects using technology are proving to be an effective strategy.

**References**


NOTICE

REPRODUCTION BASIS

☐ This document is covered by a signed "Reproduction Release (Blanket) form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

☐ This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").