This paper discusses a one-page curriculum outline for a preschool mathematics program in Israel. The curriculum was developed in the course of preservice and inservice teacher education programs in Israel, and the mathematics activities offered to support the curriculum were developed by student teachers there. The curriculum starts from Ginsburg's (1977) recommendation to focus directly on numbers, not on readiness for learning numbers, and also draws on the work of Payne and Rathmell (1975), Gelman and Gallistel (1986), Resnick (1983), and Barrata-Lorton (1972, 1979). The curriculum integrates the NCTM (National Council of Teachers of Mathematics) Standards (1989) with specific learning expectations for children age 2 to 5. Within the curriculum, three assumptions state how young children learn mathematics: (1) by confronting tasks/problems which offer a variety of solution strategies; (2) by engaging in meaningful conversation with partners and in small groups about the tasks/problems: describing, explaining, deciding, considering; and (3) by encountering the mathematics in familiar situations: stories, songs, familiar games. The curriculum outline itself is a mapping of the conceptual contents: what young children learn about numbers. This extends from the earliest stages of exploring numbers to the formal addition and subtraction of the standard first-grade curriculum. Following a presentation of the one-page outline, the paper clarifies each subheading and offers examples of appropriate activities. (EV)
MAKING CONNECTIONS: A "NUMBER CURRICULUM" FOR PRESCHOOLERS

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Ruth Shane  
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
We were riding in our car on a summer vacation and wanted to entertain our two younger children, Michelle aged five and a half and Nadia just four. I suggested the game, "I'm thinking of a number between one and ten and you have to guess it. You can only ask yes-no questions like, 'is it larger than _? is it smaller than _?'" Michelle started to ask questions, each based on the answer to the one before. "Is it larger than 5? (yes) Is it smaller than 8? (no) Is it 9? (no) Is it 8? (yes). Nadia, on the other hand, would ask questions that contradicted the results of the former question: "Is it larger than 5? (yes) Is it smaller than 4?"

What mental schema had Michelle developed at age 5 that Nadia hadn't yet acquired at age 4? Although I had taught courses in the methodology of elementary school mathematics, I hadn't considered what mental structures are developing in young children which allow them to make progress in their understanding of numbers. I realized I had to put more order into my own thinking about this development.

Pre-school mathematics programs offer a wealth of ideas for experiences to promote the construction of mathematical concepts. They tend to be organized by strands or units or chapter headings which focus on particular concepts or areas of interest: patterns, quantity, etc. Another approach to presenting a coherent pre-school mathematics program is one which I have developed in the course of pre-service and in-service teacher education programs in Israel. It starts from Ginsburg's recommendation (1977) to focus directly on number, not on readiness for learning numbers, and organizes all relevant activities into a convenient, compact framework.

This "one-page curriculum outline" draws on the important work of Payne and Rathmell (1975), Gelman and Gallistel (1986), Resnick (1983), and Barrata-Lorton (1972, 1979), integrating the NCTM Standards (1989) with the specific learning expectations for the ages of 2-5. First, three assumptions state how young children learn mathematics. The curriculum outline itself is a mapping of the conceptual contents: what young children learn about numbers. This extends from the earliest stages of exploring numbers to the formal addition and subtraction of the standard first-grade curriculum. Following the one-page outline, is a clarification of each sub-heading, with examples of appropriate activities.

*The activities in this article were developed by student teachers at the Kaye College of Education in Beersheva, Israel, in cooperation with the author. They have all been adapted successfully for use in a variety of home and pre-school settings, with children from the age of 2 to the end of kindergarten. The activities were found suitable for very heterogeneous groups of children, mostly from urban settings with many special needs,
HOW YOUNG CHILDREN LEARN MATHEMATICS:

- By confronting tasks/problems which offer a variety of solution strategies
- By engaging in meaningful conversation with partners and in small groups about the tasks/problems: describing, explaining, deciding, considering
- By encountering the mathematics in familiar situations: stories, songs, familiar games

WHAT YOUNG CHILDREN LEARN ABOUT NUMBERS:

Introduction to Number Concepts
1. Counting words
2. The number triangle

Two Important Number Relationships
i. The successor schema
2. Part-part whole schema

Informal addition and subtraction

Formal addition and subtraction
kibbutz children, and some suburban groups. A grant from the Research Committee of the Kaye College in 1996-97, helped support the documentation of these activities.

Introduction to Number Concepts

This introduction is an orientation into the vocabulary and symbols which are the elements of number literacy in the child’s culture.

1. Counting words (1-10)

Practicing the counting words is a language game for young children from the time they learn to talk. Most cultures have the equivalent nursery songs and games to “One, two, buckle my shoe,” or “I’ll count to ten and you run and hide!” At this stage it is not the counting of objects which is critical but the rote counting, using the correct words in the correct order.

Extensions of rote counting:

a) Counting backwards: children who have a clear mental image of the counting words should be encouraged to practice counting backwards. Subtraction will depend heavily on this skill.

b) Skip counting: Three-year-olds can play the game where they say every other number out-loud and the other numbers “silently.” (one, mmm, three, mmm, five, etc.)

c) Starting to count from numbers other than one: let’s count from four; let’s count backwards from six.

d) Connecting rote-counting to counting objects: Parents intuitively start to chant the counting words with their very young children while they walk up the steps or put out the cups on the table (“One...two...three...”).

2. The number triangle (0-10)

This basis for this diagram appeared in the 1975 NCTM Yearbook article, “Number and Numeration,” by Joseph Payne and remains one of the most succinct presentations of the meaning of number in early childhood. It represents the understanding of number as the synthesis of connections between quantity, number name, and graphic symbol (numeral). The following connections can be read from the triangle:

a) Comparing quantities (matching quantity to quantity)

b) Matching quantity and number name (counting objects)

c) Matching numeral and number name

d) Matching numerals which “look” differently: matching numeral to numeral; writing numerals

e) Matching numerals to quantities: the critical link to formal arithmetic where graphic symbols of numerals become the number and children must have developed a wealth of associations behind the written symbol
Many activities based on this triangle can be found in *Workjobs* (Baratta-Lorton, 1972).

a) Comparing quantities

This is one of the early stages of the triangle because it is not even dependent on developed counting skills. The concepts involved are: more, less, the same, enough; their use easily fits into the child’s daily routine. Two-year-olds will already prefer the bowl with two cookies to the bowl with just one. John shows his day-care group that he has six buttons in one hand and two in the other. In which hand does he have more buttons? In which hand does he have less buttons? Do we have enough spoons to put one on each plate? Too many? Not enough? These are activities which call for visual or manipulative one-to-one correspondence which children can apply to quantities that they can’t yet count successfully.

This can be extended to activities which are less manipulative and more representational. Three-year-olds can tap out a number of drumbeats, the same number as the child-leader tapped on the tambourine. In the card game of War, the card with the greater (or lesser) quantity takes the pair. An important discussion comparing quantities with four and five-year-olds can be generated from building simple graphs with the children: boys/girls in the group; brown eyes/blue eyes.

b) Matching quantity and number name (counting objects)

The research of Gelman and Gallistel (1986) have given a thorough analysis of the principles involved in well-developed counting: stable set of counting words, one-to-one correspondence between number names and objects; cardinality principle; abstraction; irrelevance-of-order. Children may have partially-developed principles or different levels of development for smaller numbers (1-4) and larger ones (5-10).

Tasks which encourage counting include many aspects of daily life: counting the fingers on one hand, the children at a table, the pockets in a child’s clothes, the candles on the birthday cake, the dolls at the tea party, the blocks in the tower. Movable objects can be counted as well as non-movable like stickers on a card. Stickers on a card can be arranged in a random fashion or an orderly one. As a more difficult task (Resnick and Ford, 1984), children can be asked to count out a certain number of objects from a box full of the objects (Please give me five blocks). Many wonderful counting books are available and provide an important link with the language of number as enriching our “reading” of pictures and stories. Likewise many outdoor games, such as “Take six giant steps. Mother may I?” generate motivation in counting.

c) Matching numeral and number name

Written numbers (numerals) are all around the child in his daily life: apartment number, address, telephone numbers, prices, calendars, bus numbers. Reading each numeral correctly is a decoding task which is essential. The pre-school environment which encourages exposure to letters and words as an orientation toward reading should also provide a variety of instances of numbers, as an orientation towards the language of mathematics. Vivian, the preschool director in a rural area found the simplest materials to
be the sturdiest: steps were painted with numbers (a chance to practice the counting words forwards and backwards); old calendar pages were used for cut-and-paste activities and for lotto boards.

A modified bingo game with numbers from 0-10 provide good practice in connecting oral names and written symbols: the children can fill in their own game boards on matrices 3x3, choosing which nine numbers to include. Children can take turns being the caller. Another useful game format uses a set of playing cards (22) where the numbers 0-10 appear twice. A group of four children are each given five cards (two cards are put aside) which they put face up in front of them. The caller spins a spinner which has all the numbers 0-10 and calls out the number name. Whoever has the card with that number turns it over. The goal for each player is to turn over all his cards. (Some children will naturally arrange their cards in sequential order; some will put number pairs next to each other).

d) Matching numeral and numeral
This is a modified visual perception skill for which children recognize the same written representation of a numeral whether in print or script, in a variety of fonts. The card game Memory is most appropriate where children find pairs of the same numeral which look different. Three-year olds are proud to string together a set of flat beads all of which have the same numeral sticker and which they have chosen from a box of other beads.

This is also an appropriate rubric for practicing the writing of numerals which should be done as much as possible in the context of the children’s need to document an experience. A child may record the outcomes of throwing a die; a group of children put together a telephone book where they write down each other’s phone number.

e) Matching numeral and quantity
This skill includes many variations, starting from the matching of cards with numerals to real quantities (put two popsicle-stick flowers in the jar on which is written the numeral 2). An interesting activity for three-year olds involves throwing two dice, one of which has the numbers 1,2,3 (twice) and one of which has a different color on each face. The children take turns throwing the two dice, and put colored stickers on their record sheet accordingly. If the dice show 2 and green, they take two green stickers. Children who get “3” are noticeable pleased, understanding that this is preferable to getting “1.”

Many kindergarten activities are focused on this connection which is the most advanced of the triangle. The numeral 6 is posted on the bulletin board and children choose from a variety of stamp pad designs to “stamp out” six shapes for the display. When a child relates to the written symbol of the numeral with a full understanding of the quantity that it represents then the child is ready to use more formal language such as 4 is greater than 2 or even 4>2.
Two Important Number Relationships

As the basic number connections of the number triangle are being made, two schemas emerge which will allow the development of informal addition and subtraction: one is the successor schema and the other is the part-part-whole schema (Resnick, 1983).

1. The successor schema: the mental number line

The successor schema or mental number line is a cognitive schema which allows a child to answer the question - which is bigger 7 or 8? - without going back into images of quantity. Based on research which shows that children are more successful in comparison tasks when there is a greater difference between the numbers, Resnick (1983) compares the process to a visual assessment of positions on a measuring stick. Children begin to "see" the numbers in order, from 0-10 or 0-20, or 0-100, without having to make a transitional thought into the quantities which those numbers represent. This is the schema which Michelle was using in the story with which this article opened and which Nadia hadn’t yet acquired. When presented with the challenge of guessing the number between one and ten, Michelle could visualize the number line in her head and ask appropriate questions.

The first step in developing the successor schema is in the direction of one-more, one-less. Two-year olds can be asked to put “one more” button in the box, and “one more” and “one more.” Gail who was three and a half, wanted to play with a basket of chestnuts. When she counted eight nuts, her mother told her she could have ten. Gail took one more and counted them all again. “Nine,” she said. Her mother said again that she could have ten. This time Gail said out loud, “9,...10. One more,” and took one more chestnut.

Many activities are appropriate for encouraging the development of this schema. Nursery songs like “Five little monkeys jumping in the bed. One rolled off and hurt his head...” reinforce the one-less aspect of the number line. Lotto-type games can be played with four and five-year-olds where the cards match a number one more or one less on the board. An interactive number line where the number cards can be mixed-up, or re-arranged by the children is an important activity center. The primary versions of the game Rummikub offer much practice in generating number sequences.

The successor schema is of major importance in addition and subtraction strategies. We know that 6 + 2 is 8 because we count up in our head on our mental number line. We know that 9-1 is 8 because we know that 8 is one place to the left of 9 in our mental number line. Even more advanced strategies for solving 8 + 3 are based on our realizing that 8 is just 2 (steps) from 10 and then one more is 11.

2. The part-part-whole schema

The second schema which is being developed is the part-part whole schema (Resnick, 1983). This represents the number as potentially partitioned in a variety of ways. Addition and subtraction are intrinsically connected in this special relationship
among triples of numbers; concrete tasks help in building an awareness of these "number families."

If I have seven beans, I can hold six in the right hand and one in the left or five in the right hand and two in the left or none in the right hand and seven in the left and so on. There are three principles at work here: 1) In every example there are still a total of seven beans; 2) Once it is known how many are in one hand it is possible to guess correctly how many are in the other; 3) There are a finite number of possibilities so one can be sure when all have been found. Three and four-year olds enjoy performing a similar activity with two-colored counters: A group of four children at a table each take a turn to spill out five counters from a can and answer the questions: How many counters do we see of each color? How many counters altogether?

The most well-developed activity center for this schema is based on the Workjobs II collection (Baratta-Lorton, 1979). The children sit in a group of four and are engaged in one of two kinds of activities:

a) Organizing quantities of counters in two colors on a workmat (e.g. red and yellow flowers to decorate a hat) - each child puts six flowers on the hat in a variety of combinations and they talk about their arrangements

b) Organizing quantities of identical counters in two spaces on a workmat (e.g. oranges on a tall tree and a short tree) - each child divides five oranges between two trees and they describe their arrangements

Another successful activity for reinforcing the part-part whole schema is based on a bus cut-out (9" x 12") and cut-outs of groups of passengers who can go on the bus. Groups can be one, two, or three faces, organized on squared paper. The parent/teacher sits with the children at a work-table, where each child or pair of children has the bus cut-out and groups of passengers. The story is told that three passengers got on the bus. Some children will choose to put on the bus the group of three, some will put a group of two and one, some will put three single passengers. Each child explains his choice. The story continues that now four passengers got on the bus, and the children choose which combination to make for four passengers. The children are then asked how many passengers are there altogether on the bus? Children will use different strategies to count the seven passengers. The activity can continue to ten or more.

Informal Addition and Subtraction

The bus activity described above offers a transition between the part-part-whole schema and the encouragement of strategies for informal addition and subtraction. Informal addition and subtraction will surface during mathematical discourse around familiar contexts. Young children are attracted to mini-store activity centers with the prices for goods between one and four dollars clearly labelled on the empty snack bags. Play money in different combinations (one, five and ten-dollar bills), stored in a cashier box, adds to a very popular, spontaneous group activity. Once a month it is a good idea to change the goods for sale and to initiate a small group discussion around a
task; for example, “I'm going to give each member of the group five single dollars. Which items can you buy? How much do you have left? Is there something else you can buy? What can't you buy?”

There is much potential for informal addition and subtraction in ball games usually played for developing motor skills. Young children enjoy bowling (light, plastic toy pins) and there are two variations which involve both counting and informal addition. In the first game, appropriate for three and four-year olds, each pin knocked down is worth one point and the children collect counters (bottlecaps) according to the number of pins they knock down each turn. After three turns the child with the highest score (number of counters), wins. At the next level, appropriate for 5-year olds, the pins are each given a value, 1, 2, or 3, which is written on it prominently. Points (counters) are then given at each turn according to the sum of the values of the pins knocked down. Much figuring and interesting strategies can be heard as the children share their thinking with the group.

Another motor activity which can be used to stimulate informal addition and subtraction is, Toss the Beanbag. Hoops can be set up one behind the other labeled with numbers 1,2, and 3 (points). At each turn, a child throws three bean bags and collects counters for the points he earned. If a bean bag lands out of the hoops then the child gets one point less. After three turns the child with the highest score wins - or the first child to get ten points in the winner and everyone else keeps playing until they also get ten points.

**Formal Addition and Subtraction**

Children who have had the benefit of a program as described come to first grade with the skills and concepts which can help them approach school mathematics. They find numbers to be user-friendly, and relate to them as familiar cultural tools. They see themselves as being able to understand the “number language” and to manipulate it logically to solve problems. They believe in themselves as “thinkers about numbers.”

As they learn the formal symbols for the operations of addition and subtraction, the equal sign and the proper form for writing a number sentence (6+3=9), they will already have handy mental references which will bring the symbolic language to life. When they learn new operations of multiplication and division they may draw on their experiences when they knocked down three bowling pins for two points each, or decided to “split” ten dollars with a friend. Besides the particular mathematical connections, they will have a basis for believing that formal arithmetic operations are indeed based on structures they “own.” They will see an arithmetic problem not as a moment of tension but as an opportunity to share ideas and strategies with partners, with small groups, with the whole class.

Michelle and Nadia are now fifteen and sixteen. By inspiring me to listen to the thinking behind their game strategies, they opened for me a door so wide into the minds of young children that it has never closed.
References:


Chapter Seven: The counting model (pp.73-82).


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