A central tenet of mathematics education reform is the integral role of technology at all grade levels. Current technological changes combined with changes in mathematics content and instructional method require elementary mathematics teachers to be able to design technology intensive lessons for exploration and discovery of these concepts through appropriate computer applications. In actual practice, however, most computer applications provided for mathematics education consist of software designed for a specific educational purpose—the "solution in a can" scenario. Furthermore, economical constraints often stand in the way of incorporating such special purpose software into an instructional setting. This paper discusses an alternative to this traditional approach which shifts the instructional focus from specific computer applications to more sophisticated uses of general purpose software. In particular, educational uses of spreadsheets are developed as an exemplar for this approach. The methods and approaches described were presented to graduate and undergraduate mathematics education majors in a continuing education course on microcomputers in the elementary mathematics classroom. (Author/AEF)
New tools for new thoughts: Effects of changing the "Tools-to-Think-With" on the elementary mathematics methods course.

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Abstract: A central tenet of mathematics education reform is the integral role of technology at all grade levels. The current technological changes combined with the changes in the mathematics content and instructional method require elementary mathematics teachers to be able to design technology intensive lessons for exploration and discovery of these concepts through appropriate computer applications. In actual practice, however, most computer applications provided for mathematics education consist of software designed for a specific educational purpose - the solution in a can scenario. Furthermore, economical constraints often stand in the way of incorporating such special purpose software into an instructional setting. In this paper we will discuss an alternative to this traditional approach which shifts the instructional focus specific computer applications to more sophisticated uses of general-purpose software. In particular educational uses of spreadsheets will be developed as an exemplar for this approach.

Introduction

Frequently, when we talk computer applications as pedagogical tools in the mathematics classroom we mean software designed for a particular educational purpose. Yet economical constraints often stand in the way of incorporating special purpose software into an instructional setting and thus challenge computer-mediated mathematics pedagogy and ongoing inservice and preservice programs. A possible way to address the financial challenge is to shift emphasis from specific computer applications as teaching and learning tools to a broader and more sophisticated use of general-purpose software. Spreadsheets, for example, seem to become more and more available in schools, colleges, and universities.

How can the retrofitting of generic tools into an educational environment (Kaput 1992), particularly the introduction of spreadsheets into elementary mathematics classroom, be achieved? What does it take for mathematics teachers to develop into technologically minded cognizing and reflective agents (Cooney 1994), capable of appreciating the field of technology-mediated mathematics education as disciplined inquiry, and skillful in incorporating spreadsheets into the practice of mathematics teaching?
Background

Traditional mathematics instruction has emphasized procedures, memorizing algorithms, and finding the "one right answer". Mathematics, as it was most often presented, was not a subject open for discussion, debate, or creative thinking, nor were students encouraged to find alternative ways to solve a problem or different procedures for carrying out an operation. Computational expertise resulted from this, but little else. Students followed the algorithms necessary to solve a problem, but could not understand why or how those algorithms answer the question at hand. Given this, it was hardly surprising that students became imbued with rigid mental representations of mathematical problems.

This is unfortunate since flexibility appears to be a characteristic valued in many domains. Researchers from many fields associate the flexible application of rules and strategies with expertise and higher levels of cognitive operations. Indeed, one key to successful learning in this newer approach to mathematics instruction is flexibility in choosing and using mental representations.

At this point a dilemma develops. While we now know a great deal about the current internal representations of people engaged in a variety of tasks, we know far less about how they arrived at those representations. In short, while we know a fair amount about what it means to be a mathematician, we still don't know very much about how to become a mathematician. If we want to achieve the goals of the NCTM Standards, it is necessary to first understand how one arrives at those goals.

Finding that competent mathematicians score well on tests of computation, for example, does not mean that we should attempt to implement a mathematics program emphasizing the basic facts. Rather we must closely examine how competent mathematicians reached that end state. A constructivist approach toward learning has been widely suggested as a way to explain how people acquire new information. In this perspective, learners create their own internal representations of the external world. This view has been widely embraced by the mathematics community, but as many have pointed out constructivism is a loosely construed concept.

Perhaps one of the more telling problems with branches of constructivism that adhere too closely to a representational perspective is that it leads to several problems. First, it falls into the learning paradox. If learning is triggered by internal representations, how does one construct internal mental representations of objects which are more complex than those which already exist in one's mind? Second, since the goal of instruction in representational positions is to provide explicit and transparent representations of mathematical relationships, when instruction fails the teacher's only recourse is to provide more and more explicit and transparent representations. This inevitably leads to its own form of reductionism that strips mathematics relationships of their meaning.

The difficulties that representational positions have in offering an adequate description of learning, illustrate that descriptions of how one learns may be far more complex than descriptions of existing mental representations. It appears that representational views of learning may, with different wording, lead back to the same problems with mathematics education discussed above. We are still left with some basic foundational questions concerning how children learn and how we can describe that learning.

We are not critiquing constructivism to distance ourselves from constructivism, but to clarify directions that offer more, and less, potential for constructivist educators to pursue. As pointed out above, constructivism is a broad umbrella under which many theoretical positions have pitched their tents, and some of those positions lead to contradictions between views of knowledge and an active learner.

Our position is similar to that of Cobb, Jaworski, & Presmeg (1996) when they describe an approach that views mathematics as both an individual and a collective activity that transcends the representational view. This view is shared in Schoenfeld (1992) who has approached the individual and the collective by suggesting that people learn mathematics by becoming apprentices in a mathematics culture. Participants in a mathematics culture view themselves as practitioner of mathematical sensemaking. He goes on to argue that one does not need to know all of the basics before one begins to think like a mathematician. It is necessary, however, that students must grapple with real problems, not routine exercises in which the territory is already mapped out. In "real problems", as opposed to routine problems, the routines, methods, and procedures one should use to resolve the problem are not clear. Instead, one must actively work through the parts of the problem to arrive at a reasonable solution.

Prior research (Connell and Bounieav 1997) suggests that, for the case of mathematics, a focus upon action allows for the formation of a natural bridge between current constructivists in mathematics education and sociocultural researchers. Since sociocultural researchers have been influenced by Activity Theory, as explicated by Leontiev (1981) and other students of Vygotsky, it was probably inevitable that
mathematics educators and sociocultural researchers would begin making connections. Most sociocultural researchers avoid theories of learning which employ a metaphor separating the mind from what is to be learned. Instead, the concern is with combining the external world with the internal world of the mind. In these approaches to learning, the Cartesian distinction between the mind and the environment is replaced with an Einsteinian view of ongoing negotiated relationships.

In a sociocultural approach, learning is a result of active participation in a culture and is co-constructed by active interactions between the learner and her culture. From this, it might be inferred that if one wants students to think like mathematicians, than one must create an active participatory mathematics community. In turn, participation in this community will develop the skills associated with experts in the mathematics community (Connell, Peck, Buxton and Kilburn 1994).

The specific approach in this paper viewed individual mental functioning inherently situated in a social context and mediated by tools and signs. Many of the ideas presented in this paper have been included to great benefit in the respective mathematics education courses for preservice and inservice elementary teachers offered by the authors. As we will later report, these ideas affected the teachers' attitudes towards technology and challenged them to extend the use of spreadsheets to other elementary mathematics topics and to incorporate the ideas in their own teaching.

Outline of the computer-mediated setting

The sociocultural approach to mind views humans as coming into contact with the learning environment through the action in which they engage. In turn, the action employs different tools and signs called mediational means. The major claim of the approach is that the mediational means shape human action in many essential ways. Thus the term mediated action reflects the fundamental relationship between the action and mediational means which it employs. Any mental action directed towards solving a mathematical problem and mediated by appropriate tools and signs may be termed as mediated mathematical action. It is this action that is the focus of the paper.

Another basic principle associated with the approach is that human mental functioning, particularly mathematical action, originates in the course of communication and thus is inherently social. In a particular sociocultural setting, a contemporary elementary mathematics classroom, a mediated mathematical action can be grounded in the appropriation of the tools of technology such as computers and semiotic devices such as mathematical symbols and notation systems of the software used. The goal of an instructional discourse in such a setting is to use the mediational means as generators of meaning that, in turn, shapes mathematical action. From the sociocultural perspective "Ia]ny true understanding is dialogic in nature" (Voloshinov, cited in Wertsch, 1991, p.54), and this claim ties meaning closely to the dialogic orientation of the discourse. As far as an introduction of a computer into the discourse is concerned, it is of paramount importance to provide an environment capable of engaging the student into a purposeful dialogic encounter with the computer. The didactic emphasis of such an environment is to prevent undesirable consequences of authoritative discourse and to allow for the so called internally persuasive discourse that awakens new meaning for a student (Wertsch, 1991).

Description of implementation and instructor observations

The methods and approaches described in this article were presented by one of us (Abramovich) to 7 graduate and 5 undergraduate students enrolled in a continuing education course Microcomputers in Elementary Mathematics Classroom. The students, mathematics education majors, ranged from preservice teachers to experienced inservice teachers. Familiarity with Excel, however, was not a prerequisite of the course and, as it turned out, only one participant of the course, a practicing teacher, was familiar with the software. The instructor's use of a computer, an overhead projection panel, and a screen made it possible at the beginning of the course to introduce the teachers to a spreadsheet and give basic directions how to format the cells, use colors, and enter numbers and formulas into the cells.

Yet, learning to retrofit a spreadsheet into elementary school mathematics discourse was backed up mostly by the teachers' excitement about its potential as mathematical/pedagogical tool. A practicing teacher who had begun the course with only basic word-processing skills affirmed that a limited computer
background "has not diminished my excitement and enthusiasm of the potential use of technology in my classroom."

In her opinion, using technology in teaching mathematics to children ...parallels the development of mathematics itself over the centuries: From manipulative, to pencil and paper abstract notations, to using these notations (formulas) to generate information on the computer, I believe the children need multiple experiences to understand these complex relationships." Looking forward to teaching mathematics with spreadsheets, she acknowledged: "I am especially excited that my school is purchasing a portable large screen which I can use in the classroom for demonstration purposes. Even though the students will not be at computers themselves they can come up and shade in the squares to answer the questions. My kids really like this.

It should be noted that all information regarding spreadsheet specifics as well as mathematical demonstrations were introduced to the teachers not as a final product, but in a real time. This allowed for their active involvement in a discourse on syntax, content and pedagogy of the environment in keeping with the sociocultural emphasis of the intervention. The focus was for participants of the course to experience teaching with technology through multivocal egalitarian conversation about the birth, development and implementation of the ideas. In our opinion, it is this intellectual milieu that allowed an inservice teacher to remark: I enjoyed our class discussions on the formulas, as we were able to try out and see the results immediately." Indeed, although the classroom where the course was conducted had only one computer for demonstration purposes, the machine was available for those teachers who wanted to try their own ideas in front of the class. Another inservice teacher affirmed that ...if computers were available for individual or small group explorations I would have learned so much more. This comment challenges the technological support of the teacher education course and suggests that the teacher has recognized the potential of computers as productive tools conducive to mediate mathematics learning. In addition, the particular emphasis on the amount of information learned by the teacher implicitly indicates the fundamental relationship between mathematical action and mediational means which, in the case of computer spreadsheets, turn out to shape this action in essential ways.

A survey conducted at the end of the course showed that among many topics taught in the course, the topic on exploring the concept of percent on a spreadsheet template had a major impact on the participants. In the words of one practicing teacher: Most likely the whole class was impressed with using the grid to study percents. The reason is that this topic is taught to students in elementary school currently for the IGAP [Illinois Goal Assessment Program] state standardized tests." This is of significant interest in light of "privileging" within a set of studied topics. This directly addresses notions such as how specific representations become accepted within the broader culture and come to have value. It also points to the manner in which the teachers arranged the topics in accordance with a hierarchy based on the power of applicability (Wertsch 1991 p.24) to a particular sociocultural setting -- elementary mathematics classroom.

One of the major advantages technology brings to the classroom was the emergence of an open ended intellectual milieu allowing for a variety of ideas to be explored. In such a setting, a teacher's role becomes one of extreme complexity for she or he is simultaneously substituted by a computer as an external authority for validation of truth and required to he an adaptive and reflective partner in advancement, capable of surviving ambiguity in a meaning-generative dynamic environment. Thus teachers' attitudes towards the so structured didactic setting may provide an additional insight into one's conceptualization of new patterns of interaction between content and pedagogy in developing contemporary mathematics teacher education programs.

For example, one inservice teacher expressed the belief that "open ended pedagogy is an essential component in a mathematics classroom. Many times a child will pick up on things no one has ever noticed because they are not as tied to viewing the world through the constraints which sometimes limits the views of a more educated person. It is always a joy to view the world through the eyes of a child! Our educational system often is a study of what mankind has learned in the past, and we, teachers, following the tradition try our best to organize and present this body of knowledge to the next generation. Mathematics, however, is a dynamic discipline and must be presented to students as such. We must not restrict their young minds."

This is in agreement with another inservice teacher opinion about patterns of interaction shaped by an egalitarian classroom discourse: "Students can become interested and want to explore an avenue or branch of the assignment that the teacher may not have thought of before. Students may inherit enthusiasm not only from the teacher but other students. This [open ended] environment definitely allows for creativity."
A particular example of such creativity was found in a collaborative work of two preservice teachers who demonstrated how rectangular grids could be employed for teaching routine topics in arithmetic. The cartoon displayed in Figure 6 shows an arithmetic task with "human sense" which, in particular, illustrates the fact that the origins of mediated agency can be found in social forms of human existence shaped by one's cultural background. All these examples definitely manifest the teachers' potential to be reflective and adaptive, capable of understanding how their students "come to know and believe what they do" (Cooney, 1994, p. 628). Better still, the teachers' own views on teaching and learning warrant their ability to harness semantic multivoicedness of technology-mediated discourse in future pedagogical orientations and decisions.

Note that assessment practices employed in technology-mediated settings can also be oriented toward developing teacher's ability to take intellectual risk in making pedagogical and/or curricula decisions. One such approach is to utilize the dialogic structure of classroom setting by encouraging preservice and inservice teachers to choose a topic for a project, formulate questions to be explored and decide on directions in which the ideas can be extended. Instead of mastering the process of making such decisions through experience in following them, the assessment of teachers' performance on computer-mediated mathematical assignments is organized along the lines of Palincsar and Brown's (1988) procedure of "reciprocal teaching" allowing them to "teach" the topic to the whole class.

Patterns of privileging found in the teachers' responses to question what did affect their choice of a topic for a final project provide yet a further insight into their belief system affected by experiencing first hand the power of spreadsheet-based technology. For example, one preservice teacher has recognized the potential of spreadsheets to reorganize the use of curricula materials: "I wanted to use Excel in a way that provides students with more than problems from a textbook." An inservice teacher had recognized how spreadsheets can be used in helping students make mathematical connections; "Using a spreadsheet for the discussion of area and perimeter is a very powerful tool because it gives students the opportunity to think about and gain intuitive knowledge about factoring."

Another inservice teacher described the role of spreadsheets in enabling students to gain a confidence as doers of mathematics. A very powerful feature of Excel is the program's ability to give students the opportunity to work with problems that are very complex and not usually presented until a higher-grade level. Finding a solution to a very complex problem can raise a student's self esteem and motivation. Students may feel proud and excited about their accomplishments."

These and other comments by the teachers favorably indicate their potential to move the field of mathematics education toward realizing the high visibility of the NCTM Standards. The authors believe that developing such ability in teachers is one of the fundamental goals of contemporary mathematics teacher education.

References


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