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ABSTRACT

This document features an activity for estimating the distance from the earth to the moon during a solar eclipse based on calculations performed by the ancient Greek astronomer Hipparchus. Historical, mathematical, and scientific details about the calculation are provided. Internet resources for teachers to obtain more information on the subject are also listed. (WRM)

Estimating the Distance to the Moon-- its relevance to mathematics

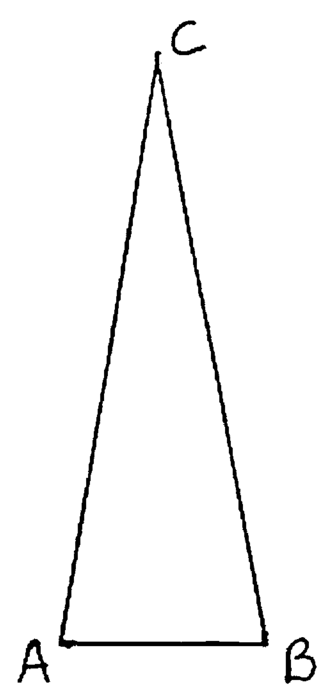
in connection with the leaflet

"The Solar Eclipse of August 11, 1999

by David P. Stern

The estimate of the distance to the moon by Hipparchus, and an earlier one by Aristarchus, involve what might be called **"Pre-Trigonometry."**

The essence of trigonometry is to use a baseline and two angles of view for estimating the distance to an unknown object. When baseline and distance are **comparable**, this requires trigonometry.



When the distance is **much larger** than the baseline--a long narrow triangle ACB, with only its base AB known--the accuracy is much worse. However, a simple approximate solution--used by the Greeks--can be used if AC = BC = R, namely to approximate AB by a small circular arc, with its center at C. Knowing the angle α it subtends, we get

$$\alpha/360^0 \approx AB/2\pi R,$$

and from that R can be found.

That approximation was used by Hipparchus in estimating the Moon's distance, except we need a small correction because AB is slanted relative to R; on top of p.4 simply states that its projection is about 3/4 its length, which is plausible and is close enough.

This and more is also contained in the web site **"From Stargazers to Starships"** at

http://www-spof.gsfc.nasa.gov/stargaze/Sintro.htm

On that site

Section (8b) gives the eclipse calculation described here
Section (8c) gives the method of Aristarchus
Section (8d) discusses the parallax
Sections (9a) and (9b) give the estimate of the size of the Sun by Aristarchus, which is related.

All these use the "pre-trig" approximation.

See also

http://www-spof.gsfc.nasa.gov/stargaze/Smath.htm

This is the introduction to an 11-part "Math refresher," whose sections (M-2) and (M-7) also give some interesting insights on the history of math.

The solar eclipse of August 11, 1999

Student Activity-- Estimating the distance to the Moon

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Wednesday, August 11, 1999, is the date of an interesting solar eclipse, visible in Europe and Asia. The Moon's shadow, inside which the Sun appears completely covered, is in this case only about 100 kilometers (60 miles) wide. However, as the Earth rotates around its axis, the shadow sweeps across the Earth, and (weather permitting) anywhere along the "path of totality" a total eclipse of the Sun may be seen at the appropriate time. On August 11, 1999, that path stretches from England through France, Germany and central Europe to Turkey and Iran, reaching India near sunset. In a much wider region on both sides of that path, a "partial eclipse" is seen, with the Moon covering only part of the Sun.

What makes this eclipse special is that, in the Near East, it comes fairly close to duplicating an ancient eclipse (probably in 129 BC) used by the Greek astronomer **Hipparchus** to estimate the distance of the Moon. That eclipse was total at the western end of the passage between the Black Sea and the Mediterranean, then known as the Hellespont (now the Dardanelles), while in Alexandria in Egypt only 4/5 of the Sun was covered.

Using that information and the position of the Sun in the sky at the time, Hipparchus was able to derive the Moon's distance from Earth. He estimated that it was between 62 and 74 times the radius of the Earth, which in its turn had been estimated fairly accurately by Erathosthenes, nearly a century earlier. The actual average distance is now known to be about 60 Earth radii.

In August 1999 the totality path (**Figure 1**) passes some 400 kilometers to the north-east of the Hellespont: had this been the eclipse observed by Hipparchus, it might have been total in or near the city of Heraclea, on the southern shore of the Black Sea. In Alexandria (then and now a large city), 71% of the Sun's diameter is covered.

Here it will be shown how students can conduct the same calculation as Hipparchus did, using those numbers as well as information about the location of the Sun in the sky. This will be a rather crude calculation, using many approximations and simplifications.

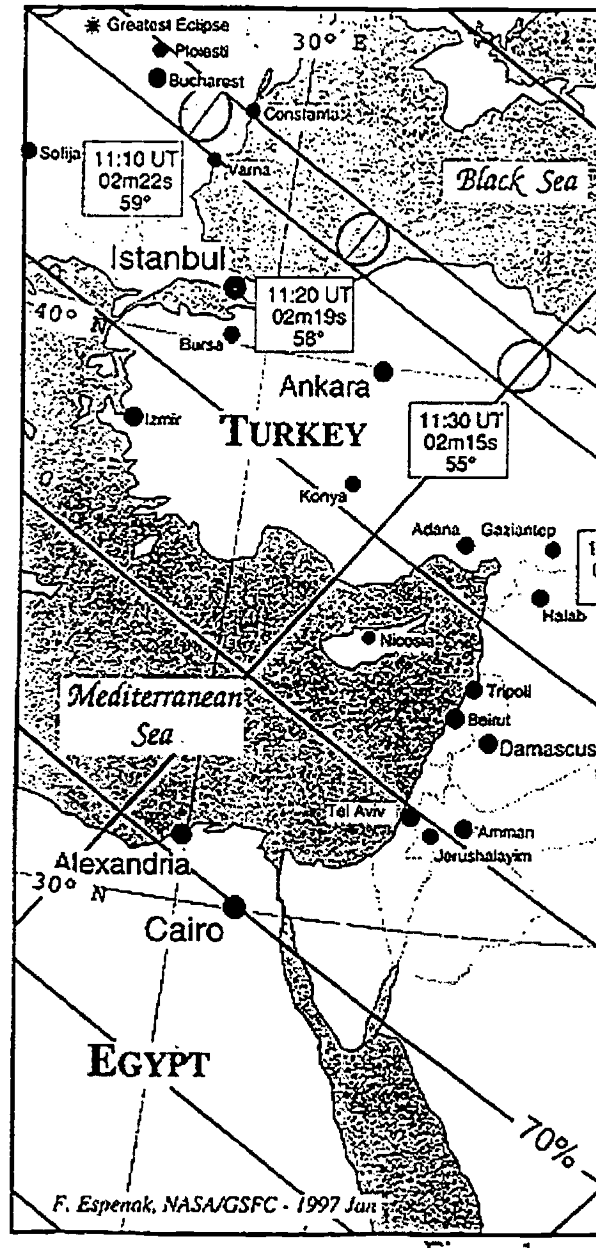


Figure 1

Hipparchus

Hipparchus was probably the greatest astronomer of ancient Greece, and yet little about him is known. He was active between 150 and 127 BC. His books were all lost, but his work is extensively quoted in the astronomical books of Claudius Ptolemy who lived nearly 300 years later.

We know that Hipparchus determined and catalogued the positions of 850 stars. He did so with the unaided eye, using simple instruments whose design we can only guess, more than 17 centuries before Galileo introduced telescopes to astronomy. Hipparchus also classified the stars by their magnitude--the brightest were assigned the 1st magnitude, the next brightest the 2nd, and so on down to the 6th, a classification from which the modern definition of star magnitudes evolved. When the European Space Agency in 1989 launched an orbiting telescope which precisely mapped the positions of stars, it was named "Hipparcos" in honor of the ancient star-mapper.

Hipparchus mapped the positions of the stars (as we still do today) by assuming they were attached to an enormous "celestial sphere" surrounding the Earth and rotating like a globe, around an axis passing two opposite "celestial poles" P and P' (**Figure 2**).

We now know, of course, that it is the Earth that rotates, while the stars keep their places, rather than the other way around. But as the figure shows, either way describes equally well the apparent motion of the stars, Sun and Moon.

Latitude

Suppose you are at the Hellespont (the Dardanelles), around north latitude of 40^0--that is, the radius from the center of the Earth to that point makes an angle of 40^0 with the equator (**Fig. 2**).

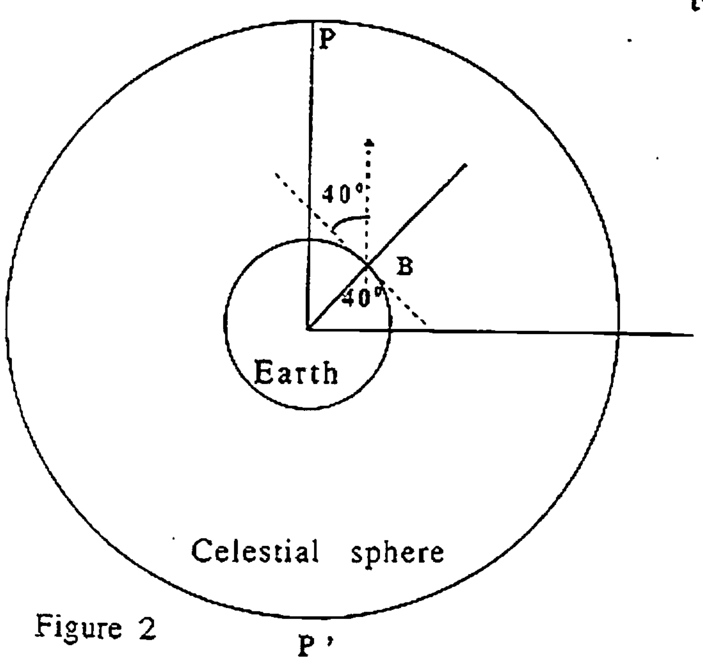


Figure 2

From that location you only see a small part of the Earth, which therefore appears as flat, along the broken line of the drawing. The direction to the celestial pole is then given by the broken arrow (remember, the pole P is <u>very</u> far!). From simple geometry--e.g. the fact that the radius is vertical and therefore perpendicular to the broken horizontal line--it then follows that the angle between the horizon (broken line) and the local direction to P (broken arrow) is also 40^0, or whatever the local latitude is.

In this century a fairly bright star is located within half a degree of P--the pole star or *Polaris*--and by observing its elevation above the horizon one can easily deduce the elevation of P and from it the local latitude (at least within half a degree!).

The pole P was in a different position in 129 BC, because of a slow periodic shift also discovered by Hipparchus ("precession of the equinoxes")--but Hipparchus must have known the pole's location from other observations. This way he might have established the latitudes of

The Hellespont as	$40\ 1/3^0$
Alexandria as	$31\ 1/3^0$

The Method

Figure 3 shows the situation during the eclipse--purely schematically, the actual angles would be much smaller and the Moon and Sun much more distant. The distance AB from Alexandria to the Hellespont can now be expressed in two ways.

First, in the triangle ABO, it can be represented in terms of the radius r = AO = BO of the Earth. If we assume that the Hellespont is exactly north of Alexandria, that triangle is determined by the latitudes of both locations.

Second, AB can also be expressed in terms of the triangle ABM, with M a point on the Moon, marking the leading edge of the Moon's disk which covers the Sun. That triangle is determined by eclipse information. If we assume the Moon is exactly overhead, AB is given in terms of the distance R = MA = MB of the Moon. By setting these two expressions of AB equal to each other, we get a relation between r and R, telling

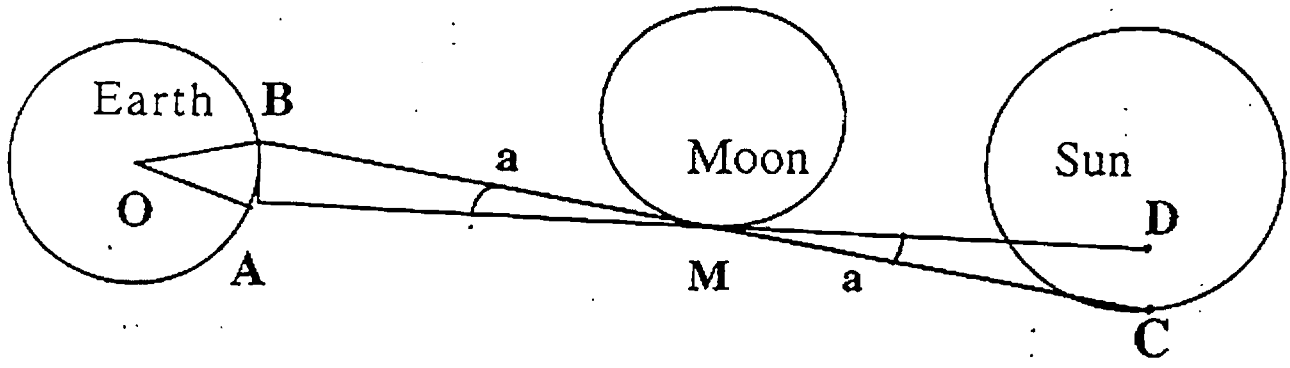


Figure 3

us how many times the Moon's distance R is larger than the Earth's radius r.

The Triangle ABO

If the Hellespont is exactly north of Alexandria, the angle AOB is

$$AOB = 40\frac{1}{3} - 31\frac{1}{3} = 9 \text{ degrees}$$

A full circle around the Earth has length $2\pi r$ --in algebraic notation, 2 times π times r, where π = 3.1415926... --and it contains 360 degrees. The distance AB is therefore

$$AB = 9\ (2\pi r/360)$$

This is the distance along the curved surface of the Earth, but because 9^0 is a rather small angle, we may approximately take it as equal to the straight-line distance AB.

The Triangle ABM

In a total solar eclipse, the Moon just barely covers the Sun. (In the eclipse of 16 February 1999, seen in Australia, its disk was too small to cover the Sun and left a complete circle of light around its edge.). Let M be the point on the Moon at the front edge of the disk covering the Sun. In figure 3 that point, when viewed from the Hellespont (point B) covers point C on the edge of the Sun's disk.

We assume that the eclipse peaks in Alexandria (point A) at the very same moment (in 1999 it actually happens about 10 minutes later). Viewed from there, the point M covers point D on the Sun, about 1/5 of the disk from the edge.

The apparent size of the Sun in the sky--the angle it covers in our field of view--is about 0.5^0. Therefore 1/5 of the disk, the angle marked by the letter a, is equal to 0.1^0.

The calculation now resembles the earlier one. If **R** is the distance to the Moon, a circle centered at M and passing through A and B has a length $2\pi R$, and the distance AB, covering 0.1^0 of that circle, has length

$$AB = 0.1\ (2\pi R/360)$$

Again, this distance is measured along the arc of a circle, but because 0.1^0 covers just a tiny part of that circle, here too no great error is committed by assuming AB is actually the straight-line distance.

Distance to the Moon--first guess

Setting the two expressions for AB equal to each other gives

$$0.1\ (2\pi R/360) = 9\ (2\pi r/360)$$

Dividing both sides by $(2\pi r/360)$ and then by 0.1:

$$R/r = 90$$

placing the Moon at a distance of 90 Earth radii, an overestimate to 50%.

A Better Guess

One reason for the overestimate (not the only one) is that our drawing placed the Sun **overhead** at the eclipse sites. Actually it never gets that far from the horizon, not even at high noon on midsummer day, when it is at its highest.

Not knowing where the Sun was in the eclipse observed by Hipparchus, one must guess. Assume that the eclipse happened near noon, and that at the same longitude the Sun was overhead at the equator (its average noontime position there). Then **figure 4** makes clear that the segment of the circle of radius R covered by the angle a is not AB but AF, which is smaller. If AF = (3/4) AB, then

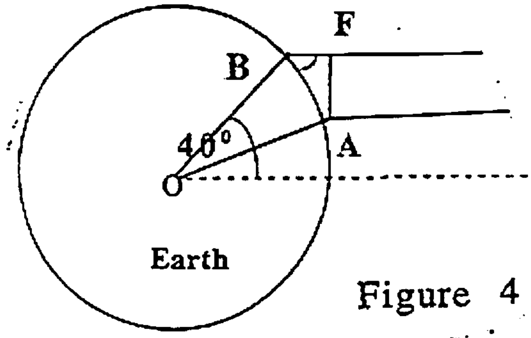


Figure 4

$$(3/4)AB = 0.1\ (2\pi R/360)$$

and we get a more accurate result, $R/r = 67.5$. One can also use trigonometry: because the marked angle in the triangle ABF equals to the latitude, we get

$$AF = AB \cos 40^0 = 0.766\ AB$$

giving approximately $R/r = 69$, well within the range obtained by Hipparchus.

The 1999 Eclipse

All these steps can be retraced with data for August 11, 1999. At the locations chosen, the peak of the eclipse occurs within about an hour and a half of noon (**Figure 1**).

The August date is about halfway between the midsummer solstice (June 21) and the fall equinox (September 23). On midsummer day the noontime Sun is overhead 23.5^0 north of the equator (on the "Tropic of Cancer"); at the solstice it is overhead on the equator. If the direction to the Sun is half-way between these values, it is overhead about 12^0 north of the equator, tilting upwards by 12^0 the lines that in Figure 4 extend to the right of A and B.

If the totality site on the shore of the Black Sea has latitude 42^0, then in that figure

$$AF = AB \cos(42^0 - 12^0) = AB \cos 30^0$$

The derivation of that cosine is part of any elementary trigonometry course and the result is

$$\cos 30^0 = \sqrt{3}/2 = 0.866....$$

As Figure 1 shows, about 71% of the Sun's diameter is covered in Alexandria. The rest of the calculation is straightforward.

Note to teachers:

(1) When presenting this unit to students not familiar with trigonometry, the proportions of the triangle ABF must be provided,
e.g. AF = 0.766 AB as estimated here,
or AF = 0.866 AB for the 1999 eclipse.
A brief course on basic trigonometry is also included at the "Stargazers" site (see below).

(2) In using the 1999 eclipse to estimate the distance to the Moon, or in the calculations of the next item below, calculators may be used.

(3) Additional related material is found in section (8c) of the "Stargazers" site (below), which expands on the concept of the parallax. Two examples are given: **(a)** Using an extended thumb for estimating distances outdoors and **(b)** The distance to nearby stars, using the diameter of the Earth's orbit as baseline.
In (a) students can practice the method and also use a yardstick to measure the actual ratio of the distance to the thumb and the separation of the eyes. In (b) students familiar with scientific notation for large numbers can calculate the number of kilometers or miles equal to one parsec and to one light year.

Web Resources:

This unit closely follows section (8b) of his site "From Stargazers to Starships" on the world-wide web, with its home page at **http://www-spof.gsfc.nasa.gov/stargaze/Sintro.htm.** Sect. (8c) there describes how Hipparchus also estimated the Moon's distance using a lunar eclipse, and sect. (20) tells about the role played by the Moon's distance in Newton's theory of universal gravitation.

Websites on the 1999 eclipse :

http://sunearth.gsfc.nasa.gov/eclipse/TSE1999/TSE1999.html

http://umbra.nascom.nasa.gov/eclipse/990811/rp.html

By the way, the Sun will rise partially eclipsed in the northeastern US--29% in New York, 60% in Boston

Sites on the Hipparcos mission (home page and educational):

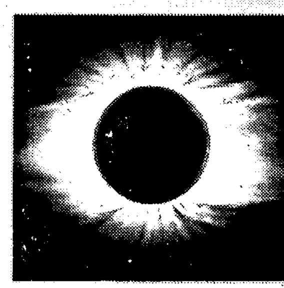
http://astro.estec.esa.nl/Hipparcos/hipparcos.html .... **/education-extra.html**

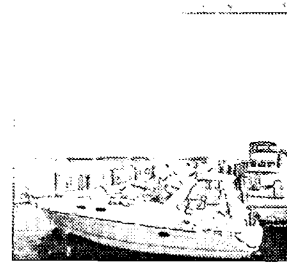
The historical information about Hipparchus is taken from "A History of Astronomy" by A. Pannekoek, Interscience 1961.

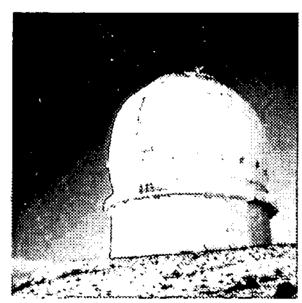
99 eclipse

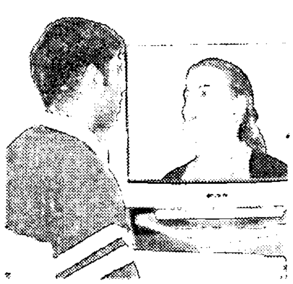
the last eclipse of the millenium

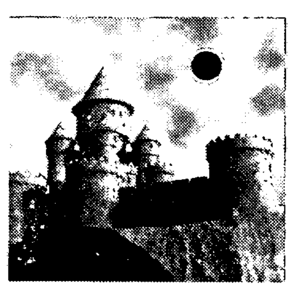
Wednesday, August 11, 1999, is the date of the "last solar eclipse of the millennium." The total eclipse is visible only in Europe and Asia, so "eclipse tours" to the Black Sea were sold out a year in advance. The Moon's shadow, traveling at nearly 1 km/sec, will traverse the center of Europe, continue through the Black Sea, and cross Turkey, Iraq, Iran, and Pakistan to India. The event will be seen by millions and will, no doubt, be the most publicized astronomical event ever - at least in Europe. Though North Americans will not be able to view the phenomenon directly they will have ringside seats to view the event through the media coverage which is planned, including both NASA Select TV broadcasts and internet coverage.

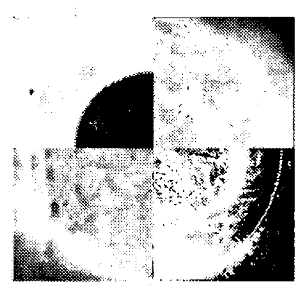






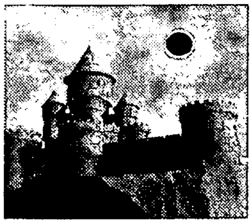


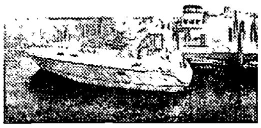




The Sun-Earth Connection Education Forum

http://sunearth.gsfc.nasa.gov/eclipse/



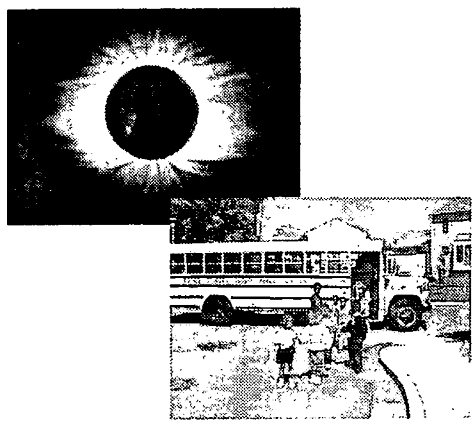


EDUCATION RESOURCES

The event takes place in the summer when school is out so it is an excellent summer learning experience for students and teachers. Even if you miss the eclipse, the resulting coverage will be archived on the website for educational enrichment at any time. These activities and background materials will be a useful adjunct to teaching about the Sun. All you need is a standard computer with a web browser connected to the network. If you have access to the NASA Select TV channel (check with your local cable provider) the experience will be enhanced by the eclipse broadcast. Some of the materials for the program will come from the cooperative efforts of the Sun-Earth Connection Education Forum (SECEF) and the Passport To Knowledge (PTK) series of interactive learning adventures which recently broadcast the first part of Live From the Sun. The second hour-long video is scheduled for April 13, 1999 at 1:00 PM, and may be seen on participating PBS stations and NASA Select-TV. A Teacher's Guide, Kit and Web site providing background on recent discoveries, current missions, and the Sun in human history, are available.

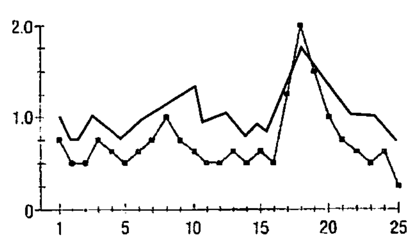
LIVE COVERAGE

Live TV and internet coverage from the cruise ship *Olympic Countess* in the Black Sea; images of the eclipse as it occurs via the NASA Select; astronaut Ron Parise and archaeo-astronomer Anthony Aveni relate their experiences as the shadow passes; Exploratorium broadcasts from ancient Harput Castle in Elazig, Turkey.



LIVE DATA

Featuring: split-screen imagery of the eclipse on the internet, views of the ship and its passengers, GOES satellite images of the shadow's progress across the Earth, automated onboard weather station readings of temperature, wind speed, etc., courtesy of NASA's OMNI (Operating Missions as Nodes on the Internet) project.

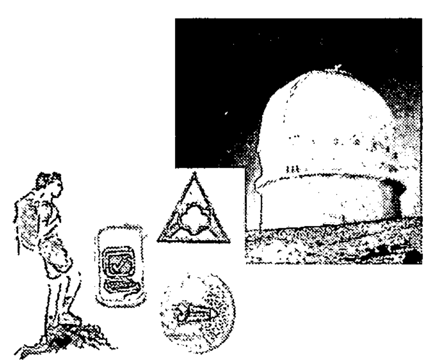


LIVE CHAT

Interact with scientists and other observers to understand what can be learned from eclipses

MUSEUMS

This is also an excellent opportunity to take advantage of the increasing popularity of "sleepovers" at science centers and planetariums. Girl Scouts are taking part in one such event at the Smithsonian's National Air and Spaçe Museum in Washington DC, and other locations. These nighttime events are intended to bring the excitement of a live viewing of the eclipse to the general public. Check the website in coming weeks and months.



http://sunearth.gsfc.nasa.gov/eclipse/
Dr. James Thieman *call* 301.286.9790

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