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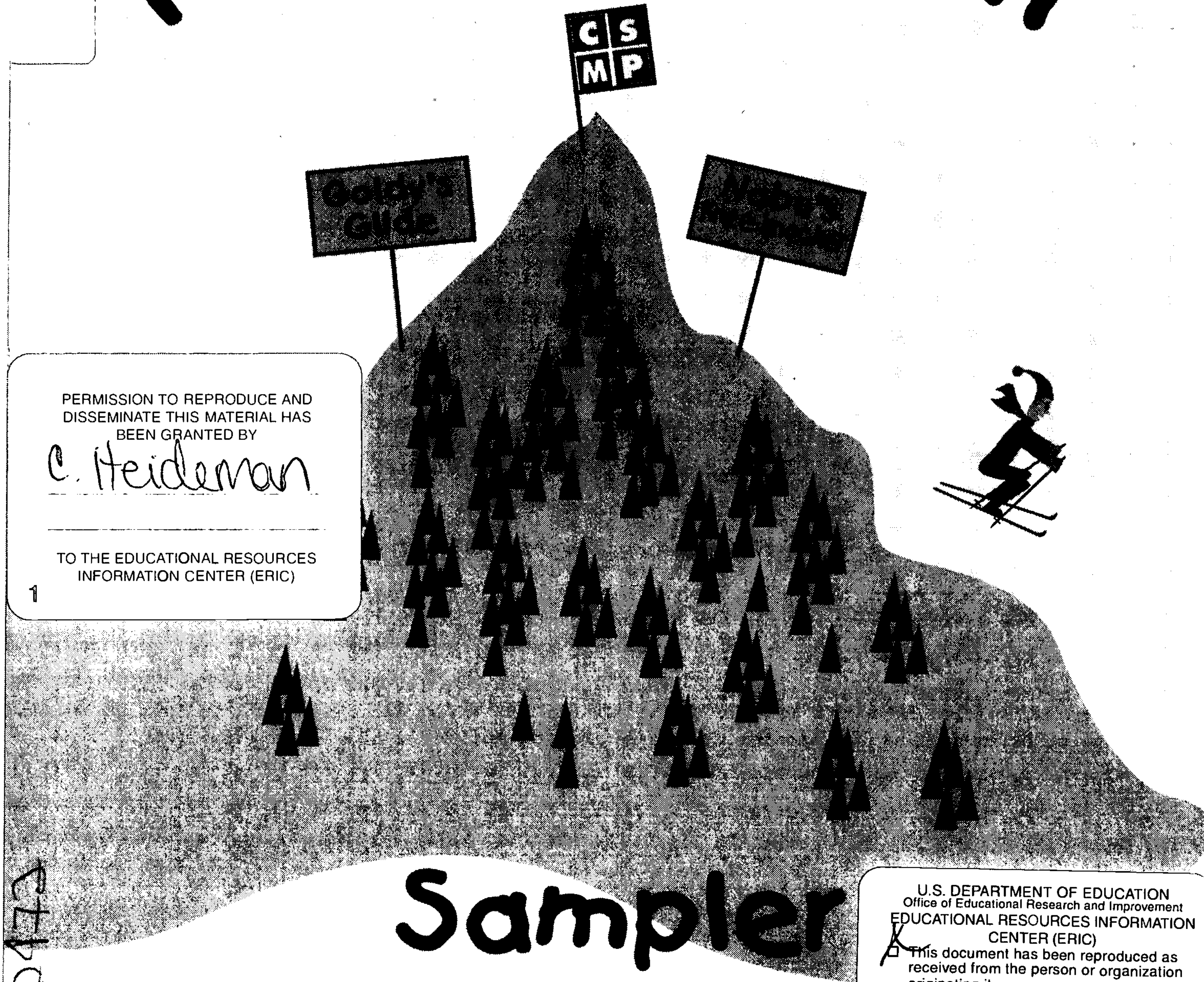
ABSTRACT

Math Mountain is a special section on the Comprehensive School Mathematics Program (CSMP) web site just for kids. It features challenging problems for primary and intermediate grade students. This booklet contains problems featured in Math Mountain over the past year and can be used as a source of additional problems to challenge students or to spark student interest in mathematics problem solving on the Web. (ASK)

Math Mountain



Sampler

Where is Math Mountain?

Somewhere in cyberspace, it's snowing. But that won't stop Goldy the Goldfish or a little boy named Nabu. They're out to challenge themselves on the slopes of Math Mountain. And you can, too.

Grab your snowboard and glide over to the CSMP Web site—http://www.mcrel.org/csmp/. Here you'll find sample lessons, a video, evaluation data—all kinds of introductory information about CSMP, the Comprehensive School Mathematics Program. Current CSMP users can check out upcoming workshops, read software reviews, obtain order forms, and more.

Math Mountain is a special section on the CSMP Web site just for kids, with challenging problems for primary and intermediate grade students. The URL http://www.mcrel.org/csmp/mtindex.asp will take you right to the top of the mountain where you can schuss down Goldy's Glide or, if you're feeling really daring, Nabu's Knee-knocker.

What's in this booklet?

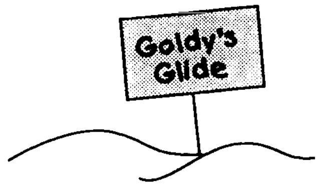
This booklet contains problems featured in Math Mountain over the past year. All are archived on the CSMP Web site and presented as in this booklet, with minor modifications. On the Web you may find a bit more description in the "Hints" sections and, in some cases, additional detail for "Solutions." Also, the graphics are in color on the Web; here we use shades of gray.

You can use this booklet as a source of additional problems to challenge students or to spark students' interest in mathematics problem solving on the Web, perhaps as recreational mathematics.

What is CSMP?

The Comprehensive School Mathematics Program (CSMP) is an exciting and powerful K–6 elementary mathematics program that focuses on problem solving and concept development. Its unique approach allows even very young children to grasp mathematical concepts and ideas. A variety of situational teaching methods; graphic, non-verbal "languages"; colorful and unusual manipulatives—even fantasy stories—activate the imaginations of young children and engage them in a fascinating exploration of mathematics, from developing basic skills to solving complex problems. This comprehensive curriculum is proven effective with all types of students at all ability levels.

February '98



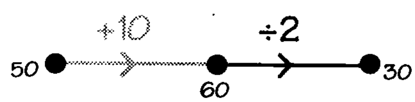
Maggie was very careless doing a math calculation. First she added 10 when she was supposed to subtract 10. Next, she divided by 2 when she was supposed to multiply by 2. The answer Maggie got was 30. Can you find the answer Maggie should have gotten?

Hint

Try to find the number Maggie started with. Then do the correct calculations.

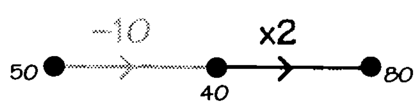
Solution

What Maggie did to get 30:

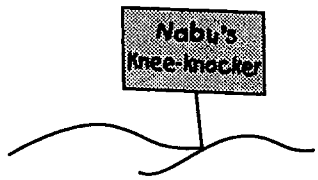


So, Maggie started with 50.

What Maggie should have done:



Maggie's answer should have been 80.

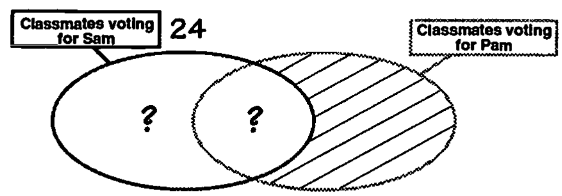


Pam is running for class president and Sam is running for class secretary. They receive votes from a total of 24 classmates, but Sam receives twice as many votes as Pam. Can you determine how many votes each student receives?

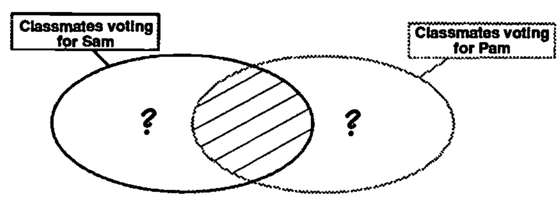
Note: There are several solutions to this problem assuming some classmates could vote for both Pam and Sam. Try to find all the possible solutions.

Hint

Could all 24 classmates vote for Sam?
Then how many would also vote for Pam?

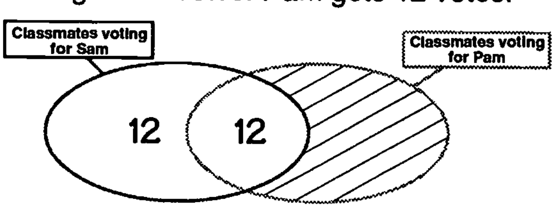


What if no one votes for both Sam and Pam?

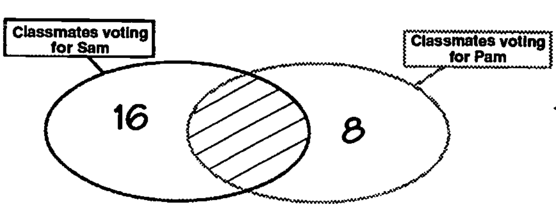


Solution

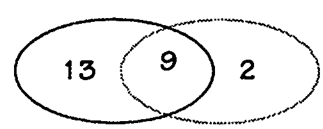
Sam gets 24 votes. Pam gets 12 votes.



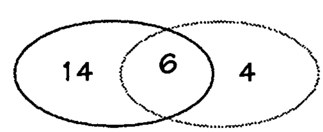
Sam gets 16 votes. Pam gets 8 votes.



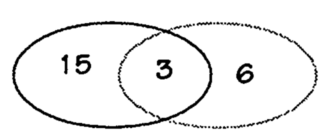
Sam gets 22 votes.
Pam gets 11 votes.

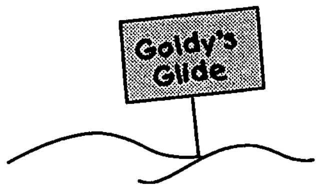


Sam gets 20 votes.
Pam gets 10 votes.



Sam gets 18 votes.
Pam gets 9 votes.





Use the even digits 2, 4, 6, and 8 to get names (expressions) for the odd numbers 1, 3, 5, 7, and 9. In each expression use each even digit once and use operations (+, −, x, ÷) as often as you like.

For example,

$$1 = (28 \div 4) - 6$$

Hint

You will need to use division (÷) at least once in each expression.

Solution

There are many solutions. Here are two possible expressions for each number.

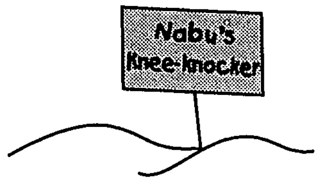
$$1 = (4 - 2) \div (8 - 6)$$
$$1 = 8 - (42 \div 6)$$

$$3 = 6 - (24 \div 8)$$
$$3 = (8 + 4) \div (6 - 2)$$

$$5 = (6 \div 2) + (8 \div 4)$$
$$5 = 6 - [(2 \times 4) \div 8]$$

$$7 = (6 \div 2) + (8 - 4)$$
$$7 = 6 + [(8 \div 4) \div 2]$$

$$9 = (24 \div 8) + 6$$
$$9 = [(4 + 2) \div 6] + 8$$



At Booker's Bakery they sell cookies at these prices:

Oatmeal Cookies: 9¢
Peanut Butter Cookies: 12¢
Chocolate Chip Cookies: 15¢

1. *What could you get in a $1.50 bag of cookies? Try to find several possibilities.*
2. *What is the maximum number of cookies you could get in a $1.50 bag?*
3. *Could you get a $1.50 bag of cookies with no chocolate chip cookies?*
4. *Could you get a $1.50 bag of cookies with an equal number of all three kinds of cookies?*

Hint

- You may notice that ten chocolate chip cookies could be in a $1.50 bag.
- You can "trade" three 15¢ chocolate chip cookies for five 9¢ oatmeal cookies without changing the price.
- Or, you can trade four 15¢ chocolate chip cookies for five 12¢ peanut butter cookies.
- Or, you can trade two 15¢ chocolate chip cookies for one 12¢ peanut butter cookie and two 9¢ oatmeal cookies.
- Also, you can trade three 12¢ peanut butter cookies for four 9¢ oatmeal cookies.

Solution

1. There are 26 solutions* to the problem "What could you get in a $1.50 bag?" For example, 10 chocolate chip (15¢), or 10 oatmeal (9¢) and 4 chocolate chip (15¢).
2. The maximum number of cookies you could get in a $1.50 bag is 16. There are two ways to get 16 cookies in a $1.50 bag:

   15 oatmeal (9¢) and 1 chocolate chip (15¢)
   14 oatmeal (9¢) and 2 peanut butter (12¢)

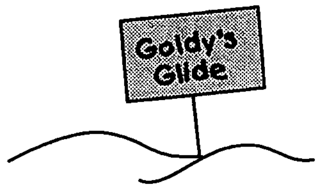
3. There are four possibilities to get a $1.50 bag with no chocolate chip cookies.

14 oatmeal (9¢) and 2 peanut butter (12¢)	6 oatmeal (9¢) and 8 peanut butter (12¢)
10 oatmeal (9¢) and 5 peanut butter (12¢)	2 oatmeal (9¢) and 11 peanut butter (12¢)

4. It is not possible to get a $1.50 bag of cookies with an equal number of all three kinds of cookies.

**A complete list of all 26 solutions can be found in Math Mountain on the CSMP Web site.*

April '98



Tory's uncle has some coins in his pocket. He tells Tory she can have them if she can say exactly the amount of money in his pocket. He gives Tory these clues. Can you help Tory?

Clues:

- *All nickels and dimes, but not all one kind of coin.*
- *Between 7 and 10 coins.*
- *More dimes than nickels.*
- *Could have the same amount of money with quarters only.*

Hint

Look at the possible amounts with eight or nine coins. Remember that there are more dimes than nickels, and not all dimes.

Solution

75¢ is the amount of money in the pocket.

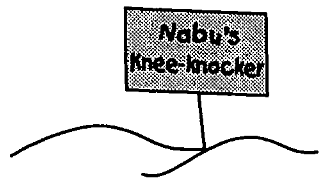
This table shows amounts satisfying the first three clues.

The only amount you could have with quarters only is 75¢.

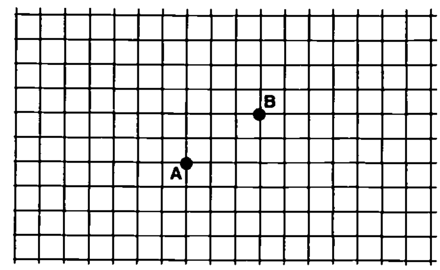
	Nickels	Dimes	Amount
eight coins	3	5	65¢
	2	6	70¢
	1	7	75¢
nine coins	4	5	70¢
	3	6	75¢
	2	7	80¢
	1	8	85¢

Note: If you also considered seven or ten coins, you found no other amount that could be made with quarters only.

	Nickels	Dimes	Amount
seven coins	3	4	55¢
	2	5	60¢
	1	6	65¢
ten coins	4	6	80¢
	3	7	85¢
	2	8	90¢
	1	9	95¢



This is the map of a neighborhood where all the streets are east–west or north–south. The "taxi–distance" between two intersections in this neighborhood is the length (measured in blocks) of a shortest route between two points. All routes must follow the streets like a taxi–cab would go. For example, the taxi–distance between points **A** *and* **B** *is 5 blocks.*



*The Library in this neighborhood is at an intersection. Here are clues about the location of the Library (***L***).*

Clues:

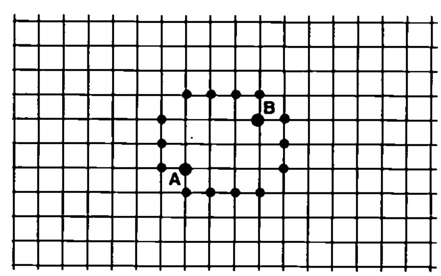
- *Taxi–distance from* **A** *to* **L** + *Taxi–distance from* **B** *to* **L** = *7.*
- *The Library is closer to* **A***.*
- **B** *is closer to the Library than to* **A***.*
- *When you are at* **A** *and go to the Library, you do not need to make any turns.*

Locate the Library.

Hint

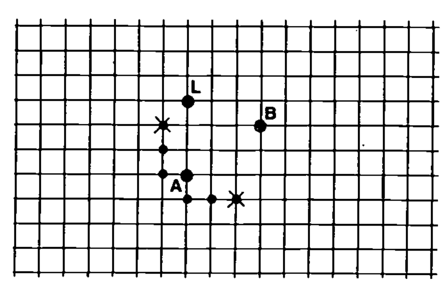
The points that fit the first clue are shown here. All the unlabeled dots could be for the Library (**L**).

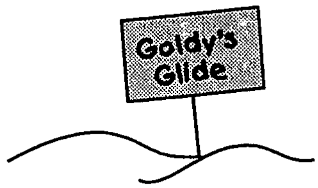
Now, decide which points fit the second clue, and so on.



Solution

The hint shows all the places that fit the first clue. **L** as well as the smaller dots fit the first and second clues. **L** as well as the × dots fit the first, second, and third clues. Only **L** fits all the clues.





Find two whole numbers Flip and Slip such that:

- *Flip is 100 less than Slip.*
- *Slip is five times Flip.*

Hint

Flip is 100 less than Slip
+100

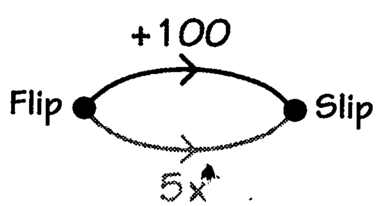
Flip	Slip
0	100
3	103
10	110
12	112
20	120
25	125

and so on.

Slip is 5 times Flip
5x

Flip	Slip
0	0
2	10
5	25
8	40
20	100
25	125

and so on.



Solution

Flip is 25 and Slip is 125.

- 25 is 100 less than 125.
- 125 is five times 25.

Nabu's Knee-knocker

May '98

Find two whole numbers □ and △ such that:

- *□ is two more than a multiple of 7.*
- *△ is three more than a multiple of 7.*
- *The product □ x △ is as close as possible to 500.*

Hint

Consider some possible products □ x △ when

□ : 2, 9, 16, 23, 30, 37, 44, 51, ··· (two more than a multiple of 7)

△: 3, 10, 17, 24, 31, 38, 45, 52, ··· (three more than a multiple of 7)

What can you say about the product □ x △?

□ x △: 6, 20, 27, 34, 48, 62, 69, 76, 90, 104, 111, ···

What numbers close to 500 fit the pattern for □ x △?

Solution

□ is two more than a multiple of 7 and △ is three more than a multiple of 7, so the product □ x △ is six more (or one less) than a multiple of 7.

But, not all numbers with this property can be □ x △.

For example, 13 = 7 + 6 or 14 – 1, but 13 is a prime number. It cannot be the product □ x △ where □ = 2 + a multiple of 7 and △ = 3 + a multiple of 7.

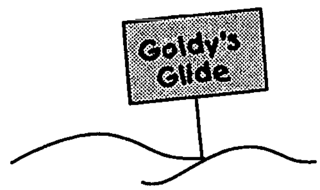
The numbers close to 500 that fit the pattern for □ x △ are:

496 (496 = 497 – 1 and 497 = 7 x 71) and 503 (503 = 504 – 1 and 504 = 7 x 72)

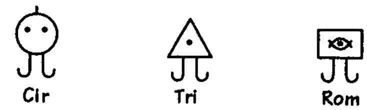
503 is a prime number so it can<u>not</u> be □ x △.

496 = 2 x 248 (□ could be 2 and △ could be 248)

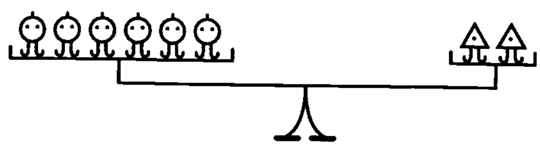
496 = 16 x 31 (□ could be 16 and △ could be 31)



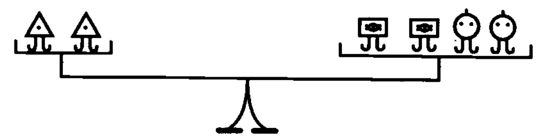
On the planet of FIG the inhabitants like to play teeter–totter.



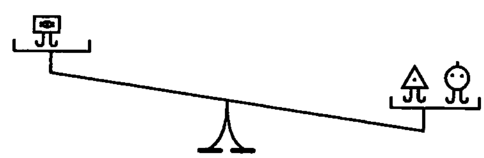
6 Cir balance 2 Tri



2 Tri balance 2 Rom and 2 Cir together



What could join 1 Rom to balance 1 Cir and 1 Tri together?

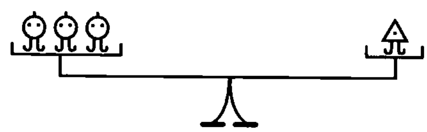


Hint

Find some other balance situations.

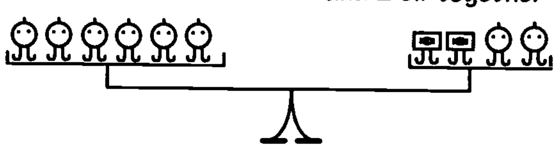
For example, taking half off both sides of the first balance works.

3 Cir balance 1 Tri

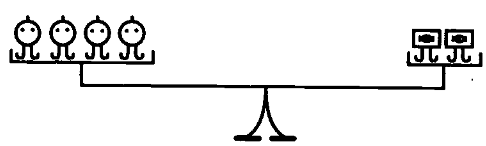


So does replacing 2 Tri with 6 Cir on the second balance, and then taking 2 Cir off each side.

6 Cir balance 2 Rom and 2 Cir together

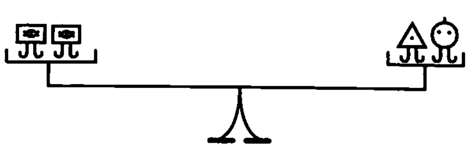


4 Cir balance 2 Rom



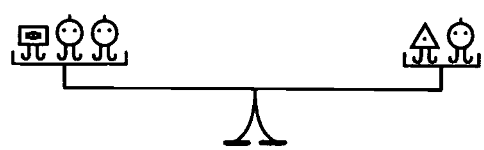
Solution

1 Rom could join 1 Rom to balance
1 Cir and 1 Tri together

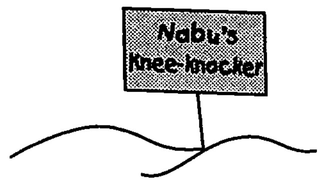


or

2 Cir could join 1 Rom to balance
1 Cir and 1 Tri together



Go to Math Mountain on the CSMP Web site to see one way to find these solutions.



Bonnie has three types of weights:

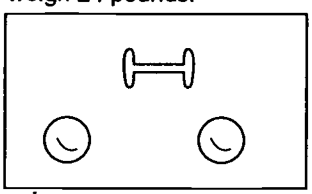
Barbells ( ) all have the same weight.

Donuts ( ) all have the same weight.

Spheres ( ) all have the same weight.

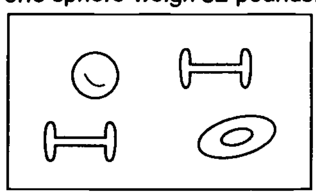
Bonnie does not know how much each type weighs, but she does know this:

A barbell and two spheres weigh 24 pounds.



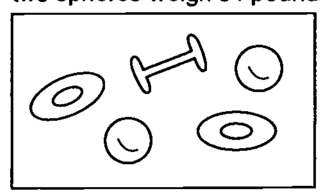
24 pounds

Two barbells, one donut, and one sphere weigh 32 pounds.



32 pounds

A barbell, two donuts, and two spheres weigh 34 pounds.



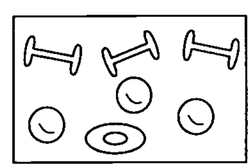
34 pounds

Can you help Bonnie determine the individual weights?

Hint

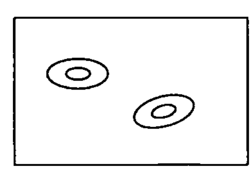
Find the weights of some different combinations.

For example, putting the 24 pound and 32 pound combinations together gives you a 56 pound combination.



56 pounds

Also, taking a 24 pound combination out of the 34 pound combination gives you a 10 pound combination.



10 pounds

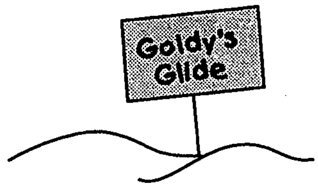
Solution

Barbells: 10 pounds

Donuts: 5 pounds

Spheres: 7 pounds

Go to Math Mountain on the CSMP Web site to see one way to find this solution.



Allie, Brad, Fran, and Gus each have a different favorite number. Their favorite numbers are:

10	18	7	15

Use these clues to match the children with their favorite numbers.

- *Allie's number is an even number.*
- *Brad's number is more than Gus' number.*
- *Fran's number is less than one–half of Allie's number.*

Hint

Use the last clue first: Fran's number is less than one-half of Allie's number.

- One-half of 10 is 5, and no choice is less than 5.
- One-half of 18 is 9, so Fran's number could be 7 (which is less than 9) when Allie's number is 18.
- 15 is an odd number, so it cannot be Allie's number.

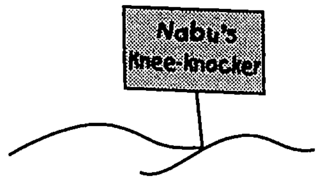
Solution

From the first clue (Allie's number is even) and the last clue (Fran's number is less than one-half of Allie's):

Allie's number is 18. One-half of 18 is 9, and Fran's number (7) is less than 9.

Then the choices for Brad and Gus are 10 and 15.
Brad's number is 15 because it is more than Gus' number.

ALLIE	BRAD	FRAN	GUS
18	15	7	10



Cody, Ellen, Nell, and Oscar each choose a different whole number between 0 and 20.

Use these clues to find each person's number.

- *Cody's number is a multiple of 5.*
- *Ellen's number is more than Cody's number.*
- *Nell's number is exactly two times Ellen's number.*
- *Oscar's number is one–third of Nell's number, but it is not the smallest.*

Hint

Use the third clue—Nell's number is exactly two times Ellen's number— to decide that Ellen's number is less than 10.

Then, use this information as you consider possibilities for Cody's number (a multiple of 5 and less than Ellen's number).

Solution

Cody's number is a multiple of 5, so the choices are 5, 10, or 15 (multiples of 5 between 0 and 20).

Nell's number is exactly two times Ellen's number, so Ellen's number is less than 10 (otherwise Nell's number would not be between 0 and 20).

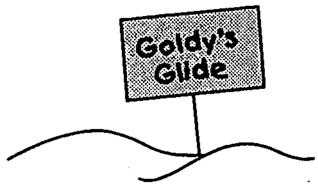
Since Ellen's number is more than Cody's, Cody's number must be 5 and Ellen's number could be 6, 7, 8, or 9. Nell's number (two times Ellen's) could be 12, 14, 16, or 18.

The last clue says:

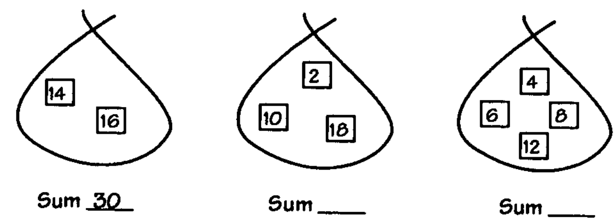
> Oscar's number is one-third of Nell's number, but is not the smallest.

From the choices for Nell's number, Oscar's number could be 4 (one-third of 12) or 6 (one-third of 18). Since it is not the smallest, we can eliminate 4.

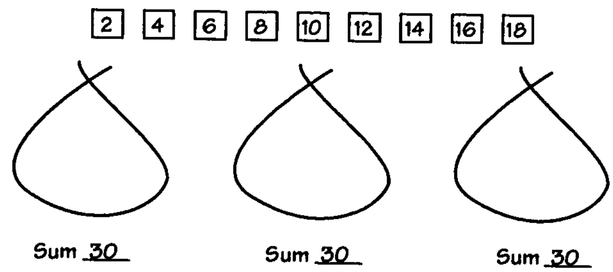
CODY	ELLEN	NELL	OSCAR
5	9	18	6



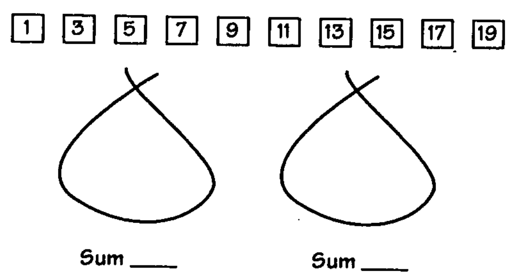
Add the numbers in each bag to get a sum.



Problem 1. *Put three even numbers in each bag so the sums are still 30. Use the nine even numbers (2 to 18) each once.*



Problem 2. *Put the ten odd numbers 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19 in these two bags so the bags have the same sum (the sums are equal). Use each number once.*



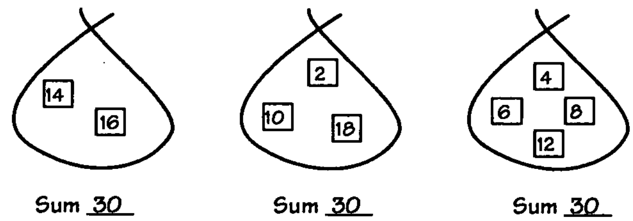
Hint

Problem 1. Decide which two even numbers you can put in the same bag with 18. Remember, the sum must be 30; that is, 18 + ____ + ____ = 30.

Do the same for 16, and then for 14.

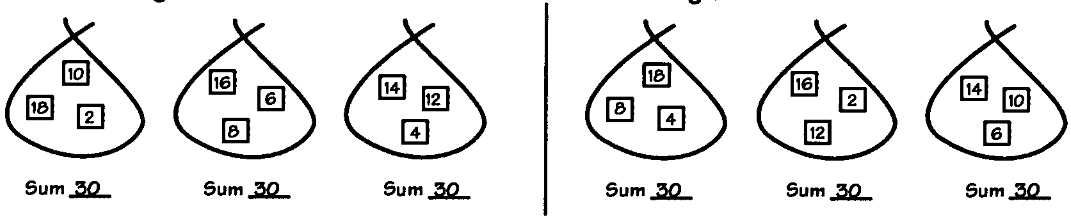
Problem 2. Add the ten odd numbers 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 to find the total. Then divide the total in half to find the sum for each bag.

Solution



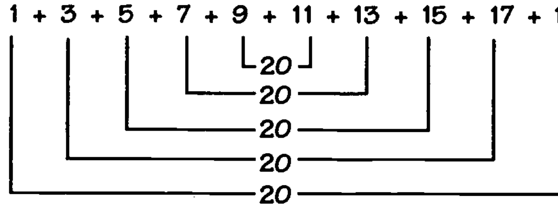
Problem 1. There are two solutions for this problem.

18 can be in the same bag with 10 and 2 **or** **18 can be in the same bag with 8 and 4**

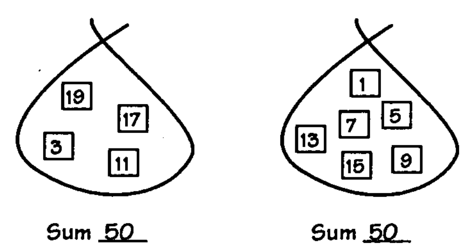


Problem 2. Add:

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$

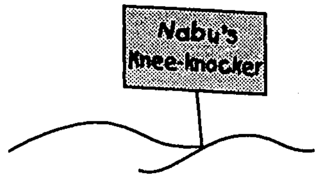


Each bag must have sum 50. For example,



Other solutions are possible. There will always be four numbers in one bag and six numbers in the other bag.

August '98

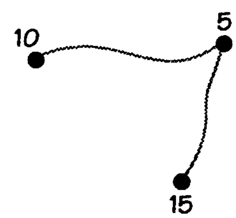


In the World of Whole Numbers, a new rule has been enacted at school to try to reduce the talking.

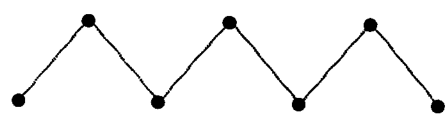
> Two numbers may talk to each other
> if and only if
> one of them is a multiple of the other.

For example,

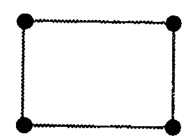
10 can talk with 5, because 10 is a multiple of 5; and 5 can talk with 15, because 15 is a multiple of 5; but 10 cannot talk with 15 (directly), because 10 is not a multiple of 15 and 15 is not a multiple of 10.



Problem 1. *Try to find seven whole numbers that can talk in this way, but when there is no gray cord (connection) between two numbers (dots), the numbers cannot talk to each other.*



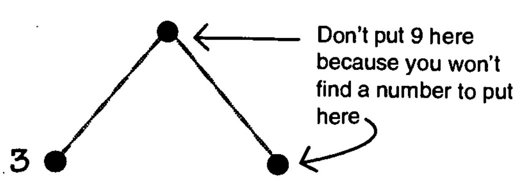
Problem 2. *Try to find four whole numbers that can talk in this way, but so that the diagonal numbers cannot talk (there is no gray cord between them).*

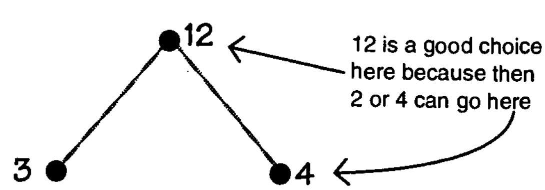


Hint

Do not choose 0 or 1 for any of the numbers. 0 and 1 both can talk to every whole number. 0 is a multiple of every whole number, and every whole number is a multiple of 1.

Problem 1. Start with any number you like (except 0 or 1) and find a multiple. Be careful to choose a number that is a multiple of several different numbers as well. For example,

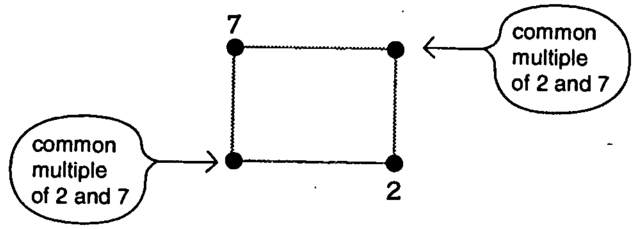




18

Problem 2. Choose a pair of diagonal numbers that cannot talk. Then find common multiples. Be careful not to use the least common multiple.

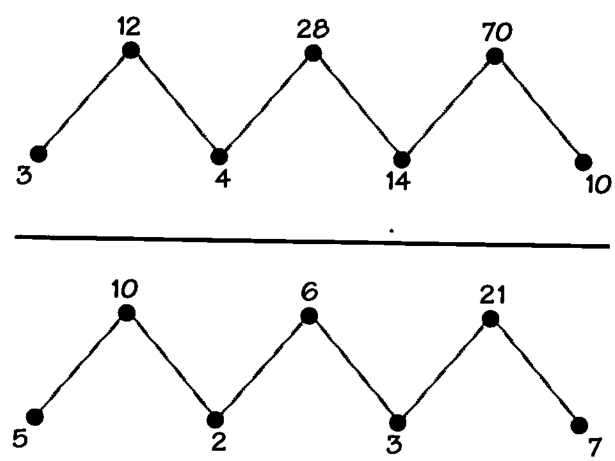
For example,



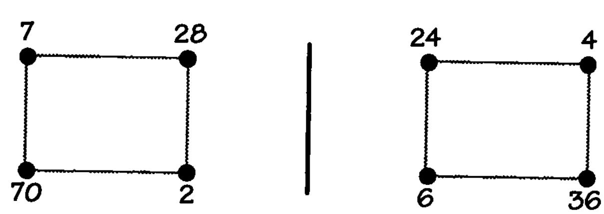
Solution

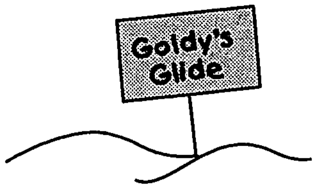
There are many solutions to both problems. Here we will give two different solutions to each problem. You are likely to have still a different solution. You may even like to find another solution.

Problem 1.

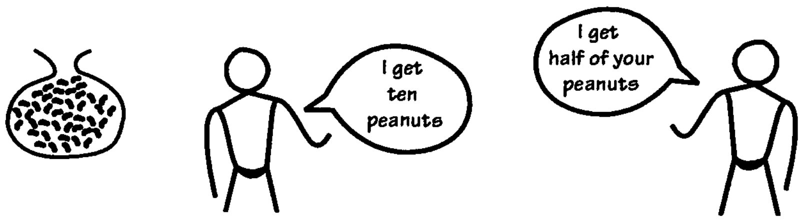


Problem 2.





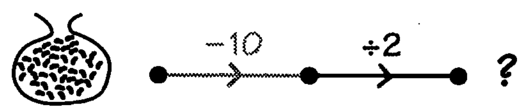
Ben has a bag of peanuts. He's not sure how many, but he knows there are more than 25. Also, Ben makes promises to two friends. He promises to give ten peanuts to one friend and to share his peanuts equally with the other friend. Ben cannot decide which friend to visit first. Ben would like to keep as many peanuts as possible for himself, and still keep his promises. What advice would you give Ben?



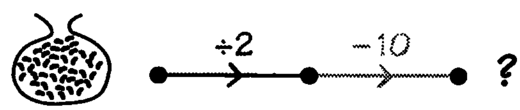
Hint

Think about some number of peanuts for Ben. Check how many peanuts Ben would keep for himself in each case.

Case 1: First visit the friend who gets ten peanuts.



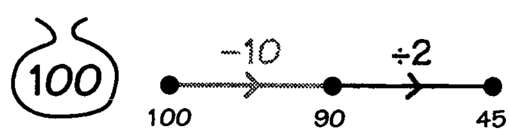
Case 2: First visit the friend who gets half the peanuts.



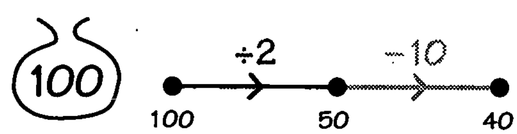
Solution

Ben should visit the friend who gets 10 peanuts first. Then he should go to the other friend with whom he will share equally.

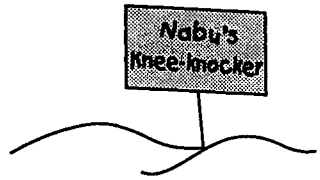
For example, if Ben had 100 peanuts, he would end up with 45.

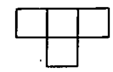


Going the other way, he would end up with only 40.



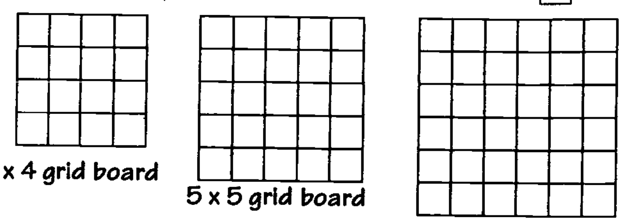
In fact, you will notice that Ben always ends up with five more peanuts when he first visits the friend getting 10 peanuts.



Shapes formed by joining four identical grid squares into a "T" are called T–tetrominoes. 

Show that several T–tetrominoes can be used to cover a 4 x 4 grid board, but not a 5 x 5 grid board and not a 6 x 6 grid board.

Note: A cover should just cover the squares of the grid board with no extra squares.



4 x 4 grid board

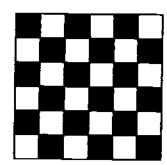
5 x 5 grid board

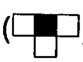
6 x 6 grid board

Hint

Count the number of grid squares on each grid board.

- How many T-tetrominoes are needed to cover a 4 x 4 grid board?
- How many grid squares are on a 5 x 5 grid board? Remember that T-tetrominoes each have four grid squares.
- View the 6 x 6 grid board colored as a checkerboard.

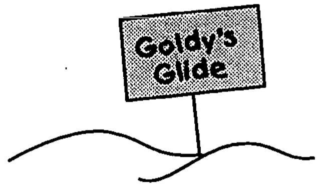


A T-tetromino would cover 1 white and 3 black squares () or 1 black and 3 white squares ().
The number of black and white squares is the same, so a cover would need half the T-tetrominoes covering 1 white and 3 black and half covering 3 white and 1 black.
Can we do that?

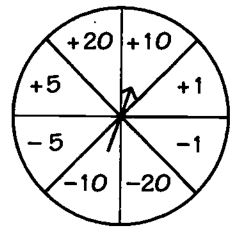
Solution

- A 4 x 4 grid board has 16 squares, so you need to use four T-tetrominoes to cover it. *Visit Math Mountain on the CSMP Web site to see one way to do this cover.*
- A 5 x 5 grid board has 25 squares. Six T-tetrominoes would cover 24 squares and seven T-tetrominoes would cover 28 squares. We can't cover just 25 squares.
- View the 6 x 6 grid board colored as a checkerboard with black and white squares. (See the hint.) A 6 x 6 grid contains 36 grid squares and would need nine T-tetrominoes to cover it (4 x 9 = 36). Half the T-tetrominoes would cover 1 white and 3 black squares, and half would cover 3 white and 1 black. (See the hint.) But 9 is an odd number and cannot be divided in half (with whole numbers), so you cannot cover a 6 x 6 grid board with T-tetrominoes.

October '98



Start with a score of 0. Imagine you spin this spinner four times. The result shows what you add or subtract from your score.



- *What is the greatest final score you could have?*
- *Could your final score be 0?*
- *Some of the scores below are possible after four spins. Show how you could get them.*

100 50 44 37 35 30 23 12

Hint

For the greatest possible score, start at 0 and add the greatest amount for each of four spins.

For each of the given scores, look for ways to write that number with only the amounts on the spinner—adding or subtracting four times.

For example, $50 = 0 + 10 + 20 + 10 + 10$

Solution

- The greatest possible score is 80. $80 = 0 + 20 + 20 + 20 + 20$
- Yes, the final score could be 0. For example, $0 = 0 + 5 - 5 + 10 - 10$
  There are many ways to get a final score of 0.
- All the given scores are possible except 100 and 37.
  Here we show one way to get each score. There are many other ways.

~~100~~ The greatest possible score is 80.

$50 = 0 + 20 + 5 + 20 + 5$

$44 = 0 + 20 + 20 + 5 - 1$

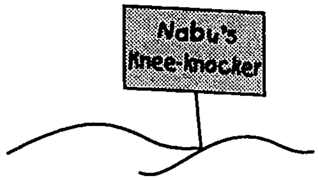
~~37~~ This score is possible with five spins, but not four. $37 = 0 + 20 + 10 + 5 + 1 + 1$
or
$37 = 0 + 20 + 20 - 1 - 1 - 1$

$35 = 0 + 5 + 5 + 5 + 20$

$30 = 0 + 10 + 10 + 5 + 5$

$23 = 0 + 20 + 5 - 1 - 1$

$12 = 0 + 5 + 1 + 5 + 1$



Here is a name for 1000 using only the symbols **1** *and* **+**.

1000 = 111 + 111 + 111 + 111 + 111 + 111 + 111 + 111 + 111 + 1

*This name for 1000 uses 37 symbols (***1***s and* **+***s).*

- *Write a name for 1000 using only the symbols* **8** *and* **+**.
  Try to use as few symbols as possible.
- *Write a name for 1000 using only the symbols* **5** *and* **+**.
  Try to use as few symbols as possible.
- *Write a name for 1000 using only the symbols* **4** *and* **+**.
  Try to use as few symbols as possible.

Hint

Try to get as close as possible to 1000 before using +. Consider how many addends will be needed by looking at the digit in the one's place (8, 5, or 4).

Solution

- **1000 = 888 + 88 + 8 + 8 + 8**
  (12 symbols)

  There are other ways to write a name for 1000 using only **8**s and **+**s.

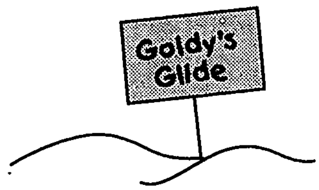
- **1000 = 555 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 55 + 5**
  (29 symbols)

  There are other ways to write a name for 1000 using only **5**s and **+**s.

- **1000 = 444 + 444 + 44 + 44 + 4 + 4 + 4 + 4 + 4 + 4**
  (25 symbols)

  There are other ways to write a name for 1000 using only **4**s and **+**s.

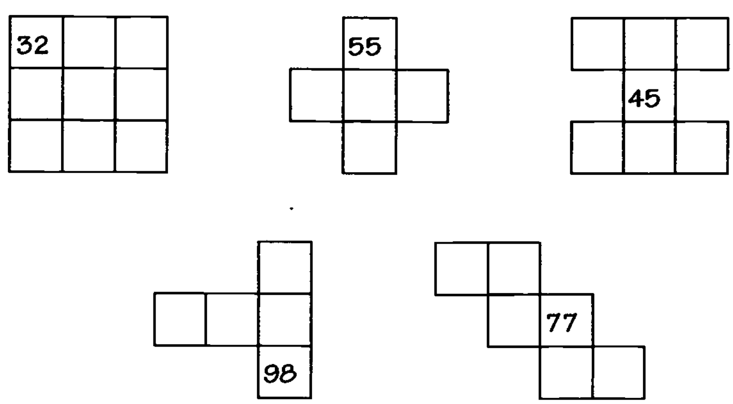
November '98



Ms. Evenodd decided to display the whole numbers from 0 to 109 in a different way. Here is part of her numeral chart.
Can you complete the next three rows of Ms. Evenodd's chart?

0	2	4	6	8
1	3	5	7	9
10	12	14	16	18
11	13	15	17	19
20	22	24	26	28
21	23	25	27	29
30	32	34	36	38
⋮	⋮	⋮	⋮	⋮

Complete these parts of Ms. Evenodd's 0–109 Chart.



Hint

Look for patterns in the chart. For example, rows of numbers alternate even and odd. In a row that has an even number, all the numbers are even. In a row that has an odd number, all the numbers are odd. Columns of numbers have both even and odd numbers. In a column all the even numbers have the same digit in the one's place, and all the odd numbers have the same digit in the one's place. You can find other patterns.

Solution

The next three rows are completed in this chart.

0	2	4	6	8
1	3	5	7	9
10	12	14	16	18
11	13	15	17	19
20	22	24	26	28
21	23	25	27	29
30	32	34	36	38
31	33	35	37	39
40	42	44	46	48
41	43	45	47	49
⋮	⋮	⋮	⋮	⋮

Here are parts of Ms. Evenodd's 0–109 Numeral Chart completed.

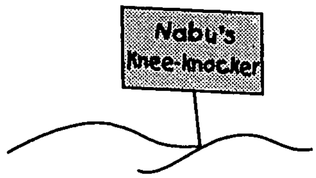
32	34	36
33	35	37
42	44	46

	55	
62	64	66
	65	

42	44	46
	45	
52	54	56

		88
85	87	89
		98

72	74		
	75	77	
		86	88



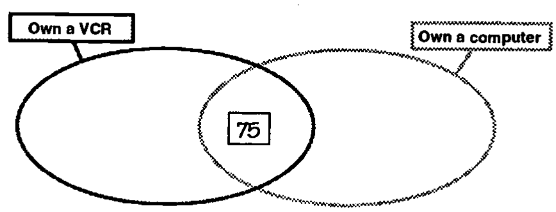
1. *A market research survey of 200 people finds that 80% of the people own a VCR and only 50% of the people own a computer. In this survey, 75 people own both a VCR and a computer. How many people in the survey own neither a VCR nor a computer?*
   Explain your answer. ______________________________

2. *In the same survey, the researchers find that one-half of the people owning a VCR use their VCR to watch movies they rent and one-half use the VCR to tape TV shows. Still, 20% of the VCR owners say they never use their VCR. Explain how this can be true.*

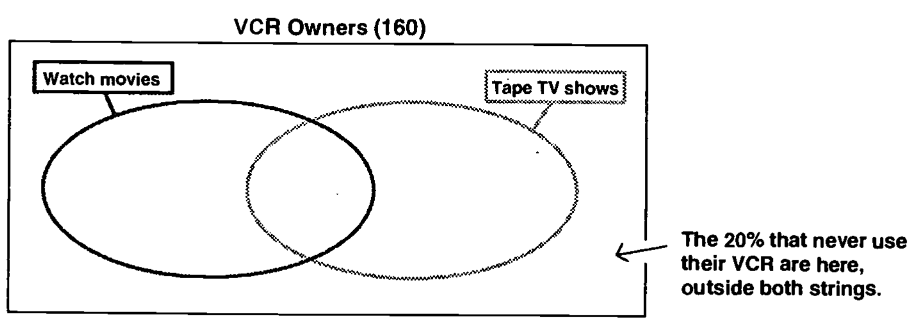
Hint

1. First calculate
   80% of 200 (number of people who own a VCR)
   50% of 200 (number of people who own a computer)

   Then consider the people who own both a VCR and a computer. A string picture (Venn Diagram) might help.



2. There are 160 people (80% of 200 = 160) who own a VCR. Since half (80) use their VCR to watch movies, half (80) use their VCR to tape TV shows, and 20% do neither, there must be some people who use their VCR to do both. A string picture (Venn Diagram) might help.

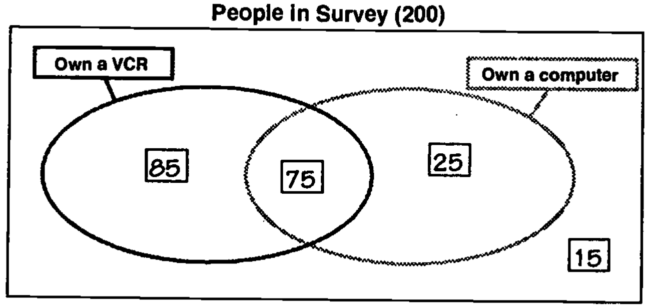


Also, compare the total number of VCR owners to the number that actually use their VCRs.

Solution

1. There are 15 people in the survey who own neither a VCR nor a computer. The picture below shows how to classify the 200 people in the survey.

People in Survey (200)



80% of 200 = 160; 160 people own a VCR.
50% of 200 = 100; 100 people own a computer.
75 people own both a VCR and a computer.

Notice that 185 people are inside the strings (85 + 75 + 25 = 185), so 15 people are outside; that is, 15 people own neither a VCR nor a computer.

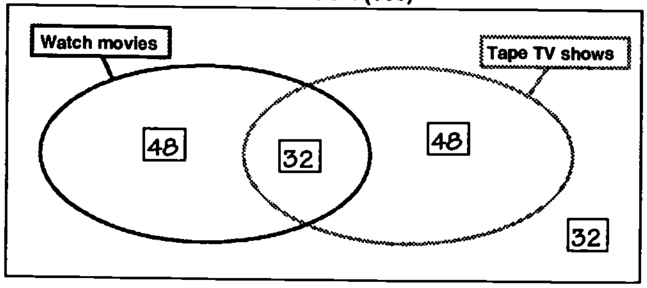
2. 160 people in the survey own a VCR (see problem #1).

Half (80) watch movies
Half (80) tape TV shows } Some (32) watch movies and tape TV shows.

20% of 160 = 32 Some (32) never use their VCR.

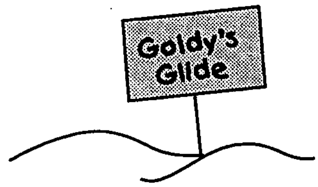
The picture below shows how to classify the 160 people who own a VCR.

Own a VCR (160)

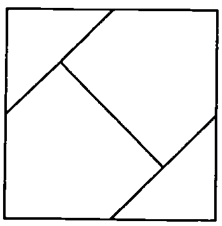


48 + 32 = 80
80 watch movies

32 + 48 = 80
80 tape TV shows

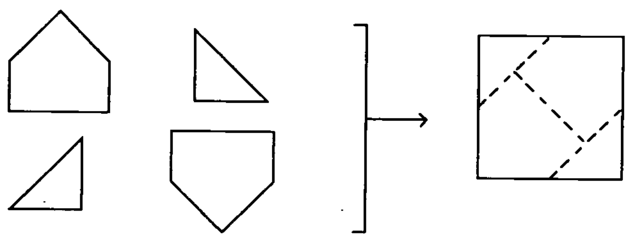


Make a Tangram-like square puzzle with four pieces, as shown below.

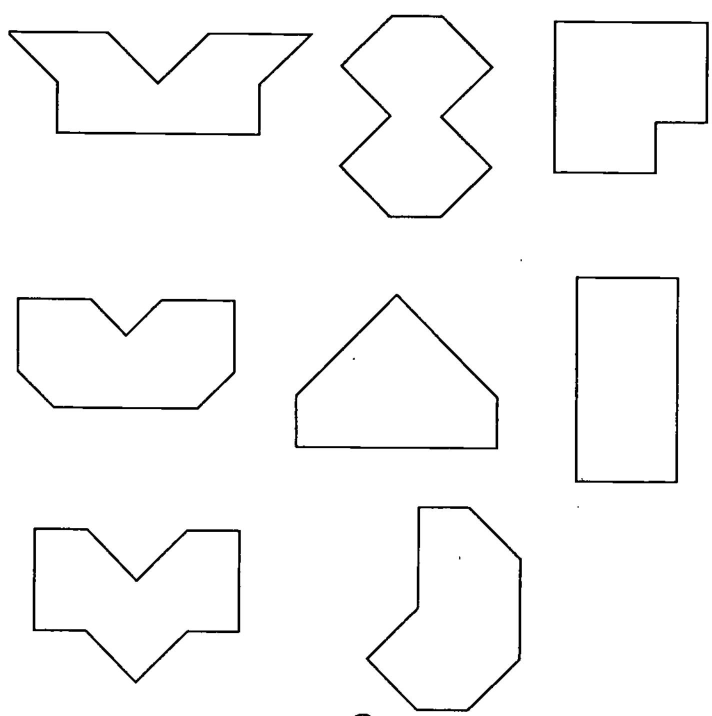


<u>Construction Suggestion:</u>
Use an 8 cm square cut from cardstock.
Find the midpoint of each side of the square. Cut along lines drawn between adjacent midpoints to get the triangles.
Fold the remaining piece in half and cut along the fold line.

You can, of course, put the four pieces back together to make a square.

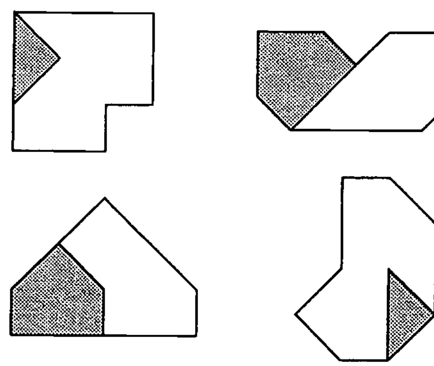


Use the four pieces to make each of the shapes below.



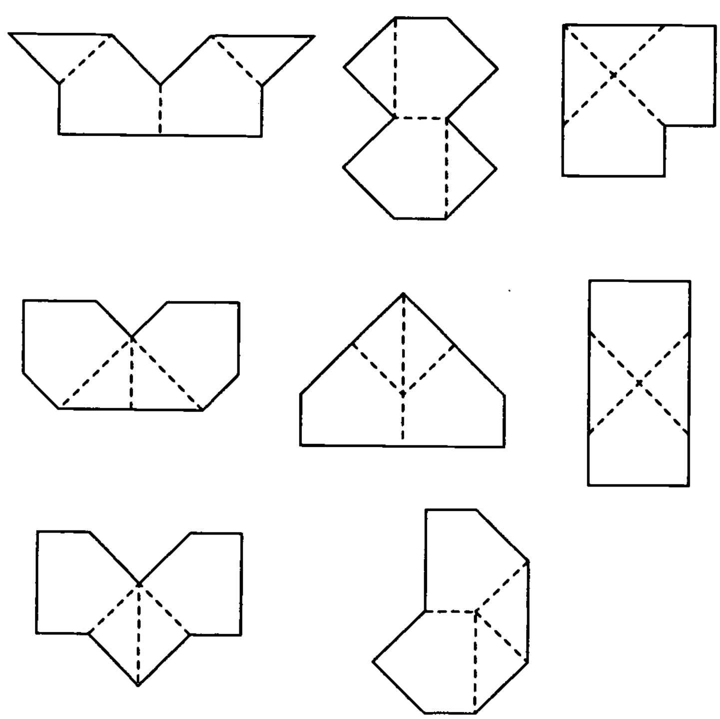
Hint

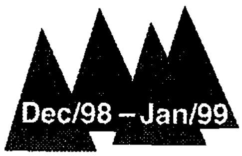
Some shapes may be harder than others to see how to arrange the four pieces. The pictures here show where to place one of the four pieces in some of the shapes.

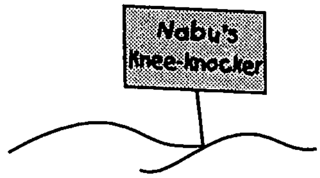


Solution

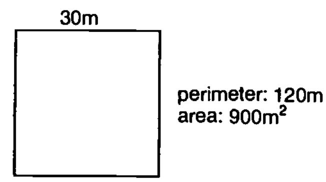
Dotted lines show how to place the four pieces for each shape. There may be more than one solution for some shapes.



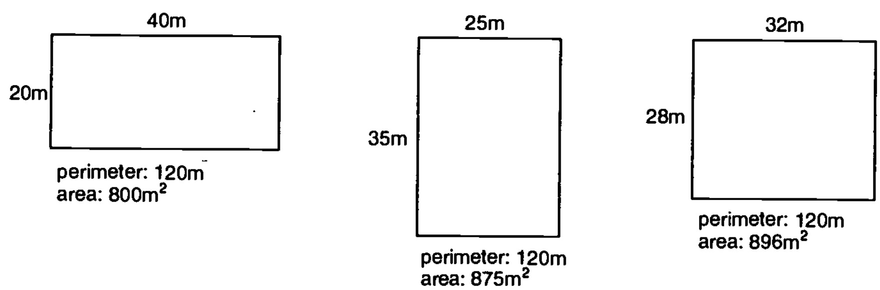




Perhaps you know that the largest (in area) rectangle with perimeter 120 meters is a square with 30 meter sides. The area is 900 square meters (m²).



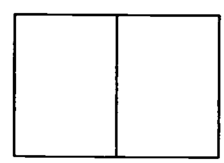
You can check the area of other rectangles with perimeter 120 meters.



So, if you have 120 meters of fencing to build a rectangular pen and you want the pen to have as large an area as possible, you'd make a square pen with 30 meter sides.

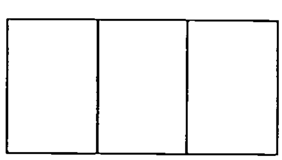
Problems

1) *Suppose you have 120 meters of fencing and want to build two congruent rectangular pens with one side in common. "Congruent" means the rectangular pens look exactly alike. The picture shows an example.*

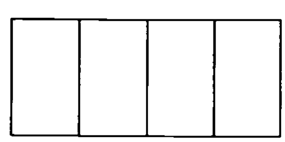


Also, suppose you want the total area to be as large as possible. What dimensions do you make the rectangular pens?

2) *Consider the same problem for making three or four rectangular pens with 120 meters of fencing.*



Use 120 meters of fencing.
Make total area as large as possible.



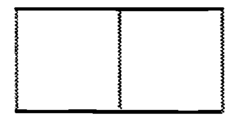
Use 120 meters of fencing.
Make total area as large as possible.

What dimensions do you make the rectangular pens?

3) *Can you describe how to build several rectangular pens (any number) with 120 meters of fencing and with as large a total area as possible?*

Hint

In this picture, the three vertical (gray) segments are the same length, and the two horizontal (black) segments are the same length.



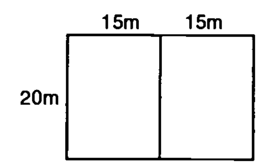
The length of three gray plus two black is 120 meters (the amount of fencing). The total area is gray length x black length. Investigate some possible lengths for the gray and black segments. This table shows some possibilities and the resulting total areas.

	∘∘∘	• •	Area
(3 x 10) + (2 x 45) = 120	10m	45m	450m^2
(3 x 20) + (2 x 30) = 120	20m	30m	600m^2
(3 x 30) + (2 x 15) = 120	30m	15m	450m^2
(3 x 25) + (2 x 22$\frac{1}{2}$) = 120	25m	22.5m	562.5m^2
(3 x 18) + (2 x 33) = 120	18m	33m	594m^2

Solution

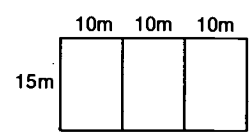
1) TWO PENS

The largest possible total area is 600 m^2. Each pen is 20 m by 15 m. (See the table of other possibilities in the hint.)



2) THREE PENS

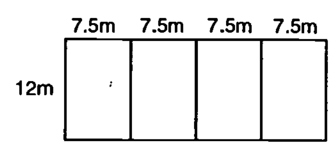
The largest possible total area is 450 m^2. Each pen is 15 m by 10 m.



Math Mountain on the CSMP Web site includes a table showing other possible dimensions and the resulting total areas.

3) FOUR PENS

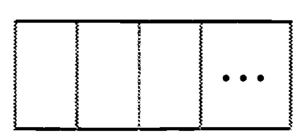
The largest possible total area is 360 m^2.
Each pen is 12 m by 7.5 m.



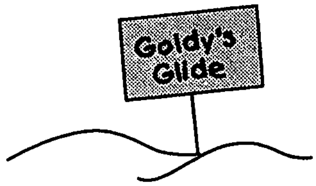
Math Mountain on the CSMP Web site includes a table showing other possible dimensions and the resulting total areas.

4) To build several rectangular pens with 120 meters of fencing and with as large a total area as possible:

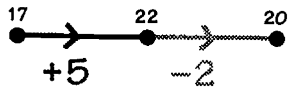
- use half of the fencing for the common sides and corresponding ends (gray segments in picture). There is one more such side than the number of pens, so divide 60 by (n+1)—n being the number of pens.



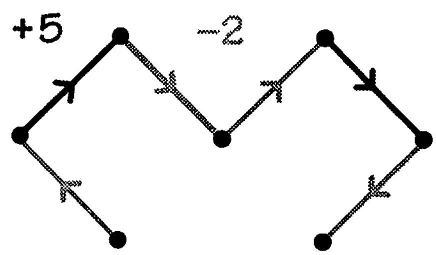
- use half of the fencing for the non-common sides (black segments in picture). So, the total length of the pens will always by 30 meters.



In this arrow picture, all dark arrows are for +5 and all light arrows are for −2.

Example: 

because 17 + 5 = 22 and 22 − 2 = 20.



11
8
13
9
14
10
12

Label the dots in this picture with the numbers in the box (at the right). Use each number only once (at one dot), and use only the given numbers.

Hint

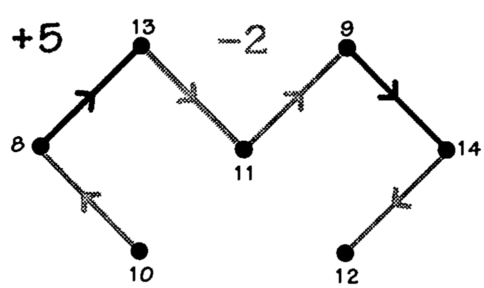
There are many ways you could start working on this problem. Here is one way.

Notice how the leftmost (second) dot and the rightmost (sixth) dot are related: (__+5 −2 −2 +5 __)

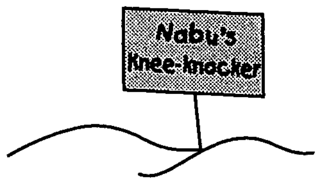
If you add 5 twice and subtract 2 twice, the same thing would be to add 6.

Check for numbers in the box that are 6 apart. The only choice is 8 and 14.

Solution

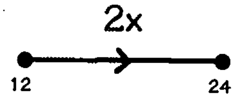


~~11~~
~~8~~
~~13~~
~~9~~
~~14~~
~~10~~
~~12~~



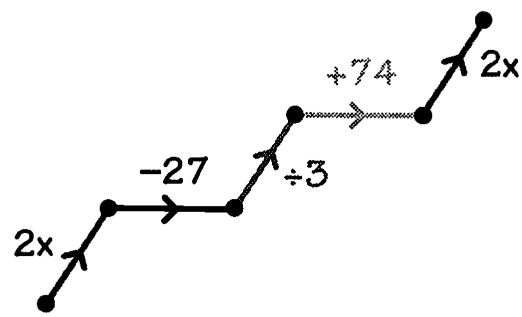
In this arrow picture, each arrow has its own label.

Example:



because 2 x 12 = 24.

129
117
78
234
43
156



Label the dots in this picture with the numbers in the box (at the left). Use each number only once (at one dot), and use only the given numbers.

Hint

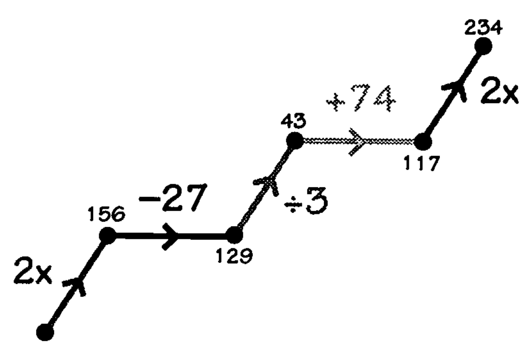
There are several ways you could start working on this problem. Here is one way.

Look for doubles among the numbers in the box: 117 and 234; 78 and 156.
These are choices for the dots at the beginning and end of a 2x arrow.

Try one of these choices for the first 2x arrow. Notice that if you make the first choice, you will then need to subtract 27 from 234. The end of the −27 arrow would be 207, and that number is not available. So you must choose 78 and 156 for the first 2x arrow.

Solution

~~129~~
~~117~~
~~78~~
~~234~~
~~43~~
~~156~~



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