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ABSTRACT

The ability to use algebra in describing and analyzing real-world situations is becoming a basic skill for all students. The main focus of this book is on patterns, functions, and relationships. Information and ideas on how algebraic thinking can be developed in the years leading to a formal algebra course are included. K-8 teachers are offered a series of appropriate activities that will excite students about mathematics in general and algebra in particular. Activities, grouped by grades K-2, 3-5, and 6-8, are organized around the instructional process and include a list of necessary materials and background information for students and instructors. (ASK)

K-8 Building Blocks for Algebra

Patterns
Functions
Relationships

Developed by

Sandra Critchfield • Nora Hall • Deborah Pittman

In Collaboration with the

Eisenhower Regional Consortium for Mathematics and Science Education

AEL

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About the Authors

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Introduction

The ability to use algebra in describing and analyzing real-world situations is becoming a basic skill for all students. The algebra that is appropriate for all students moves away from a focus on manipulating symbols toward a greater emphasis on conceptual understanding, on algebra as a means of representation, and on algebraic methods as a problem-solving tool. The key ideas of this book are patterns, functions, and relationships. These building blocks of algebra help develop a child's ability to think logically, form generalizations, and predict future events. Patterns, functions, and relationships bring the real world into the mathematics classroom and help children make sense of the relationships between numbers. The focus of this guide is on algebraic thought rather than algebraic symbol manipulation.

This book helps students develop the concept of function. Functional relationships provide a powerful way to express patterns and order. These activities offer students opportunities to investigate variables and functions in new ways.

For students to succeed in a formal algebra course, the foundation must be built in elementary school. The building blocks of a foundation include seeing patterns and relationships as well as making generalizations. While this means introducing some new concepts, it mainly requires a new focus on what is currently being taught.

This book gives information and ideas about how algebraic thinking can be developed in the years leading to a formal algebra course. It also offers K—8 teachers a series of appropriate activities that will excite students about mathematics in general, and algebra in particular.

To assist teachers in using this book, activities are grouped for students in grades K-2, 3-5, and 6-8. While these grade level groupings consider the development level of students, it is important to remember that these activities build upon each other. For example, students in grades 3-5 should have worked with at least some of the K-2 activities before being introduced to the activities in the grades 3-5 section.

The activities are organized around the instructional process and include a list of necessary materials and background information for students and instructors. The step-by-step progression through each lesson offers teachers a structure that they may use as presented or adapt to a particular classroom situation.

No matter what level you teach, you are part of a team charged with teaching children mathematics. This book offers a way to weave algebraic concepts into the K-8 curriculum.

—the Authors

K-2 Lessons

Overview of K-2 Lessons

Mathematics is the science of pattern and order. Mary Baratta-Lorton (Burns, 1992) says that "looking for patterns trains the mind to search out and discover the similarities that bind seemingly unrelated information together in a whole. When children expect things to make sense they look for the sense in things. This leads to understanding. Students see that mathematical ideas are related and connected, rather than discrete, separate, and unrelated."

Implicit in the K-2 lessons are the notions of repetition and regularity; therefore, patterns are the focus of each activity in this section. Patterns are the foundation upon which other mathematical concepts are built. Tools such as physical objects and drawings help students think about patterns and order. Functional relationships provide a powerful way to express patterns and order. Functions allow us to begin using mathematical symbols as a tool for thinking.

Students take a concrete pattern and translate it into different forms. These forms may be other concrete media, such as letters or numbers. Students communicate, both orally and in writing, and predict patterns and relationships. Once students communicate patterns and relationships, the next level is to translate quantities from a problem-solving situation into symbols. They should begin to make and manipulate expressions. Students record data from the concrete patterns in chart form, thereby relating two quantities. At this point, students describe the numerical pattern and predict what comes ahead.

Enjoy the activities presented. Although they may be taught separately, it is better to teach lessons as presented to guarantee logical development of concepts. A conceptual foundation needs to be set so students will continue to have success in the upcoming 3-8 mathematical experiences.

Burns, Marilyn (1992). About teaching mathematics: A K-8 resource. Sausalito, CA: Math Solutions Publications.

"Counting On" Activity

Students visualize number patterns through colors. The relationship between numbers is developed by building a number on the Counting On frame.

Prerequisites

- ❑ number recognition to 10
- ❑ one-to-one correspondence

Materials

- ❑ Unifix cubes
- ❑ Counting On frame
- ❑ 10 two-color counters
- ❑ yellow and red crayons
- ❑ number cards
- ❑ overhead projector
- ❑ *Anno's Counting Book*
- ❑ Counting On frame (transparency
- ❑ yellow and red markers
- ❑ recording sheet

Background for the Instructor

This activity develops an awareness of relationships between numbers through "counting on". When completing a number sentence with a missing addend, students use two-color counters for filling in the Counting On frame. A record is made of the number sentence combination by coloring in the Counting On frame when counters are removed. This creates a color pattern showing the missing addend and completes the number combination. The recording of the missing addend lays the foundation for functional relationships.

Set the Stage

- Read *Anno's Counting Book* by Mitsumasa Anno, published by Harper and Row (1975), to the students.

- Model counting from zero to twelve with the unifix cubes. Connect the twelve cubes and point to the third cube and say, "three," and continue "counting on" to twelve. Practice "counting on" starting at different points on the group of twelve unifix cubes.

Conduct the Learning Experience

- Play "Counting On"

  Student Materials:

  √ 10 two-color counters to each student or each pair of students

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Teacher Materials:

√ overhead projector
√ transparent two-color counters
√ Counting On frame (transparency)
√ number cards

■ Model how to play the game for the students and record the results.

1. Select a number card and show students the side of the card with the lower number (e.g., "4").

2. Color the first four blocks of the frame yellow.

3. Turn the number card over and display the larger number (e.g., "7").

4. Say, "4 plus *what* equals 7?"

5. Count on saying, " 5...6...7" while pointing to the blocks on the "Counting On" frame.

6. Color these three blocks red (the two different color counters help students visualize the action of "counting on" by 3); this creates a color pattern. Save the pattern for later use with symbols (numbers and variables).

7. Record the equation.

8. Pick another number card and repeat the process. (Example: 2 on the front of the card and 5 on the back of the card; "2 plus *what* equals 5").

9. Review combinations by writing the number sentences on the overhead using the format: 4 +_ = 7 and asking 4 plus what equals 7.

10. Students can play the game using number cards.

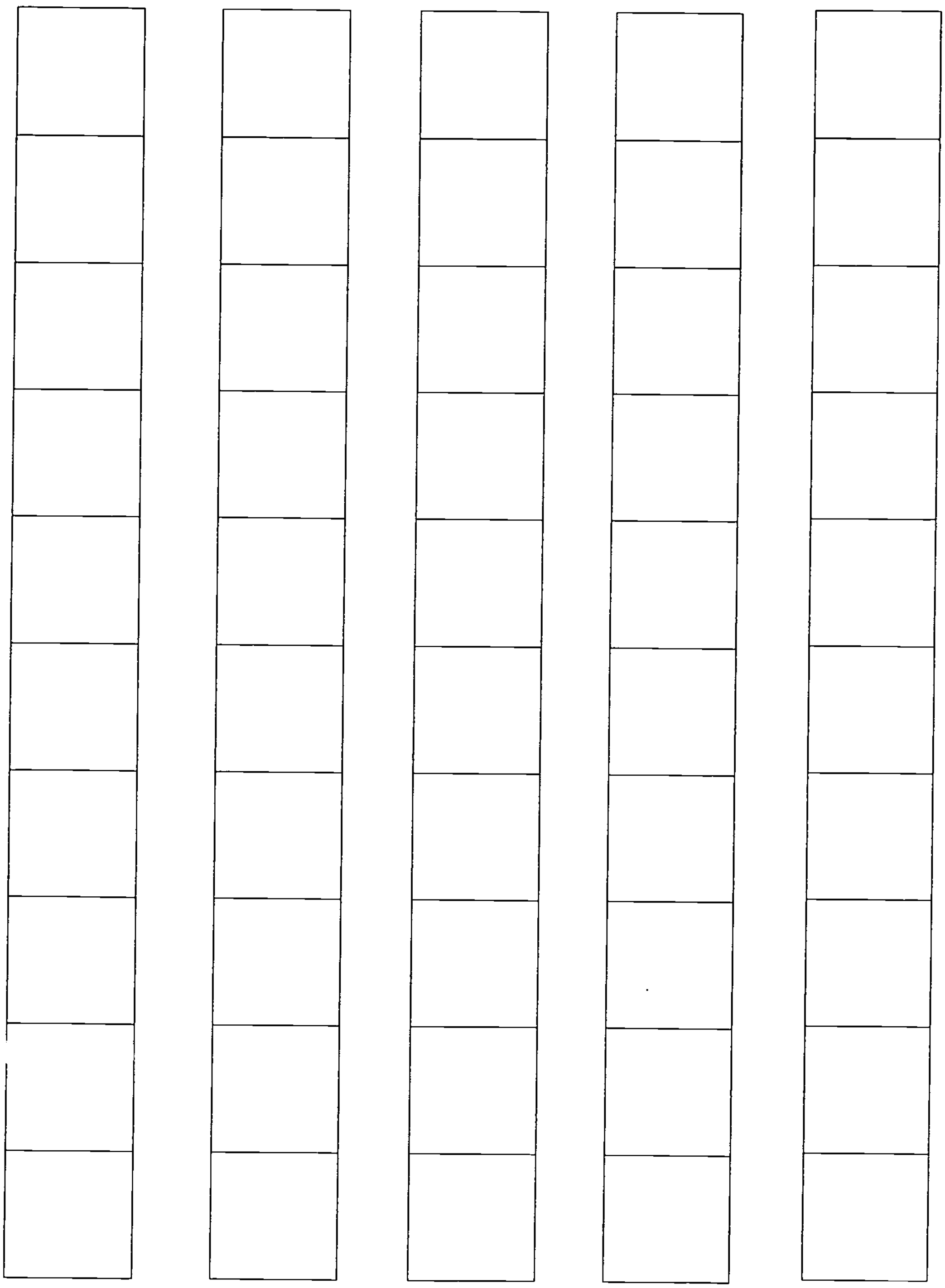
11. Students can later record their own number sentences on the recording sheet in the format: 4 + _ = 7.

12. Students can progress to record number sentences in this format:
 4 + __ =

Closure

- This lesson illustrates the relationship between numbers (functional concept) by having students "count on" for missing addends. The color pattern created (4 yellow plus X red = 7) can later be used to explore the concept of variables. The recording process is important for the students because the format for expressing changes over time from $4 + \_ = 7$ to $4 + \_\_ = 7$, then $4 + X = 7$. Students may later use these recordings to review number combinations and the process for solving for a missing variable.

"Counting On" Frames



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Number Cards

0	1	2	3
4	5	6	7
8	9	10	

1. Run off several sets on card stock for use as the front and back of cards

2. Glue the back number card to the front number card upside down

3. The set for 7 includes:

Front	*Back*
0	7
1	6
2	5
3	4

"Counting On" Recording Sheet

name ____________________

4

1. $0 + \square = 4$

2. $1 + \square = 4$

3. $2 + \square = 4$

4. $3 + \square = 4$

5. $4 + \square = 4$

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Jumping for FUNctions

These activities take students from a real graph to a pictorial graph to a symbolic graph. Students actively engage in developing functional concepts through patterning.

Prerequisites

- ❑ complete the Counting On activity

Materials

- ❑ jump ropes
- ❑ masking tape
- ❑ tagboard (cut into rectangles)
- ❑ yarn
- ❑ pencils
- ❑ glue
- ❑ math journals
- ❑ markers
- ❑ chart
- ❑ paper
- ❑ chalkboard
- ❑ chalk

Background for the Instructor

This series of activities focuses on three types of graphs: real, pictorial, and symbolic. Students actively participate in making all three types of graphs. The collected data is organized into graphs. Students look for patterns in the graphs and try to predict what comes next. A T-chart helps students to organize the data symbolically. Using math journals helps students describe the pattern. The information from the pattern is used for analyzing, making predictions, and drawing conclusions.

These lessons encourage students to explore the many ways to collect and record data. The lessons allow students to expand one data set (jump ropes and students) into three different types of graphs: real, pictorial, and symbolic. The groundwork for functional concepts is developed through patterning. Having students write in math journals illustrates that data can be organized into graphs and tables, making mathematical ideas easier to comprehend. The use of the math journal also allows teachers a window into student understandings and misunderstandings.

Set the Stage

- Pass out one jump rope to every two students.
- Have students go to the playground and practice turning the jump rope.
- Help students group themselves to form a real graph.
- Have students look for a pattern. If possible, take "instant" pictures of the students while in their real graph for use in *Lesson One: Pictorial Graph.*

- Return to the classroom and have students write in their math journals and answer the following questions you have written on the chalkboard:

  1. What is the pattern of the graph? Predict what comes next.

  2. Can you draw the pattern just made on the playground with your partner and a jump rope?

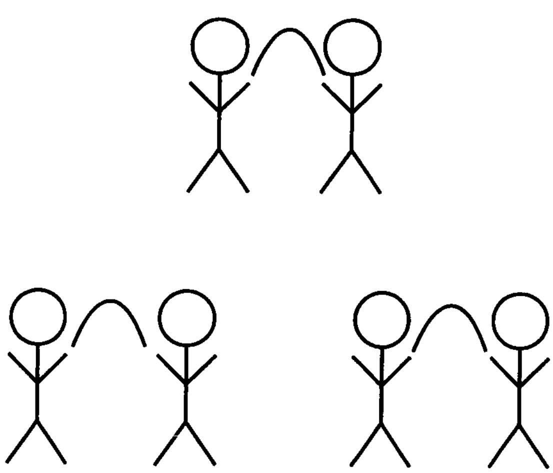
  3. Is your answer, "I will try?" If so, please come to my desk. (You may need to help students see that the elements of the pattern are one rope and two students.

Conduct the Learning Experience

Lesson One: Pictorial Graph

- Have the students meet with a partner and pick up tagboard, yarn, glue, marker, and pencil.

- Have the students make a "picture" of themselves turning the jump rope (stick people work well). If pictures of the real graphs were taken in *Set the Stage*, students can refer to those to help make their own drawings.



Ropes	Students
1	2
2	4

- Tape the pictures on the chalkboard in this pattern:

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Lesson Two: Symbolic Graph

- Have students look at their math journals and reread their description of the pattern from the playground.
- Write these questions on the chalkboard:
  1. Do you see a pattern in the pictures (picture graph) on the board?
  2. Can we put numbers in place of the pictures? Try!
- Have students read from their math journals their ideas about how to substitute numbers for the pictures.
- Record the students' ideas in a T-chart on the chalkboard (next to the picture graph).
- Example:

Ropes	Students
1	2
2	4
3	6

Closure

- Have students record information in their math journals and answer these questions:
  1. Predict what comes next.
  2. Is there a pattern in the numbers? Do you see a relationship between the 1 and 2, the 2 and 4, etc.?
  3. If we had 32 students, how many jump ropes would we need?
  4. If we had 47 students, how many jump ropes would we need?
  5. If we had 23 jump ropes, how many students would we need to turn them all?

Extension

Extend the lesson by adding a "jumper" to the two jump rope turners and one jump rope. See if the students can graph the correct number of students per jump rope and predict how many students are needed if there are eight jump ropes, etc.

MathMagical Function Box

Students use number patterns (+1, +2, +3, etc.) to develop the concept of missing addends, which leads to functional relationships. Data is then recorded on a T-chart numerically.

Prerequisites

- ❑ complete the Counting On activity
- ❑ addition and subtraction facts through ten (fact families)

Materials

- ❑ 1 shoe box
- ❑ scarf
- ❑ 20 color blocks (10 each of 2 colors)
- ❑ chart paper
- ❑ watercolor marker
- ❑ floor graphing mat
- ❑ student graphing mat
- ❑ T-chart recording sheet
- ❑ overhead calculator
- ❑ overhead projector
- ❑ blank transparency
- ❑ student calculators

Background for the Instructor

This activity develops an awareness of missing addends through number patterns. The data from the activity is graphed onto a floor graphing mat using real objects (blocks). The data is then transferred, in numeric form, to a T-chart. The T-chart allows for exploration of functional relationships. This is a lesson that may take several months at the K-2 level to completely develop.

Set the Stage

- The calculator is the student's own input-output machine. Use four-function calculators with the automatic constant function feature (allows you to enter + 2 =, and the calculator subsequently counts by two's each time you enter =).
- Pass out calculators to students and set up an overhead projector and overhead calculator.
- Tell students the calculator can be a magic machine if programmed correctly.
- Program the calculator to be a "+2" machine using the following commands, and record the number pattern on the overhead transparency.

Commands:
1. Clear
2. +
3. 2
4. =
5. =
6. =

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- Review with students the numbers generated from the calculator and recorded on the transparency.

- Ask students if they see a pattern in the numbers. You may want to give hints such as, "What is the difference between 2 and 4?" or "How many more do you need to get from 2 to 4 and from 4 to 6?"

- Enter "=" again on the calculator, and record the results.

- Ask students to predict what comes next.

- Review the concepts (input, output, number pattern) and begin the MathMagical Function Box activity.

Lesson Preparation

- Use a hinged shoe box covered with adhesive paper. Try to use a print that appears magical. (Remember, this is a MathMagical Function Box!) You'll also need a plain-colored scarf.

Conduct the Learning Experience

1. Write on the board: 3 + = 5 (+2 pattern).

2. Show students the MathMagical Function Box filled with 3 yellow blocks and tell them it is a "MathMagical" box because it does magic with numbers.

3. Cover the MathMagical Function Box with the magic scarf and chant, "abracadabra" while placing 2 red blocks into the box (+2 pattern).

4. Remove the scarf and open the box to show students what magically happened (two red blocks appeared); discuss how many blocks were in the MathMagical Function Box before the magic and how many were in the box after the magic.

5. Have students put 3 yellow blocks on the left side of the Student Graphing Mat, 2 red blocks in the center, and 3 yellow and 2 red blocks on the right side, totalling 5 blocks; students begin building the +2 pattern.

6. Discuss what happened by asking, "3 plus *what* equals 5?"
   Answer: 2

The use of a visual model helps build a child's confidence in dealing with abstract symbols.

- Repeat steps 1–6 using different number combinations for practicing the + 2 pattern (4 + □ = 6, 5 + □ = 7, 6 + □ = 8, 7 + □ = 9, 8 + □ = 10) up to the sum of 10.

- Record the pattern as shown on the floor graphing mat with the blocks. Ask students if they see a pattern. What is the rule that describes what happens in the MathMagical Function Box? (+2)

Student Graphing Mat and Floor Graphing Mat

Begin With	MathMagical Function Box Trick	End With

Students are able to infer the rule of the MathMagical Function Box and predict the result of the third or fourth event.

- Take the data from the floor graphing mat and transfer it to a T-chart using chart paper and marker.

T-chart with Symbols

Begin With	MathMagical Function Box Trick	End With
3	+2	5
4	+2	6
5	+2	7
6	__	__
7	__	__

- Discuss the T-chart using the number of blocks in the MathMagical Function Box as input and the resulting number of blocks (after the magic trick) as output. Refer to the graphing mat so students can make the connection from the real graph to the T-chart. Students are able to infer the rule of the MathMagical Function Box and predict the result of the third or fourth event.

The use of a visual model helps build a child's confidence in dealing with abstract symbols.

- Going horizontally (across), the pattern is +2. Going vertically (down), the pattern is +1 in both the input and output columns. The students may conclude that as we increase the vertical numbers by one, the horizontal numbers increase by two, or +2. This is the functional relationship of input to output:

0 + 2 = 2	2 + 2 = 4	4 + 2 = __	6 + 2 + __	8 + 2 = __
1 + 2 = 3	3 + 2 = __	5 + 2 = __	7 + 2 = __	

  Students can predict this number because they see the pattern at both the concrete (MathMagical Function Box with blocks, graphing mat) and symbolic (T-chart) levels.

- Repeat the activity the next day and subsequent days using the MathMagical Function Box, floor graphing mat, and T-chart recording sheet.

Closure

- This lesson begins to develop the concept of function through patterning. The magical appearing (or disappearing) of blocks with the MathMagical Function Box needs to be repeated using different numeric patterns (+ 2, + 3, + 4, or -2, -3, -4, etc.). The number relationships discovered from the T-chart are key to understanding the idea of a functional relationship.

Student Graphing Mat

Begin With	MathMagical Function Box Trick	End With

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T-Chart Recording Sheet

Input	Output

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A Balancing Act

Students explore different ways to represent equal quantities.

Prerequisites	Materials
❑ knowledge of addition and subtraction facts through 12 ❑ Counting On activity	❑ color counters ❑ recording sheet ❑ calculators ❑ pencils ❑ 12 color blocks for each pair of students

Background for the Instructor

The three lessons in this section need to be completed in the order presented to develop the concepts of equal quantities, missing addends, and balanced equations. Students practice equal quantities by using addition and subtraction facts learned in first grade and early second grade ($4 + 2 = 9 - 3$). The equal (=) sign means the quantity on the left is the same as the quantity on the right. The idea of balance develops when students understand "counting on", equal quantities, and missing addends ($1 + \square = 4$).

Students will balance equations using counters. These experiences help develop the concepts of equal quantities and missing addends. In order to balance equations, students deal with one or several unknown quantities. Often, when we think students understand the concept of equal, they do not. Students see addition and subtraction (+, -) as commands. The (+) tells you to add, and adding is a verb or action. The idea of $2 + 3$ as another way of writing 5 is not considered. The equal sign is an operator button, like the one on the calculator. When you press the (=), you get an answer. The (=) is often thought of as separating the problem (question) from the answer. Students need to understand the basic concept that (=) does not always mean you do this (+, -) and you get that. The concept of equal quantities is a balancing concept. Students develop these concepts as they lay the foundation for algebra.

Students need to relate what they already know (number combinations through twelve) to the concepts of equal quantities and missing addends. Building on students' previous knowledge makes the learning process easier and builds success. The format is changed to better prepare students for algebraic thinking (to $\square$ to X) and to help students make the transition from arithmetic to formal algebra.

Set the Stage

- Give each pair of students 12 blocks.
- Have each pair stand facing each other. One student in each pair will be Student One and the other will be Student Two. Have Student One place a few blocks in his or her left hand.
- Call out one of the following: Equal, More, Less.
- Have Student Two fill Student One's right hand with enough blocks to follow the directions. The right hand will have either an equal quantity, or more or less than the quantity, of blocks in the left hand.
- Have each pair stand facing each other. One student in each pair will be Student One and the other will be Student Two. Have Student One place a few blocks in his or her left hand.
- Have Student One and Student Two switch roles and repeat the activity.

Conduct the Learning Experience

Lesson One: Balanced Quantities

- Students will use their hands to balance quantities.
- Give each pair of students 12 blocks.
- Have Student One place 2 Red + 3 Yellow blocks in his or her left hand. Student Two places an equal quantity of blocks in his or her right hand. These blocks must be represented in a <u>different way</u>.
  - 2 *R*ed + 3 *Y*ellow = <u>0 *R*ed + 5 *Y*ellow</u> or
  - $2\,R + 3\,Y = \underline{1\,\mathrm{R} + 4\,Y}$ or
  - $2\,R + 3\,\mathrm{Y} = \underline{3\,R + 2\mathrm{Y}}$ or
  - $2\,R + 3\,\mathrm{Y} = \underline{4\,\mathrm{R} + 1\,Y}$ or
  - $2\,R + 3\,Y = \underline{5\,\mathrm{R} + 0\,Y}$

  (***Note:*** 2 + 3 = 2 + 3 is not included because it is not a different way to represent an equal quantity.)
- Have students continue the balancing activity for several more quantities.

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Lesson Two: Equal Quantities

- Pass out counters.

- Have students show the number "4" in as many different ways as they can, using the counters.

- Have students share their equations. You may record the students' equations on the chalkboard. Here is a sample of possible student-generated equations:

——— 4 ———

$1 + 3 = 4 + 0$	$4 - 0 = 5 - 1$
$2 + 2 = 3 + \square$	$6 - 2 = 3 + \square$
$0 + \square = 2 + 2$	$\square + \square = \square + \square$

- Have students write in their math journals, answering the following questions, which you have written on the chalkboard:

  1. What does equal mean?
  2. Write and draw a picture describing what you know about equality.

- Continue the equal quantities activity by using the solutions from the student writings and any equations not given.

Lesson Three: Missing Addends

- Repeat Lesson Two using the counters and the Balancing Act recording sheet.

- This time substitute a __ for the missing number and have students model using counters. For example: $2 + 2 = 3 + \_\_$.

- Have students share their findings while modeling equations with the counters.

- Write on the chalkboard the following questions:

  1. What does the $\square$ mean?
  2. Are the $\square$ and __ the same? How? Write an example.
  3. For the equation $2 + 3 + \square = 1 + 6 + 4$, what goes in the $\square$ and *balances* the equation?

- Have students continue, using the operations of both addition and subtraction.

Closure

- Review with the students the concepts of *equal, more*, and *less*. Have them share the different ideas of *equal* (do this action and get this: $3 + 2 = 5$) and *balance*: ($2 + 3 = 1 + 4$). Students can cite examples from the lessons. Journal entries are appropriate at this time to assess understanding and possible misunderstanding of concepts

A Balancing Act Recording Sheet	A Balancing Act Recording Sheet
=	=
=	=
=	=
=	=
=	=
=	=
=	=
=	=
=	=
=	=

name

name

Lessons

Overview of 3-5 Lessons

Mathematical content for students in grades 3–5 has traditionally focused on computation. The study of patterns, relationships, and functions can deepen a student's understanding of and facility with the more traditional mathematics topics as well as prepare students for subsequent course work in middle school. It is important for students to understand how mathematical ideas are related. Students should explore relationships between quantities before they use formal algebraic notations.

The activities in this section provide opportunities for students to model expressions in different ways. The focus is on fun ways to engage students in developing their algebraic thinking through exploration. Students begin to formalize the concept of function with hands-on input/output activities and identify connections between algebraic thinking and arithmetic. All of these activities build on the ideas and activities developed in the K–2 section and begin to formalize the notation that will be needed in the 6–8 section.

Note: The K–2 activities provide such an important foundation that it may be necessary to incorporate some of the K–2 ideas for students in grades 3–5 who have not previously been exposed to algebraic concepts.

Exploring Multiples

The hundred chart offers students a visual way to look at patterns that leads to a generalization of multiplication facts.

Prerequisites

- ❑ 1st quadrant graphing

Materials

- ❑ handouts
- ❑ unifix cubes
- ❑ scissors
- ❑ graph paper
- ❑ math journals

Background for the Instructor

The study of patterns gives students an opportunity to make conjectures about relationships. Algebra is the language used to express mathematical relationships. Students need to explore the relationships between quantities before they use formal algebraic notation. The hundred chart is used in these activities as a tool to facilitate exploration.

Set the Stage

- Making pictures on a hundred chart is a good review strategy for many mathematical topics. The example used in Activity 1 reviews place value, more than/less than vocabulary, and multiplication facts.
- Distribute "Picture This" worksheets and unifix cubes to students.
- Have students cover the appropriate number with the indicated color unifix cube as each clue is read.
- Use the Activity 1 example or make up other clues. After students are familiar with this activity, let them create their own pictures and corresponding sets of clues.

Activity 1: Clues for picture of a flower

Yellow

1 ten and 1 one	one more than 50	one less than 68	9 x 7
one more than 51	4 x 3	7 x 3	one less than 62
9 x 3	one more than 56	8 x 2	8 x 7
one more than 21	6 tens and 2 ones	one less than 18	one more than 65

Brown

3 tens and 3 ones	one more than 33	9 x 5	5 x 7
4 x 11	1 x 43		

Green

9 x 6	1 more than 63	1 less than 75	1 less than 85
9 tens and 4 ones	1 more than 84	7 tens and 6 ones	9 tens and 3 ones
2 more than 80			

Conduct the Learning Experience

- Following are several activities that develop the concept of patterns with multiples. These activities may take place over several days.

Hundred Chart Exploration

1. Give each pair of students a hundred chart, such as the "Picture This" handout. Have students identify patterns that they see on the chart. They will probably identify the place value patterns dealing with the tens and ones places.

2. Pick a row. Add the digits of consecutive numbers. For example: 30, 31, 32, 33, 34.... What pattern do you see? Does it hold in every row of the chart?

3. Give out a hundred chart that has been cut into puzzle pieces (use the "What A Cut Up" handout). Before allowing students to re-assemble the hundred chart, have them place the pieces on their desk. What clues do they see that indicate which pieces go together?
   Note: Students who need assistance with this task may be given an uncut hundred chart to use as a base on which to place the cut-up pieces. Have students assemble the hundred chart and discuss with their partners the different techniques that could be used to re-assemble the chart.

5. Have students find the piece that has the number 65 on it. Let students re-assemble the hundred chart beginning with the piece that has 65 on it. Did they use the same strategies?

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5. The "What Comes Next?" handout shows pieces of a hundred chart. Have students fill in the missing numbers. Followup with a discussion about the students' reasoning as they filled in numbers.

Exploring Multiples on a Hundred Chart

- "Skip counting" can provide practice with multiples while deepening students' understanding of multiplication facts. Students are able to observe visual patterns resulting from identifying the multiples of a number.
  Note: Use a hundred chart that begins with 1 instead of 0. (See the handout on page 42).

1. Have students skip count by 2s (beginning with the number 2) marking the multiples of 2 with a unifix cube. Students will begin placing the unifix cubes as they skip count; however, many students will soon see the pattern and begin placing the cubes using the pattern rather than the skip counting. Have students describe the pattern. Have students repeat the activity several times skip counting by 3s, 4s, 5s, etc., up to 12s. It is important that students be able to verbally describe the pattern.

2. Have students repeat the previous activity. This time have students record their findings on the Recording Sheet. After they have skip counted by 3s, for example, ask the students what number is under the fourth 3. When they respond 12, reply that four 3s is equal to 12. Continue this type of questioning so that students understand the relationship between skip counting the multiples and multiplication.

3. Look at the Recording Sheet. What are the differences and similarities among the patterns? Is 239 a multiple of 6? How do you know? If a number is a multiple of 6 is it a multiple of 2? Of 4?

- ***Extension:*** Skip count by 2s and then by 3s, marking the multiples of 2 with one color and the multiples of 3 with a different color unifix cube. Which numbers have two colors on them (6, 12, 18, ...)? Why? Why is "Common Multiples" a good name for this set of numbers? Looking at the numbers that are common multiples, which number is the Least Common Multiple?

Graphing Hundred Chart Patterns

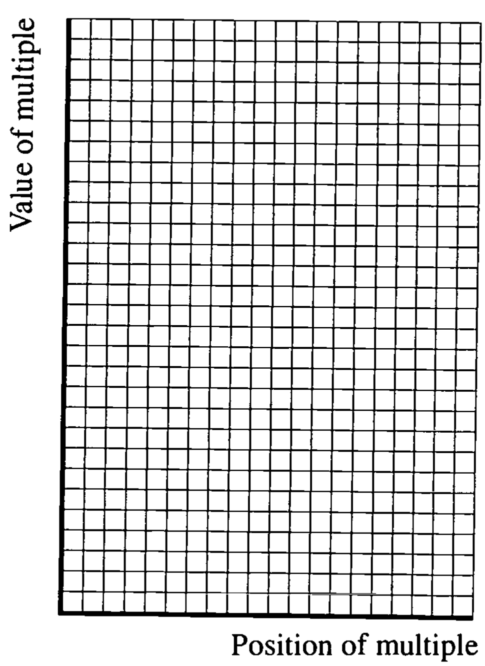
1. Look back at the recording sheets where you marked all of the multiples. Transfer these findings to the Multiplication Table worksheet (page 44).

2. The objective is to visually represent each group of multiples. Use the 3s as an example to model the process. Look back at the recording sheet where multiples of 3 were marked. The first multiple was 3, the second 6, the third 9, and so on. Record this information in a chart (page 43).

Multiple of 3	
Position of multiple	Value of multiple
1	3
2	6
3	9
4	12
5	15

3. What is the relationship between the position of the multiple and the value of the multiple? *Value = 3* x *position*
   What is the relationship between the chart and the 3s row or column on the multiplication table?

4. Graph the above information on a 1st quadrant graph where the position of the multiple is graphed on the x-axis and the value of the multiple is graphed on the y-axis. Lay a piece of spaghetti along the points to show that they are linear and can be connected with a straight line.



5. Continue making charts and graphing this information for the other multiples on the same graph. Compare graphs with the multiplication table.

6. Students should look for patterns and relationships on the graph. For example, to find common multiples on the graph, look for the times tables that have points located at 12 on the y-axis. The 2s, 3s, 4s, and 6s have 12 as a multiple. Twelve is a common multiple for those numbers. Another way to state the relationship is that all numbers on the x-axis that have points with a y value of 12 are factors of 12.

Closure

- Journal Entry: Explain what is meant by the term "multiple."

What a Cut Up!

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Hundred Chart Recording Sheet

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

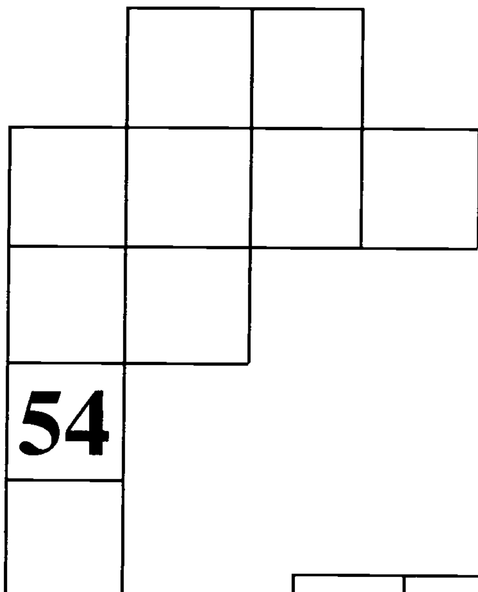
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

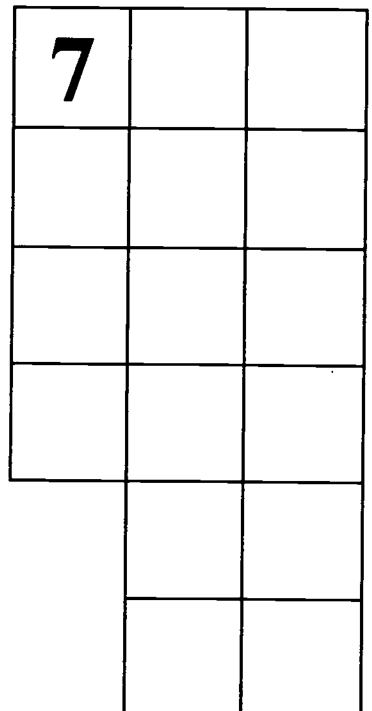
What Comes Next on a Hundred Chart?

- Here are sections of a hundred chart. As you can see, many of the numbers are missing. Your job is to fill in the numbers to complete the section.

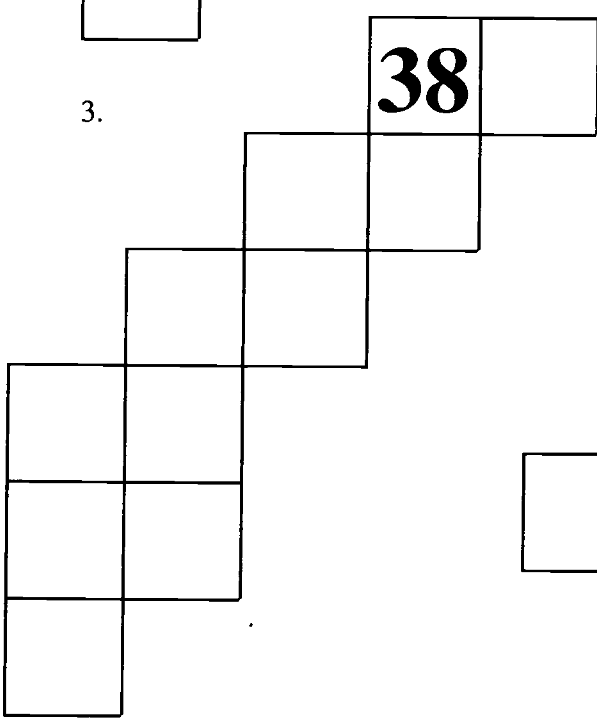
1.



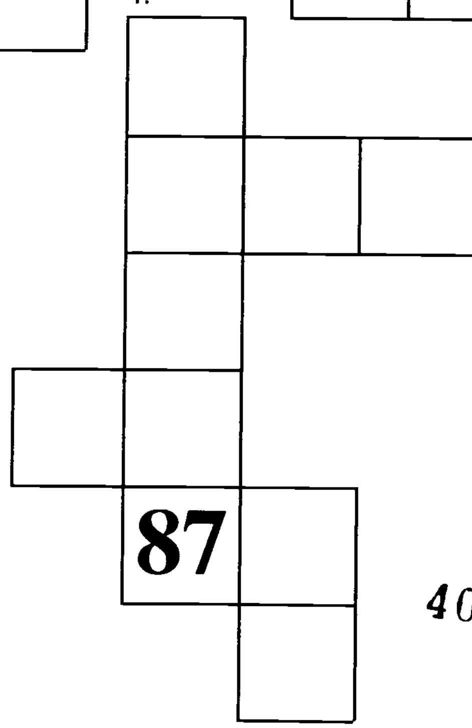
2.



3.



4.



40

Picture This

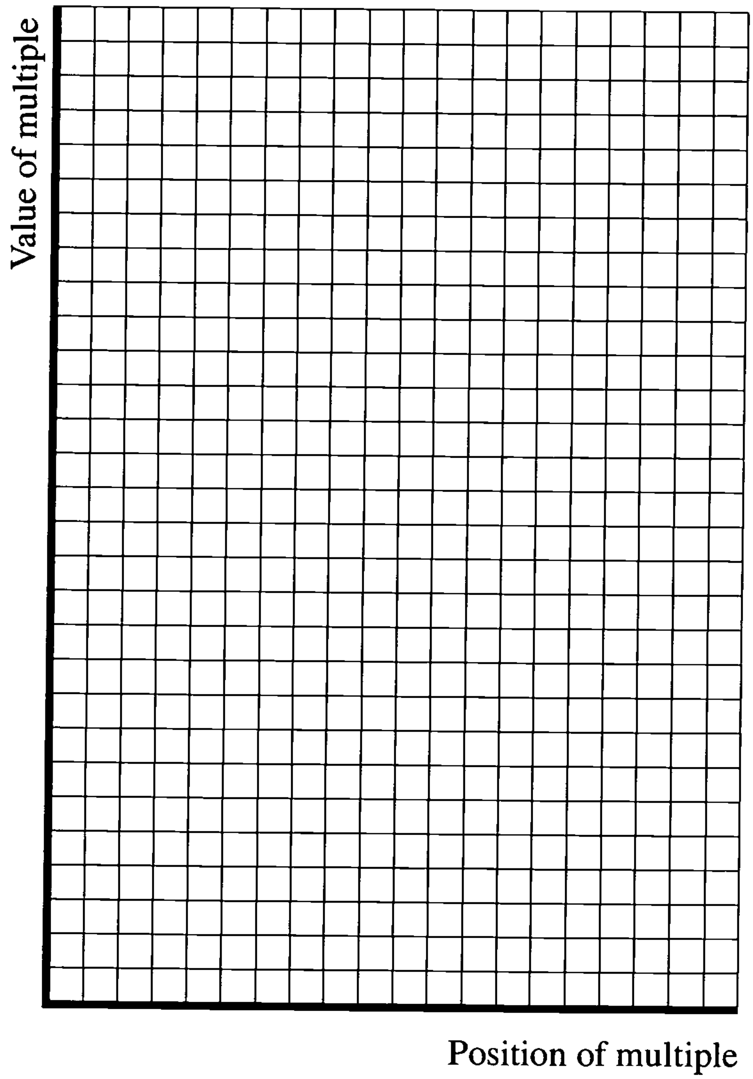
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Graphing Hundred Chart Patterns

Multiple of____	
Position of Multiple	Value of Multiple



Multiplication Table Worksheet

X	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

44

Fun with Function Machines

Students use the function machine as a visual tool for working with the concept of input and output.

Prerequisites

- ❑ MathMagical Function Box
- ❑ experience with concrete and pictorial model of a function

Materials

- ❑ function machines made from milk cartons

Background for the Instructor

A function machine is a fun way to introduce students to the concept of function. Students work with input and output numbers and the rule that defines their relationship. Function machines always follow a rule. If students have not had prior experience with the concrete and pictorial model of a function, begin with the MathMagical Function Box.

Set the Stage

- Display the actual Function Machine with its input and output slots. Discuss the terms "input" and "output." The machine takes numbers in, performs operations on the numbers according to a rule, and then outputs them. We will use the term "rule" to describe what the machine does to the input number. For example, if the machine's rule is to add 3, the number 7 will be output when the number 4 is input. Similarly, if the number 6 is input, a 9 is output; if the number 12 is input, a 15 is output, etc.

Conduct the Learning Experience

- Begin feeding a stack of previously prepared cards into the machine, showing students each number as it is input and the corresponding number that is output. Record the input and output numbers on the recording sheet. Have students look for the relationship (the rule) between the input and output numbers. While the input numbers do not have to be consecutive, it is easier for students in these beginning explorations to view the input numbers in increasing order.

- In the following example, we want students to recognize that the machine added 2 to each number that was input.

Rule:	
Input	Output
3	5
4	6
5	7
10	12

- The relationship could be described by the equation:
  Output number = input number + 2
- The rule "Add 2" should be written on the top line of the chart.

- Play the game "I Know!" This game repeats the above activity with a different set of cards. Cards are fed into the machine and the corresponding input and output numbers are recorded. When students believe they know the rule, they raise their hand and say "I know!" To test their knowledge the teacher will write a number in the input column of the recording sheet and the student must give the correct output number. Play continues until most of the class has correctly identified a corresponding pair of numbers.

- With practice, students should be able to find any one of the three parts to the problem—the input number, the output number, or the rule—when given two of the parts.

- Each set of cards uses a different rule and could include any one of the four operations. Combining operations such as 2 x the number + 4 would indicate two function machines since each machine can perform only one operation at a time.

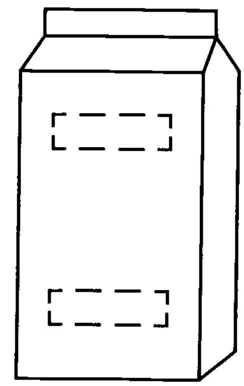
- When making cards you should have about 10 cards per rule. Extra cards can be put through the Function Machine to check the rule. Make sets of cards for these rules: +2, +3, +4, +5, -1, -2, -3, etc. As students become comfortable with addition, move to subtraction. As they become comfortable with recording data in order, shuffle the cards and use the cards out of order.

Closure

- Discuss real-life examples of function machines, such as a gum ball machine where you put in a penny and get out 2 pieces of gum. Let students suggest other machines.

Instructions for Creating a Function Machine

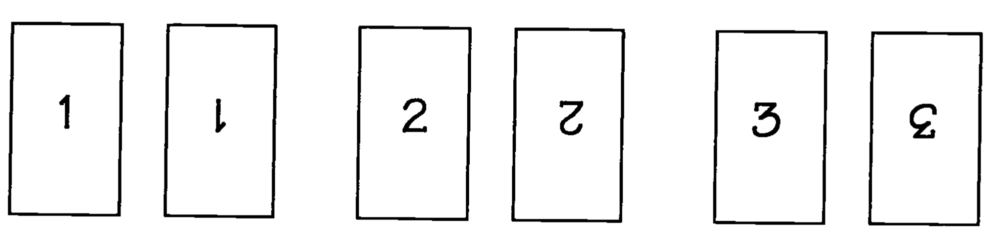
- Cut a slit across the top of a half-gallon milk carton about .5" high and 3" inches wide.



- Cut another slit across the bottom.
- Cut a piece of tagboard 3.5" wide and 8" long.
- Fold under about .5" at the top and bottom of the tagboard. This is the slide the card will travel along.
- Make sure the tagboard will go inside the box with the folded flange taped to the outside of the top slit.
- Make sure the slide curves from the top slit, touches the back of the carton, and comes out the bottom slit. The cards will be placed into the machine with the input number facing up. It is important that the card "flip" on its way down the slide so that the output number is facing up when the cards slides out of the box.
- Adjust the length of the slide if necessary before taping the bottom folded flange to the bottom slit.
- Decorate your function machine.

Construction of Cards

- Prepare cards as shown. Approximately ten cards per rule should be created. The cards should include a notation about which side is the input side. The size of the cards depends upon the size of the opening of slits of your function machine.



Recording Sheet for Fun with Function Machine

Rule:	
Input	Output

Rule:	
Input	Output

Rule:	
Input	Output

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A Calculator Function Machine

Students use the calculator as a function machine.

Prerequisites	Materials
❑ Fun with Function Machines	❑ calculators

Background for the Instructor

The calculator is a great tool to consolidate the previous work done with function machines.

Set the Stage

- Calculators with a built-in constant function can be used as a function machine.
- For example, on the TI-108, clear the calculator, press "+", "6", "=". Then, without touching any other keys, press "4" and "=" and the display will show 10. The calculator is now a "+6" machine.
- Hand the calculator to a student with instructions not to touch any key until told to do so. Invite the class to select a number. If the number is 7, instruct the student helper to enter "7" and then press "=" to display the number "13" in the calculator window.
- The student helper can continue to choose numbers to enter followed by (=) until the class predicts the rule that the calculator is using. Usually students get the rule after only one or two examples as long as only addition and subtraction are allowed.

Conduct the Learning Experience

- Group students and give each group of students a calculator.
- One student in each group will act as the leader. Leaders choose the rule and set up the calculator as a function machine by pressing "+" "a number" and "=" in that order. They then pass the calculator to another student in the

group. The calculator is passed around the group allowing each student to enter a number of their choice and find the corresponding output number. The leader records the data so that the group can analyze the data to determine the rule.

- After the rule has been determined, the calculator is given to a different student, who becomes the new leader for the next round.

Closure

- Journal entry: Have students respond to the question, "What is a function machine?"

Number Tricks

Number tricks provide an informal introduction to variables and algebraic proof.

Prerequisites	Materials
❑ none	❑ prepared overhead ❑ handouts

Background for the Instructor

These activities provide opportunities for students to explore expressions in different ways. The activities begin with students hearing expressions described orally, then seeing the expressions modeled with manipulatives, pictures, and algebraic expressions.

Set the Stage

- Prior to class, use lemon juice to draw a large number 5 on an overhead transparency. Allow to dry.
- Announce to the class that you are able to read their minds and that you are going to do an activity to prove it. Have a student go to the board or overhead projector while you position yourself so that you cannot see the student's work. As you call out instructions, the student at the board will work the problem.
- Here are the instructions for the student:

  1. Choose any whole number.
  2. Add 7.
  3. Multiply by 2. (Double it.)
  4. Subtract 4.
  5. Divide the number by 2.
  6. Subtract the original number.
  7. Write your answer on a small piece of paper.
  8. Fold the paper so that the answer is not visible.

- Explain that while you know what the hidden number is (you can read their minds), you don't want to make it too easy on yourself! Take the paper and carefully set fire to it while holding it over a large ashtray. Place the burning paper into the ashtray. (Obviously students should not be too close and you should not toss any burning object into the trash can!) After the flames are out, dump the ashes onto the prepared overhead transparency (that has had the number 5 drawn on it with lemon juice) and turn the overhead on. As you rub the ashes across the overhead transparency the students are amazed as the number 5 begins to appear.

- The obvious question is: How did you do that? The following activities lead students through a visual, pictorial, and algebraic explanation.

Conduct the Learning Experience

Number Magic Explanation - Manipulatives

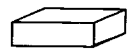
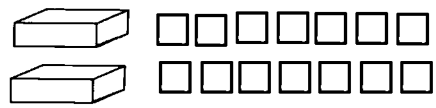
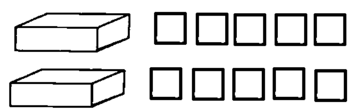
- After seeing the number trick performed, students are eager to know how the trick works. Have the class follow the instructions of the number trick with each person choosing a number of their choice so that they see that the answer is always 5. Have students make observations and discuss their ideas about why everyone got the same answer.

- Use manipulatives to model the instructions. Provide students with manipulatives so that they may follow along. The following example will use boxes ( ) and color tiles (□). Students must understand that the box holds a certain number of tiles, and the number of tiles in the box can vary from problem to problem. However, in a given problem, once a number of tiles has been assigned to a box, that number cannot be changed, and every box in that problem must contain the same number of tiles.

- Give students the instruction, "Choose any whole number." Upon receiving this instruction, the students will put a number of color tiles in the box. The number of color tiles that they use is their choice. Every box used in this problem however, must have the same number of color tiles in it. Tiles cannot be added to or taken away from inside the box during this problem.

- After students are comfortable with the concept of the box taking on different values for different problems, encourage them to remember the value of the box rather than always putting the tiles inside. Eventually we want students to think of the box as "some number" rather than a specific number.

- Following is a re-enactment of the magic trick problem with a picture of the manipulatives that the students will be using.

1. Choose any whole number. 
2. Add 7. 
3. Multiply by 2. (Double it.) 
4. Subtract 4. 
5. Divide the number by 2. 
6. Subtract the original number. 

- Students are able to see that the box (the number they chose) is not in the solution because they have a picture that shows that no matter what number they start with, they always end up with the number 5.

Mind-reading Explanation - Visual

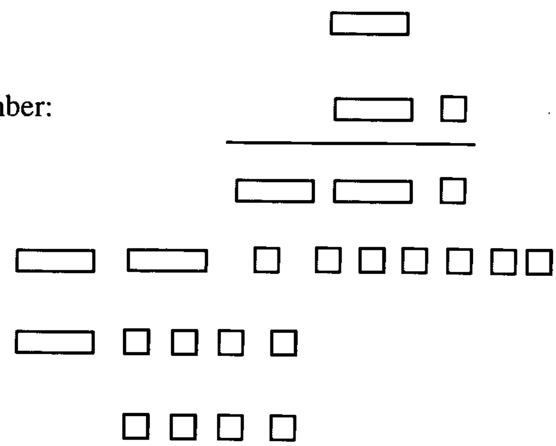
- Once students are proficient with modeling, it is time to move to drawing visual representations of the modeling. This step is the link between the modeling with manipulatives that we did in the previous activity and the abstract algebraic method that we will use in the next activity. This is a very important step. Rectangles (▭) will be used to represent the boxes and squares (□) will be used to represent the color tiles.

- Consider this number trick:

1. Choose any whole number.
2. Add the next consecutive number.
3. Add 7.
4. Divide the sum by 2.
5. Subtract your first number.

- Have the students model with their manipulatives. Show how to draw a picture that will visually represent the actions that have just been completed. Here is the same example represented pictorially:

1. Choose any whole number:
2. Add the next consecutive number:
3. The result is:
4. Add 7:
5. Divide the sum by 2:
6. Subtract the original number:

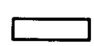


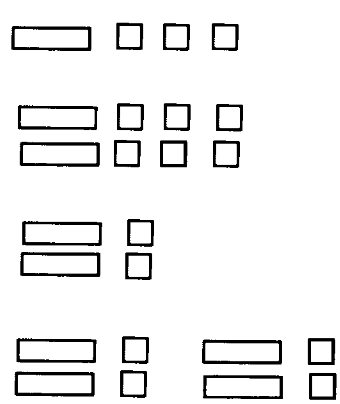
- Overhead transparencies can be made from the Number Magic charts provided. These can be useful as you show students how to move between the visual, verbal, and numerical models.

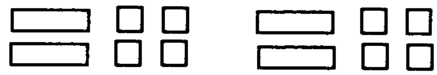
Mind-reading Explanation - Algebraic

- The final step that the students must undertake is using algebraic notation. The transition to the algebraic representation is easily made from the picture of the box holding an unknown number of tiles. The box can be symbolically represented by the letter n (called the variable). The individual tiles that were used outside of the box can be represented by a number (called a constant).

- ***Example:***

1. Choose any whole number:  n
2. Add 3: $n + 3$
3. Multiply by 2: $2(n+3) = 2n + 6$
4. Subtract 4: $2n + 6 - 4 = 2n + 2$
5. Double it: $4n + 4$



6. Add 4:  $4n + 4 + 4 = 4n + 8$

7. Divide by 4:  $n + 2$

8. Subtract the original number:  $n + 2 - n = 2$

- Some discussion should take place concerning vocabulary. For example, consider the different ways that instructions could be given:

  1. Double the number and take away two.
  2. Take twice the number, then minus two.
  3. Multiply by two, subtract two.

  These activities offer teachers an interesting way to teach vocabulary. Have students state the instructions in as many different ways as they can.

- The Number Tricks worksheet gives some additional tricks to try.

Closure

- Write the clues to a number trick. Show how your trick works using numbers, pictures, and algebra.

Number Tricks

1. Write down any number.	n
Add 4.	n + 4
Multiply by 2.	2(n + 4) = 2n + 8
Subtract 4.	2n + 8 - 4 = 2n + 4
Divide by 2.	(2n + 4) /2 = n + 2
Subtract your original number.	n + 2 - n = 2
Write down your answer.	2
2. Write down any number.	n
Multiply by 3.	3n
Add 6.	3n + 6
Subtract the number you originally wrote down.	3n + 6 - n = 2n + 6
Divide by 2.	(2n + 6)/2 = n + 3
Subtract 3.	n + 3 - 3
Write your answer.	n
3. Think of a number.	n
Add 2 to your number.	n + 2
Double the amount you now have.	2n + 4
Add 6.	2n + 4 + 6 = 2n + 10
Divide by 2.	(2n + 10)/2 = n + 5
Subtract your original number.	n + 5 - n
What is the result?	5
4. Write down any number.	n
Add 10.	n + 10
Multiply by 2.	2n + 20
Divide by 4.	(2n + 20) / 4 = (1/2 n + 5
Subtract 5.	(1/2)n + 5 - 5 = (1/2)n
Multiply the difference by 2.	2* (1/2)n
What is your answer?	n
5. Choose a number between 1 and 9.	n
Multiply by 5.	5n
Add 3.	5n + 3
Multiply by 2.	10n + 6
Choose another number between 1 and 9 and add to the product.	10n + 6 + x
Subtract 6.	10n + x
What is your answer?	(2 digit number where the 10s digit is the number and the ones digit is the second number.

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6. Choose any number.	n
Add the number of days in November.	$n + 30$
Multiply by the number of days in a school week.	$5n + 150$
Subtract the number of years in a century.	$5n + 150 - 100 = 5n + 50$
Double the answer.	$10n + 100$
Delete the "0" in the ones place.	$n + 10$
Subtract the original number.	$n + 10 - n$
What is the result?	10

7. Write down the day of your birth.	n
Multiply by 5.	$5n$
Add 18.	$5n + 18$
Multiply the sum by 4	$20n + 72$
Add 1.	$20n + 73$
Multiply the sum by 5.	$100n + 365$
Add the month of your birth.	$100n + 365 + x$
Subtract 365.	$100n + x$
What is your answer?	(A number showing your day and month of birth.)

8. Choose any whole number.	n
Add the next consecutive number.	$n + n + 1$
Add 9.	$2n + 1 + 9 = 2n + 10$
Divide the sum by 2.	$n + 5$
Subtract your first number.	$n + 5 - n$
What is your number?	5

9. Pick a number from 1 to 10.	n
Multiply it by 9.	$9n$
Add the digits together.	9
Subtract 5 from the sum.	4

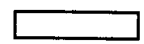
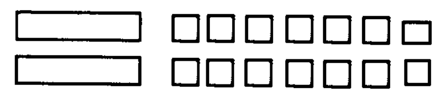
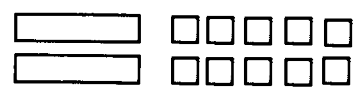
Number Magic for Mind-Reading Trick

Fill in the Blank Columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.			
Add 7.			
Multiply by 2 (double it).			
Subtract 4.			
Find 1/2 of the results.			
Subtract the original number.			
Write your answer.			

Number Magic for Mind-reading Trick

Answer Key

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.	7		n
Add 7.	14		n + 7
Multiply by 2 (double it).	28		2n + 14
Subtract 4.	24		2n + 10
Find 1/2 of the results.	12		n + 5
Subtract the original number.	5		n + 5 - n
Write your answer.	5		5

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Number Magic for Mind-Reading Trick

Fill in the Blank Columns

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
Think of a number from 1 to 10.			
Multiply your number by 6.			
Add 12 to the result.			
Take half.			
Subtract 6.			
Divide by 3.			
Write your answer.			

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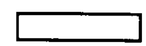
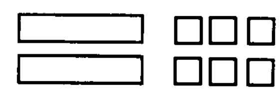
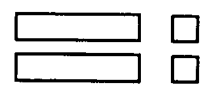
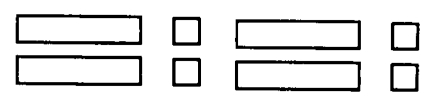
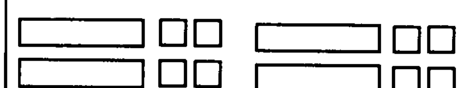
Number Magic for Mind-Reading Trick

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)
			
			
			
			
			
			
			
64			65

Number Magic for Mind-Reading Trick

VERBAL (Mental Math)	NUMERICAL (Example)	VISUAL (Manipulative/Pictorial)	ALGEBRAIC (Abstract)

Toothpick Patterns

Students investigate a pattern. The extension of this pattern leads to the introduction of variables and the development of formulas.

Prerequisites

- ❑ experience in patterning

Materials

- ❑ 15 toothpicks per person
- ❑ overhead of toothpicks
- ❑ picture of old/young woman
- ❑ math journals

Background for the Instructor

Students have had experience exploring many different types of patterns. This activity shows students that there are many ways to look at patterns and that algebra is simply a generalization of arithmetic.

Set the Stage

- Give the picture of the old/young woman to each group.
- Have each student look for the old and the young woman in the picture. Some students will see the young woman more easily—others will see the old woman more easily. We can look at the same picture and see things differently. This is also true for patterns. Students should understand that patterns may be expressed in different ways. Each student may see the pattern differently.
- Explain that just as everyone saw at least one of the views of the woman, everyone will be successful with the following problem and algebra will be used to generalize the solutions.

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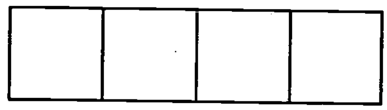
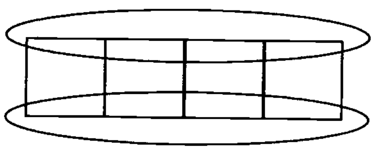
Conduct the Learning Experience

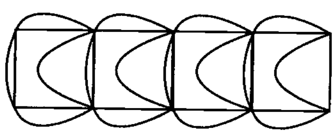
- Distribute toothpicks to each student and have them form 4 squares in a row as shown below.



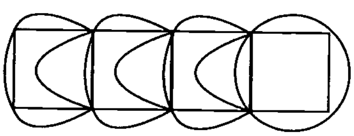
- Have the students find a way to find the total number of toothpicks used without counting by ones. As students describe their method, name the method after the student, e.g. Brittany's method. Here are some possible solutions:



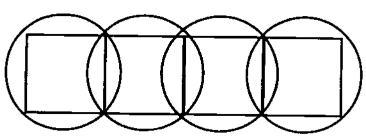
<u>Two</u> groups of 4 and **5** singles:
$2(4) + \mathbf{5} = 13$



<u>Four</u> groups of 3 and **1** single:
$\mathbf{4}(3) + \mathbf{1} = 13$



<u>Three</u> groups of 3 and 1 group of **4**:
$3(3) + \mathbf{4} = 13$



<u>Four</u> groups of 4 minus the **3** counted twice:
$\mathbf{4}(4) - \mathbf{3} = 13$

- Ask students to consider a row of 9 squares. Without building the toothpick pattern, determine how many toothpicks will be used. Students should extend the methods developed with 4 squares above to the 9-square problem.

Example

Two groups of 9 and 10 singles:	$2(9) + 10 = 28$
Nine groups of 3 and 1 single:	$9(3) + 1 = 28$
Eight groups of 3 and 1 group of 4:	$8(3) + 4 = 28$
Nine groups of 4 minus the 8 counted twice:	$9(4) - 8 = 28$

- Using the same technique, have students find the number of toothpicks in
  - 25 squares
  - 52 squares
  - 100 squares

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- Have students generalize the number of toothpicks for an unknown number of squares

  1. by writing a description in words showing how they would find the total number.

     Example

     Two times the number of squares (the top and bottom) plus the number of squares plus 1
     Three times the number of squares plus 1 single
     Three times one less than the number of squares plus 4
     Four times the number of squares minus one less than the number of squares

  2. by writing a description in symbols.
     Let s = the number of squares

     Example

     $2s + (s + 1)$
     $3s + 1$
     $3 (s - 1) + 4$
     $4s - (s-1)$

Closure

- Journal Entry: Describe how you would find the number of toothpicks you would use to make a strip pattern of 5 triangles.

What's Your Method?

6 8

Lessons

Overview of 6-8 Lessons

In grades K-2 students were encouraged to recognize and describe **patterns** using concrete materials. In grades 3-5 students were introduced to the concept of variables and asked to extend and refine their knowledge of patterns to include using a variable to represent a given mathematical **relationship**. In grades 6-8 students will expand their use of pattern-based thinking to explore, analyze, and make conjectures about the general rule for a mathematical relationship. One of the goals of the activities in this section is to help students view rules and input/output situations as **functions**. The ultimate goal is for students to build mathematical models to predict the behavior of real-world phenomena based on an observed pattern.

These activities help students to generalize and describe patterns and functions in many ways. The focus is on understanding different representations of functions (tables, graphs, equations, verbal explanations) and how to move among and choose the best representation for the situation. As students move among the different representations, they develop an understanding that functions are composed of variables that have a dynamic relationship—changes in one variable result in changes in another.

Patterns, relationships, and functions will become powerful problem-solving strategies. Students who are faced with non-routine problems often give up, because they do not know how to get started. However, students with the ability to discover and analyze patterns may move forward. Even if they are uncertain what they are looking for, they are able to organize and analyze their data. Often patterns emerge that help students to solve problems.

These experiences with various representations of functions will help students build bridges to later work in algebra. The identification of special characteristics of a relationship, such as the point when the input is zero, lays the foundation for later work in algebra; in this case, the y-intercept.

Activities in this section will ask students to use letters to represent variables when they are trying to discover a rule. The rules will involve more than one operation. Students are asked to describe patterns that they see presented in table form. Some students will be more comfortable starting with manipulatives or pictorial representations, recording their findings in tables, and finally writing a symbolic equation.

Numberless Graphs

Students experience visual patterns in life situations in preparation for their work with functions. The focus is on understanding relationships among quantities whose values change.

Prerequisites	Materials
❑ experience with real, pictorial, and symbolic graphs and the notion of variables	❑ copies of Numberless Graph handouts

Background for the Instructor

Graphing can present a visual representation of relationships, and is interwoven with the concept of functions. In this activity, students are asked to interpret graphs depicting real-life situations. Essentially students are being asked to compare information from a geometric perspective. The focus is on understanding quantities whose values change. This activity helps students to extend the notion of a variable from "a letter standing for a number" or "an unknown value in an equation" to thinking about a variable as a quantity that changes as the situations in which it occurs changes.

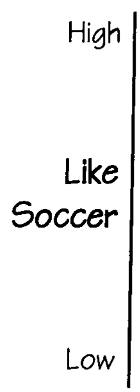
The purpose of this activity is to help students see that graphing is a means of recording information, as are tabular forms and equations. Since many students are visual learners, graphing makes patterns clearer than recording information in a table or an equation. The emphasis is on making graphs and interpreting information, not on coordinate graphing. This activity sets the stage for formal training in slope and equations of lines in algebra.

Numberless graphs help students associate mathematics with their daily lives. With the increased availability of graphing calculators, students need to be able to interpret graphs and to see the connection between a graph and a verbal expression, table of data, or an algebraic expression.

If you have access to graphing calculators and motion detectors, ask the students to create the distance and time graphs depicted by their motion.

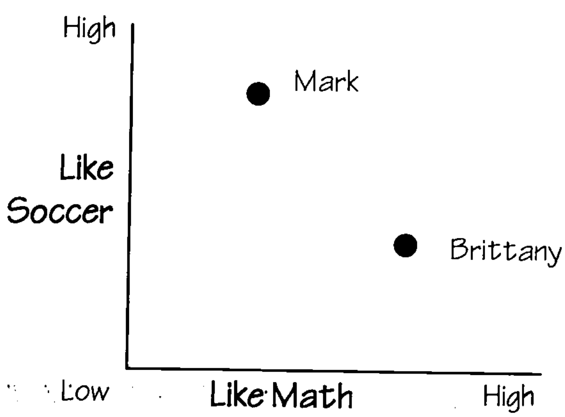
Set the Stage

- nvolve the students in human graphs by asking a group of four to five students to line up according to height. Record this information on a horizontal number line. Ask these same students to line up based on how much they like math, soccer or something popular in your area. Record this information on a vertical number line. Note there are no numbers on the number lines. Data is to be recorded in perspective.

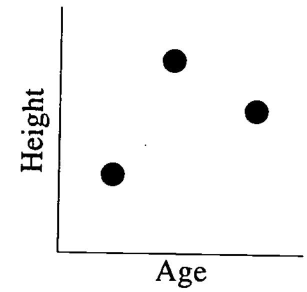


Conduct the Learning Experience

- Using these two pieces of information about the students, construct a graph. For example, the following graph shows that Mark likes soccer more than math, while Brittany likes math more than soccer.



- Construct a numberless graph using the people in a row to depict their height and ages. Ask the students to stand in a line. Represent each person with a point on the graph. Ask the members of the class to label the points with the names of the students.



- Now make the transition from points to lines. Show the students graphs such as those in *Numberless Graphs 1* and ask them to make comparisons, contrasts, and conclusions. Discuss what is happening in the graph and then ask students to answer the questions about the graph.

- Be sure that students understand that in these examples, as the points on the line move to the right more time is elapsing, and as the points move up there is a greater height (water level or temperature).

- In *Numberless Graphs 2*, be sure that the students understand how time and speed are represented on the axes. Talk about what is happening when the line goes up or down or when it is horizontal or vertical. Ask students to work in pairs on two graphs at a time. Ask each pair to share with another pair and come to agreement on their answers. Ask each quad to share with the class. Continue in this fashion through all the graphs.

- In *Numberless Graphs 3*, students are asked to write a scenario describing the motion of the object represented in the graph. This is usually more difficult for the students and shows whether they understand the concepts involved. Be sure the students understand the following:

  1. An object moving at a steady pace will be a straight line graph, with positive slope if the distance is increasing and negative slope if the distance is decreasing.
  2. An object at rest is represented by a horizontal line.
  3. An object that moves toward you is decreasing the distance and thus the line slopes down.
  4. The graph of an object that changes directions changes from sloping down to sloping up or vice versa.

Closure

- Ask students to create their own numberless graphs to share with the class.

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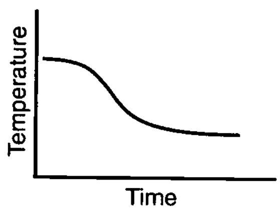
Possible Answers to Numberless Graphs

Numberless Graphs 1

1. a. B b. E c. D d. G
2. endless possibilities
3.



Numberless Graphs 2

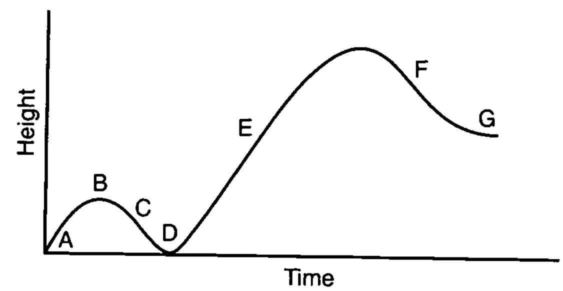
1. d
2. g
3. a
4. f
5. b
6. e
7. c

Numberless Graphs 3

1. A car approaches your house.
2. A car leaves your house.
3. A friend leaves your house walking. (Because there is no scale on the axes, you do not know that the slope of this line is less than the slope of the line for #2. However, it appears that the distance increases more slowly.)
4. A friend leaves your house, walks to the bus stop and waits for the bus.
5. A friend is waiting at the bus stop, runs towards you to catch a frisbee, runs back to the bus stop, and continues waiting.
6. A friend in a car approaches an intersection, stops, and continues toward your house.
7. A friend throws a ball into the air and catches it.
8. A child is riding the elevator up and down between two floors.
9. A friend is riding on a Ferris wheel.
10. A yo-yo completes one cycle.
11. A flag is being raised.
12. A toy car moves away from you, hits a wall, turns, and moves back toward you.

Numberless Graphs 1
Miscellaneous

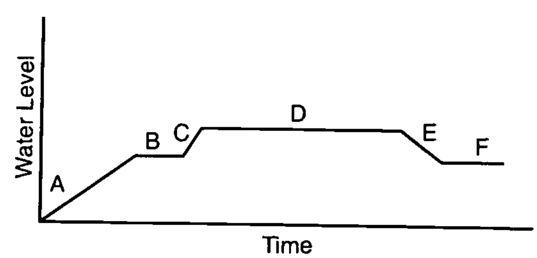
1. This graph shows the height of a football above the ground from the time the ball was snapped for a field goal attempt.



- Identify the letter where the following events occurred.

   a. The holder received the snap.
   b. The ball was increasing in height.
   c. The ball was kicked.
   d. The ball lands in the stands behind the goal post.

2. The graph below shows the water level in a bathtub over time.



- Discuss different situations the graph might depict. For example:

   a. The tub is being filled to a certain level.
   b. The faucet is turned off.
   c. You get in the tub (water level rises).
   d. You soak.
   e. The phone rings—you get out of the tub, etc.

- Draw graphs depicting other "bathtub" problems.

3. Draw a graph that shows how the temperature of a can of soda changes after it is moved from a shelf in the kitchen pantry into the refrigerator and left for three hours.

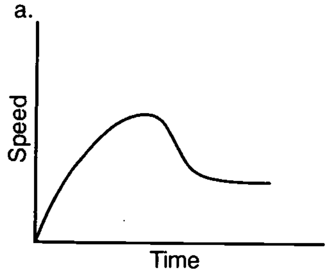
Numberless Graphs 2

Speed and Time

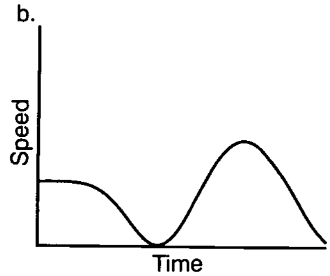
- In the following graphs, time is represented on the horizontal axis. Speed is represented on the vertical axis. Discuss the relation between the two quantities as they change and select the best graph to represent the following situations.

1. You ride your bike at a steady pace and then let it coast down a hill.
2. You ride your bike downhill and then up a hill, stopping at the top of the hill.
3. You ride your bike down a hill and then at a steady pace on level ground.
4. You are riding your bike at a steady pace when suddenly you crash into something and stop.
5. You ride your bike up a hill, stop, and then coast down the hill, stopping at your house.
6. A friend riding his bike stops in front of your house to pick you up.
7. You race your bike on an oval track.

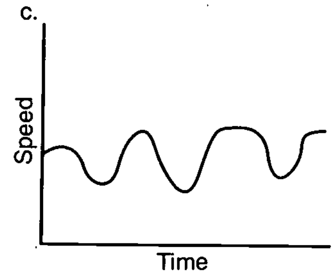
a.



b.



c.



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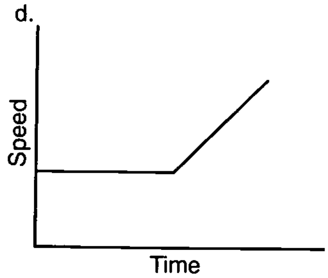
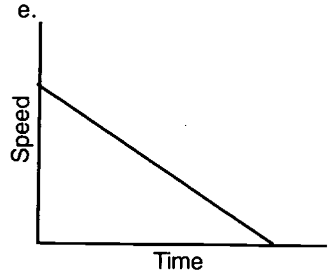
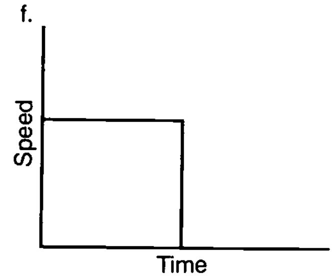


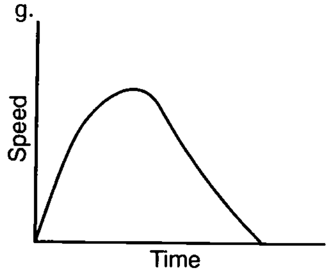
e.



f.



g.



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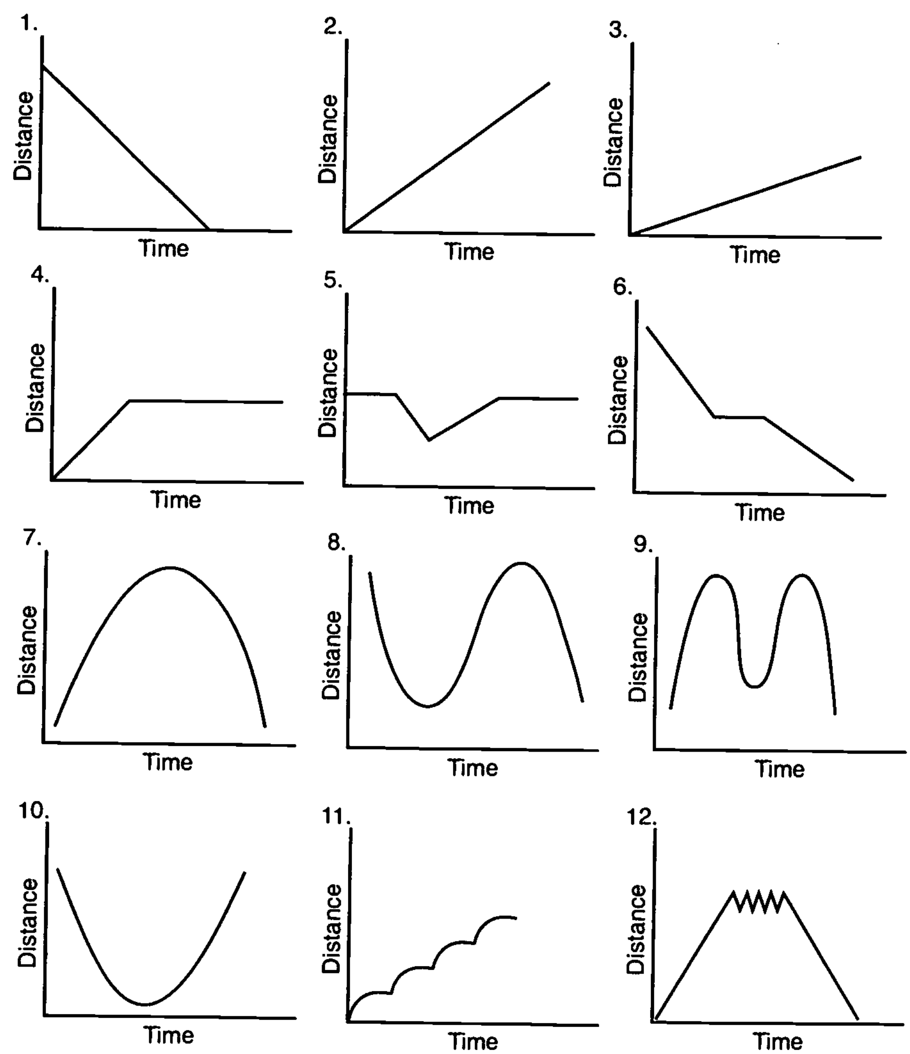
Numberless Graphs 3

Distance and Time

- In the following graphs, time is represented on the horizontal axis. Distance is represented on the vertical axis. Consider what the graph will look like under the following conditions:

  - an object moving at a steady pace
  - an object at rest
  - an object that starts far away from you and moves towards you
  - an object that changes directions

- Discuss the relation between the two quantities as they change and describe the motion of an object(s) represented by the following graphs.



What's My Pattern?

To develop an understanding of the relationship between patterns and functions, this activity describes linear relationships from number sequences and generates linear graphs of physical data.

Prerequisites

- ❑ "Fun with Function Machines"
- ❑ building patterns with concrete materials

Materials

- ❑ handouts or transparencies of number sequences and "Straight Line Graphs of Physical Data"
- ❑ rice
- ❑ cylindrical jars and cans
- ❑ bicycle
- ❑ paper cups
- ❑ pennies
- ❑ rulers
- ❑ graph paper
- ❑ Balance

Background for the Instructor

The study of patterns actively involves students in constructing their mathematical knowledge. It provides the opportunity for conjecture, analysis, and application of mathematics. The long-term goal is that students will be able to take a real-world event, translate it into mathematical symbols, and apply it back to the real world via a generalization. In the short term, students are motivated by patterns—they spark their curiosity.

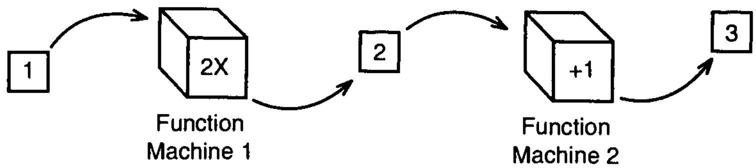
The focus of this lesson is on uncovering one of the big ideas of algebra—functions. Functions are based on patterns. As students study patterns, they are developing an understanding of this relationship between patterns and functions. Please do not force students to memorize; rather, encourage them to understand. You may wish to begin by asking students to build relationships with hands-on models such as cubes or pattern blocks. These types of activities are readily available in supplementary mathematics materials relating to patterns or algebra.

If students have not had experience with function machines, you should lead the activity "Fun with Function Machines" prior to this lesson. This will help the students to connect the numbers in the table with input/output numbers from the function machine. Because you are dealing with two operations, use two function machines. For example, for the function $y = 2x + 1$, the first function machine doubles the input ($2x$) and the second machine adds one to the input ($2x + 1$). This helps the students to see the two processes involved in the function.

Because students are familiar with the function machine and identifying the pattern, they are sometimes perplexed that they cannot see the pattern as quickly in the more complex functions. The following activity gives them a strategy to use to help identify the pattern.

For example, in the sequence 3, 5, 7, 9, 11, students may recognize that you add 2 to get the next term and have no strategy for getting the 50th term other

than to generate all 50 terms. This activity is designed to help them find a strategy to generate a *relational* formula rather than a *recursive* formula. Help students to see that the numbers in the sequence are "one more than the multiples of 2." Use two function machines to demonstrate the following relationships until students understand the rule y = 2x + 1.



Set the Stage

- Surprisingly, students are motivated by number sequence activities. The object is to find the general rule by which the sequence is generated. Observe the students closely to determine the processes they use. Most students begin with trial-and-error, yet some will develop strategies. A side benefit to this activity is the reinforcement of basic facts as students perform many computations on the path to discovering the general rule.

— Journal Entry —
How do patterns help us to solve problems? Think of patterns of all kinds, make a list of ideas, and then use search engines on the World Wide Web to locate web sites that are based on patterns.

- Students should extend patterns in three ways based on their previous experiences. The most concrete way is to build the pattern with manipulatives (such as cubes), count, and record. Next, the students are likely to notice a pattern in the output values (typically called the *y*-values) and use this pattern to continue the number sequence. Finally, students will find a general rule that relates any number in the first list (input values or *x*-values) to the corresponding number in the second list (output values or *y*-values). This last method of recording the general rule using a mathematical symbol will help students make the connection to algebra.

Conduct the Learning Experience

- Because the goal is to emphasize patterns, the number sequences are listed in tables or ordered pairs. The student is to discover a relationship between the *x*- and y-terms in the table. To facilitate finding a pattern, the numbers are listed in increasing consecutive order. Continue the pattern as far as necessary until students discover a relationship. The relationship is then printed in the rule box in equation form (for example y = x + 2).

- It is important for students to understand that you are asking for a *relational* formula—one that relates the output value to the input value in the sequence. In other words, we can find an output for any given place in the sequence—the 5th term, 100th term, or *n*th term. We are using a *recursive* relationship (one that requires knowing the prior term) as an aid in determining the type of relational formula (linear or quadratic). Consider the following data:

Rule: y = x + 2	
x	y
1	3
2	4
3	5
4	6
5	7

Rule: y = 2x + 1	
x	y
1	3
2	5
3	7
4	9
5	11

Rule: y = 3x - 2	
x	y
1	1
2	4
3	7
4	10
5	13

Rule: y = 5x + 6	
x	y
1	11
2	16
3	21
4	26
5	31

- In this activity, students are asked to discover a relationship between the *x*- and *y*-values (a relational or horizontal relationship in the table). One way to approach this is to look at the patterns that emerge when you subtract successive *y*-values in the sequence (a recursive or vertical relationship). These patterns are based on using the tables above:

y	Difference
3	1
4	1
5	1
6	1
7	
.	
.	
1x + 2	

y	Difference
3	2
5	2
7	2
9	2
11	
.	
.	
2x + 1	

y	Difference
1	3
4	3
7	3
10	3
13	
.	
.	
3x - 2	

y	Difference
11	5
16	5
21	5
26	5
31	
.	
.	
5x + 6	

- As an aid to help students discover the relationship between *x* and *y*, point out that *if* there is a common difference in the first subtraction, a linear (first degree) expression describes the relationship. A linear relationship produces a straight line graph. The general rule for this first degree expression is $ax + b$, where *a* is the constant difference between the *y*-values. This is comparable with the operation of the first function machine.

- To find the operation of the second function machine, help students to find the constant term *b* by working backwards in the table. Using the difference pattern in the *y*-values, identify the output (*y*-value) for a zero input and help them realize why this would be the constant term. For example:

x	y	Difference
0	2	1
1	3	
2	4	
3	5	
4	6	
5	7	
.	.	
.	.	
x	$1x + 2$	

- Students will often equate the output value for $x = 1$ to be the constant; rather than the output value for $x = 0$. In algebra, the output value for $x = 0$ is called the *y*-intercept. This work with patterns helps to lay the groundwork for this important idea in algebra.

- Generate as many more linear (first degree) number sequences as you feel are necessary to insure that the students understand the concept. To help students see how linear relationships are modeled in the real world, complete the activity, *Straight Line Graphs of Physical Data* at the end of this lesson.

- When students are comfortable with linear relationship patterns, look at some non-linear relationships. It is important to expose students to examples of relationships that are not linear. Otherwise, they tend to think all relationships are linear. With the advent of technology, it is no longer necessary to restrict instruction in functional relationships to simple, linear functions. See the activity *Numberless Graphs* as an example.

- Give the students data that is not defined by a linear relationship. Have them note that the common differences are not constant. Help them to discover that a common difference will show up in the second subtraction.

x	y	Difference	
0	0	1	
1	1	3	2
2	4	5	2
3	9	7	2
4	16	11	2
5	25		
.	.		
.	.		
x	x^2		

x	y	Difference	
0	5	1	
1	6	3	2
2	9	5	2
3	14	7	2
4	21	9	2
5	30		
.	.		
.	.		
x	$x^2 + 5$		

- Again, take the common differences of the successive terms in the output. It will be necessary to create two columns of differences to find a constant difference. Help students to find the constant term b by working backwards in the table. Using the pattern, identify the output for a zero input and help them realize why this would be the constant term.

- As time permits, consider other rules where the numerical coefficient of the x^2 term is something other than one. Students will discover that the common difference seems to be *twice* the coefficient of the x^2 term. For example:

x	y	Difference	
1	2	6	
2	8	10	4
3	18	14	4
4	32	18	4
5	50		
.	.		
.	.		
x	2 + 2		

x	y	Difference	
1	7	6	
2	13	10	4
3	23	14	4
4	37	18	4
5	55		
.	.		
.	.		
x	$2x^2 + 5$		

Closure

It appears that a pattern has been found. How do we know that it will always follow this pattern?

It is true that if we get a common difference in the *second subtraction*, the general rule is a *second degree expression*. It is also true that the common difference is always twice the coefficient of the x^2 term. In fact there are patterns that hold for common differences in the third subtraction for third degree expressions and in the fourth subtraction for fourth degree expressions.

To ensure a unique formula for each relationship be sure that there are a sufficient number of terms in the sequence to establish a pattern of *common* differences, and assume that the pattern continues infinitely. If either of these criteria are not met, your formula may not be unique to the sequence.

The general formula for a quadratic (second degree) equation can be tested. This is a good exercise for algebra students; however, it is beyond the scope of this lesson.

Straight Line Graphs of Physical Data

Teacher Instructions

- These activities allow students to experience collecting data that generates straight line graphs. In general the manipulated variable goes on the horizontal axis and the responding variable on the vertical axis. These are sometimes referred to as independent (x-axis) and dependent (y-axis) variables. Students should collect at least the four data points called for in the activity. They may collect additional ones as time permits.

- Ask students to analyze their graph and write a description of what the graph tells them. Look for indications of a direct variation (for example, as one variable increases, the other variable increases).

- Students will work in groups to perform the four activities described on page 88.

Straight Line Graphs of Physical Data

- Analyze your graph and write a description of what the graph indicates. Look for indications of a direct variation (for example, as one variable increases, the other variable increases) as you work in groups to perform the following activity.

1. Measure in a tall cylindrical jar the height of material for the number of measures poured in and graph the height against the number of measures.

   √ Pour 3 level measures of rice in an olive jar
   √ Measure height of the rice to the nearest inch
   √ Repeat for 5, 7, and 9 measures

2. Graph the distance rolled by a cylindrical can in terms of the number of revolutions.

   √ Put a mark on the can and a mark on the floor
   √ Roll the can 1 revolution and mark the floor
   √ Measure the distance to the nearest 1/8 inch
   √ Repeat for 2, 3, and 4 rolls

3. Graph the number of revolutions of a bicycle wheel against the number of revolutions of the pedals.

   √ Put the pedals in straight up-and-down position and mark the rear tire
   √ Turn the pedals slowly 1 complete turn and count how many turns are made by the rear wheel
   √ Fractions of a turn can be calculated by using the spokes
   √ Repeat for 3, 5, and 7 turns of the pedals

4. Balance different numbers of paper clips with the appropriate number of pennies.

   √ Place 5 pennies in one cup and balance by putting paper clips in the other cup on a balance
   √ Repeat for 10, 15, and 20 pennies

Functional Relationships

A linear relationship is explored via a graphing calculator by collecting data.

Prerequisites

- familiarity with the graphing calculator functions of List, Window, and Scatterplot

Materials

- stopwatch
- graphing calculators
- overhead graphing calculator

Background for the Instructor

The function concept is perhaps as important as any concept in mathematics. It permeates all of mathematics, from the middle school years through calculus and beyond. This activity provides an early introduction to the function concept without undue emphasis on formal notation and definitions. Notations should be introduced only after concepts are well developed and students see a need for symbolism. In this activity students will graph data collected from doing "The Wave." This linear graph offers students opportunities to interpret mathematics based on their experience and understanding rather than on memorization of abstract symbols. The Wave provides an activity in which the students may represent a function in a tabular, graphic, algebraic, and verbal format.

Set the Stage

- Explain how to do The Wave (a movement often performed by crowd at football games) and demonstrate with a group of five students. Assign one student to be the timekeeper and give him or her a stopwatch. When the student using a stopwatch says, "Go," the first student stands up, raises her arms and sits down in sequence. The second student does the same. When the last person sits down, he says, "Stop," and the timekeeper records the elapsed time.

- Students should be familiar with the graphing calculator functions of List, Window, Scatterplot, and Linear Regression (optional). If they are not, you may wish to review these procedures prior to doing The Wave.

Conduct the Learning Experience

- We expect that the duration of the wave will be a linear function (straight line) of the number of people performing it. The number of people is the independent variable, x, and the time needed to complete the wave is the dependent variable, y. The independent variable is manipulated to produce changes in the dependent variable.

- Record the data from the demonstration with the group of five students. Repeat the wave with groups of 8, 10, 15, or 18 students.

- Data should be recorded on a graphing calculator. See below for specific instructions for the TI-83 graphing calculator. Remember the line of best fit is not always linear. The line of best fit is the regression equation whose correlation coefficient, r, has an absolute value closest to 1. You may wish to check this r value on the TI-83 graphing calculator.

1. Begin by clearing any data that is already in memory.
   - Press $\boxed{\text{STAT}}$
     — Select the EDIT submenu
     — Select 1:Edit
     — You will see three columns
     — Move the cursor to highlight L_1
   - Press $\boxed{\text{CLEAR}}$
   - Press $\boxed{\text{ENTER}}$
   - Repeat for columns L_2 and L_3

2. To enter data:
   - Press $\boxed{\text{STAT}}$
     — Select the EDIT submenu
     — Select 1:Edit
     — Enter your data, x values in L_1 and y values in L_2
   - Press $\boxed{\text{ENTER}}$ after each entry
   - Use arrow keys to move from one list to the other

L_1	L_2
8	recorded time
10	recorded time
15	recorded time
18	recorded time
Keep going ...	

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3. To set the window:

   - Press $\boxed{\text{WINDOW}}$
     — Set the following parameters:
     $X_{min} = 0$ $Y_{min} = -10$
     $X_{max} = 30$ $Y_{max} = 30$
     $X_{scl} = 5$ $Y_{scl} = 5$

4. To draw a plot:

   - Press $\boxed{\text{STATPLOT}}$ (by pressing 2nd y =)
     — Select PLOT1, select ON (by pressing Enter)
     — Arrow down to TYPE: select scatter plot (the first one)
     — Arrow down to Xlist: select L_1 (by pressing 2nd 1)
     — Arrow down to Ylist: select L_2 (by pressing 2nd 2)
     — Arrow down to Mark: select the first symbol
   - Press $\boxed{\text{GRAPH}}$

■ Ask the students to predict how long it would take 20 people to do The Wave. To ensure that students can interpret the data and apply their conclusions about the functional relationship, ask questions such as these:

1. How long will it take 30 people to make a wave?
2. How many students are needed for a 30-second wave?
3. Would the graph still be a straight line if the students clapped twice before sitting down?
4. How would the graph change?
5. How would the graph change if only the first person claps four times before the wave starts?

■ Ask different students to share how they arrived at their response to ensure that the students know how to use the tabular, graphical, and algebraic representations to answer the questions.

■ Depending on your students' familiarity with the graphing calculator, you may wish to extend the lesson by viewing the regression line. You may demonstrate this if the students are not familiar with these functions on the graphing calculator.

1. To find the Regression Equation that is most closely correlated to the data:

   - Press $\boxed{\text{STAT}}$
     — Select the CALC submenu

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   — Select LinReg(ax+b)
   — Press L_1 (2nd 1), Press comma (above the 7 key), Press L_2 (2nd 2)
- Press ENTER

2. To import the chosen regression equation to your graphing screen:
   - Press Y = CLEAR VARS
     — Select 5: Statistics
     — Arrow over to EQ
     — Select 1: RegEq
   - Press GRAPH

3. To compare the actual time values in L_2 with the values calculated by the regression equation:
   - Press STAT
     — Under the EDIT submenu, select 1: Edit
   - Press ENTER
     — Move cursor ▷ and △ to cover the L_3 label

4. To define all values in L_3 as a function of L_1 as predicted by the regression equation:
   - Press VARS
     — Select Y-VARS
     — Select 1: Function
     — Select 1: Y1
   - Press (L1) and ENTER

5. To predict how long it would take 20 people to do The Wave using data presented in a table:
   - Press STATPLOT (2nd Y=)
     — Select 1: Plot 1
     — Arrow over the Off
   - Press ENTER
   - Press TblSet (2nd Window)
     — TblMin=20
     — △ Tbl = 1
   - Press TABLE (2nd Graph)

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6.) To predict how long it would take 20 people to do The Wave using data presented in a graph:

- Press $\boxed{\text{CALC}}$ (2nd TRACE)
  — Select 1: Value
  — X = 20
- Press $\boxed{\text{ENTER}}$

Closure

- Ask students to name other ways they might change The Wave to have an effect on the graph.

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Connections!

To make the connection among the various ways to describe a linear relationship, students match cards with patterns represented as word phrases, equations, tables, and graphs.

Prerequisites

- ❑ graphing
- ❑ identifying a pattern
- ❑ expressing a pattern using a variable, expression, or sentence

Materials

- ❑ linear relationship cards
- ❑ chart
- ❑ scissors
- ❑ tape

Background for the Instructor

Students with experience in generalizing patterns, working informally with open sentences, and representing numerical situations using verbal phrases, tables of data, equations, and graphs are prepared to make the connections among the various ways to describe linear relationships. This activity helps students to connect three categories for recording functional relations using mathematical language: geometric—graphs, arithmetical—tables, and algebraic—equations.

Algebra is a language for describing patterns. Previous activities have helped students progress from concrete to a more abstract representation of data. Students should have had prior experiences in writing generalizations in words as well as in equations. Students should understand that variables can be used to represent unknown numbers in equations, as a generalized number in pattern rules or formulas, and as characteristics to be graphed.

Set the Stage

- Generate a discussion on linear relationships by asking what makes data linear in nature.

- Continue the discussion by asking, "If the data is linear, how can you use that information to make predictions?" Help students extend and refine their thinking about linear relationships by determining if there is a "best way" (verbal explanation, equation, table, graph) to represent a linear relationship in a given situation. If students are familiar with graphing calculators, ask them to recall situations when they preferred to use the table format or the graph format.

- Play "I Have, Who Has?" with cards similar to the examples at the end of this lesson. Use the cards describing data in table format and one other type of card to get students warmed up for the activity. Then use the table format cards with another type of card. This will help the students to see that the data in the table (a pattern) is the key that links to multiple representations—word phrases, equations, and graphs.

Conduct the Learning Experience

- Distribute the pages containing the word phrase, equation, table, and graph. Also distribute scissors and tape.

- Ask each student to cut out each word phrase, equation, table, and graph. Match those cards containing expressions that are equivalent to each other.

- When a student completes the task, ask him or her to pair with another student to check their work. Then ask the pair to share with another pair to check their work.

Closure

- Ask students to respond to the following prompts in their journal:

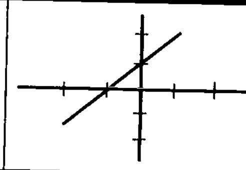
  1. What makes data linear in nature?
  2. If the data is linear, how can you use that information to make predictions from the data?
  3. Is it possible to make predictions if the data is not linear? If so, what would be different?

- Question three helps students to realize that all data is not linear. This is important for them to know, even though they are not ready to work with quadratic relationships. Often, students think that all lines are straight.

- You may wish the students to practice this activity again by using the cards in a Jeopardy format.

Connections! Linear Relationships Game

- Cut out each graph, table, equation, and word phrase from the following three pages. Match the descriptions that are equivalent (one from each group). Tape them to the chart below. (Continue on back.)

Word Phrase	Equation	Table	Graph
Example: One more than x	$y = x + 1$	x | -3 -2 0 1 y | -2 -1 1 2	
1.			
2.			
3.			
4.			
5.			

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Connections! Linear Relationships Game

Word Phrase, Equation, Table, and Graph

- Cut out each word phrase, equation, table and graph. Match the cards that contain expressions that are equivalent to each other.

$y = 2(x+1)$	$y = -x$	$y = 2x + 1$	$y = 2$
$x = 2$	$y = x - 2$	$y = 2x$	$y = x + 2$
$x + y = 2$	$2y = x$	y is 12 less than x	y is twice x
y is 1 more than twice x	x is twice y	y is the opposite of x	y is always 2
the sum of x and y is 2	x is always 2	y is twice the sum of x and 1	y is 2 more than x

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Connections! Linear Relationships Game

x	y
-2	-2
-1	0
0	2
1	4
2	6

x	y
-2	-3
-1	-1
0	1
1	3
2	5

x	y
-2	2
-1	1
0	0
1	-1
2	-2

x	y
-2	2
-1	2
0	2
1	2
2	2

x	y
-2	-4
-1	-3
0	-2
1	-1
2	0

x	y
-2	-2
-1	-1
0	0
1	1
2	2

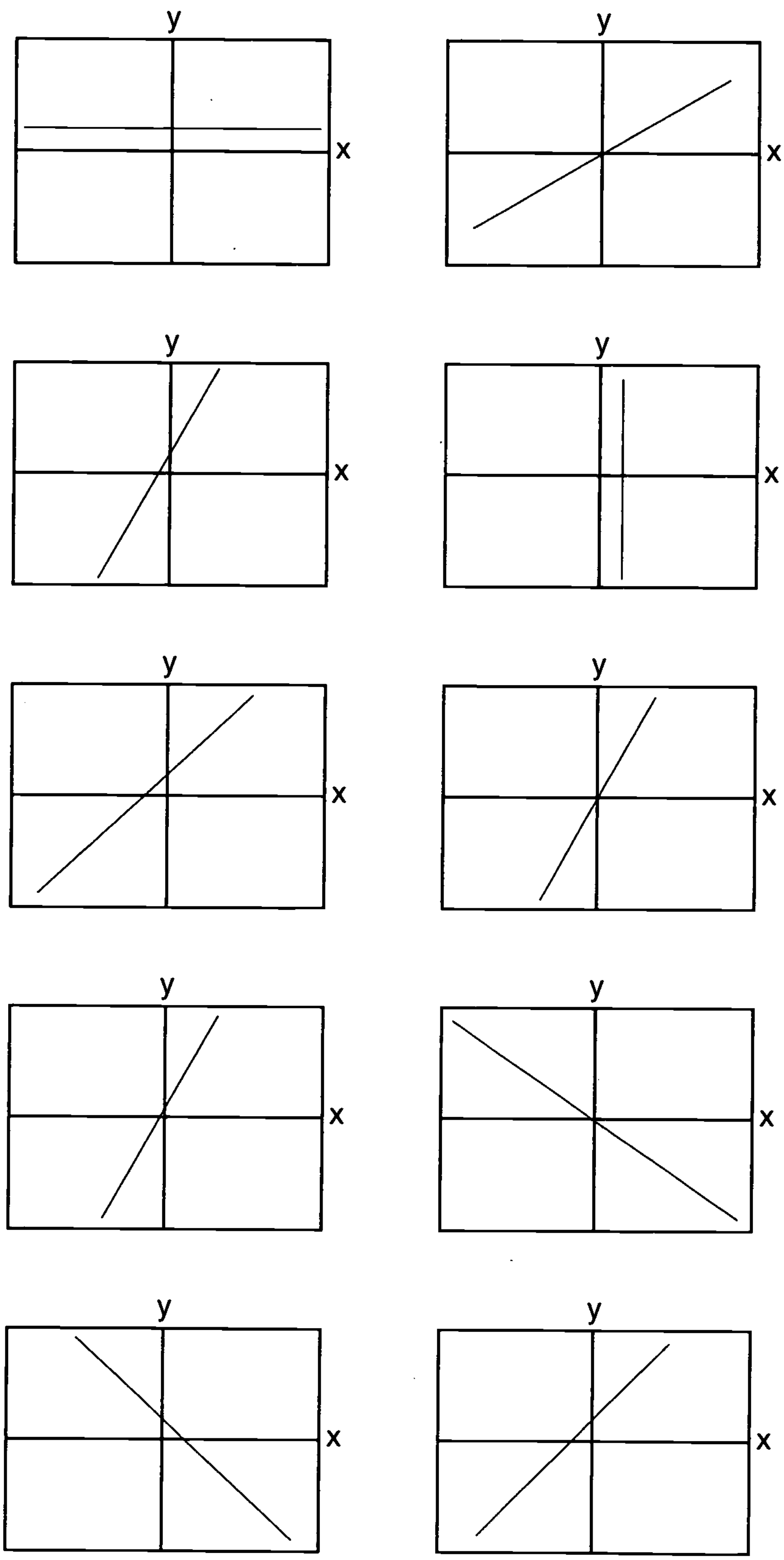
x	y
-2	-4
-1	-2
0	0
1	2
2	4

x	y
-2	0
-1	1
0	2
1	3
2	4

x	y
-2	4
-1	3
0	2
1	1
2	0

x	y
-2	-2
-1	-1
0	0
1	1
2	2

Connections! Linear Relationships



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