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ABSTRACT

Factor analysis has been characterized as being at the heart of the score validation process. In virtually all applications of exploratory factor analysis, factors are rotated to better meet L. Thurstone's simple structure criteria. Two major rotation strategies are available: orthogonal and oblique. This paper reviews the numerous rotation options available in the factor analysis literature, examining the pros and cons of various analytic choices. A heuristic data set was examined to make the discussion concrete. Some guidelines are also offered for resolving differences in the analytic choices so that the appropriate rotation methods can be selected. (Contains 10 tables and 16 references.) (Author/SLD)

Orthogonal Versus Oblique Factor Rotation: A Review of the Literature Regarding the Pros and Cons

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Abstract

Factor analysis has been characterized as being at the heart of the score validation process. In virtually all applications of exploratory factor analysis, factors are rotated to better meet Thurstone's simple structure criteria. Two major rotation strategies are available: orthogonal and oblique. The present paper reviewed the numerous rotation options available in factor analysis, and in particular examined the pros and cons of various analytic choices. A heuristic data set was examined to make the discussion concrete. Further, some guidelines are offered for resolving differences in the analytic choices so that the appropriate rotation methods can be selected.

Orthogonal Versus Oblique Factor Rotation: A Review of the Literature Regarding the Pros and Cons

The utilization of factor analytic techniques in the social sciences has been integral to the development of theories and the evaluation of the construct validity of scores. It is not surprising, therefore, that Pedhazur and Schmelkin (1991, p. 66) noted, "Of the various approaches to studying the internal structure of a set or indicators, probably the most useful is some variant of factor analysis." The current accessibility and widespread applicability of factor analytic techniques have rendered these methods popular among researchers exploring data structure and the construct validity of scores.

One variant of factor analysis is termed exploratory factor analysis. In this application of factor analysis, the primary concern is with the development of theories or the generation of alternative explanations for commonly accepted theories about a phenomenon of interest. As noted by Tinsley and Tinsley (1987),

> Factor analysis is an analytic technique that permits the reduction of a large number of interrelated variables to a smaller number of latent or hidden dimensions. The goal of factor analysis is to achieve parsimony by using the smallest number of explanatory concepts to explain the maximum amount of common variance in a correlation matrix. (p. 414)

Thus, EFA is a data reduction technique that permits the reduction of a large number of variables (e.g., test items, individuals) into constituent components by examining the amount of variance that can be reproduced by the latent or synthetic variables underlying the observed or measured variables.

The purpose of the present paper was to briefly explicate the concept of exploratory factor analysis with an emphasis on a conceptual understanding of the factor rotation process and the two types of rotation strategies available to researchers. Examples were provided using the Holzinger and Swineford (1939) data set that has been utilized extensively in illustrating factor analytic principles (cf. Gorsuch, 1983). Further, the relative merits of utilizing different rotation strategies were explored, and guidelines for their appropriate application were provided.

Brief Overview of Exploratory Factor Analysis

Factor analysis has been conceptually available to researchers since the turn of the century (Thompson & Daniel, 1996), but unfortunately due to the complex nature of the mathematical manipulations necessary to perform computations, has not been used extensively until the advent of the computer. However, a renewed interest in factor analytic techniques was evidenced in the middle of the 20th century following the delineation of test standards by the American Psychological Association (Thompson & Daniel, 1996). Following the introduction of these test standards, researchers began utilizing factor analysis to demonstrate the validity of scores generated by their instruments.

EFA has generally been utilized for two general purposes in the social sciences. The first general purpose has been to better understand the structure of a data set when no previous information on the data structure is available. This is an example of utilizing EFA as a tool in the generation of theories about phenomena of interest, and is an example of an appropriate application of this analytic technique. The second general application of EFA has been to reexamine patterns in data sets when the tenability of the emergent factors in previous research has been questioned. The latter example use of EFA is inappropriate, as a model-to-data fit can be directly evaluated through another variant of factor analysis termed confirmatory factor analysis (CFA). Thus, the primary application of EFA is to explore the factor structure of a set of indicators (e.g., variables, test items, individuals, occasions) when no previous research is available.

Basics of Exploratory Factor Analysis

Factor rotation is just one of several steps completed when conducting an EFA. Further, each individual step often involves a decision based on the intrinsic values of the researcher or the goals of the analysis. Consequently, there are many different ways in which to conduct a factor analysis, and each different approach may render distinct results when certain conditions are satisfied (cf. Gorsuch, 1983). The one consistent element in conducting an EFA, however, is that the results of the analysis are based solely on the mechanics and mathematics of the method and not on the a priori theoretical considerations of the researcher (Daniel, 1989). To provide a context for the discussion of factor rotation, a few of the steps involved in conducting a factor analysis are briefly reviewed.

Matrices of Association. One of the first decisions to be made in performing an EFA is to determine the manner in which the data matrix will be represented in the analysis. Since all statistical analyses are correlational (Cohen, 1968; Knapp, 1978; Thompson, 1997a), the focus of every statistical analysis is on the relationship among a set of variables or other entities (e.g., people) that may be factored. Matrices of association (e.g., correlation matrices, variance-covariance matrices) are arrays of numbers that are utilized to concisely express the linear relationships between a larger set of variables. The most common matrix of association utilized in EFA is the correlation matrix (in which values of 1.0 are on the main diagonal and bivariate correlation coefficients between the variables are on the off-diagonals), perhaps partly because this is the default in most statistical software packages.

Factor Extraction. After the researcher has chosen which matrix of association will be utilized in the analysis, the researcher must then determine which extraction method to employ in conducting the analysis. Factor extraction refers to removing the common variance that is shared among a set of variables. There are currently several different techniques available for the extraction of common variance (e.g., principal components analysis and principal factors analysis), and the results generated by the analysis can differ based on the particular method of extraction utilized.

Of the techniques available, principal components analysis and principal factors analysis are the two most widely used extraction methods in EFA. Although some researchers have argued that the difference between these extraction methods is negligible (cf. Thompson, 1992), other researchers have contended that the difference is substantial enough to warrant careful consideration of the extraction method utilized (cf. Gorsuch, 1983). The interested reader is referred to Gorsuch (1983), Stevens (1996) or Tinsley and Tinsley (1987) for a more thorough treatment of this material.

Factor and Coefficient Generation. One advantage in employing a factor analysis is that each latent or synthetic variable (factor) extracted from the analysis is perfectly uncorrelated with all of the other factors. This is often advantageous when the purpose of the EFA is theory generation as the interpretation of the extracted factors is thereby greatly simplified.

When extracting factors, only a certain portion of the variance for any given variable will be reproduced by the factors. Two matrices are formed as a result of the factor extraction procedure, the factor pattern matrix (comprised of weights that are identical to ß weights in multiple regression analysis and that indicate the relative importance of a given variable to the extracted factors with the influence of the other variables removed) and the factor structure matrix (coefficients that represent the bivariate correlation of measured/observed variables with scores on the extracted latent/synthetic factors). Since the extracted factors are always initially perfectly uncorrelated, the factor pattern matrix and the factor structure matrix are exactly equal; thus these two matrices can be simplified into one matrix termed the "factor pattern/structure matrix."

Variance-Accounted-For Statistics

After the factor pattern/structure matrix is generated, it is possible to derive two variance-accounted-for statistics that help the researcher determine the amount of variance that is reproduced by the latent constructs. The first of these is the communality coefficient, $\underline{h}^2$, which can be defined as the amount of variable variance that is reproduced by the factors. This value is calculated by summing the squared pattern/structure coefficients across the row for each variable. The resultant coefficient is an index of the proportion of total variance for a given variable that is reproduced by the extracted factors. Since it is a squared statistic, it can range from 0 to 1.0.

Another variance-accounted-for statistic that is generated from the factor pattern/structure matrix is the eigenvalue. An eigenvalue represents the amount of variance in the original data set that is reproduced by a given factor. For the principal components case, eigenvalues can be computed by summing the squared factor pattern/structure coefficients down the columns of the matrix. Eigenvalues represent the amount of factor-reproduced variance and their values can range from 0.0 to the total number of variables in the analysis. Eigenvalues can also serve as effect size measures, as each eigenvalue can be divided by the number of total variables in the analysis and a percentage of the total variance for a given factor can be computed.

Factor Retention. After the factor pattern/structure matrices and variance-accounted-for statistics and have been computed, the researcher must decide the number of factors to retain in the analysis. Since different retention methods can often generate divergent results, it is generally important to examine more than one factor retention method.

Two popular methods of determining the number of factors to retain is the eigenvalue greater than 1.0 rule (Kaiser, 1960) and the scree test (Cattell, 1966). It is important to carefully examine all of the eigenvalues, however, as previous research has reported that in certain situations the eigenvalues greater than 1.0 rule and scree test can underestimate or overestimate the number of factors that should be retained (cf. Hetzel, 1996; Zwick & Velicer, 1986).

Interpretation of Results. After the appropriate number of factors are retained in the analysis, it is necessary to interpret the results. It is often difficult to interpret the initial factor/pattern structure matrix as many of the variables typically manifest noteworthy coefficient magnitudes on many of the retained factors (coefficients greater than |0.60| are often considered large and coefficients of |0.35| are often considered moderate) and especially on the first factor.

Rotation of Factor Analytic Results

After the factor analysis is completed, it is usually necessary to rotate the factors to formulate a better solution that is more interpretable (i.e., has better "simple structure" (Thurstone, 1947)). Thompson (1984, pp. 31-34) demonstrated how the unrotated pattern/structure matrix actually misrepresents the true nature of the factors, and how factor rotation resolves this misrepresentation. Pedhazur and Schmelkin (1991) commented on the need to rotate factors:

> While the results of a factor analysis may produce a good fitting solution it is not necessarily susceptible to a meaningful interpretation. It is in attempts to improve the interpretability of results that factors are rotated.... Because there is an infinite number of ways

> in which factors can be transformed or rotated, the question arises: Are some rotations better than others?.... The reason that rotations are resorted to for the purpose of improving interpretability of factor analytic results and interpretability, is by its very nature, inextricably intertwined with theory. (p. 611)

Interpretation of the factor analytic results is therefore almost always aided by the rotation of the factor solution, as it is possible to redistribute the common variance across the factors to achieve a more parsimonious solution. It is important to note, however, that factor rotation is not "cheating" and does not generate or discover more common variance; rather, factor rotation merely redistributes the variance that has been previously explained by the extracted factors.

After a factor solution is rotated, the first unrotated factor may not account for the largest portion of the variance and thus may not have the largest variance-accounted-for value. Since the variance has been redistributed throughout the factors, any of the factors could account for the largest proportion of the total variance. Additionally, after the rotation is conducted, eigenvalues are no longer termed as such; rather, after rotation, the variance-accounted-for statistic for the factors (columns of the factor pattern/structure matrix) is termed "trace." One of the most commonly committed mistakes by researchers is believing that the eigenvalue for a given factor before extraction informs judgement regarding the variance accounted for by the factor after the factor solution is rotated (Hetzel, 1996).

Objectives of Factor Rotation. The objective of factor rotation is to achieve the most parsimonious and simple structure possible through the manipulation of the factor pattern matrix. Thurstone's (1947) guidelines for rotating to simple structure have largely influenced the development of various rotational strategies. The most parsimonious solution, or simple structure, has been explained by Gorsuch (1983, pp. 178-179) in terms of five principles of factor rotation:

1. Each variable should have at least one zero loading.
2. Each factor should have a set of linearly independent variables whose factor loadings are zero.
3. For every pair of factors, there should be several variables whose loadings are zero for one factor but not the other.
4. For every pair of factors, a large proportion of variables should have zero loadings on both factors whenever more than about four factors are extracted.
5. For every pair of factors, there should only be a small number of variables with nonzero loadings on both.

Thus, factor rotation is devised to shift the factors in their factor space so that each variable in the analysis has a large factor pattern coefficient on only one factor and has very small or zero factor pattern coefficients on the other extracted latent constructs.

The primary goal in rotating to simple structure is to produce better fitting solutions that are more replicable across studies. As stated by Gorsuch (1983), "Thurstone showed that [simple structure] rotation leads to a position being identified for each factor that would be independent of the number of variables defining it. Therefore, a simple structure factor should be relatively invariant across studies" (p. 177).

Types of Factor Rotation

Orthogonal Factor Rotation

Two types of factor rotation are available: orthogonal and oblique. Orthogonal rotation shifts the factors in the factor space maintaining 90 degree angles of the factors to one another to achieve the best simple structure. Since the cosine of the angles between vectors of unit length equals r, and the cosine of a 90 degree angle is zero, this rotation strategy maintains the perfectly uncorrelated nature of the factors after the solution is rotated and often aids in the interpretation process since uncorrelated factors are easier to interpret. In theory, the results of an orthogonal rotation are likely to be replicated in future studies since there is less sampling error in the orthogonal rotation due to less capitalization on chance that would occur if more parameters were estimated, as is the case in oblique rotation.

Varimax Rotation. Of the orthogonal rotation strategies available, one of the most popular orthogonal rotation technique is rotation to the varimax criterion developed by Kaiser (1960). In this technique the factors are "cleaned up" so that every observed variable has a large factor pattern/structure coefficient on only one of the factors. Varimax rotation produces factors that have large pattern/structure coefficients for a small number of variables and near-zero or very low pattern/structure coefficients with the other group of variables.

Quartimax Rotation. Another popular orthogonal rotation strategy is quartimax. In this rotation strategy, the factor pattern of a variable is simplified by forcing the variable to correlate highly with one main factor (the so-called G factor of early IQ studies) and very little with the remaining factors (Stevens, 1996). The variables are much easier to interpret in this case, but the factors are more difficult to interpret since all variables are primarily associated with a singe factor.

Since the emphasis of varimax rotation is on easing the interpretability of the factors, it may seem logical to utilize varimax rotation when employing an orthogonal rotation strategy. However, researchers must carefully examine the objectives of an analysis, as quartimax rotation would be preferred in an analysis in which the researcher expected one general (i.e., the so-called "G") factor. Although the results between quartimax and varimax tend to be similar (Stevens, 1996), thoughtful researchers will examine their expectations of the analysis and choose orthogonal rotation strategies accordingly.

Advantages and Limitations of Orthogonal Rotation. There are several advantages to employing orthogonal rotation strategies. First, the factors remain perfectly uncorrelated with one another and are inherently easier to interpret. Secondly, the factor pattern matrix and the factor structure matrix are equivalent and thus, only one matrix of association must be estimated. This means that the solution is more parsimonious (i.e., fewer parameters are estimated) and thus, in theory, is more replicable.

Orthogonal rotation strategies do, however, have limitations. Orthogonal rotations often do not honor a given researcher's view of reality as the researcher may believe that two or more of the extracted and retained factors are correlated. Secondly, orthogonal rotation of factor solutions may oversimplify the relationships between the variables and the factors and may not always accurately represent these relationships.

Oblique Factor Rotation

The second type of factor rotation is termed oblique rotation. This method of rotation provides for correlations among the latent constructs. This rotation strategy is termed oblique because the angles between the factors becomes greater or less than the 90 degree angle.

Direct Oblimin. One type of oblique rotation strategy is direct oblimin. This strategy is moderated by a delta value (chosen by the researcher), in which positive values of delta produce higher correlations between factors and negative values of delta produce smaller correlations between factors. The direct oblimin strategy more closely honors the nature of reality and demands careful consideration by the researcher, as the correlation between the factors must be set prior to analysis.

The preset default value of delta can have serious consequences on the results of the direct oblimin factor rotation. Gorsuch (1983) indicated that at delta = -4 the factors actually become orthogonal, and he further recommended varying delta from 1 to -4 to help researchers ascertain a clearer conception of their data structure. Researchers have objected to the use of this strategy, however, as the technique appears very subjective in nature (regarding the preset correlation of the factors).

Promax. One of the most popular oblique rotation strategies is promax (see Hetzel, 1996). In this technique, the researcher is attempting to achieve the most parsimonious simple structure given that the factors are allowed to be correlated with one another. Promax rotation has three distinct steps, the first of which is to rotate the factors orthogonally. Next, a target matrix is contrived by raising the factor pattern/structure coefficients to an exponent greater than two (typically exponents of three or four are used). The coefficients in the target matrix become smaller, but the absolute distance between them actually increases. For example, if 0.9 and 0.3 are two factor pattern/structure coefficients, the first coefficient is three times larger than the second; however, if both values are squared, the resultant value of 0.81 is nine times larger than 0.09. Thus, the result of transforming the factor pattern/structure matrix is that many moderate coefficients (i.e., |0.30| or smaller) approach zero more quickly than the large coefficients (i.e., |0.60| or larger).

The final step in a promax rotation involves the "Procrustean" rotation of the original matrix to a best fit position with the target matrix. Promax is often the oblique rotation strategy of choice, as it is relatively easy to use, typically provides good solutions, and tends to generate more replicable results than the direct oblimin rotations.

Advantages and Limitations of Oblique Rotations. Oblique rotation strategies can be useful to researchers for a variety of reasons. One advantage of using an oblique rotation strategy is that the solution more closely honors the researcher's view of reality. Unfortunately, oblique rotations may be difficult to interpret, especially if there is a high degree of correlation among the factors. Since the factor pattern and factor structure matrices are not equal, both have to be interpreted in conjunction with the other.

However, an oblique factor solution inherently tends to be less parsimonious. For example, if 5 factors for 100 factored entities (e.g., variables) are extracted and orthogonally rotated, only 500 factor pattern/structure coefficients are estimated (the 5 x 5 factor correlation matrix is not estimated, since it is constrained to have 1's on the diagonal and 0's everywhere else). If the same EFA factors are rotated obliquely, *1,010* coefficients (500 factor pattern coefficients, plus 500 factor structure coefficients, plus 10 factor correlation coefficients (the 10 non-redundant off-diagonal entries in the 5 x 5 factor correlation matrix)) are estimated. [It might be argued, however, that only *510* coefficients are estimated in this case, since with either the 10 unique factor correlation coefficients, and either the 500 pattern or the 500 structure coefficients, the remaining 500 pattern or structure coefficients are fully determined.]

The fact that more parameters are estimated in an oblique rotation means that oblique solutions almost always better fit sample data than do orthogonal solutions. However, some of this fit involves "overfitting" sampling error variance. This means that orthogonal solutions, though they may tend to somewhat fit sample data less well, are generally more replicable in future samples, since orthogonal solutions capitalize on less sampling error. Usually, at least in EFA, somewhat poorer fit is deemed an acceptable tradeoff for better solution replicability (i.e., factor invariance). As the degree of correlation between the factors decreases, both orthogonal and oblique solutions will tend to provide increasingly similar results. Given that oblique solutions are less parsimonious and therefore less replicable, an oblique rotation would therefore only be employed when the benefits of simpler, more interpretable structure outweigh the costs of less replicability (i.e., when the orthogonal factors are not readily interpretable, and the oblique factors are fairly highly correlated but more interpretable).

<u>Which Type of Rotation is Better</u>?

The decision to rotate orthogonally or obliquely is often difficult for researchers and is largely based on the goal of the analysis. If the goal of the analysis is to generate results that best fit the data, then oblique rotation seems to be the logical choice. Conversely, if the replicability of the factor analytic results is the primary focus of the analysis, then an orthogonal rotation might be preferable since results from orthogonal rotation tend to be more parsimonious. The decision to rotate orthogonally or obliquely was discussed by Pedhazur and Schmelkin (1991):

> the decision whether to rotate factors orthogonally or obliquely reflect's one's conception regarding the structure of the construct under investigation. It boils down to the question: Are aspects of the a postulated multidimensional construct intercorrelated?... The preferred course of action is, in our opinion, to rotate both orthogonally and obliquely. When, on the basis of the latter, it is concluded that the correlations among the factors are negligible, the interpretation of the simpler orthogonal solution becomes tenable. (p. 615)

Researchers (Thurstone, 1947; Cattell, 1966; 1978) have challenged the utility of orthogonal rotation in preference for the utilization of oblique strategies. Thurstone (1947, p. 139) contended that the use of orthogonal rotation indicates "...our ignorance of the nature of the underlying structure... The reason for using uncorrelated [factors] can be understood, but it cannot be justified." Similarly, Cattell (1978, p. 128) argued in regard to researchers performing orthogonal rotation, "...in half of [the] cases it is done in ignorance of the issue rather than by deliberate intent." Consequently, even though orthogonal rotation eases the interpretability of the factor solution, it may not accurately portray the relationships between the variables and the emergent factors. However, many researcher do not blindly employ orthogonal strategies and instead utilize these approaches to ease result interpretation and to generate a more parsimonious result.

Two elements typically whether orthogonal and oblique rotation strategies will generate similar or identical results: (a) the factor to variable ratio; and (b) the degree of correlation between the factors. When the ratio of variables to factors is small, both rotation strategies will produce similar results, as simple structure will tend to be the same regardless of the type of rotation. Further, if the correlation between the factors is small (i.e., factor correlation coefficients closer to zero), then orthogonal and oblique rotation strategies will generally produce similar, if not identical, results.

In sum, choosing a rotation strategy to employ in factor analysis is not an arbitrary decision; rather, the appropriate choice of either an orthogonal or oblique rotation largely depends on the goals of the analysis (best fit to data or replicability of the analysis), the factor to variable ratio, and the degree of correlation between the factors. The guidelines stated by Pedhazur and Schmelkin (1991) appear to contend appropriately with the issues, as these authors suggested conducting both strategies and then comparing the results of the different rotations. If the difference between the two results is negligible, then the researcher can interpret the orthogonal rotation. Conversely, if the differences between the two rotations is noteworthy, then the researcher must consider interpreting the oblique rotation.

Heuristic Example of Differences in Rotation Strategies

To demonstrate the differences in orthogonal and oblique rotation strategies, a heuristic example was utilized in which tests T1, T8, T9, T11, T12, T14, T15 and T16 from the original Holzinger and Swineford (1939) data set were factor analyzed. Further, the factor pattern and structure matrices in each of three cases (no rotation, orthogonal rotation and oblique rotation) were compared. The means, standard deviations and labels for the raw data are presented in Table 1 and the variable correlation matrix is presented in Table 2.

Insert Table 1 and 2 About Here.

Principal components analysis (PCA) was chosen to extract the variables since this method yields results similar to principal factors analysis as the number of factored entities increases (Thompson & Daniel, 1996). The results of the factor extraction are presented in Tables 3 and 4.

Insert Table 3 and 4 About Here.

Notice that two matrices are contrived as a result of the PCA: one is a factor pattern matrix (Table 3) and the other is a factor structure matrix (Table 4). Since PCA was utilized to extract the factors, the extracted factors are perfectly uncorrelated and, consequently, the factor pattern and structure matrices are exactly equal. Thus, the factor pattern matrix and the factor structure matrix can be combined into one factor pattern/structure matrix since all of the values are identical and no information will be lost.

The factor correlation matrix is presented in Table 5. The table indicates that each of the factors correlates perfectly with itself but does not correlate with any of the other factors (i.e., the factors are perfectly uncorrelated). This is the universal result of initial factor extraction.

Insert Table 5 About Here.

An examination of Table 4 reveals that the factor saturation (which observed variables have large coefficients on which latent constructs) is complex, and it is difficult to interpret the factor pattern/structure matrix in its present form. In Table 4, all eight of the variables have pattern/structure coefficients greater than |.35|, the criterion generally indicative of a moderately large coefficient. Thus, to more easily interpret the results, the three factor solution could be rotated to different criteria. The three-factor solution was first rotated to the varimax criterion. The results of the varimax rotated solution are presented in Table 6.

Insert Table 6 About Here.

An examination of the results presented for the four factor solution rotated to the varimax criterion reveals that ascertaining which variables are associated with which factors has been greatly facilitated by the rotation procedure. Factor I is most highly saturated with tests T1, T8, and T9 (perhaps indicating a strong verbal component to the tests). Factor II is most highly saturated with tests T14, T15 and T16 (likely indicating a strong memory component to those three tests). Finally, Factor III is most highly saturated with T11 and T12 (appearing to indicate a speed component common to both tests). Thus, the rotation of the results to the varimax criterion enabled the easier interpretation of the results.

It is important to notice that the communality coefficients for the varimax rotated solution are identical to the communality coefficients in the unrotated four factor solution. The reason is that the variable variance reproduced by a given factor is only <u>redistributed</u> in the rotated solution, and no new variance is ever generated through a rotation procedure.

The first factor still accounts for the majority of the variance after rotation, even though after rotation other factors could account for the majority of the variance. It is important to note that the total variance-accounted-for the by the three-factor solution before rotation (64.4%) is exactly equal to the total variance-accounted-for after rotation. Notice, however, that the factors are still perfectly uncorrelated as presented in Table 7.

Insert Table 7 About Here.

The unrotated factor pattern and factor structure matrices in Tables 3 and 4 were rotated to the direct oblimin criterion with delta equal to zero (see Gorsuch, 1983 for a more detailed explanation of the effects of varying the value of delta) to compare the parsimony of the two rotation strategies. The results of the oblique rotation are presented in Tables 8 and 9.

Insert Table 8 and 9 About Here.

When interpreting the results of an oblique rotation, it is necessary to interpret two separate factor association matrices since the factor pattern matrix is no longer identical to the factor structure matrix. Similarly to multiple linear regression, however, it is *critically* important to interpret *both* pattern coefficients (standardized weights) and structure coefficients as each can provide only one piece of information regarding the larger relationship (Thompson, 1997b).

By examining the variance-accounted-for by each factor (trace), the results of the direct oblimin rotation appear to closely resemble the results generated by the varimax rotation. As a matter of fact, the oblique rotation strategy only increased the variance-accounted-for by the first factor by one-tenth of a percent. Thus, the results of orthogonal and oblique rotation strategies in this case are essentially equal and will lead to almost identical conclusions. However, there are some issues to consider in interpreting the results of the oblique rotation.

Since the factors are allowed to be correlated in the oblique case, it is important to consider the correlation of the factors when interpreting the solution. The factor correlation matrix for the direct oblimin rotation is presented in Table 10. Factors I and III share approximately 10% common variance and factors I and II share over 5% of their variance. Since the common variance between the factors is minimal in this case, the results of the oblique rotation would be interpreted much in the same manner as the orthogonal rotation. Since both solutions are similar and the orthogonal rotation is interpretable and more parsimonious, most researcher would select this as the preferred solution.

Insert Table 10 About Here.

Other items in the oblique rotation deserve comment as well. Notice that the communality coefficients in the oblique case are exactly equal to the results attained in the unrotated and varimax rotated factor solutions, thus again illustrating only the redistribution of common variance. The sum of the communality coefficients is still equal to 64.4%, as in all of the prior analyses. The trace and communality coefficients are computed slightly differently, however, as it is necessary to multiple a given factor pattern coefficient by the corresponding factor structure coefficient and then to sum down the columns or across the rows to derive the various variance-accounted-for estimate (see the note on Tables 8 and 9 for a more detailed explanation).

Summary of Heuristic Example

The primary difference between these two rotation strategies in this heuristic example is the correlation between the factors, and one of the difficulties incurred in interpreting oblique rotations is how to explicate a high degree of correlation among the factors. Most of the correlations in the present example were relatively small and posed little difficulty in the present analysis. However, in other analyses a high degree of correlation between the factors could result in very different interpretations of the factor solutions between the orthogonal and oblique cases.

Summary of Exploratory Factor Analysis and Rotation Strategies

The present heuristic example of EFA has demonstrated the differences (or lack thereof) typically encountered in EFA. Using orthogonal versus oblique rotation strategies demands careful consideration, and each strategy may be preferable in certain situations. However, as the factor to variable ratio and the factor correlation decrease, the results from orthogonal and oblique rotations tend to become more similar. Thus, the researcher must decide, based on intrinsic judgments and expectancies of the analysis, which rotation strategy will generate the most appropriate results.

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Table 1

Means and Standard Deviation of 8 Variables From Holzinger and Swineford, (1939)

Variable	Mean	SD	Label
T1	29.61462	7.00459	Visual Perception Test
T8	26.12625	5.67544	Word Classification
T9	15.29900	7.66922	Worded Meaning Test
T11	69.16279	15.67025	Speeded Code Test
T12	110.54153	20.25230	Speeded Counting Dots
T14	175.15282	11.50753	Memory of Target Words
T15	90.00997	7.72937	Memory of Target Numbers
T16	102.52492	7.63306	Memory of Target Shapes

Table 2

Correlation Matrix for Example Data

	T1	T8	T9	T11	T12	T14	T15	T16
T1	1.0000							
T8	.3310	1.0000						
T9	.3568	.5816	1.0000					
T11	.2859	.3133	.2902	1.0000				
T12	.2239	.1842	.1496	.3977	1.0000			
T14	.1289	.1786	.1721	.2248	.0385	1.0000		
T15	.1845	.0527	.0519	.1400	.0779	.3967	1.0000	
T16	.3646	.2924	.2528	.3048	.1463	.3875	.3382	1.0000

Table 3

Unrotated Factor Pattern Matrix

Variable	I	II	III	h^2
T1	.63601	-.13656	-.02894	.42399
T8	.66835	-.38257	-.35789	.72114
T9	.64966	-.39131	-.41078	.74392
T11	.64121	-.15033	.42783	.61679
T12	.43541	-.27421	.73125	.79949
T14	.51078	.58907	-.12257	.62293
T15	.41950	.68773	.07930	.65524
T16	.67430	.33096	-.05772	.56755
				(Sum =)
Eigenvalues	2.76551	1.34543	1.04007	5.15115
% of Variance	34.6	16.8	13.0	64.4

Table 4

Unrotated Factor Structure Matrix

Variable	I	II	III	h^2
T1	.63601	-.13656	-.02894	.42399
T8	.66835	-.38257	-.35789	.72114
T9	.64966	-.39131	-.41078	.74392
T11	.64121	-.15033	.42783	.61679
T12	.43541	-.27421	.73125	.79949
T14	.51078	.58907	-.12257	.62293
T15	.41950	.68773	.07930	.65524
T16	.67430	.33096	-.05772	.56755
				(Sum =)
Eigenvalues	2.76551	1.34543	1.04007	5.15115
% of Variance	34.6	16.8	13.0	64.4

Table 5

Factor Correlation Matrix

	I	II	III
Factor 1	1.00000		
Factor 2	.00000	1.00000	
Factor 3	.00000	.00000	1.00000

Table 6

Factor Pattern/Structure Matrix Rotated to the Varimax Criterion

Variable	I	II	III	h^2
T1	.52093	.23462	.31236	.42399
T8	.83904	.06217	.11527	.72114
T9	.85882	.04716	.06422	.74392
T11	.28515	.20428	.70267	.61679
T12	.03921	-.02612	.89291	.79949
T14	.13507	.77746	-.01540	.62293
T15	-.08429	.80052	.08542	.65524
T16	.33755	.64787	.18403	.56755
				(Sum =)
Trace	1.93514	1.76860	1.44741	5.15115
% of Variance	24.2	22.1	18.1	64.4

Note. Coefficients greater than |.35| are underlined.

Table 7

Factor Correlation Matrix After Rotation to Varimax Criterion

	I	II	III
Factor 1	1.00000		
Factor 2	.00000	1.00000	
Factor 3	.00000	.00000	1.00000

Table 8

Factor Pattern Matrix Rotated to the Oblimin Criterion

Variable	I	II	III	h^2
T1	.46919	.16137	.23464	.42399
T8	.85781	-.03373	-.00270	.72114
T9	.88892	-.04698	-.05788	.74392
T11	.15958	.12135	.68095	.61679
T12	-.10360	-.10761	.93555	.79949
T14	.05184	.79198	-.10216	.62293
T15	-.19885	.83029	.03285	.65524
T16	.24771	.61946	.09006	.56755

Note. Communality coefficients are now computed differently. To compute the h^2 for variable T1, each pattern coefficient is multiplied by its corresponding structure coefficient and then summed across the rows. For T1, h^2 = (.46919)(.58274) + (.16137)(.32857) + (.23464)(.41574) = .42399. The new communality coefficient is identical to the value attained in the Table 4 analysis.

Table 9

Factor Structure Matrix Rotated to Oblimin Criterion

Variable	I	II	III	h^2
T1	.58274	.32857	.41574	.42399
T8	.84855	.17963	.25808	.72114
T9	.85912	.16234	.20978	.74392
T11	.40259	.30678	.75676	.61679
T12	.16184	.06663	.88017	.79949
T14	.21744	.78306	.08341	.62293
T15	.01849	.78772	.14830	.65524
T16	.43034	.70050	.29993	.56755
				(Sum =)
Trace	1.92663	1.77798	1.44683	5.15115
% of Variance	24.1	22.2	18.1	64.4

Note. Trace are computed differently than eigenvalues. To compute the trace for Factor I, each pattern coefficient is multiplied by its corresponding structure coefficient and then summed down the rows. For Factor I, Trace = (.46919)(.58274)+(.85781)(.84855)+(.88892)(.85912)+(.15958)(.40259)+(-.10360)(.16184)+(.05184)(.21744)+(-.19885)(.01849)+(.24771)(.43034) = 5.15115. The trace still sum to 5.15115, the sum of communality coefficients in the Table 4 analysis.

Table 10

Factor Correlation Matrix after Rotation to the Oblimin Criterion

	I	II	III
Factor 1	1.00000		
Factor 2	.24940	1.00000	
Factor 3	.31242	.21387	1.00000

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