With the increasing recognition that connections are an important component in the pedagogy of school mathematics (National Council of Teachers of Mathematics, 1989), there is a need for a theoretical framework that addresses the ways in which the real experiences and cultural practices of students may be connected with mathematics classroom pedagogy. In this paper, the objective is to construct such a theoretical framework, drawing on literature from semiotics and ethnomathematics. Examples and evidence that suggest the efficacy of this framework in connecting school mathematics and mathematical ideas constructed from cultural practice are drawn from the literature and from data collected in a research project in a multicultural high school mathematics class. Seven high school students from African American, Caucasian, Asian American, and Hispanic American cultural backgrounds described their lives and cultural heritages as bases for the development of culturally responsive mathematics curricula. (Contains 1 figure and 12 references.) (SLD)
A SEMIOTIC FRAMEWORK FOR LINKING CULTURAL PRACTICE AND CLASSROOM MATHEMATICS

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With the increasing recognition that connections are an important component in the pedagogy of school mathematics (National Council of Teachers of Mathematics, 1989), there is a need for a theoretical framework which addresses the ways in which real experiences and cultural practices of students may be connected with mathematics classroom pedagogy. In this paper, the objective is to construct such a theoretical framework, drawing on literature from semiotics and ethnomathematics. Examples and some evidence which suggests the efficacy of this framework in connecting school mathematics and mathematical ideas constructed from cultural practice, are drawn from the literature and from data collected in a research project in a multicultural high school mathematics class.

Abstraction and generalization are fundamental components of academic mathematics, defined as "the science of detachable relational insights" (Thomas, 1996). At the same time, mathematics is a cultural product (Bishop, 1988), and there is a growing literature suggesting that the potential for constructing mathematical ideas is present in everyday practices in all cultures. In multicultural classrooms, the cultural heritages of students may be viewed as a rich resource for learning and for fostering a classroom climate which promotes equity (Nieto, 1996). It is necessary, then, to reconcile the specificity of cultural practice with the generality of academic mathematics, the concreteness of many out-of-school activities with the abstraction of this mathematics, if everyday practice is to be useful in mathematical classroom pedagogy. It is argued in this paper that a semiotic framework (Whitson, 1994) provides connections between these two aspects of the construction of mathematical ideas. Symbolism and structure are key elements in the connecting of the domains of everyday practice and academic mathematics, and a science which addresses signs, their connections and meanings (i.e., semiotics), is eminently suitable for the development of a connecting framework.

Modes of inquiry
Firstly, two examples from ethnomathematics literature will be used to show the capacity of a semiotic framework to connect cultural practices and formal academic mathematics. The first example is an extension of Marcia Ascher’s (1991) analysis of the kinship relations of the Warlpiri of Australia as a dihedral group of order 8. The second example is Paulus Gerdes’ (1986) mathematical treatment of Angolan Tchokwe sand drawings, which will be discussed in the presentation although lack of space precludes its inclusion here.

Secondly, evidence for the need for such a theoretical framework will be drawn from data collected in an ethnomathematics research project with high school students from diverse cultural backgrounds.

Theory development and evidence

In the Warlpiri system of kinship relations (Ascher, 1991), the population is divided into eight sections, simply numbered 1-8 by Ascher. Persons in section 1 may only marry spouses in section 5, those in 2 may marry those in 6, 3 in 7, and 4 in 8. The section of children from a marriage is determined by the section of the mother, according to the rule

\[ 1 \rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 1 \]
\[ 5 \rightarrow 7 \rightarrow 6 \rightarrow 8 \rightarrow 5. \]

This means that children of both sexes from a mother in section 1 will be in section 4, those from a mother in section 4 will be in 2, and so on. In this way the population is divided into two matrimoities or cycles, consisting of sections \{1, 4, 2, 3\} and \{5, 7, 6, 8\}. There are four patricycles, i.e., \{1, 7\}, \{2, 8\}, \{3, 6\}, and \{4, 5\}. If a boy is in section 1, his father is in section 7, and his grandfather is again in section 1, and so on. This system, which seems complex, and specific to the Warlpiri cultural practice, has a structure which is isomorphic to five of the eight symmetries of the square, if each side of the square is linked to a specific section of the
matrimoity in order, clockwise. (This notation differs from Ascher’s, in which the vertices symbolized the sections, although it gives a characterization analogous to the torus suggested by Ascher & Ascher in Powell & Frankenstein, 1997.) The symmetries used are as follows: starting from a particular individual who is in, say, section 1, four counterclockwise rotations are used for the relation “is the mother of”, and a flip about a horizontal axis for the relation “is the spouse of”. This symbolism takes all the relationships into account. Ascher showed further that if a table is constructed linking each of the eight sections through their relationships, a dihedral group of order eight is formed. Note that the “mathematical ideas” implicit in the structure belong to the Warlpiri and are intrinsic to their cultural practice. A “dihedral group of order eight”, on the other hand, belongs to Ascher’s mathematics, as she is the first to admit: “A Warlpiri, of course, does not go through this analysis. ... A variety of diagrams were used to describe the Warlpiri kin system. The system is theirs but the diagrams were ours” (Ascher, 1991, p. 77). In this paper, a further construct, the “chaining of signifiers” is introduced from semiotics. This chain also belongs to a culture other than that of the Warlpiri; but this in no way diminishes the recognition of the value of the Warlpiri structure or its complexity. It merely serves a different purpose: in fact it could be characterized as “wonderful” that constructs for different purposes in two very different cultures may be connected in this way. The feeling evoked in me is awe at the unity of humanity.

The increasingly abstract systems of symbolism in this example illustrate a chaining of signifiers (Walkerdine, 1988) which may be derived from the semiotic system of Jacques Lacan (Whitson, 1994; Presmeg, 1997). Unlike Charles S. Peirce who constructed a triadic theory of semiotics in the USA, Lacan’s system was developed from the diadic theory of Swiss linguist Ferdinand Saussure, which addresses the relationships between signifiers and signifieds. The following figure illustrates a chaining of signifiers in the Warlpiri example.
Dihedral group of order 8

Table of combinations of possible relationships

Five of the eight symmetries of a square

Warlpiri kinship relations

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Figure 1: Chaining of signifiers in a progression of generalizations from the Warlpiri kinship system to a dihedral group of order 8.

Now one might question, 'Where does mathematics start in this chaining of signifiers?'

The answer to this question hinges on a culture’s definition of mathematics. The Warlpiri, if questioned, would in all likelihood not consider their kinship system to be mathematics, even though some definitions of ethnomathematics would include their practice as it is (Powell & Frankenstein, 1997). My position is that the Warlpiri are not “doing mathematics” merely by practising their kinship system; but when they or others recognize the structure of their system as a structure, explain it to others for example by encoding it in a diagram, or in some other semiotic form, then there is mathematics. The definition of ethnomathematics which I use is simply “the mathematics of cultural practice” (Presmeg, 1996), which includes what Ascher (1991) calls “mathematical ideas” used by the Warlpiri, as well as those of so-called academic mathematics, which is arguably a culture of its own. Discussion of definitions of ethnomathematics by writers such as Ascher, Pompeu, Borba, and others, could constitute a paper in its own right. Some of these definitions may be found in Powell & Frankenstein (1997). The definition of ethnomathematics given by D’Ambrosio (1985, also published as Chapter 1 in Powell &
Frankenstein, 1997) is “the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on” (p. 45). D'Ambrosio based his definition on a “ceaseless cycle” involving an individual in a model with three components, reality, individual, action, going back to reality, and so on. The intellectual action of the individual is an essential element, in a process which he called reification, used by sociobiologists as “the mental activity in which hazily perceived and relatively intangible phenomena, such as complex arrays of objects or activities, are given a factitiously concrete form, simplified and labeled with words or other symbols” (D'Ambrosio, 1985, p. 46). This characterization suggests the role of signification and symbolism, which can provide connections between cultural practice and academic mathematics in a semiotic framework, consonant with the theoretical position formulated in this paper.

In the use of the sides and symmetries of a physical square, say, made of cardboard (signifier) to illustrate the structure of the Warlpiri system (signified), the symbolism may not yet be of a level of generality to satisfy some definitions of academic mathematics. In the next link of the chain, the concrete square gives way to more abstract symbolism in a table. Finally, a generalized structure called 'a dihedral group of order 8' becomes the signifier for this specific table, which is now no longer the signifier, but the signified, in an academic mathematical structure. In this way, semiotic processes may be used to illustrate cultural connections as symbol systems are constructed in a bridge between cultures. In this way, symbolism provides possible connections between mathematical ideas “frozen” in practices (Gerdes, 1986), and academic mathematics. Different symbolism would facilitate the construction of different mathematical concepts.

A high school research project
The need for a theoretical model such as the one developed and illustrated in this paper was strikingly highlighted in a research project to investigate possible ways of introducing ethnomathematics in a high school mathematics classroom. The purpose of the project was to work with a group of students and their teacher to develop viable ways of using the cultural and ethnic backgrounds of the students as a resource for the learning of mathematics. The seven students involved in the project were from African American, Caucasian, Asian, and Hispanic cultural backgrounds. In video or audio recorded interviews, they described rich activities based on four “h”s: their hobbies, hopes (career aspirations), homes and cultural heritages. These activities were an integral part of their lives. Other issues which were discussed were the nature of mathematics, the work done by their parents (and whether mathematics was involved in this), their achievement in and feelings towards school mathematics, and perceived links between mathematics and other subjects in the school curriculum. These students, and their mathematics teacher, with their current beliefs about the nature of mathematics, could not readily develop mathematical ideas from these practices. However, the research project (more fully reported in Presmeg, 1996) did give strong evidence for the richness of the experiences and activities in the lives and cultural heritages of the students. According to Nieto (1996), and strongly suggested in Bishop (1988), it should be possible for teachers to use such experiences and activities to facilitate students’ construction of mathematical ideas, with a consequent affirming of cultural diversity. The present paper begins to illustrate how symbol systems are a connecting bridge in this endeavor. A semiotic framework thus has the potential to provide a basis for culturally relevant pedagogy in multicultural mathematics classrooms.

References


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