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ABSTRACT

The purpose of this study was to evaluate the effects of manipulatives on student achievement in a high school Algebra I class. The study was conducted during the third nine weeks grading period in the spring of 1997 at a high school in Lewisburg, West Virginia. The study groups used in the study were two Algebra I classes. One class had an enrollment of 24 students and the other an enrollment of 25 students. The classes were composed of sophomores and juniors. The groups and instructional strategies used included: (1) Group A (Control Group)--students were taught using the traditional teaching method of lecture, homework, and in-class worksheets; and (2) Group B (Experimental Group)--students were taught using the traditional teaching method of lecture and homework but instead of in-class worksheets, students worked with the manipulative Algeblocks. Both groups were taught at the same rate and by the same method except for the use of the manipulative. A pretest was administered to each class at the beginning of the study and the results tested to be certain that the groups were homogeneous. The results were analyzed using a two-sample t-test and at the .05 level of significance, no significant difference was identified in the achievement levels of the two groups. A posttest identical to the pretest was given to both groups at the conclusion of the study in order to determine if the groups were homogeneous. The results of the posttest were also analyzed using two sample t-test. At the .05 level of significance there was a significant difference in the achievement levels of the two groups at the conclusion of the study. When comparing the mean scores of the posttest, it was discovered that the mean score of Group A was higher than that of Group B which would indicate that the students taught using the traditional method of lecture, homework, and in-class worksheets outperformed the students taught using the manipulatives. (Contains 42 references.) (Author/NB)

A STUDY ON THE USE OF MANIPULATIVES AND THEIR EFFECT ON STUDENT ACHIEVEMENT IN A HIGH SCHOOL ALGEBRA I CLASS

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By

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ABSTRACT

The purpose of this study was to evaluate the effects of manipulatives on student achievement in a high school Algebra I class. The study was conducted during the third nine weeks grading period in the spring of 1997 at Greenbrier East High School in Lewisburg, West Virginia. The study groups used in the study were two algebra I classes taught by the researcher. One class had an enrollment of 24 students and the other an enrollment of 25 students. The classes were composed of sophomores and juniors. The groups and instructional strategies used were:

1. Group A (Control Group): The students were taught using the traditional teaching method of lecture, homework, and in class worksheets.
2. Group B (Experimental Group): The students were taught using the traditional teaching method of lecture and homework, but instead of in class worksheets the students in Group B worked with the manipulative Algeblocks.

Both groups were taught at the same rate and by the same method except for the use of the manipulative.

A pretest was administered to each class at the beginning of the study and the results tested to be certain that the groups were homogeneous. The results were analyzed using a two-sample t-test and at the .05 level of significance no significant difference was identified in the achievement levels of the two groups at the beginning of the study.

A posttest, that was identical to the pretest, was given to both groups at the

conclusion of the study in order to determine if the groups were homogeneous at the conclusion of the study. The results of the Posttest were also analyzed using a two-sample t-test. At the .05 level of significance there was a significant difference in the achievement levels of the two groups at the conclusion of the study.

When comparing the mean scores of the posttest it was discovered that Group A's mean score was higher than Group B's mean score, which would indicate that the students taught using the traditional method of lecture, homework and in class worksheets outperformed the students taught using the manipulatives.

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CHAPTER ONE
THE PROBLEM AND IT'S SETTING

Introduction/Background

Algebra is, and always has been, an important part of a child's education. Algebra serves as a gatekeeper between arithmetic and higher level math courses such as geometry, trigonometry, and calculus. Success in algebra is needed if the student hopes to be successful in a higher-level math course. Algebra also teaches invaluable problem solving skills that are needed by all students. Algebra is described in the NCTM's Curriculum and Evaluation Standard's for School Mathematics as " the language through which most of mathematics is communicated" (Schultz, 1989, p. 34). In this context it is appropriate to think of algebra as "a cohesive body of concepts, closely related to other branches of mathematics" (Schultz, 1989, p. 34).

It is for this reason that many educators feel that all students, regardless of their future plans, should complete an algebra course prior to graduation from high school. This creates a problem. How do we best teach algebra to the lower level math students who must take it, without "watering down" the course to the point that it is nothing more than a basic arithmetic course?

Because of the importance of having a strong algebra background, it is necessary for the algebra teacher to employ instructional strategies that will produce the optimal results for all students.

One strategy is to incorporate the use of manipulatives. Manipulatives are used extensively in the lower grades but decrease in use as grade level increases, and are almost non-existent on the high school level (Chester, Davis, & Reglin, 1989, p. 9).

Manipulatives are objects that appeal to several of the senses. They are objects that students are able to see, touch, handle, and move. The senses are stimulated as the students touch the manipulatives, move them about, rearrange them, and/or see them in various patterns and groupings. Manipulatives assist students in bridging the gap from their own concrete sensory environment to the more abstract levels of mathematics. There are different types of manipulatives including, dry models (using concrete objects or representations of objects), length models (using rods or number lines), and area models (using tiles or pictures). The manipulative focused on in this study was an area model, sold commercially as, Algeblocks. Area models were chosen as the focus of this study because they seem to be the most appropriate manipulative for use in Algebra. Area models generalize from discrete situations, involving the arithmetic of whole numbers, to continuous situations involving decimals, fractions, percents, probability, algebra, as well as mathematics that is more advanced.

The uses of manipulatives are rooted in the theory of Jean Piaget (Phillips & Soltis, 1985, p. 41). Piaget was a Swiss child psychologist who spent most of his lifetime studying children and their developmental stages. Piaget's major contribution to learning is his basic theory of cognitive development.

Piaget observed that children, at different age levels, answer questions and view the world in characteristic and predictable ways, and seem to reason in different ways (Phillips & Soltis, 1985, p.42).

Piaget describes the development in terms of a series of stages through which each child passes. Each stage is marked by strikingly different perceptions of the world and adaptations to it; each is the product of learning that occurred during the previous stage and a preparation for the stage that follows. The 4 stages of Piagetian development are listed in Table 1.

Table 1: Descriptions of Piaget's Stages of Learning

Stage	Approximate Age	Major Characteristics
Sensorimotor	Birth – Two years	Motoric intelligence. No language thought, notion of objective reality at beginning of stage.
Preoperational	Two – Seven Years	Egocentric thought. Reason dominated by perception. Intuitive rather than logical solutions.
Concrete Operations	Seven – Twelve Years	Logic of classes and relations. Understanding of numbers. Thinking bound to concrete.
Formal Operations	Twelve Years – Adult	Complete generality of Thought. Ability to deal with the hypothetical.

Individuals at each stage have a predictable way of thinking. The development of each stage is a necessity for advancement to the next stage. The concrete operational stage is the basis for the use of manipulatives. The major acquisition of the concrete operational stage is the ability to think operationally. An operation is a mental action, or more precisely, an operation performed on ideas according to certain rules of logic. This logic is tied to real, concrete objects and events. In this stage, the child is unable to reason logically about hypothetical situations and can not go from the real to possible or from the possible to the actual. Thought at this stage is bound to the real, concrete world. Therefore, manipulatives can, should, and are used extensively during this period of development. Toward the end of this stage, the use of manipulatives tend to decrease and are seldom used once the child enters the formal operational stage, which is characterized by the ability to manipulate abstract ideas (Phillips & Soltis, 1985, p. 43).

In general, Piaget felt that it was a waste of time to teach children things that they could not experience through their senses. He felt that children must be allowed to manipulate objects, try different experiments, pose questions, and test their findings against other children's perceptions. Only after children have had a great deal of sensory experience are they ready to comprehend abstract concepts. Piaget was critical of teaching conceptualization without offering children the opportunity to experience through their senses (Phillips & Soltis, 1985, p, 44).

Statement of the Problem

Is there any significant difference in the achievement of students in two Algebra I groups, when one group is given traditional instruction, and the other group is given traditional instruction supplemented by the use of a manipulative?

Hypothesis

H_o: There will be no significant difference in achievement levels between the group using manipulatives (Algeblocks) and those taught using a more traditional method at the conclusion of the study.

H_1: There will be a significant difference in achievement levels between the group using manipulatives (Algeblocks) and those taught using a more traditional method at the conclusion of the study.

Purpose of the Study

The purpose of this study was to investigate the effects of two instructional strategies on student achievement in two Algebra I classes. One group was taught using a traditional instructional strategy where the teacher in class modeled problems. The students were given examples to work in class, homework/worksheets were given, and students were tested for mastery. The other group was taught using the traditional method, but this method was supplemented by the use of a manipulative (Algeblocks).

Significance of the Study

It is important that studies be conducted on instructional strategies. Research makes teachers aware of various techniques being tried and gives them an idea of their effectiveness. There have been very few studies conducted on the use of algebra manipulatives at the secondary level. Most studies deal with the use of mathematical manipulatives at the elementary level. This study will increase the body of knowledge on the use of manipulatives at the secondary level. This study will inform teachers of the effectiveness of manipulatives when used in Algebra I classes at the secondary level, and may help teachers decide whether to use them in their classrooms.

Limitations of the Study

1. The study was limited to one topic, Polynomials.

2. The study was limited to relatively small sample groups of 23 and 24 students.

The Assumptions

1. The instruments, which included the pretest and posttest, were so constructed that they were good indicators of student achievement levels.
2. The group members were homogeneous: therefore, results of the study were valid.
3. The sample size of the students was adequate.

CHAPTER TWO
REVIEW OF THE RELATED LITERATURE

Introduction

In the review of related literature, it was found that the studies and research on the use of manipulatives were geared toward the primary and middle school level. After an extensive search was conducted, very little literature was found to exist on the use of manipulatives at the secondary level. However, the literature that was found concluded that student's understanding of mathematical concepts increased when manipulatives were used correctly in the classroom.

History of Teaching with Manipulatives

Manipulatives have been used effectively for many years to help students understand mathematical concepts. Van Engen (1949, p. 397) claims that experiences with concrete models should precede the experiences with symbolic models. Van Engen stated that the "meaning of words cannot be thrown back on the meaning of other words. When the child has seen the action and performed the act for himself, he is ready for the symbol for the act." Friedrich Frobel, inventor of the term kindergarten, realized that objects from the most concrete part of mathematics should be used to introduce children to the world of mathematics (cited in Isaacs, 1972, p. 11). According to Isaacs (1972, p. 12) Frobel suggested the use of balls, cubes, and cylinders. In his analysis of this concept Frobel presented children with trays covered by tiles, which he found helped the children move to a more advanced level of abstraction. Using a collection of sticks that varied in length, the children could place the sticks in designs that would later relate to number patterns. Frobels' concept of using concrete materials to help children understand mathematical concepts is still evident in kindergarten classes of the present with blocks for stacking and tiles used for creating patterns. In the late 1950s and early 1960s, much emphasis was placed on using concrete objects in mathematics instruction. At the *Conference on Cognitive Research and Curriculum Development* held at Cornell University in 1964, Piaget stated that "teaching means creating situations where structures can be discovered; it does not mean transmitting structures which may be assimilated at nothing other than a verbal level… The teacher must provide the instruments that the children can use to decide things by themselves. Children themselves must verify, experimentally in physics, deductively in mathematics. A ready made truth is only a half-truth" (Duckworth, 1964, p.496).

In 1964, psychologist Jerome Bruner hypothesized four major benefits from using just such an approach as described above. He claims that there will be: (1) "enhancement of the memory processing, (2) an increase in general intellectual potency, (3) an increase in motivation via a shift from extrinsic to intrinsic motivation and, (4) the acquisition of the heuristics of discovery"(Bruner. 1961, p. 21-22). The appearance of manipulatives being used in the classroom accelerated in the late 1960s when Zolton Dienes and Jerome Bruner published theoretical justification for the use of manipulatives (Thompson, 1994, p.556). During this period educators helped the manipulative movement by conducting staff workshops for teachers, writing teacher and student curriculum materials, writing research reports, integrating mathematics manipulatives into teacher education courses, and supervising doctoral research into manipulatives.

Shulman and Keislar (1966, p. 37) stated in *The Meaning of Discovery and Learning* that the "student, confronted with a set of concrete experiences is led to scan through a range of stored models of different kinds and try to fit these models to the data or problem at hand. What is discovered is not something outside the learned but an appropriate internal structure for handling the problem. What discovery does is allow the individual to translate the solution into his/her own language forms rather than some outside formal language forms. Thus verbal cues tell the student what to retrieve."

Although research indicates that manipulatives increase children's understanding of mathematical concepts in the primary and middle school level, further research is being conducted (Suydam, 1984 p. 439).

Use of Manipulatives

Research indicates that manipulatives have been used by mathematics teachers in elementary schools for years and with "varying degrees of success" (Ross & Kurtz, 1993, p.256).

Gilbert and Bush (1988, p. 461) surveyed a group of elementary teachers and found that primary grade teachers were familiar with manipulatives and that various manipulatives were available to them. The teachers also revealed that as the grade level increased the use of manipulatives decreased.

In a similar study by Hatfield (1994, p. 304), a questionnaire was sent to 106 kindergarten through sixth grade teachers who were serving as cooperative teachers for student teachers from a large university in the southwestern United States. Thirteen manipulative devices were listed and teachers were instructed to check which manipulative(s) they were familiar with, used for mathematics instruction, and the number of times per week/month each device was used. In comparing the grade level to the manipulative use, Hatfield concluded that manipulative use declined as the grade level increased from kindergarten through sixth grade.

Gilbert and Bush (1988) conducted a two-part study "to ascertain through teacher's self-reporting the degree to which primary grade teachers were using manipulative devices to teach mathematics." The first part of the study involved compiling a list of recommended manipulative devices and the second part of the study involved teachers of grades one to three, from eleven different states to complete a survey to report their familiarity, use, and availability of a particular set of manipulative devices. The teachers who responded to the study had an average of 13.2 years of experience in teaching elementary school mathematics, approximately 86% of the teachers taught one class of mathematics per day, 10% taught two classes, and the remaining teachers taught three to four mathematics classes daily. The results of the study revealed that "the use of manipulative devices was low given the current availability of information and materials." The conclusion of the study revealed that teachers were familiar with selected manipulatives and that most of the materials were available to them but that the teacher simply do not use them as often as is recommended (Bush, 1988, p. 467)

Availability and Teacher Competency

Many teachers admit that the reason they do not use manipulatives is that manipulatives are not readily available (Hatfield, 1994, p. 305). Hatfield found that availability of manipulatives was a factor to consider by 81% of the teachers who participated in her survey, and availability ranked number one on the list as factors to consider when deciding whether to use manipulatives in their classroom. Eighty percent of the teachers reported that they consider teacher competency of teaching mathematics using manipulatives as another factor. In a survey by Scott (1987), teachers were asked whether they wanted in-service training on the use of mathematics materials. The majority responded that they would like to have training on the use of manipulatives. Workshops and other assistance programs at many schools were then dedicated to helping the teachers learn how to effectively use mathematics manipulatives and courses at many universities were also offered (p. 21). In Gilbert and Bush's (1988) study, over three-fourths of the teachers participating in the study reported availability of manipulative devices, expect for fraction bars and math balances, as a factor for hindering the use of manipulatives. It was also revealed that experienced teachers tend to use manipulative devices less often than inexperienced teachers (p. 464).

Tooke, Hyatt, Leigh, Snyder, and Borda (1992) interviewed thirty teachers of the 4th through 8th grades about their attitudes towards manipulatives and the confidence they had when using manipulatives to teach mathematics. Their findings were in direct opposition to the ones by Scott (1987). The participating teachers in Tooke's study stated that the reason for their refusal to use manipulative instruction was that many of them did not know how to use manipulatives, much less what concepts, skills, or abilities the manipulatives were to be used to teach. However, not one teacher was willing to learn the use and purpose of manipulatives if they had to spend their own money and time, and definitely "had no interest in enrolling in a university course to learn" (Tooke et al, 1992, p. 64)

Attitudes

Many of the negative attitudes towards the use of manipulatives by teachers of mathematics are because teachers feel that: (1) manipulative instruction is inappropriate for students above the fourth grade; (2) the students are confused by manipulatives and, (3) many teachers say that manipulatives are not worth the expense (Tooke, et al, 1992 p. 62).

Teachers also claim that there is not enough time to use manipulatives, using manipulatives is too much like playing games, and they are difficult to manage with large numbers of students (Herbert, 1989, p. 4)

Prospective teachers resist using manipulatives in the classroom for two reasons: (1)"a lack of confidence in their ability to use manipulative materials correctly and, (2) the general belief that children will become too dependent on these materials and as a result, will not master basic computational algorithms and related concepts" (Trueblood, 1986, p. 51)

Hands- on Math, a developmental project funded from 1985-1987 by the State of Georgia through a grant by the 1985 Quality Basic Education Act (QBE), has 5 objectives: (1) to cause teachers to believe in the importance of concrete experiences in mathematics learning; (2) increase hand-on activities in mathematics classrooms; (3) improvement achievement in students in k-2 "as measured by criterion- and norm-referenced tests"; (4) select and correlate concrete manipulatives to existing state and local curriculum; and (5) to increase communication with parents regarding manipulative experiences they can offer at home to support the school program.(Fielder, 1989, p. 14). Four elementary schools in a Georgia district were chosen to participate in an experimental program. At the beginning of the project teachers and the mathematics coordinator worked together to develop criteria for selecting materials so that both state and local objectives were met. A two-day summer in-service workshop was conducted for all participating teachers and their principals. A third day of training was held immediately before the school year began. During the year, observations by the mathematics coordinator and the principals were conducted as part of the evaluation design.

The teachers reported that "the program had made students enjoy mathematics more than ever", and "their students experienced more success in math which improved their self-concepts." Several teachers indicated that the effect of the project on their own teaching behaviors was that using the manipulatives had helped them become teachers that are more creative. This study proved that "in order to bring about a desirable change in teachers' behaviors, teachers needed to be convinced that the change would make their job of teaching easier and more productive" (Fielder, 1989, p.15-16)

Effects of Teaching with Manuplatives

Using mathematics manipulatives helps students learn from concrete examples and they are then better equipped to deal with abstract concepts (Phillips & Soltis, 1985, p. 42).

Research indicates that use of manipulative devices produce greater mathematics achievement than a lesson not incorporating manipulatives (Suydam and Higgins, 1977, p.2). In a meta-analysis of sixty-four students, Parham (1983, p. 96) found that achievement scores of elementary students who had used manipulatives were decidedly greater than those of the students who had not. Mathematics manipulatives are adaptive at every grade level, achievement level, ability level, and across a wide variety of topics (Suydam, 1984, p. 27).

Research supports that manipulatives will increase the level of understanding of mathematics, and literature clearly advocates the advantages of an environment rich with hands-on experiences for all levels of learners (Tooke, et al, 1992, p. 62).

"Using manipulative aids and devices makes the classroom a more interesting and exciting place for both teachers and students." Many teachers are aware that manipultives aid in "solidifying concept development in primary mathematics" but according to Williams (1986, p. 42) "they are also useful to the teacher of pre-algebra and algebra." Williams (1986, p. 44) also states that "It is as necessary to involve students physically in active learning experiences in an algebra class as it is in a first-grade classroom."

The National Council of Teachers of Mathematics Curriculum and Evaluation Standards (1989, p.372) states that "knowing mathematics is doing mathematics." When mathematics is consistent with this notion, focus is provided on a variety of "mathematical experiences" across a broad range of topics in which students are encouraged to explore "concrete situations and problems" (Lave, Smith, and Butler, 1988). This allows the students to construct personal meaning individually and in groups (NCTM, 1989, p. 373).

Baxter, Shavelson, Herman, Brown, and Valadez (1993, p. 190) reports the findings from their study in which they developed and evaluated mathematics performance assessments that were aligned with manipulatives. The subjects involved in the study were sixth-grade students from two types of mathematics curricula: hands-on and traditional, from regular mathematics classrooms, the students were all from the same socioeconomic-status levels, and the same school district. The results of the study showed that students who had received hands-on mathematics instruction scored higher, on average, than did students in a traditional curriculum.

In Sowell's (1989, p. 505) study the results of sixty studies were combined to determine the effectiveness of mathematics instruction with manipulative materials. The students involved in the study ranged in age from kindergartners to college-age adults and they studied a wide variety of mathematics topics. The results showed that mathematics achievement increases through the long-term use of concrete instructional material, and students' attitudes toward mathematics are improved when they have instruction with concrete materials provided by teachers who are knowledgeable about their use.

In another report (Parham, 1983, p. 95) there was a great difference in the achievement scores of students who had used manipulative materials and those who had not. The results from the analysis of sixty-four research studies, conducted at the elementary school level, showed that the students who used manipulative materials scored in the eighty-fifth percentile.

Suydam (1985) concludes, from the extensive amount of research she has conducted, that the use of manipulative materials appears to be of definite importance to how well children understand and achieve in mathematics (p. 34).

Manipulative use in mathematics classrooms also allows teachers to better assess and meet the individual needs of elementary school children as they construct "personal mathematical knowledge" (Ross and Kurtz, 1993, p.257).

At-Risk and Targeted Students

The level of cognitive development varies with each child and their needs must be considered when using manipulatives (Suydam, 1984, p.437). Suydam states "...we need to begin with the student, assessing learning styles, interests, and talents, and attempting to pinpoint the mathematical ideas with which difficulty exists. Diagnosis is imperative."

In a study by Bryant (1992, p. 27) at-risk and targeted students in grades four through six were not doing well in identified mathematics objectives. It was noted that the number one reason why at-risk and targeted students were not achieving in mathematics was that the teachers did not use mathematics manipulatives to stimulate critical thinking and/or problem solving solutions. More in-servicing to familiarize teachers on the "effectiveness and practicality" of the usage of mathematics manipulatives at the intermediate level would be helpful (Bryant, 1992, p. 10). The Research Advisory Committee of the National Council of Teachers of Mathematics (1989) states that "Mathematics has become a critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate. Equity has become an economic necessity" (Bryant, 1992, p. 12).

In At-Risk Youth Can Succeed, Green (1989, p. 15) lists several ways that at-risk students can succeed. These include increased parent involvement, in-service training for classroom teachers, community partnership with schools, a strong emphasis on teaching students critical thinking/logical reasoning, goal setting, and problem solving techniques. No mention of manipulative usage was made.

It was reported in the Phi Delta Kappan that in 1983 the SAT scores were rising for black students (Bryant, 1992, p. 14). Researchers investigated the data and found that the reasons for the rise in the scores were that black students were taking more mathematics classes, attending more private schools, and the income level of the students' parents was above average, again, manipulative use was not mentioned as a reason.

In a similar study (Jones, Burton, & Davenport, 1984, p. 156) many causes were cited as to why certain students do not do well in mathematics: parental contribution, low expectations of minorities, and the courses to which minorities are assigned but there is nothing mentioned about the use of manipulatives.

In Bryant's (1992, p. 36) study it is concluded that an "effective in-service for teachers on the appropriate use of manipulatives, peer tutoring, collaborative teaching methods, and computer use, were successful solutions to improving mathematics achievement for at-risk and targeted students.

Representational Models

Using concrete materials helps students make the transition from concrete to abstract mathematical symbols.

Heddens (1986, p. 17) states that "teachers need to orchestrate mathematical concept development very carefully to provide a smooth transition…and… the need for a careful sequencing of activities to lessen students' dependence on the concrete level and increase their facility with the abstract level is crucial."

Schultz (1986, p. 54) classifies the types of representational models into concrete, pictorial, and symbolic. She states that " the earlier phase of concept development in mathematics instruction is strengthened by the provision of concrete models, then a transition to pictorial, and finally, the symbolic models."

Concrete models are anything that the students can actually feel, touch, and move around. Some examples of concrete models are blocks, sticks, chips, Cuisenaire rods, and Dienes blocks. Pictorial models are simply pictures of objects, or visual aids such as pictures of blocks, sticks, and chips. The pictorial models can be pictures on worksheets, textbook pages, cards, and bulletin boards. Symbolic models are numerals on worksheets, textbook pages, chalkboards, or cards (Schultz, 1986, p. 54). Each model is used in agreement with its meaningfulness. The directly meaningful model is when the actual items that the problem is about are used. For example, if the students were studying about addition of money, the students would use coins. Indirectly meaningful models are representations of real objects. Toy cars would be used to represent full-sized cars in problems about cars. Non-meaningful models, according to Schultz, are any objects used to represent other things; for example, bottle caps would be used to represent people in problems about people.

According to Driscoll (1984, p. 35) "research has shown that the sensible use of concrete materials is effective in teaching mathematics." While manipulatives are being used at the elementary levels some research indicates that older students could benefit from the use of manipulatives as well.

The 1984 National Assessment of Educational Progress (NAEP) showed that "only about 40% of the 17-years-olds appear to have mastered basic fraction computation" (Driscoll, 1984, p. 34). In one study, less than 20% of seventh graders were able to locate the number 1/5 on a typical number line (Larson, 1980). The researchers who reported on NAEP results concluded that most thirteen-year- olds see different interpretations of rational numbers as separate, unrelated topics (Carpenter et al, 1981). Progress from concrete work with rational numbers to the ability to talk about them is not easy for most children (James, 1980, p.39).

According to Harrison, Brindly, and Bry 1980, (Driscoll, 1984, p. 35) "seventh graders used a process approach to investigate fractions they experimented with concrete materials." After three months, their achievement level exceeded that of the control group and they enjoyed working with fractions considerably more than the control group did.

As activities for interpretation of rational numbers Kieren (1975) suggests making and measuring scale drawings on graph paper, to develop an understanding of the interpretation of rational numbers as ratios, activities such as folding strips of adding machine tape to compare equivalent ratios, is beneficial to students.

Driscoll (1984, p. 46) states that "some well-chosen models, a willingness to use manipulatives, and a dedication to exploring the connections are all that are needed" to helps students understand mathematics at all levels.

Suggestions for Success with Manipulatives

"Manipulative materials must be used at the right time and in the right way, if they are to be effective" (Suydam, 1985, p. 2). The materials must be selected with the mathematical purpose in mind. Suydam claims that it is important that the child is focused on the objective, and encouraged to "think along" as they use the manipulative materials.

Bohan and Shawaker (1994, p. 246) recommend using the manipulatives in the context of transfer of learning. This means that studying topic A will help in understanding topic B. Bohan and Shawaker state that "two important conditions have to be met in order for the transfer to occur; common elements must exist between two topics, and the learner must be aware of the existence of the common elements."

The manipulatives chosen should support the lesson's objective and involve participation of each student. A system of evaluation must be developed that reflects an emphasis on the development of reasoning skills, organize students into groups of four to reduce the amount of materials on the table which allows for less clutter and maximum learning (Ross & Kurtz, 1993, p. 256).

Suydam (1984, p. 32) suggests that teachers practice using the materials before the lesson to become familiar with them. There should be sufficient material, in good working order, for each student to use; provide ample time for using the material; encourage the students to think for themselves-do not provide all the answers for the students; allow for and encourage group interaction, and provide follow-up question and answer time.

One example of success with manipulatives is from Ross and Kurtz' (1993) article *Making Manipulatives Work: A Strategy for Success*. A group of second-graders was taught mathematics, with much success, using manipulatives. The class consisted of twenty-four students with varying abilities and backgrounds. Their teacher created various centers and stations in which the children were free to choose which station they wanted to work, each station was directed towards achieving classroom objectives. There were many baskets, bags, and boxes filled with mathematics manipulatives on countertops and tables so that they were easily accessible to the students when "counting, classifying, patterning, constructing, and exploring" (p. 254). The teacher had been teaching place-value and used base-ten blocks in a game called "get to a hundred". The students were divided into groups of four and the materials needed for the game were the base-ten blocks, place-value board, and die. The object of the game was to reach "100" by trading, and the first player to get a flat (10 longs) would win. Using an overhead projector to model the game, the teacher played against the class until he was satisfied that all the students understood the rules and time was allowed for the students to ask questions before students proceeded. As the students "played", the teacher walked around the room, watching and listening to the interaction among the students. He also used the time to evaluate students' progress from the comments the students made as well as the strategies used to reach the goal. The teacher found "that time spent reteaching and remediating is greatly reduced when he allows his students the time to build and reflect on their own personal knowledge." The teacher spent time at the end of the lesson to discuss some of his observations with the students, and then asked students to use pencil and paper to write answers to one of his questions. In this, he was assessing which students needed further conceptual development, which students reflected an understanding of concept, and which others indicated advanced development. The "assessment of students' writings, oral comments, and teacher's observations allows the teacher to address the needs of individuals by directly questioning during class…or by working with a small group to facilitate understanding" (Driscoll, 1993, p. 255). When planning lessons in which manipulatives will be used, Driscoll (1993, p. 256) lists some suggestions: (1) manipulatives chosen will support the lesson's objectives; (2) significant plans have been made to orient students to the manipulatives and corresponding classroom procedures; (3) the lesson involves the active participation of each student; and (4) the lesson plan includes procedures for evaluation that reflect an emphasis on the development of reasoning skills." The effective use of manipulatives, according to Driscoll, also depends on "the adequate preparation of the students and the materials:. In addition, every student must be kept actively involved in order to achieve success with manipulatives. In order for this to happen students should: (1) work in pairs, (2) have a mental objective at the beginning of the lesson, (3) use visual signals, such as "thumbs up or thumbs down", to promote active participation, and (4) ask students to reflect on the mathematical thinking involved in their lessons and to respond in writing" (Driscoll, 1993, p. 2567-257).

Reforms in Mathematics

In order to meet the needs of a rapidly changing technological society, mathematics in the classroom must change. Reports have addressed the seriousness of students' poor performance in math "as well as their poor preparation to meet the needs of business and industry in the 21st Century" (Cauley and Seyfarth, 1995, p. 22).

The Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) and the *Professional Standards for School Mathematics (NCTM 1991)* have created "blueprints for curricular and instructional reform in mathematics" and it has received resounding acceptance throughout the country. The STANDARDS, not actual curriculum, does provide a framework for "specific aspects of the curriculum and about instruction and evaluation against which districts, schools, and teachers can judge their own programs" (Cauley and Seyfarth, 1995, p. 23). Some of the reforms include less attention to arithmetic computation, "especially in the mastery of complex paper and pencil graphing of equations." Although all students will be exposed to the same core topics, there will be differentiation for student ability, degree of difficulty of exercises and applications, level of abstraction, and instructional pace" (Cauley and Seyfarth, 1995, p. 24). The middle level curriculum for grades seven and eight need to be broadened to "expand students' knowledge of numbers, computation, estimation, measurement, geometry, statistics and probability, patterns and functions, and the fundamental concepts of algebra" (NCTM, 1989, p. 65). Calculators are wonderful manipulatives for the older student to use. Students who have not learned basic computational skills by middle level should not be held back from more advanced mathematics when the use of calculators can help them move forward. This confirms what Suydam (1982, p. 3) discovered after reviewing over 150 studies on the effects of calculator use. About half of these studies had one goal: "to ascertain whether using calculators would harm students mathematical achievement." In all but a few instances, "achievement scores were as high or higher when calculators were used for mathematics instruction as when they were not used." Hembree (1985), also reviewed 79 studies and confirmed that (except for one grade level) "use of calculators improved students' basic skills with paper and pencil. Moreover, better attitudes toward mathematics and an especially better self-concept in mathematics were found." These changes are more challenging and better preparation for students than the way that most adults were taught (Cauley and Seyfarth, 1995, p. 24).

Since reasoning is integral to mathematics as problem solving, Cauley and Seyfarth (1995, p. 25) state that "students learn that mathematics is not a collection of arbitrary rules but a system that makes sense and can be figured out". No longer can the middle level or secondary teacher rely on instructional sequence of "review, homework, introduce and explain new material, assign problems for seatwork and homework" because the emphasis is placed on problem solving, reasoning, communication, and connections." Therefore, a different type of instruction is required. Students need to discuss problems with other students, be involved in cooperative learning groups, and justify their choice of strategy. The teachers for the middle/secondary level "need to use manipulatives, calculators, and computers as an integral part of instruction."

There is no "model" provided for how to implement the changes that the NCTM STANDARD argues for, therefore, the state departments of education in three states have developed instructional materials that incorporate new approaches to teaching mathematics while "some universities are developing projects that integrate technology with secondary mathematics" (Cauley and Seyfarth, 1995, p.27).

A list of suggestions (Cauley and Seyfarth, 1995, p. 28-29) for principals to use will give support to teachers in preparing students for the twenty-first century: (1) principals will need to convince teachers and parents that change is essential; (2) provide long-term professional development to allow teachers to explore mathematics reform; (3) facilitate "collegial coaching" among mathematics teachers and a network that supports change; (4) facilitate communication between mathematics and other teachers so that students can learn true problem solving and reasoning in all areas; (5) provide support and opportunity to review the structure of the school day, teacher planning, etc.; (6) provide resources to acquire instructional materials "manipulatives, graphing calculators, computers, and computer software are all tools essential to implementing the STANDARDS"; (7) "de-emphasizing the importance of current standardized test, and most important, convey the message to the community." The STANDARDS advocate "fundamental changes in the mathematics curriculum, in mathematics instruction, and in assessment so that the mathematics classrooms of the twenty-first century will bear little similarity to the mathematics classrooms of today" (Cauley and Seyfarth, 1995, p. 23). These "fundamental changes" that are required cannot be implemented in one faculty meeting. It will require a sustained effort over a number of years from both teachers and administrators (Cauley and Seyfarth, 1995, p. 29).

CHAPTER THREE
METHODOLOGY

Description of the Population

The study took place at Greenbrier East High School, which is located in Lewisburg West Virginia. East draws its population from seven different communities. They are Alderson, Frankfort, Lewisburg, Renick, Ronceverte, Williamsburg, and White Sulphur Springs. Each community contains its own elementary school. The occupations of the student's parents range from such blue-collar careers as loggers, coal miners, and farmers, to such professional careers as doctors, lawyers and teachers. A significant percentage of students have parents who are unemployed or disabled. Most of the student's families fall in the middle to low-middle class.

The students in the Algebra I classes are traditionally lower level math students, with a few above average students blended in. The majority of the mid- to upper- level math students take Algebra I in the eighth or ninth grade.

The study consisted of two Algebra I classes. One class was designated as the control group and labeled Group A. This class met from 8:15a.m. to 9:05a.m. Monday through Friday. The class contained 24 students at the beginning of the study, however one male student moved during the study and therefore was not counted, bringing the class total to 23, with 10 boys and 13 girls. There were 17 sophomores, 6 juniors and 0 seniors. The second class was designated as the experimental group and labeled as Group B. This class met from 10:15 a.m. to 11:05a.m. Monday through Friday. The class contained 25 students at the beginning of the study, but one male student moved out of this class during the study as well, which brought the total down to 24 . It contained 11 boys and 13 girls. There were 21 sophomores, 3 juniors and 0 seniors. Both classes had the same instructor.

Research Question

Does the use of a manipulative (Algeblocks) affect the achievement level of students in a high school Algebra I class?

The Hypothesis

H_0: There will be no significant difference in achievement levels between the group using manipulatives (Algeblocks) and those taught using a more traditional method at the conclusion of the study.

H_1: There will be a significant difference in achievement levels between the group using manipulatives (Algeblocks) and those taught using a more traditional method at the conclusion of the study.

Instrumentation

The instruments used in this study included a pretest, constructed by the instructor, which covered the learning outcomes that would be tested during the study and a posttest, which was identical to the pretest. A copy of the pretest/posttest is included in the appendix.

Research Design

The nature of the study was descriptive in that it presented the data using descriptive statistical methods to organize, compare, and summarize.

Two Algebra I classes, were used in the study. Students were randomly assigned to the classes by computer. A pretest was administered to each class at the beginning of the study and the results tested to be certain that the groups were homogeneous. Group A was then taught using the traditional teaching method of lecture, homework and in class worksheets. Group B was taught using the traditional teaching method of lecture and homework, but instead of in class worksheets, the students in Group B worked with the manipulative (Algeblocks).

The procedure followed during the study was explained to the students at the beginning of the study. Both groups covered the same material at the same rate. On the days that Group A was doing the in class worksheets, Group B was doing the Algeblock labs. Both Group A and Group B students were placed in cooperative learning groups consisting of four students per group.

The Group A students completed their worksheets in their cooperative groups and the Group B students completed their Algeblock Labs in their cooperative groups.

At the conclusion of the study, the posttest was administered and the results were tested to determine if there was any significant difference in achievement levels between the two groups.

The algebra unit covered during the study was Polynomials.

Collection Procedures

A pretest and a posttest were administered to obtain the data needed. The pretest and the posttest were tests for the two groups. The pretest was identical to the posttest. The students were not informed until after they had completed the pretest that their scores on the pretest would not count against their grade in the class. The students also were not told that the pretest and posttest were identical.

Analysis of the data

A two-sample t-test at the .05 significance level was used to analyze the data collected from both the pretest and the posttest. A significant difference that is less than or equal to the alpha level of .05 would result in the rejection of the null hypothesis and would indicate that there was reason to believe that the use of the manipulative (Algeblocks) has a influence on the learning of algebra. A positive significant difference will be essential for choosing manipulatives (Algeblocks).

CHAPTER FOUR
STATISTICAL ANALYSIS

Research Question

The research question answered by this study was, does the use of a manipulative (Algeblocks) affect the achievement level of students in a high school Algebra I class?

To determine if the groups were homogeneous at the beginning of the study a pretest was administered to each group. The students were not given any information as to what types of questions would be asked on the pretest, which covered basic algebra concepts such as multiplying monomials, dividing monomials, arranging polynomials in ascending or descending order, multiplying two binomials and multiplying a monomial by a binomial or a trinomial.

At the conclusion of the study, a posttest was administered to each group to determine if the groups were at equal achievement levels at the conclusion of the study. The posttest was identical to the pretest. The students were not told that the posttest was identical to the pretest.

Description of the Population

The study took place at Greenbrier East High School, which is located in Lewisburg, West Virginia. Lewisburg, the county seat of Greenbrier County, is a rural community located in the southeastern portion of West Virginia. Greenbrier East High School has a student enrollment of approximately 1,000 students in grades 10, 11, and 12. The school has one feeder junior high school, Eastern Greenbrier Junior High School with an enrollment of approximately 1,200 students in grades 7, 8, and 9. The sample groups consisted of 23 students in Group A and 24 students in Group B.

Descriptive Statistics

The mean score for Group A and Group B, the pooled estimate squared ($\hat{S}^2_{pooled}$), the square of the estimate of the population standard deviation of the difference of the means (est. $\sigma^2_{\bar{A}-\bar{B}}$) and the estimate of the population standard deviation of the difference of means (est. $\sigma_{\bar{A}-\bar{B}}$), were computed for the pretest and are listed in Table 2.

Table 2: Pretest Descriptive Statistics

Group A Mean	Group B Mean	$\hat{S}^2_{pooled}$	est. $\sigma^2_{\bar{A}-\bar{B}}$	est. $\sigma_{\bar{A}-\bar{B}}$
17	12	277.07	23.59	4.87

The mean score on the pretest for Group A, which was the control group, was 17%. The mean score on the pretest for Group B, which was the experimental group, was 12%. The pooled estimate squared ($\hat{S}^2_{pooled}$), for the pretest was calculated to be 277.07. The square of the estimate of the population standard deviation of the difference of the means (est. $\sigma^2_{\bar{A}-\bar{B}}$), was found to be 23.59 and the estimate of the population standard deviation of the difference of the means (est. $\sigma_{\bar{A}-\bar{B}}$) was 4.87.

The mean score for Group A and Group B, the pooled estimate squared ($\hat{S}^2_{pooled}$), the square of the estimate of the population standard deviation of the difference of the means (est. $\sigma^2_{\bar{A}-\bar{B}}$) and the estimate of the population standard deviation of the difference of the means (est. $\sigma_{\bar{A}-\bar{B}}$), were also calculated for the posttest and are listed in Table 3.

Table 3: Posttest Descriptive Statistics

Group A Mean	Group B Mean	$\hat{S}^2_{pooled}$	Est. $\sigma^2_{\bar{A}-\bar{B}}$	Est. $\sigma_{\bar{A}-\bar{B}}$
70	52	645.91	54.99	7.42

The mean score on the posttest for Group A, which was the control group, was 70%. The mean score on the posttest for Group B, which was the experimental group, was 52%. The pooled estimate squared ($\hat{S}^2_{pooled}$), for the posttest was calculated to be 645.91. The square of the estimate of the population standard deviation of the difference of the means (est. $\sigma^2_{\bar{A}-\bar{B}}$), was found to be 54.99 and the estimate of the population standard deviation of the difference of the means ($\sigma_{\bar{A}-\bar{B}}$) was 7.42.

t-tests

A two sample t-test was used to analyze the mean scores of the pretest to determine if the groups were at equal achievement levels at the beginning of the study. The t-value, critical t, degrees of freedom (ν) and alpha level (α), are listed in Table 4.

Table 4: Summary of T-test Calculations Performed on the Pretest.

t-value	Critical t	ν	α
1.03	2.021	45	.05

The results showed a t-value of 1.03, critical t was 2.021 with 45 degrees of freedom. at the .05 level of significance. The groups, therefore were determined to be at equal achievement levels at the beginning of the study, due to the fact that $1.03 < 2.021$ at the .05 level of significance.

A two-sample t-test was also used to analyze the mean scores of the posttest to determine if the groups were at equal achievement levels at the conclusion of the study.

The t-value, critical t, degrees of freedom (ν) and alpha level (α) are listed in Table 5.

Table 5: Summary of T-test Calculations Performed on the Posttest

t-value	Critical t	ν	α
2.43	2.021	45	.05

The results showed a t-value of 2.43, critical t was 2.021 with 45 degrees of freedom at the .05 level of significance. The groups were therefore determined to **not** be at equal achievement levels at the conclusion of the study due to the fact that 2.43 > 2.021 at the .05 level of significance.

Hypothesis

The null hypothesis that was tested follows: No significant difference would be found in achievement levels between the groups using manipulatives (Algeblocks) and those being taught by a more traditional method. Based on the results of the t-test the null hypothesis must be rejected and the alternate hypothesis accepted. The alternate hypothesis that is accepted follows: There will be significant difference in achievement levels between the groups using manipulatives (Algeblocks) and those being taught by a more traditional method.

CHAPTER FIVE
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this study was to investigate the effects of two instructional strategies on student achievement in two Algebra I classes.

The two instructional methods used were:

1. The control group (Group A) was taught using the traditional method of lecture, homework, and was placed in cooperative learning groups to work on worksheets.
2. The experimental group (Group B) was taught using the traditional method of lecture and homework, but instead of in class worksheets the students worked in cooperative group with the manipulative Algeblocks.

A pretest was given at the beginning of the study to determine if the groups were at equal achievement levels at the beginning of the study. A two-sample t-test was used to analyze the pretest scores. At the .05 level of significance there was no significant difference in achievement at the beginning of the study.

A posttest equivalent to the pretest was given at the conclusion of the study to determine if the groups were at equal achievement levels at the conclusion of the study. A two-sample t-test was used to analyze the posttest scores. At the .05 level of significance there **was** a difference in achievement level at the conclusion of the study. This resulted in the null hypothesis being rejected and the alternate hypothesis being accepted.

Conclusions

The analysis of the data did indicate a significant difference in achievement levels of

Group A and Group B, however in examining the mean scores of the Group A and Group B posttest, the mean score of Group A was higher than the mean score of Group B. This implied that the students in Group B, who used manipulatives, achieved at a significantly lower level than those students in Group A, who did not use the manipulatives. The results of this study would therefore seem to indicate that the use of manipulatives in Algebra I, at the high school, is not beneficial and may in fact be detrimental to student achievement.

Piaget may explain the reason for this. According to Piaget, the concrete operational stage is the basis for the use of manipulatives. The concrete operational stage begins at seven and goes to age twelve. The major characteristics of this stage are, logic of classes and relations, understanding of numbers, and thinking bound to the concrete (Phillips & Soltis, 1995 p.43).

The age of the students in the study groups ranged from fifteen years of age to seventeen years of age. These students have moved out of the concrete operational stage and have moved into the formal operational stage of development. In the formal operational stage, which begins at age twelve and continues through adult life, students have complete generality of thought, and the ability to deal with the hypothetical (Phillips & Soltis, 1985 p. 43). Manipulatives may not be as effective for students who have already entered the formal operations stage because their thinking is not bound to the concrete.

Another factor that may have played a major role in the results of the study is the fact that while the experimental group worked in class with the manipulative, they were not allowed to use the manipulative on the posttest.

Yet another factor that may have been responsible for the results of the study is the fact that the instructor was new to the concept of using manipulatives and did not acquire sufficient knowledge of the manipulative before the study began. The instructor therefore did not properly incorporate the manipulative into the curriculum.

It is also possible that the study fell victim to the "halo effect". This is a situation in which the instructor may have subconsciously given the control group extra help because of a feeling that the control group was being deprived of a valuable instruction aid.

Recommendations

Further research is needed on the effects of using manipulatives at the high school level. The following recommendations are made for further study:

1. Conduct the study over one whole semester, including many topics instead of only one topic.
2. The manipulative was introduced to the students before the study was begun; however, a few more days on familiarization of the manipulative may have been helpful.
3. The instructor should attend as many workshops/training sessions that deal with the manipulative as possible, so that the students are familiar and comfortable with the manipulative.
4. Students in the experimental group should be allowed to use the manipulative on the posttest.
5. A larger sampler size should be used.

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APPENDIX A
PRETEST / POSTTEST

Algebra I Test: Polynomials

Simplify. Assume no denominator is equal to zero.

1. $a^2 * a^3 * b^4 * b^5$ 2. $(-12abc)(4a^2b^3)$ 3. $(9a)^2$

4. $(-3a)^4(a^5b)$ 5.. $(-5a^2)(-6b)^2$ 6. $(5a)^2b + 7a^2b$

7. $\frac{63a^2bc}{9abc}$ 8. $\frac{14ab^{-3}}{21a^2b^{-5}}$ 9. $\frac{y^3x}{yx}$

10. $\frac{y^{11}}{y^6}$ 11. $\frac{48a^2bc^5}{(3ab^3c^2)^2}$ 12. $\frac{10a^2bc}{20a^{-1}b^{-1}c}$

Arrange the terms of each polynomial so that the powers of x are in descending order.

13. $5x^2 - 3 + x^3 + 5x$ 14. $5 - xy^3 + x^3y^2 - x^2$

Simplify.

15. $(a + 5)^2$ 16. $(2x - 5)(7x + 3)$ 17. $3x^2y^3(2x - xy^2)$

18. $-4xy(5x^2 - 6xy^3 + 2y^2)$ 19. $(4x^2 - y^2)(4x^2 + y^2)$ 20. $(2a^2b + b^2)^2$

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