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ABSTRACT

This study examines how teachers' frameworks for human development, curriculum, and mathematics influence how they interpret children's mathematical thinking. The teachers in the study were participants in a professional development project to help them implement cognitively-guided instruction (CGI) in their mathematics instruction. Data on teacher beliefs, interpretations of children's solutions to mathematics problems, and instructional decision making were gathered. Results suggest that teachers focus most frequently and very consistently on the curriculum framework. The increasing importance of the developmental framework appears to be due to the increased attention paid by teachers to the different kinds of solution strategies used by students. (Author/ASK)

Teachers' Frameworks for Understanding Children's Mathematical Thinking

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In this study we examined how teacher's frameworks for human development, curriculum, and mathematics influenced how they interpreted children's mathematical thinking. These frameworks would presumably act as filters for deciding what aspects of students' explanations need to be addressed and what aspects could be ignored.

The teachers in the study were participants in a professional development project to help them implement cognitively guided instruction (CGI) in their mathematics instruction. In implementing CGI, teachers learn to assess students' thinking, primarily through listening to students explain solutions to mathematics problems, and then use that knowledge to plan instruction (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996). As part of this project, we are gathering data on teachers' beliefs, interpretations of children's solutions to mathematics problems, and instructional decision making. Here we analyze the frameworks that seemed to have been used by project participants as they analyzed a transcript of interactions between a first grade teacher and three of her students. This study covered the time period from the beginning of implementation of CGI to the end of the second year of that implementation (May 1995 - June 1997).

Human development. Most teachers seem to be aware of general characteristics of children's cognitive development. For example, the general stages of development often ascribed to Piaget are quite familiar to teachers. However, teachers often "know" these general stages only as they apply to children as a group rather than to children as individuals.

A potentially more useful framework for understanding learning has been suggested by Fosnot (1996, pp. 29-30) since it speaks to the connections between learning and teaching.

- Learning is not the result of development; learning *is* development. It requires invention and self-organization on the part of the learner. Thus teachers need to allow learners to raise their own questions, generate their own hypotheses and models as possibilities, and test them for viability.

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- Disequilibrium facilitates learning. "Errors" need to be perceived as a result of learners' conceptions and therefore not minimized or avoided. Challenging, open-ended investigations in realistic, meaningful contexts need to be offered, thus allowing learners to explore and generate many possibilities, both affirming and contradictory. Contradictions, in particular, need to be illuminated, explored, and discussed.
- Reflective abstraction is the driving force of learning. As meaning-makers, humans seek to organize and generalize across experiences in a representational form. Allowing reflection time through journal writing, representation in multisymbolic form, and/or discussion of connections across experiences or strategies may facilitate reflective abstraction.
- Dialogue within a community engenders further thinking. The classroom needs to be seen as a "community of discourse engaged in activity, reflection and conversation".... The learners (rather than the teacher) are responsible for defending, proving, justifying, and communicating their ideas to the classroom community. Ideas are accepted as truth only insofar as they make sense to the community and thus rise to the level of "taken-as-shared."
- Learning proceeds toward the development of structures. As learners struggle to make meaning, progressive structural shifts in perspective are constructed - in a sense, "big ideas".... These "big ideas are learner-constructed, central organizing principles that can be generalized across experiences and that often require the undoing or reorganizing of earlier conceptions. This process continues throughout development.

This framework seems consistent with the goals of CGI. Further, its underlying philosophy seems supportive of teachers' developing specific understanding of individual children.

The Levels of Cognitively Guided Instruction and Levels of Teachers' Cognitively Guided Beliefs (Fennema, et al., 1996) speak specifically to this issue. At the lowest level, a teacher "provides few, if any, opportunities for children to engage in problem solving or to share their thinking" (p. 412) and "does not believe that children can solve problems without instruction" (p. 413). At the highest level, a teacher "provides opportunities for children to be involved in a variety of problem-solving activities [as well as] elicits children's thinking, attends to children sharing their thinking, and adapts instruction according to what is shared" (p. 412). Further, the teacher "believes that children can solve problems without instruction across mathematics content domains and that what he or she knows about children's thinking should inform his or her decision making, both regarding interactions with the students and curriculum design" (p. 413). Further, as teachers became more expert at implementing CGI, their understanding of "CGI knowledge" in the form of taxonomies of problem types (e.g., join change unknown) and children's solution strategies for those problems (e.g., direct modeling, counting, derived facts) becomes richer and more accessible. It would be reasonable to expect, therefore, that teachers' interpretations of a child's development might shift from a Pigetian framework to the more specifically mathematically-oriented framework evident in CGI.

These differences might be manifest in teachers' examination of children's verbalizations in response to questions asking them to explain their thinking. What teachers see as important in those responses might reveal teachers' thinking about the nature of both mathematics and children's thinking about mathematics. Thus, from one perspective, this study can be viewed as being designed to determine whether teachers used general frameworks (e.g., Pigetian) that speak to children as a group or more specific frameworks that speak to understanding children as individuals.

Curriculum. In North Carolina, where this study was conducted, there is a *Standard Course of Study* (SCOS) for mathematics, grades K-8, approved by the State Board of Education. This document contains mathematics objectives for each grade level which teachers are expected to "cover" during a year's instruction. In grades 3-8, there are end-of-grade tests which are designed to determine whether students have acquired the prescribed mathematics knowledge.

Because teachers are responsible for teaching the content in the SCOS, they are usually cognizant of the objectives for their grades and for the grade that immediately follows. This seems especially true for teachers who have taught the same grade for multiple years. The curriculum framework provided by the SCOS, then, might be chosen by teachers as one way of interpreting whether students have acquired the appropriate kinds of mathematics knowledge.

Teachers who are implementing CGI might begin with a strong curriculum framework for understanding students' thinking. However, as noted by Fennema et al. (1996), as teachers move toward higher levels of CGI instruction and beliefs, they might be expected to move away from abstract curriculum as a framework and more toward identification of individual students' strategies as an alternate way of interpreting children's thinking.

Mathematics. There are two major views of mathematics that teachers might adopt. One is "mathematics as a static discipline developed abstractly" (Dossey, 1992, p. 39) which would presumably be that adopted by teachers who do a lot of modeling and demonstrating for students. An alternate view is "mathematics as a dynamic discipline, constatntly changing as a result of new discoveries" (Dossey, 1992, p. 39). This view seems more in line not only with notions of social construction of shared knowedge but also with the view of the NCTM's *Standards* documents. As teachers implement CGI and focus more and more of their attention and their students' attention on understanding each other's solution strategies, the teachers might move toward a "socially constructed" view of mathematics.

Method

Subjects and instrumentation. The study was conducted during the first two years of a five-year (calendar years 1995 - 1999), National Science Foundation, Teacher Enhancement project in which elementary teachers are learning to use CGI as a basis of mathematics instruction. Teachers and mathematics educators from different regions in the state formed five teams; each team was composed of 2 teacher educators (i.e., team co-leaders) and 6 elementary teachers. The data reported here come only from the elementary teachers who were participants in the project.

All subjects completed a transcript analysis instrument (Appendix 1). The instrument contains three teacher-and-student (Mac, Tom, and Sue) dialogues that occurred while a group of 23 first-grade students worked individually on 5 written problems. The teacher interacted with Mac after he had completed the problem: *If frog's sandwiches cost 10 cents, and he had 15 sandwiches, how much did frog's sandwiches cost altogether?* As the teacher moved to Tom's desk, Tom was working on the same problem. The teacher's interaction with Sue occurred as she was working on a different problem: *Frog had 15 sandwiches. If each sandwich cost 5 cents, how much do all the sandwiches cost altogether?* After reading the dialogues, subjects were asked to state their conclusions about the children's (a) levels of thinking and (b) mathematical understanding. Subjects were also asked to identify specific evidence from the transcript that was important to them in making those conclusions. No definitions for the phrases "levels of thinking" or "mathematical understanding" were requested or provided during administration of this instrument.

Transcript analyses were completed on the first day of the project's first workshop (May 1995), on the first day of the second summer workshop (June 1996), and on the first day of the third summer workshop (July 1997). Subjects worked individually, without discussion with any other participants. Complete data were available for 20 female teachers: 4 in kindergarten, 5 in grade 1, 4 in grade 2, and 7 in grades 3/4. (One teacher taught grade 4, so that teacher is included in the grade 3/4 group.)

Between the first and second administrations, project teachers participated in two workshops (3 days in May 1995; 10 days in July 1995) and began CGI implementation in their own teaching during 1995-96. During the school year each team met after school approximately once a month to discuss their progress, each teacher was visited approximately once a month during mathematics instruction by one of her team's co-leaders, and each teacher was visited during mathematics instruction once each semester by project staff. The purpose of the visits was to support teachers as they began implementing CGI; visits were never used to "evaluate" teachers.

Between the second and third administration, project teachers participated in one workshop (8 days in June 1996) and continued CGI implementation in their teaching during 1996-97. Teams continued to meet monthly and teachers were visited by the team co-leaders and by the project staff.

Analysis of responses. Five frameworks were discussed by the authors, until agreement was reached on characterizations of these frameworks and on the nature of evidence that would be accepted for categorizing responses according to these frameworks. Teachers' responses were then coded independently by the authors according to these frameworks. The codings were discussed and discrepancies were negotiated. These frameworks, along with brief quotes from participants' responses, are presented below:

1. developmental (e.g., Piaget): "Mac .. solved the problem in an abstract way .. Sue needs to work on a very concrete level." "Sue is at a direct modeling level of thinking. Mac and Tom are approaching derived facts or higher levels of problem-solving/critical thinking."
2. taxonomic (e.g., Bloom): "[Sue] has a grasp of some mathematical concepts but she is unable to apply the skills."
3. problem solving (e.g., Polya): "Mac and Tom understood what the problem was asking.. Sue does not know what she needs to find out."
4. curriculum (e.g., focus on specific content or on grade-appropriate content knowledge): "[Mac] knew to add a 0 at the end of the 15 because he was counting by 10's." "[Mac's] understanding of math goes beyond what one would expect of a first grader."
5. deficiency (e.g., lack of prerequisites): "Sue has not mastered the concept of skip counting."

Results

Teachers' rankings of children's thinking. The teachers were quite explicit (and in almost universal agreement) that Mac and Tom exhibited higher levels of thinking and better mathematical understanding than Sue. Teachers did not always explicitly order the students by their levels of thinking, but comments universally referenced greater understanding by Mac and Tom than by Sue. Often, teachers suggested that Mac exhibited higher levels of thinking than Tom, though sometimes teachers' comments suggested that they thought that Mac and Tom were at about the same level of thinking.

Teachers often defined "levels of thinking" either *relatively* as higher level thinker versus lower level thinker or advanced thinker versus less advanced thinker or *absolutely* in terms of a developmental framework. Less precise descriptions included "good thinker" and "independent thinker." In the first administration of the transcript analysis, there was consensus among the teachers that use of concrete objects necessarily indicated lower level thinking while use of mental math and visualization showed higher level thinking (Bowman, Bright, & Vacc, 1996).

Teachers' frameworks. The categorizations of teachers' apparent frameworks for interpretation of students' responses are given in Table 1 and summarized by grade in Table 2. Because of the classification of multiple frameworks for most responses, the number of categorizations exceeds the number of teachers.

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Insert Tables 1 and 2 about here

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The average number of categories coded per teacher generally increased across the three administrations (Table 3), though the pattern of change varied by grade level. Across administrations, the teachers in grade K showed the greatest variability and the teachers in grade 2 showed the least variability. In the second and third administrations, every teacher was coded with at least two categories. Two teachers, both in grade 3, were coded identically in each of the three administrations.

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Insert Table 3 about here

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As a follow-up analysis, the percentages at each grade for each administration were ranked. Across all grades and across all three administrations, the curriculum framework was the most important (or tied for most important) relative to the other frameworks. Across all grades and across all three administrations, the developmental framework increased in importance relative to the other frameworks. With one exception (i.e., grade 1, administrations 1 and 2), across all grades and across all three administrations, the problem solving framework diminished in importance relative to the other frameworks. Across the three administrations, the deficiency framework seemed to increase slightly for teachers in grades K and 1, but not for teachers in grades 2 and 3/4.

Eighteen of the teachers were classified as using a common framework across all three administrations. In 16 instances the common framework was curriculum (3 at grade K, 4 at grade 1, 4 at grade 2, and 5 at grades 3/4), in 6 instances the common framework was problem solving

(2 at grade 1, 1 at grade 2, and 3 at grades 3/4), in 6 instances the common framework was deficiency (1 at grade 1, 1 at grade 2, and 4 at grades 3/4), in 1 instance the common framework was developmental (grade 3), and in 1 instance the common framework was taxonomic (grade 3). The teachers at grade K displayed the least amount of, and least variety in, commonly used frameworks, and the teachers at grade 3 displayed the greatest amount of, and greatest variety in, commonly used frameworks.

Between successive administrations, most teachers either "gave up" or "took on" one or more frameworks; that is, a framework identified in the first of the successive administrations was not identified in the second, or a framework identified in the second of the second administrations was not identified in the first. From 1995 to 1996, 2 teachers each gave up two frameworks, 8 teachers each gave up one framework, 12 teachers each took on one framework, and 3 teachers each took on two frameworks. In particular, 1 teacher gave up the developmental framework and 7 teachers took it on; 1 teacher gave up the taxonomic framework and 0 teachers took it on; 6 teachers gave up the problem solving framework and 4 teachers took it on; 1 teacher gave up the curriculum framework and 2 teachers took it on; and 3 teachers gave up the deficiency framework and 5 teachers took it on. From 1996 to 1997, 8 teachers each gave up one framework, 11 teachers each took on one framework, 5 teachers each took on two frameworks, and 1 teacher took on three frameworks. In particular, 1 teacher gave up the developmental framework and 7 teachers took it on; 1 teacher gave up the taxonomic framework and 5 teachers took it on; 3 teachers gave up the problem solving framework and 3 teachers took it on; 0 teachers gave up the curriculum framework and 2 teachers took it on; and 3 teachers gave up the deficiency framework and 7 teachers took it on.

Discussion

Teachers focused most frequently, and very consistently, on the curriculum framework. This is not surprising, and it may simply reflect the deep familiarity of the teachers with the state curriculum. Because of the high stakes character of end-of-grade tests in North Carolina, the curriculum framework was expected to be commonly used in interpreting children's thinking. The project certainly was not trying to de-emphasize the importance of the proscribed curriculum. Rather, we were trying to help teachers understand their students' thinking so that the curriculum objectives mandated by the state could be learned better by students.

The increasing importance of the developmental framework seems to be due to the increased attention paid by teachers to the different kinds of solution strategies (e.g., direct modeling, counting, derived facts) used by students. We interpret the teachers' focus on the mathematically related development of students as a direct outcome of the professional development provided during the project, since CGI professional development focuses a lot of attention on understanding children's thinking.

We view the diminishing in importance of the problem solving framework as positive. The problem solving framework can, at some level, be thought of as being related to teachers' view of curriculum, at least as instantiated in textbooks. Over the past 10-20 years, many elementary school textbooks have used a set of "stages" for problem solving derived from Polya's (1945) four steps for problem solving: understand the problem, devise a plan, carry out the plan, look back. It is not surprising, therefore, that teachers would think about these stages in trying to interpret students' solutions to word problems. It is encouraging, however, that across the first year of implementing CGI, teachers would put more emphasis on students' demonstration of understanding of specific content (e.g., as instantiated in the state curriculum) rather than on generic stages of problem solving. Teachers in the project seem to be trying to understand the specifics of students' solutions rather than generic thinking skills.

We are unsure how to interpret the fact that the teachers in grade K were classified as using the least number of frameworks and had the least variation in consistently coded frameworks. Perhaps this reflects the limited amount of content specified for grade K in the North Carolina curriculum. Perhaps teachers in grade K have to deal with less variation in student thinking, so interpreting students' thinking in terms of the curriculum is adequate. At the same time, the teachers in grade K showed the greatest gain across the three administrations in the number of frameworks categorized. Perhaps these teachers began to "internalize" more of the information from the workshops than other teachers.

In contrast, teachers in grade 2 showed the least variation in number of frameworks coded. In North Carolina, grade 2 is often viewed as a "transition" from relatively open-ended instruction for which there are no end-of-grade tests to more traditional instruction in grade 3 where end-of-grade tests are given for the first time. Further investigation seems warranted on the relationship between use of frameworks and teachers' knowledge about curriculum.

The increased relative importance of the deficiency framework for teachers in grades K and 1 might be related to this speculation about less variation in very young children's thinking. Teachers in grades K and 1 may have become more aware of the richness of children's mathematical thinking and may have used deficiency as a way to account for some of the newly recognized variation in children's thinking. The increase in numbers of teachers using this framework, however, surprised us.

On average, for each pair of successive administrations, each teacher either gave up or took on approximately 1.5 frameworks; for each pair of successive administrations, the range of number of changes was from 0 to 4. This seems like a substantial amount of change and might reflect the changes in philosophy of mathematics teaching that the project teacher participants are likely to have made during their struggles to implement CGI (e.g., Fennema et al., 1996). These data are consistent with the view that the project's strong focus on understanding children's thinking in ways that were quite unfamiliar to teachers caused them to re-conceptualize what they "knew" about students. One might expect the greatest amount of change in a project such as this to be during the first year (between the first two administrations), but in fact the amount of changes was slightly greater during the second year (during the second two administrations). It is a little surprising that change continues at a fairly constant pace. It will be interesting to determine from data gathered in subsequent years of the project whether the amount of change in thinking is different or whether teachers' views tend to stabilize in configurations that are different from those at the beginning of the project.

The categorization of frameworks undoubtedly under-estimates the actual use of multiple frameworks by teachers, since classifications were made only on the basis of what the teachers wrote. Additional research that triangulates written responses against data gathered in other ways (e.g., interviews) seems warranted.

To date, the project appears to have created an environment in which teachers seem to use an increasing number of interpretive frameworks for understanding children's mathematical performance. It is not intuitively obvious, however, that simply increasing the range of teachers' interpretive options will necessarily improve the quality of mathematics instruction. It is probably more important to try to determine how teachers are using their new interpretive skills in planning and delivering instruction.

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Table 1. Categorization of Teachers by Framework for Each Administration

Grade	May 95	July 96	July 97
K	3 4	1 4	1 2 3 4 5
K	3	1 5	1 2 4 5
K	3 4	4 5	1 2 4 5
K	4	1 3 4	1 4 5
1	1 3 4 5	3 4	3 4 5
1	1 3 5	1 3 4	3 4 5
1	4 5	3 4 5	1 2 4 5
1	3 4	1 4	1 4 5
1	4	4 5	1 3 4 5
2	4 5	3 4 5	1 3 4 5
2	2 3 4 5	1 2 4	1 4 5
2	3 4 5	4 5	2 4
2	2 3 4	3 4 5	1 3 4
3	2 3 4	1 2 3 4	1 2 3 4 5
3		4 5	1 3 4 5
3	4 5	4 5	4 5
3	1 3 4 5	1 3 4 5	1 3 4 5
3	4 5	3 5	3 4 5
3	3 4 5	1 3 4 5	1 4 5
4	3 4 5	3 4 5	1 3 4

NOTE: Framework codes are as follows:

1 = Developmental
2 = Taxonomic
3 = Problem Solving
4 = Curriculum
5 = Deficiency

Table 2. Numbers (Percentages) of Teachers Categorized by Framework for Each Administration

Framework	Administration 1995	1996	1997
Grade K (N = 4)			
Developmental	0 (00)	3 (75)	4 (100)
Taxonomic	0 (00)	0 (00)	3 (75)
Problem Solving	3 (75)	1 (25)	1 (25)
Curriculum	3 (75)	3 (75)	4 (100)
Deficiency	0 (00)	2 (50)	4 (100)
Grade 1 (N = 5)			
Developmental	2 (40)	2 (40)	3 (60)
Taxonomic	0 (00)	0 (00)	1 (20)
Problem Solving	3 (60)	3 (60)	3 (60)
Curriculum	4 (80)	5 (100)	5 (100)
Deficiency	3 (60)	2 (40)	5 (100)
Grade 2 (N = 4)			
Developmental	0 (00)	1 (25)	3 (75)
Taxonomic	2 (50)	1 (25)	1 (25)
Problem Solving	3 (75)	2 (50)	2 (50)
Curriculum	4 (100)	4 (100)	4 (100)
Deficiency	3 (75)	3 (75)	2 (50)
Grade 3/4 (N = 7)			
Developmental	1 (14)	3 (43)	5 (71)
Taxonomic	1 (14)	1 (14)	1 (14)
Problem Solving	4 (57)	5 (71)	5 (71)
Curriculum	6 (86)	6 (86)	7 (100)
Deficiency	5 (71)	6 (86)	6 (86)
Total (N = 20)			
Developmental	3 (15)	9 (45)	15 (75)
Taxonomic	3 (15)	2 (10)	6 (30)
Problem Solving	13 (65)	11 (55)	11 (55)
Curriculum	17 (85)	18 (90)	20 (100)
Deficiency	11 (55)	13 (65)	17 (85)

Table 3. Average Number of Frameworks Coded per Teacher for Each Administration

Grade	1995	1996	1997
K	1.50	2.25	3.00
1	2.40	2.40	3.40
2	3.00	2.75	3.00
3/4	2.43	3.00	3.43
Total	2.35	2.65	3.45

Appendix 1. Transcript Analysis Instrument

Directions. Please read the attached transcript and answer the following questions:

1. What conclusions can you make about the levels of thinking of the three children?
2. What conclusions can you make about the mathematical understanding of the three children?

Please identify the specific evidence from the transcript that is important to you in making those conclusions. (You may write on the transcript.)

The following protocol is from an actual first-grade classroom of 23 students. The teacher (T) has given a sheet of 5 written problems to all her students. The problems have been read to the students and the students are at their desks working individually on them. As the students work, the teacher stops and talks with them about the problems they are solving. The actual problems are written in **bold** print.

Mac: I'll show you, mmmm, this one and this one. I'll do this one first. **If frog's sandwiches cost 10 cents, and he had 15 sandwiches, how much did frog's sandwiches cost altogether?** I didn't need to use the number chart because all I did was, I did a 15 and then I put a 0 in the back and then I put a zero at the end ..

T: Go ahead. I'm just going to write what you're telling me.

Mac: I'll wait for you.

T: [T writes on Mac's paper: I wrote a 15 and put a 0 in the back. Go on.

Mac: And then I, I, I, I looked at the number in my head.

T: Yes, go on.

Mac: And I saw that it looked, I saw that it was a 150, a dollar fifty. You should put, if you're doing with money, and you put a period after the one.

T: [T writes on Mac's paper: I looked at the number in my head and saw that it was 150 -- $1.50 if it's money.] Mac, is there another way you could solve that?

Mac: Yeah, I could count by tens. Like this, I could go, I could keep my finger on how many tens [referring to the number chart].

T: Okay.

Mac: Like this: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 [as he counts he points to the numbers 1-10 on the number chart]. And then I'd do it again, until I get to 50. 110, 120, wait a minute, 120, 130, 140, 150. [He continues pointing to the numbers 11-15 as he counts.]

T: Why did you stop there?

Mac: Because a hundred, because that's 15 tens.

T: Okay, and how did you know you need 15 tens?

Mac: Because frog ate 15 sandwiches and each of them cost 10 cents.

T: That's very good.

[Teacher moves on to Tom's desk. Tom is solving the same problem as Mac.]

T: Tom, tell me about this one right here. It says if frog's sandwiches cost 10 cents how much would frog's sandwiches cost altogether? How many sandwiches did frog eat?

Tom: 15. So I think it would cost a dollar and fifty cents. [He has written 150¢ on his paper.

T: And why do you think that?

Tom: Because, I'll show you what I do. Um, you see, first I went like 10, 20, 30, 40, 50, 60, 70, 80, 90, 100. And 100 pennies I know is a dollar, but ..

T: [interrupting] Very good.

Tom: Yeah, and um, and ten groups of pennies, ten groups of ten pennies that would be a dollar, too.

T: Correct.

Tom: Then I just went 10, 20, 30, 40, 50. And the first one I did was a dollar and the next one was 50 cents.

T: Uh-huh.

Tom: So it should have been a dollar fifty cents.

T: Yes, very nice. Now, Tom, do you see what you did over here where you put the dollar sign? [For problem 3 he had written $130.]

Tom: Yeah.

T: [writes $1.50 as she talks.] This is really a better way to write a dollar fifty cents, than this way [writes 150]. Actually, we never even use this sign [points to ¢] right here when we have a dollar. We use this sign [points to $] instead, which is a dollar, and then we put a dot right after the dollars and before the cents. And the dot helps us know cents. So now we read it one dollar and fifty cents. Okay?

Tom: Yeah.

T: So what do you need to add here? [Points to the $130.]

Tom: Just the dollar sign like I did and now you got to put that [points to the "." in $1.50, then adds the decimal point -- $1.30].

T: Exactly. Would you do that please? And you will have done perfectly. [T writes on Tom's paper: counted by 10s 15 times.]

[T moves on to work with Sue. She is solving a different problem on the same page of problems.]

T: Okay, you've got to solve this before you do science today. **Frog had 15 sandwiches. If each sandwich cost 5 cents, how much do all the sandwiches cost altogether?** So how many sandwiches do you have there?

Sue: 15. [She has 15 cubes in front of her.]

[Sue starts to count out more cubes.]

T: Now wait a minute, wait a minute, you tell me these were the 15 sandwiches right here. This one costs 5 cents, this one costs 5 cents, this one costs 5 cents, what's this one cost [pointing to each cube as she talks]?

Sue: 5 cents

T: And this one?

Sue: 5 cents

T: Every one of them costs 5 cents. How much is it going to cost to buy these 15 sandwiches?

Sue: Ah, 5 cents.

T: 5 cents? 5 cents buys only one of them. We want to buy them all.

Sue: 6 cents?

T: You're guessing. Is there a way to figure it out?

Sue: 25?

T: [interrupting] Oops, try again, you missed one. [T helps Sue count by fives using the cubes to keep track of how many should be counted. After 45 T says the numbers with Sue as they both point to the cubes.]

Sue: 75.

T: 75. What does it cost? 75 M&M's? 75 what? pause] Cents. What's cents? What does cents mean?

Sue: What does cents mean?

T: Yea.

Sue: It means, cents, quarter, or dimes ..

T: [interrupting] Money, right?

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