Strategies and programs for improving mathematics instruction should be derived from sound educational theory. The sociocultural learning theories of Vygotsky may offer guidance in developing technology-based mathematics curriculum materials consonant with the NCTM (National Council of Teachers of Mathematics) goals and objectives. Vygotsky’s theories, especially when they are applied to the use of current and emerging technologies, have the potential to help classroom teachers narrow the gap between current practices and the vision of mathematics education represented in the NCTM standards. Vygotskian theory incorporates several key ideas relative to levels and types of concept formation, the role of collaborative interchange in concept development, and instructional configurations that guide and promote concept development. Issues related to technology’s role in mathematics education include content contextualization in the negotiation of meaning and reasoning skills, collaboration, communication, and language in the process of concept development. Mindtools (i.e., computer-based tools and learning environments that have been adapted or developed to function as intellectual partners with the learner in order to engage and facilitate critical thinking and higher-order learning), databases, spreadsheets, and computer-mediated communication are all tools which, when appropriately implemented, may promote high level thinking skills and support concept development. (Contains 26 references.) (AEF)
Improving Mathematics Instruction Using Technology: A Vygotskian Perspective

Francis A. Harvey
Lehigh University

Christina Wotell Charnitski
College Misericordia

Introduction

The need for reform in mathematics education is well documented (International Association for the Evaluation of Educational Achievement, 1996; National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1989, 1991, 1995). To address this need, the National Council of Teachers of Mathematics (NCTM), in concert with other professional organizations, developed the NCTM Standards (National Council of Teachers of Mathematics, 1989, 1991, 1995). These Standards redefined the parameters of mathematical literacy, the scope of mathematics instruction, and the areas of content emphasis in the K-12 mathematics curriculum. The implications of these changes for the classroom teacher are considerable; compliance with the Standards may require the classroom teacher to reevaluate his or her role and the roles of his or her students in the teaching/learning process. Modifying classroom roles and monitoring the resulting changes may prove to be a formidable undertaking.

The Professional Standards for Teaching Mathematics (National Council of Teachers of Mathematics, 1991) identified five major pedagogical shifts that are necessary to achieve the NCTM goals in today's mathematics classroom. These shifts will directly affect not only the classroom environment, but also the roles of students and teachers in the learning process. The 1991 Standards proposed:

- a shift toward classrooms as mathematical communities away from classrooms as simply a collection of individuals;
- a shift toward logic and mathematical evidence as verification away from the teacher as the sole authority for right answers;
- a shift toward mathematical reasoning away from merely memorizing procedures;
- a shift toward conjecturing and problem solving away from an emphasis on mechanistic answer-finding;
- a shift toward connecting mathematics, its ideas, and its applications away from treating mathematics as a body of isolated concepts and procedures. (p.3)

The Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) pointed out that advances in technology have extended the boundaries of mathematics and underscored the importance of the integration of technology in the mathematics curriculum. In a summary of the Third International Mathematics and Science Survey, Beaton, Mullis, Martin, Gonzalez, Kelly, and Smith (1996) asserted that technology is a key factor in determining the economic health of a country. They also pointed out that the overriding presence of technology makes new demands on educational communities to prepare students with high levels of technical competence and flexible thinking. For these reasons the International Association for Evaluation of Educational Achievement (IEA) (1996) recommended appropriate integration of technology in the mathematics curriculum. The Board of Governors of the Mathematical Association of America (MAA) maintained that computing has a profound and permanent impact on the practice of mathematics at all levels (Steen, 1992). The MAA recommended that technology be incorporated into the mathematics curriculum in a way that permits students to connect mathematics with diverse fields and applications, and to put mathematical ideas into action.

The NCTM Standards do not imply that the use of technology guarantees mathematical literacy, or that the use of calculators eliminates the need for students to learn algorithms. Rather, the standards indicate how technology can be used to facilitate the learning process and to emphasize the fundamental mathematics students will need.

The NCTM Curriculum and Evaluation Standards stated three overarching goals: (a) students should become mathematical problem solvers; (b) students should learn to communicate mathematically; and (c) students...
should learn to reason mathematically (pp. 5-6). These goals assume active student learning, and emphasize the necessity for students to develop an understanding of mathematical models, structures, and simulations so that they may apply their mathematical knowledge flexibly across situations and disciplines. In this context, mathematics instruction becomes process-driven rather than product-driven, and in many respects becomes a collaborative endeavor.

A report by the Council of Chief State School Officers (CCSSO) on state-by-state trends in teaching practices and content emphases in instruction (Blank, & Gruebel, 1995) presented data representing the extent to which mathematics classes in grades 4 and 8 included weekly instructional practices that are consistent with the NCTM standards. The four types of instructional practices were: (a) work in small groups, (b) write reports or do mathematics projects, (c) use manipulative objects, and (d) use calculators in class. The CCSSO survey found that on a weekly basis 65% of the mathematics classes nationally have students work in small groups, 1% of the classes include math projects and/or reports, 46% employ math manipulatives with instruction, and 18% use calculators. These findings suggest that change is still needed in many United States classrooms in order for mathematics instruction to comply with the NCTM standards.

The Professional Standards for Teaching Mathematics and the Assessment Standards for School Mathematics (National Council of Teachers of Mathematics, 1995) recognized that teachers are primary figures in changing the ways in which mathematics is taught and assessed in schools, and that technology is an important component of that change. It becomes apparent that teachers and other educators must bear the major responsibility for realizing the NCTM Standards in practice.

Strategies and programs for improving mathematics instruction should be derived from sound educational theory. The sociocultural learning theories of L. S. Vygotsky (1986) may offer guidance in developing technology-based mathematics curriculum materials consonant with the NCTM goals and objectives. Vygotsky's theories, especially when they are applied to the use of current and emerging technologies, have the potential to help classroom teachers narrow the gap between current practices and the vision of mathematics education represented in the NCTM Standards. Vygotsky's theories appear particularly useful for defining and guiding the changing roles of the teacher and students.

Vygotsky's Theories of Sociocultural Learning

Vygotsky's sociocultural theory of mind serves as a middle ground between formalism and constructivism (Kozulin, 1990; Moll, 1990; Vygotsky, 1986; Wertsch, 1985). Concept formation, according to Vygotsky's theory, is a dynamic interactive action between the concrete and the abstract that does not require the child to reinvent information (as does constructivism), nor does it expect the child to conceptualize abstractions without having first engaged in concrete activities that support the formation of mental models. Vygotskian theory incorporates several key ideas relative to levels and types of concept formation, the role of collaborative interchange in concept development, and instructional configurations that guide and promote concept development.

Language

Language, and its relationship to thinking, is at the foundation of Vygotsky's sociocultural theories; he contended that language is an indispensable requisite for all intellectual growth. Vygotsky (1986) contended that a concept is not "...an isolated, ossified, changeless formation but an active part of the intellectual process, constantly engaged in serving communication, understanding, and problem-solving." (p.98) He argued that the language of adults and others which surrounds the child is a major factor in the child's cultural and social environment, and that this language influences the content of the child's thoughts as well as how the child thinks. Vygotsky did not isolate verbal interchanges with adults as a separate component in the process of a child's concept formation, but rather stressed the essential interrelationship between sensory material and words as mutual components of a child's concept development.

The child and the adult may share many word meanings, but Vygotsky (1986) cautioned against using this as evidence that the mental operations of the child and adult are one and the same. He maintained that it is through verbal interactions with adults and sensory interaction with their culture that children negotiate meaning and form concepts. Vygotsky viewed conceptual thinking as being purposeful in that it arose in answer to a problem:

Memorizing words and connecting them with objects does not in itself lead to concept formation; for the process to begin a problem must arise that cannot be solved otherwise than through the formation of new concepts. (p. 100)
According to Vygotsky, attainment of real concepts is impossible without words (Vygotsky, 1986; Vygotsky & Luria, 1993). Therefore, he considered the central moment in concept formation to be the point at which the child is able to use words as functional tools. He considered words to be the tools or means that direct mental operations and focus their course toward the solution of a problem.

**Spontaneous and Scientific Concepts**

Vygotsky recognized the need to connect formal concepts learned in school with children's understanding that resulted from their everyday experiences (Davydov, 1990; Ratner, 1991; Schmittau, 1993). The crux of the Vygotskian approach to instruction lies in the differentiation Vygotsky made between spontaneous and scientific concepts. Spontaneous concepts are formed by the child through everyday exposure to the cultural environment; they are unsystematic and highly contextualized. Scientific concepts are mediated through formal instruction; are characterized by hierarchical, logical organization; and typically are derived from the highly structured activity of classroom instruction. Spontaneous and scientific concepts are characteristically different, yet they are not independent of each other.

While Vygotsky (1986) asserted that scientific and spontaneous concepts develop in reverse directions, he maintained that they are closely related, with each supporting the formation of the other. He perceived spontaneous concepts as bottom-up constructs that must be sufficiently developed before the child is able to absorb a related scientific concept. In Vygotsky's words:

> In working its slow way upward, an everyday concept clears a path for the scientific concept and its downward development. It creates a series of structures necessary for the evolution of a concept's more primitive elementary aspects, which give it body and vitality. Scientific concepts, in turn, supply structures for the upward development of the child's spontaneous concepts toward consciousness and deliberate use. (p.194)

**Zone of Proximal Development**

Vygotsky (1986) argued against traditional cognitivist theory, which suggests that a child's developmental level should lead instruction. He contended that the child's current developmental level should be viewed as a starting point rather than as an ending point; a point at which instruction leads rather than follows development. Vygotsky proposed that the appropriate level of instruction lies between a child's actual mental age (i.e., his or her level of independent functioning) and the level of problem solving that the child is able to reach with assistance.

Vygotsky labeled this domain the zone of proximal development (ZPD). With adult support, the child is able to accomplish more than he or she is able to accomplish independently. The child's state of development is the limiting factor in this process. Mediation leads a child through his ZPD via verbal communication and collaborative interchange with an adult or more competent peer. As the child becomes more capable of independent thinking, the child's state of development rises and mediation can reach toward a higher instructional level, thus leading development.

The child's movement through the ZPD is characterized by concept development and intellectual movement which are subject to the child's own developmental laws. Movement through the ZPD involves intelligent, conscious imitation, which requires that the child first understand the field structure and relationships between objects. This conscious imitation should not be confused with automatic imitative behavior that shows no signs of conscious understanding.

Vygotsky's theories and the tenets of the NCTM standards appear to have many areas of commonality. Both imply multidimensional thinking rather than one-dimensional learning. Both contend that intelligent, conscious imitative behavior, rather than automatic imitative behavior, leads to concept development. Vygotsky's theories and the NCTM Standards agree on the value of cultural contextualization in the negotiation of meaning and reasoning skills, and on the importance of collaboration, communication and language in the process of concept development. Thus, it appears that Vygotsky's theories may offer appropriate and useful guidelines identifying the characteristics of effective uses of various technologies in the mathematics curriculum.
The Role of Technology in the Mathematics Curriculum

The Standards are purposefully general about specific types of technology and their uses. In doing so, they recognize the dynamic state of technological advancement and assume that ongoing change in technology will continue to impact the nature of mathematics. The focus of the standards is on the goals and objectives of the mathematics curriculum, relegateing technology to the role of a facilitating tool for concept formation. It, therefore, seems appropriate to explore available technologies and to investigate their potential supportive application in the mathematics curriculum. Issues related to technology's role in mathematics education include content contextualization in the negotiation of meaning and reasoning skills, collaboration, communication and language in the process of concept development.

Mindtools

Jonassen (1996) discussed using "mindtools," which are computer-based tools and learning environments that have been "...adapted or developed to function as intellectual partners with the learner in order to engage and facilitate critical thinking and higher-order learning" (p.9). The functional role of mindtools is twofold: (a) to extend the learner's cognitive functioning during the learning process; and (b) to engage the learner in operations while constructing knowledge that they would not have been able to accomplish otherwise. According to Jonassen, mindtools support a learning environment in which students are able to process information intentionally and meaningfully, to build on prior learning, to elaborate on new knowledge, to interrelate new knowledge with their prior knowledge, and to consciously reflect on their learning. He further suggested that mindtools might be enhanced through collaborative and cooperative efforts between and among students and teachers in the learning community. Mindtools, therefore, appear to offer a supportive structure to a Vygotskian approach to mathematics learning. Among the applications Jonassen lists as mindtools are databases, spreadsheets, and computer mediated communication. Each of these will be discussed in the following sections.

Databases

Databases are computerized record-keeping applications which allow easy entry and fast, flexible retrieval of information. The two basic components of a database are the field entries, which are the individual categories under which data is entered, and the record, which is a compilation of the representative fields. The database is a collection of records within which like categories of information are stored. This is similar to having a collection of index cards on which one would store the names, addresses, and phone numbers of all friends and family. The collection of index cards would represent the database; each index card is equivalent to a record, while each piece of information on the card equates to a field. Electronic databases are much more flexible than a system based on index cards, since the records may be sorted and organized by any part of the information in separate fields. This allows quick retrieval of any particular kind of information the student may wish to access.

Construction of databases requires that learners identify and classify the important underlying constructs of their information and the contextual relationships within the content. Students must analyze and comprehend the information they wish to store. A database management system can help students organize, interrelate and synthesize content information in a more meaningful manner. Using databases extends critical, creative and complex thinking skills (Jonassen, 1996).

Classification involves forming an organizational schema among elements in which noticing similarities and differences of objects is foundational (Sovichik, 1996). The ability to classify and to sort is fundamental in the hierarchy of mathematical thinking skills (Holmes, 1995; Kennedy, & Tipps, 1996; Troutman & Lichtenberg, 1995). Hallmark to Vygotsky's theory of concept development is the child's movement from the ability to form relationships between and among physical objects on various levels to the ability to abstract properties independent of the physical object (Vygotsky, 1986). According to Vygotsky, the child's classifications progress from (a) an initial level of forming concrete and factual bonds which are based on external likeness, to (b) grouping by contrast where objects are categorized on the basis of a single trait on which the objects differ and thus complement one another, to (c) linking meanings in a chain-like manner where meaning is carried over from one link to the next, and finally to (d) fluid connections between and among objects.

Databases allow visual representation and provide a dynamic repository for information that may support the child from the most elementary through the most complex stages of classification and abstractions. Further, the construction, maintenance and use of databases encourage collaborative enterprises.

In a school setting, students could use databases with increasing complexity from the early grades throughout their secondary education. Databases can be used to reflect authentic information gathered by the
students, and to provide a focus for the analysis and interpretation of the information stored. For younger students, this information may revolve around personal and concrete information (e.g., tracking weather information by day) while for older students it may contain data relative to more complex and more abstract subject matter (e.g., categorizing mathematical functions).

**Spreadsheets**

Electronic spreadsheets are a tool for organizing and manipulating numerical data; therefore they can help students manage numerical information and relationships. The spreadsheet is a two-dimensional matrix of cells. Columns are identified by letters, and rows are identified by numbers. Each cell may contain text, a numerical formula, a value or a function. Spreadsheets have three main functions that make them particularly conducive to the integration and extension of the mathematics curriculum: (a) storing information, (b) calculating functions which relate information, and (c) presenting the results of these calculations as well as the underlying information (Grabe, & Grabe, 1998; Jonassen, 1996; Roblyer, Edwards, & Havriluk, 1997). Many spreadsheets have the additional capability of easily converting numerical information into various graphical representations.

At the most fundamental level, developing and using a spreadsheet as a mindtool requires students to identify rules and to understand relationships. On more advanced levels, students generate new rules that describe and organize relationships (Jonassen, 1996). Jonassen cautioned, however, that if students merely engage in data entry they are engaging in lower level learning on the knowledge and comprehension levels. On the other hand, answering complex questions elevates learning to the application level. Students who are identifying, describing and classifying relationships in terms of higher order rules are participating in higher levels of critical thinking.

Vygotsky (1986) identified stages of abstraction that coincided with the various levels of classification abilities during the process of concept formation. At the most elementary level of concept development, Vygotsky maintained that the child is able to abstract whole groups of properties based on functional similarities (e.g., spoon, plate, cup). The child then progresses to abstracting a single property (e.g., cup and glass for drinking), and eventually to abstracting physical properties, impressions, and meaning or other attributes not tied to physical attributes. Vygotsky's contention supports Bruner's (1960) advocacy of teaching mathematical concepts on a continuum that progresses from the enactive (concrete representations) through the ikonic (pictorial and graphical representations) and finally to the symbolic (characterized by the ability to use words and other symbols as abstractions).

By nature, spreadsheets are much more abstract than databases; their appropriate use is limited to quantitative problem solving (Grabe, & Grabe, 1998; Jonassen, 1996; Newby, Stepich, Lehman, & Russell, 1996). A spreadsheet, when appropriately integrated into instruction, can be a higher level organizational and functional tool that may augment and support the child's conceptual progression through his or her ZPD. As with databases, spreadsheets may also promote increased collaboration and communication among students.

A classroom teacher may effectively and flexibly use a spreadsheet by structuring the content to appropriately reflect the students' ZPDs. For example, younger students could use more concrete experiences as data sources (e.g., by preparing different soap solutions and collecting data on the bubbles produced) while more advanced students might use more sophisticated data (e.g., survey information) from which they may draw abstract inferences. Used in this manner, spreadsheets offer promise both as an instructional medium and as an alternative assessment tool.

**Computer Mediated Communication**

Computer mediated communication (CMC) mediates communication between and among individuals and groups of individuals. Through computer technology and networks, individuals who may be as close as an adjacent room or as geographically distant as in another state or country may readily communicate either synchronously or asynchronously. Synchronous communication requires that participants communicate with each other at the same time and requires that connections be established for both receiving and transmitting information. Presently synchronous CMC requires more sophisticated linkages and is more expensive than asynchronous CMC. For these reasons, access to synchronous CMC is more limited in school settings.

Asynchronous communication does not require that those communicating be connected at the same time. Because the linkages are simplified and less expensive, it is more commonly found in school settings. Asynchronous communication may take one of three forms: (a) one-to-one, as in e-mail or use of search engines; (b) one-to-many, as in bulletin boards; or (c) many-to-many, as in conferencing or listservs. All of these uses permit communications to transcend time and space limitations.
Bednar, Cunningham, Duffy, and Perry (1992) contended that identifying and applying a particular learning theory to instructional design is crucial when using CMC. Khan (1997) developed a list of attributes that characterize CMC implemented over the World Wide Web. Many of these characteristics would lend themselves to meaningful inclusion in designing effective instruction that is consonant with Vygotsky’s learning theories and with the NCTM Standards. Among those attributes that relate to instruction that is consistent with both Vygotsky and the Standards are:

1. Interactivity, which allows students to interact with teachers, peers, and online resources, and when asynchronous, also allows time for reflective learning and self-pacing;
2. Multimedia, which extends the potential to address all learning styles through the incorporation of various media including text, video, animation, audio, graphics and simulations;
3. Open systems, which permit the learner to move outside of his or her immediate environment and to experience other resources, perspectives, and cultures;
4. Distributed resources, which make available relevant information, a variety of materials, and easily attainable multimedia representations of a wide range of subject matter;
5. Collaborative learning, which emphasizes cooperative efforts and communication, and which engages students in the negotiation and validation of meaning;
6. Formal and informal environments, which support instruction on a continuum from instructor driven to student-centered discovery; and
7. Authentic resources, which give students access to real world problems and to real world data.

Bannan and Milheim (1997) discussed general instructional methods that are useful with web-based instruction. The particular methods they presented take advantage of the characteristics listed by Khan, and also are consonant with a Vygotskian approach to instruction. Particular methodologies Bannan and Milheim described that directly support Vygotsky’s theories include: (a) facilitation, (b) inside and outside collaboration, (c) apprenticeship, (d) content generation, (e) role-playing, and (f) modeling. The authors went on to discuss general activities that could be adapted to support group communication, course interaction, discussion, and facilitation. All of these are applicable to mathematics learning.

Asynchronous communication has given rise to more than just ideas; it has enabled a plethora of actual classroom projects sponsored by organizations such as the Eisenhower Clearinghouse, NASA, and PBS. Several of these will be described in the following section.

In 1996 and 1997, among the various virtual field trips offered by NASA, classrooms could enroll in a virtual expedition to Antarctica. While an actual scientific expedition was on site in Antarctica, students in the United States communicated via e-mail with the scientists in Antarctica and with other classes involved in the project. During the project, the scientists e-mailed to participating students not only their personal journals but also reports on activities, weather conditions, and problems they encountered. Students could also access these updates on a NASA-sponsored World Wide Web. The web site included relevant project information on scientific projects in graphics, audio, and video formats. The project provided a contextualized background for extension activities that gave students an opportunity to extend mathematical concepts by manipulating and interpreting data on various levels.

Additional asynchronous communications sites sponsored by diverse organizations, such as Scholastic, Family Math, and various colleges of education are available on the Internet or the World Wide Web. These sites engage students in on-line problem solving projects and in other math-related activities. They permit students to access databases of relevant information. Students may share their ideas and respond to others' participants' ideas about particular questions or topics by subscribing to and participating in a listserv. A listserv is a common mailbox that receives mail and then distributes that mail to all subscribers. Topical sessions may be facilitated and moderated by an expert who guides discussions by providing pertinent questions. In this venue students can participate in collaborative processes to negotiate meaning, to validate methods, to resolve misconceptions, and to form and extend concepts.

As access to asynchronous CMC becomes more widespread among school communities, possibilities for supporting connectivity increase. Partnerships between classrooms could be formed with children in geographically distant schools. The children could share information about their communities, and compare data from their partner schools with their local data. Students could organize these data and develop databases and spreadsheets. Analyzing data from various locations could provide a basis for forming inferences about topics such as topography,
industry, and population. Students would then be able to share their analyses with partner schools in order to validate and/or correct their inferences.

Conclusion

Vygotsky's sociocultural theories appear to support the change in direction of mathematics education that is advocated by the NCTM Standards. The standards call for students to become literate in a mathematical language through which they can articulate, communicate and solve problems. The change in instructional focus from a product to a process orientation finds affirmation in Vygotsky's theories of concept development, and the change in the roles of both teacher and student are consonant with Vygotsky's views of mediation and the ZPD.

 Appropriately implemented, technology offers tools to the classroom that may promote high level thinking skills and support concept development. Asynchronous computer mediated communication allows students access to other individuals, information, and experiences that were previously unavailable in the classroom setting. CMC may be implemented in a number of ways to support the mathematics curriculum through increased collaboration and mediation.

 As access to CMC becomes more universal, new ways of assuring supportive connectivity between and among classroom communities should be explored. Through technology, the perceived and functional distance between learning communities is becoming smaller. This new potential for communicating is important not only for students, but also for the educators who implement the curriculum. As we open the confines of time and space and virtually remove the walls that isolate the classroom, we will move into unexplored ways of communicating, learning, and teaching with technology. In this process, we must keep in mind that the nature of the child does not change and that our efforts to educate should be consonant with a sound theoretical base.

References


NOTICE

REPRODUCTION BASIS

X This document is covered by a signed “Reproduction Release (Blanket)” form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a “Specific Document” Release form.

☐ This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either “Specific Document” or “Blanket”).