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ABSTRACT

Contemporary reform in mathematics education emphasizes interaction among students as a source of learning. There are several insights that sociological theory and research can supply to researchers, staff developers, and curriculum developers in mathematics education who are using small groups. This paper investigates four questions concerning the use of small groups in mathematics classrooms. Group work from a mathematics classroom is presented and discussed. Elements of collaborative situations such as the importance of the assigned task, fostering interaction in small groups, and inequalities within groups are also touched upon by presenting research results from the field. (Contains 31 references.) (ASK)

A Sociologist Looks at Talking and Working Together in the Mathematics Classroom

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Address to the SIG of Mathematics Education

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Contemporary reform in mathematics education emphasizes interaction among students as a source of learning. Silver (1994) describes the reform vision of mathematics classrooms as places in which students engage actively with mathematics they are asked to learn, in which discourse is a prominent feature of classroom activity, and in which personal meaning-making and understanding are important goals of the socially-situated classroom activity. The curriculum and evaluation standards for mathematical communication developed the National Council of Teachers of Mathematics(1989) include the following statement:

> Small-group work, large group discussions, and the presentation of individual and group reports -- both written and oral -- create an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies. Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism, and summarize their discoveries in writing. P. 78.

Following the vision of the reformers, many of the newer curricula in mathematics recommend that teachers use small groups so that students can construct their own mathematical knowledge through talking and working together. For example, the Interactive Mathematics Program is a four-year program of problem-based mathematics that replaces the traditional algebra through pre-calculus sequence and that is designed to exemplify the reform called for by the National Council. Extensive use is made of groups in units such as The Game of the Pig (Fendel & Resek, 1997; Fendel et al., 1996). Other examples of curricula featuring the use of small groups for each level of schooling include the Core-Plus Mathematics Project for high school (Hirsch et al., 1997); Connected Mathematics for Grades 6-8 (Lappan et al., 1996); Number Power for Grades K-6 (Robertson et al., 1996); and Investigations in Number, Data, and Space for Grades 3-4 (Russell, et al., 1994). Davidson (1990) provides an excellent overview of cooperative learning in mathematics.

However, the use of small groups involves fundamental change in the organization of the classroom. For example, when students are working in small groups, the teacher is no longer the sole transmitter of knowledge and the direct supervisor of all student work. Instead, the successful teacher of small groups delegates authority to the students while holding both groups and individuals accountable for their work.

Teachers who are excellent classroom managers do not intuitively know how to change their roles when students are in groups. The students, in turn, have had little experience with taking responsibility for their own behavior in groups. Moreover, students do not naturally construct knowledge when you put them in groups. Their conversation may center on procedural rather than substantive issues. They may obsess over the "right" answer. They may even call each other names. From a sociological perspective, the use of groupwork in the classroom is no minor change, but one that requires careful preparation of both students and teachers.

Beyond these challenges arising from the reorganization of classroom rules and roles, there is an additional challenge that arises from within the small groups. When students work together, sharp inequalities appear in the rates of participation and influence of the different members of the group. Some members do too much of the talking and too much of the final decision-making. Others participate very little and what they do say appears to fall on deaf ears. This pattern is characteristic of cooperative groups in mathematics classrooms as well as in classrooms using other subject matters. One can observe these inequalities in interaction in groups of second graders, as well as in groups of middle school, high school, or adult students.

In this paper, I will answer four basic questions concerning the use of small groups in mathematics classrooms:

1. What can one expect to result from a process of students talking and working together?
2. How do these results depend on the nature of the task the teacher assigns to the group?
3. How can the teacher foster high-level discourse in small groups?
4. What are the causes and consequences of inequalities in participation and influence and what can be done about these inequalities?

My answers to these questions come from theory, from research results based on classrooms using complex instruction, and from practical experience acquired in working with groups in the mathematics classroom. Complex instruction (CI) is a set of instructional strategies that enables teachers to teach at a very demanding intellectual level in classrooms that are multilingual, multiethnic, and are heterogeneous academically. Since 1979, the staff of the Program for Complex Instruction has carried out a vigorous program of research and development. The strategies developed by the Program have been widely disseminated in the United States, in Israel, Scandinavia, and in Europe.

Process of talking and working together

Let me start by painting a picture of students talking and working together in a complex instruction classroom. Students are using each other as resources in heterogeneous groups of four or five. They help one another to understand the activity card which poses discussion questions and requests a specific group product. Learning tasks in CI are challenging, open-ended, and require many different intellectual abilities (not just reading and computing). The group product typically requires creative input and exchange from every member of the group.

To illustrate such an open-ended, problem-solving process, the following is an activity from a seventh grade unit on "Area and Perimeter," adapted at Stanford:

> We have three rather large tables with the following dimensions (in meters): 2.5L X 1.2W X 0.75H. How many guests can we invite to a festive Thanksgiving dinner?

Rachel Lotan(in press) describes the deliberative process for the group tackling this problem as follows:

> ---before they can figure out the number of guests that can be comfortably accommodated, the groups have to make some decisions. Do the hosts prefer a sit-down or a buffet-style dinner? Will they place the tables in one straight line, in a T- or in a U-shape? How much elbow room (specified in centimeters) will they allow for each guest? Will youngsters need less room than adults or possibly more? Only after clarifying. deliberating, drawing, or building models while carefully considering the dimensions of the tables, are students

ready to proceed and calculate the answer to the problem as posed on the activity card. Although each group presents one correct answer, *the answer* varies from group to group, since each group has specified different pre-conditions. Furthermore, the process of problem solving continues as groups listen to each others' reported solutions and discuss the advantages and disadvantages of prior decisions.

In addition to the group product, every student must write up an individual report. Students may help one another in writing up this required report.

In the group, each student is assigned a different role such as facilitator, reporter, materials manager, measurer, safety officer etc. Roles vary somewhat with subject matter and age of students, but they are always procedural rather than substantive. In other words, each student is expected to make a contribution to the solution of the group's problem in addition to helping the group move through the task by playing his or her role. Roles always rotate so that everyone gets to play every role.

After an initial whole class orientation, the class breaks up into groups and goes to their learning stations where there are a variety of manipulatives. Each group completes a different task with a different set of materials.

> "Group activities of a CI unit are organized around a big idea or a central concept of a discipline. As groups of students rotate through the activities they have multiple opportunities to grapple with the concept, to understand the idea in different settings, and to recognize its multiple representations." R. Lotan, in press.

While students are in groups, the teacher is circulating around the room with a clipboard taking notes. She does not intervene very often except when a group is hopelessly lost or in severe conflict. She insists that the group take responsibility for what is happening. She holds them accountable through insisting that they play their roles, by expecting a respectable group product that will be presented to the class as a whole, and by checking individual reports.

The class ends with a wrap-up in which the reporters with the assistance of their groups, present their group product which may be a poster, a role play, or a physical model. During the course of the class discussion, the teacher provides feedback and asks questions. She makes connections with the underlying concepts and directs students as to what they should concentrate on solving when they get to this task in turn.

In their first year of implementation, teachers use multiple ability curriculum materials that have been specifically constructed or adapted for the complex instruction classrooms. In elementary schools, we have used *Finding Out/Descubrimiento* (DeAvila & Duncan, 1982), a bilingual curriculum using concepts from mathematics and science. In the middle schools, Ruth Cossey developed and adapted special units for complex instruction in mathematics, stressing the use of oral and written mathematical communication. The extended example above is taken from her materials (Cossey, 1997).

Student Interaction and Learning

Given the setting I have just described, CI researchers have consistently observed very strong relationships between the rate of talking and working together and gains in achievement, including mathematics (Cohen, Lotan, & Holthuis, 1995). For example, the higher the percentage of students talking and working together in an elementary school complex instruction classroom, the greater were the average gains in CTBS (California Test of Basic Skills) scales for computation and for concepts and applications (Cohen, Lotan, & Leechor, 1989). Similarly, there was a strong and significant correlation between the percentage of students talking and working together in the middle school and gains on a test of mathematical communication (Spearman R = .77)(Cossey. 1997). The test Cossey used was a set of 9 items selected from the test developed by the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) Project (Silver & Lane, 1995).

These findings on the relationship of interaction to learning also hold at the individual level. In elementary classrooms using the Finding Out/Descubrimiento curriculum, the more that a student talked and worked together the higher was his or her individual gain score in CTBS computation and concepts and application (Cohen, Lotan, & Holthuis). Similarly, Cossey (1997) found that the more air time individual seventh and eighth grade students had in mathematics conversations, the more they gained on the QUASAR test. Finally, in a recent study of complex instruction in elementary classrooms in Israel (Ben Ari, in press), interaction was related to gains in individual cognitive development, as measured by a standardized test of reasoning.

However, this robust relationship between interaction and achievement will not hold under all conditions. In her studies of collaborative seatwork in mathematics, Webb (1983, 1991) has never found a consistent relationship between interaction and achievement.

Importance of the Assigned Task

In order to understand this apparent contradiction it is necessary to analyze differences in the task and task instructions in the two settings. In Webb's studies, students were given a typical math problem that one might find in a text and told to help each other. This is what I call collaborative seatwork. It does not require a two-way exchange of ideas. Only the students who know the answers will need to explain well to those who don't. This is undoubtedly why Webb finds that the two major predictors of learning gains are the giving of explanations (a positive predictor) and failing to receive an explanation after asking for one (a negative predictor).

In an analysis of productivity of R & D teams in industry and groups of children working together in classrooms, Cohen and Cohen (1991) conclude that it is *only* under the conditions of a true group task and an ill-structured problem that interaction is vital to productivity. The collaborative seatwork used in the Webb studies meets neither of these conditions. In contrast, complex instruction tasks fit the definition of a true group task. They require resources (information, knowledge, heuristic problem-solving strategies, materials, and skills) that no single individual possesses so that no single individual is likely to solve the problem or accomplish the task objectives without at least some input from others (Cohen & Arechavala-Vargas, 1987). Complex instruction tasks are also ill-structured problems in that they are open-ended non-routine problems for which there are no standard procedures. In contrast, the tasks in Webb's studies are computational or require the application of an algorithm. Thus the importance of interaction for learning holds only for specified kinds of tasks.

Figure I contrasts collaborative seatwork and complex instruction.

In designing classroom settings for mathematics curricula, the following proposition will prove useful :

> Given a true group task requiring two-way exchange and a problem with an ill-structured solution, more talking and working together leads to more learning.

There are two major implications of this proposition for mathematics educators and researchers. When studying the source of learning in mathematics classrooms using groups, the first thing to do is to analyze what kind of a task has been given to the group. Many of the new constructivist curricula are using true group tasks and problems with ill-structured solutions. However, in the more typical case where cooperative learning is being used in mathematics classrooms, it is closer to the collaborative seatwork model.

Having analyzed the type of task at hand, the second implication of this proposition is that staff development should vary accordingly. There will be different features of the small groups that will predict the learning in the two types of tasks. In studying or in designing constructivist classrooms where groups are working on true group tasks with problems with ill-structured solutions, it will be of vital importance to foster interaction among the students. If the analysis reveals a collaborative seatwork situation, then it would be wise to follow Webb's recommendation and teach students how to give elaborated explanations. I would moreover recommend that students be taught that they have the duty to assist those who ask for help.

Fostering Interaction in Small Groups

How does one foster interaction among the students who are working on uncertain tasks? The sociological answer to this question is that with the increase in uncertainty of the task from

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the student's perspective, the teacher must delegate authority and there must be increased use of lateral relations, i.e. interaction among the students. The research on complex instruction has found both at the elementary and middle school levels that if the teacher fails to delegate authority, there will be less interaction between the students (Cohen, Lotan & Leechor, 1989; Cohen, Lotan, & Holthuis, 1995). With less interaction there will be less organizational effectiveness, or in this case learning.

I often find failure to delegate authority while groups are in operation in classrooms using challenging group tasks. The teacher may be at the front of the room supposedly guiding the discovery process as each group works with identical materials in identical ways, but actually permitting little spontaneous interaction or discovery. Or the teacher frequently interrupts the work of the groups telling them better ways to accomplish their tasks. Some teachers routinely run to the rescue of any group at the first sign of trouble. And other teachers try to make things simpler and clearer for students by giving them incredibly detailed step-by-step instructions. Any of these strategies will have the net effect of diminishing group interaction and therefore learning gains.

Changing the Teacher's Role

In complex instruction, we stress the changed role of the teacher. Teachers learn to delegate authority to groups of learners without giving up accountability. Staff developers stress the "No hovering" rule; teachers should avoid leaning over groups, manipulating materials, or telling the group in detail how they should be going about their task. They should, if possible, avoid direct instruction while groups are in operation. The time for direct instruction is in the orientation prior to groupwork or during the wrap-up period. With older students, the groupwork can no longer carry the main burden of instruction. The groupwork used in the complex instruction model is ideal for developing a grasp of abstract concepts of mathematics, but students will often need preparation with whole class methods or with group methods that are less open-ended and are designed to provide practice on particular skills.

Activity Cards. Preparation of activity cards for the group can help the teacher foster interaction while delegating authority. The activity card substitutes for verbal instructions from the teacher, while the group takes the responsibility for everyone understanding the activity card. Activity cards are not recipes or step-by-step instructions, but raise general questions or problems, leaving it up to the group to invent strategies for solutions. In the middle school, complex instruction curricula supply additional intellectual resources for the group with a resource card and resource material. With the activity and resource cards available, the group has less reason to turn to the teacher for additional information. Students' past experience has led them to expect that the answer to the questions raised on the activity card will be available somewhere in the written material the teacher has provided, but they soon learn that this is not the case.

Accountability. Delegation of authority will not work without accountability of groups and individuals. I sometimes observe teachers who accept anything a group produces with enthusiasm. Delegating authority to groups should not be confused with uncritical acceptance of the group product. Students need specific, critical, but constructive feedback on their efforts. The next group to attempt this task needs guidance on where they should focus their efforts to do a better job. In addition, individuals need feedback on their individual reports.

I am currently working with a group of teachers on increasing accountability by having the

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teachers develop rubrics for evaluation of group and individual products. Students learn these rubrics so that they have clarity on the standards for evaluation. They evaluate their own individual reports and group products. Teachers can see where individual students are having real difficulty or where the entire class is having a problem with some of the objectives. One of the advantages of this process is that teachers grapple with what they expect students to gain from confronting an uncertain and challenging task. Careful observation of groups lacking certainty about what they are supposed to gain from open-ended tasks will reveal a concentration on procedural rather than substantive discussion and sometimes a level of frustration that will cause group members to turn on one another.

Roles. The use of roles is a major technique of delegating authority to groups; roles help manage the group process and can do much to foster interaction. Unfortunately, the use of roles is rather widely misunderstood as a restriction of intellectual autonomy on the part of the students. The contrary is actually true. Because roles take over much of the managerial function of the teacher, the absence of roles makes it necessary for teachers to interfere in the operation of the groups far more frequently.

Teachers delegate authority to groups by assigning roles that help manage the group process. One key role is that of facilitator who sees to it that each person gets the help that he or she needs and that everyone participates. The facilitator can be the only person in the group who has the right to summon the teacher. This occurs only after it becomes clear that no one in the group has the answer to the question. Facilitation is a frequent and natural behavior for teachers; much of our staff development focuses on helping the teacher to see that if properly trained, encouraged, and held accountable, groups can solve problems for themselves.

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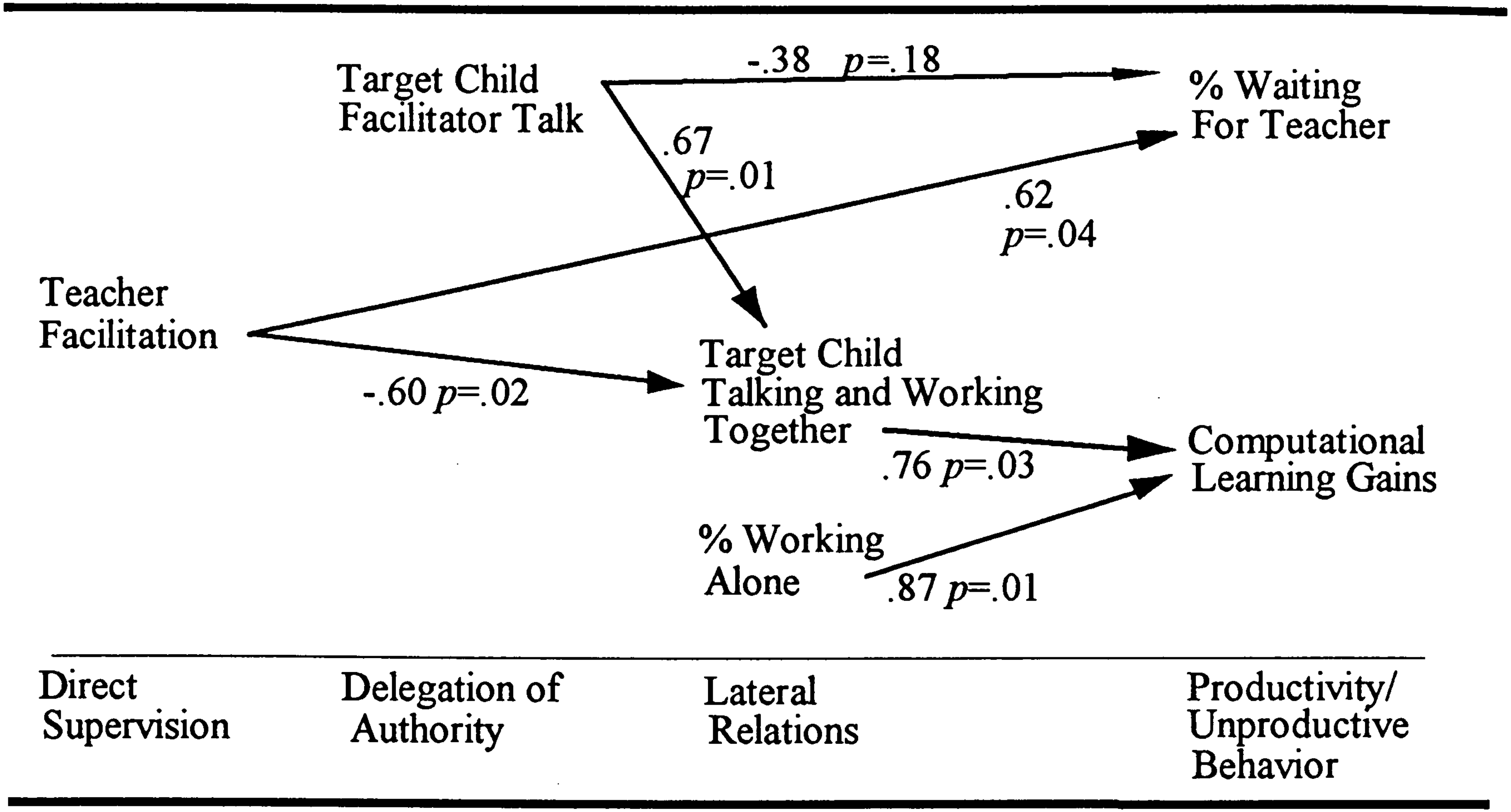
Another role is that of reporter, a role that requires careful development. If left to their own devices, groups will not even discuss the final report, but will leave it to the reporter to manage on his or her own. The presentation of a group product should involve the group in the preparation as well as in the presentation itself. The job of the reporter is to coordinate these efforts and to lead the discussion of what will be relayed to the rest of the class.

Materials manager is another key role in classes where groups are using a variety of manipulative materials. The use of this role avoids having a large number of students floating around the classroom getting materials. The materials manager can also direct clean-up activities.

Everyone has a role and roles rotate. I am recommending the use of procedural roles such as Materials Manager, Reporter, Facilitator, or Harmonizer and not substantive roles such as computer or graphics specialist. Procedural roles do nothing to micromanage the thought processes of the group, whereas a division of labor with substantive roles not only specifies who is supposed to have the decision rights over particular features of the task, but does away with the necessity for much of the desired interaction.

Studies of complex instruction direct evidence on the effectiveness of the student role of the facilitator and the ineffectiveness of the same role played by the teacher. Figure 2 illustrates this paradox in a path model based on systematic observations of teachers' behavior while students were in groups and on timed observations of target students as they worked in groups (Ehrlich & Zack, in press). In this path model, Zack hypothesized that when teachers did less facilitating for the students, while more students simultaneously facilitated for one another, there would be an increase in the amount of students' talking and working together. Student facilitation was a positive indicator of the theoretical concept of delegation of authority, while teacher facilitation

Figure 4.1: Teacher and Student Facilitation, Implementation, and Productivity (Zack, 1988)



was a negative measure. In Zack's view, teacher facilitation should lead to a higher percentage of students waiting for the teacher. To the extent that the teacher is telling the students how to do their task, students will believe that the teacher is the only source of authoritative knowledge. They will therefore seek her help before moving on with their task. To the extent that the group becomes an alternative source of knowledge, there should be less use of hierarchical channels.

For this analysis, Zack aggregated observations of target students to the classroom level. The analysis is based on 13 elementary school classrooms using the Finding Out/Descubrimiento curriculum. To measure student facilitative talk, Zack scored target students for talk such as directing others what to do, asking one person to help another, making general announcements to the group, and asking questions regarding members' progress. Teachers were scored as facilitating if they tried to help the students move through their task while the students were in groups. Of special interest to mathematics educators is the inclusion in the path model of class-level performance on CTBS computation.

As predicted, there was a strong negative effect of teacher facilitation (path coefficient of -.60, $p = .02$) and an equally strong positive effect of student facilitation (path coefficient of .67, $p = .01$) on the average rate of target children talking and working together. Also, as hypothesized, the teacher's facilitation had the effect of increasing the percentage of students waiting for the teacher (path coefficient of .62, $p = .04$). As we always find in complex instruction settings, the rate of talking and working together had a strong positive effect on learning gains, in this case the computation subscale.

The path model demonstrates several important points for this discussion of delegation of authority. Firstly, the students' use of the facilitator role acts to foster interaction among the students which in turn leads to improved mathematics test scores. Secondly, when the teacher tries to play the role of the facilitator, it has the opposite effect of lowering the rate of interaction and the learning outcomes. Thirdly, the teacher's insistence on playing the role of the facilitator has the additional side effect of increasing the number of students waiting for the teacher.

Norms for Discourse. Another way to foster interaction is to train students in advance in skills for high-level discourse. One cannot expect high quality conversations to occur in groups in mathematics classes without a lot of work. If the goal is for students to use patterns, conjectures, and fully articulated reasons for their ideas, students will need some training. I think some of the curriculum developers today have naive and romantic notions about student interaction. Students are placed in groups and given problems to solve with no preparation for how to participate in a creative problem-solving process. As a result, an observer is more likely to hear one student ask another," What's the answer dummy?" rather than an impressive question such as "What would happen if---?"

Working with the students to improve the level of discourse is time well-spent. Ruth Cossey(1997) found that the more that seventh and eighth grade students were exposed to high quality mathematical conversations in their groups, the greater were their gains on the QUASAR instrument. She measured the quality of conversations with classroom observers who scored for behaviors such as pattern-seeking, conjectures, observations, and giving reasons for ideas.

These features of discourse are behaviors and students can learn these new behaviors through practice and reinforcement. "Giving reasons for ideas" can become a norm, a rule for behavior. If the behavior becomes an internalized norm, students will feel that they ought to provide reasons and will demand reasons from others in the group.

In addition to the norms for discourse, some of which may be specific to the mathematics classroom, students also need to learn norms specific to the cooperative setting such as "You have the right to ask any one else for assistance" and "You have the duty to assist those who ask for help." (For a full discussion of norms and skillbuilders, see Cohen, 1994).

In complex instruction, we develop norms (rules for behavior) in groups by the use of skill builders. In designing these preparatory activities, we employ the principles of social learning theory developed by Bandura (1977).

(1) The new behavior is given a name that is used consistently.

(2) The students are given consistent practice with the new behavior.

(3) The teacher reinforces the occurrence of the desired new behavior during the course of the skillbuilder.

Figure 3 presents Rainbow Logic, an adaptation of an activity developed by Family Math (Stenmark, Thompson, & Cossey, 1987) that we use as a skill builder for mathematics classrooms. "Discuss and decide" and "Give reasons for your suggestions" are new norms that are introduced in the exercise. Students have adequate practice with the new behaviors as they discuss what questions to ask in their group. Moreover, the use of an observer in the group who watches and reports how well the group is discussing and deciding allows students to recognize the desirable new behaviors when they occur. The teacher who circulates while the groups are in operation is in a position to provide reinforcement for the new behaviors as she or he overhears them in the group. After the skill builders, when groups are actually dealing with problems in mathematics, the new norms are further reinforced by teachers who move around with clipboards taking notes on group processes and providing feedback during wrap-up.

Inequalities within Groups

Use of small academically heterogeneous groups and collective tasks in the mathematics classroom will activate status problems. Status problems occur when some students dominate the group discussion, monopolize the manipulative materials, and/or take over the work and the decision-making of the group. Within the same group other students fail to participate or, if they do attempt to make a contribution, are ignored. Sometimes the group physically shuts out one person and acts if he or she were invisible. Teachers who have used small groups in their classrooms often report observing this problem. They ask,"How can you prevent one student from taking over and doing all the work?" or "What do I do about the student whom no one wants in the group and who says practically nothing during the discussion?"

Systematic observation of small groups of students working within classrooms reveals a self-fulfilling prophecy at work. Students who are not seen as good at math and science and students who are infrequently chosen as friends talk less in the small groups than students who are seen as good in math and science and who are frequently chosen as friends by their classmates. Even with multiple ability open-ended tasks, such as those in the Finding Out/Descubrimiento curriculum, researchers found that students who were high on academic and peer status talked more and learned more than low status students (Cohen, 1984).

This phenomenon is called status generalization and the process is carefully described by Status Characteristic Theory (Berger, Cohen, & Zelditch, 1966; Berger, Cohen, & Zelditch, 1972). Within the class, there is a rank order on academic and peer status in which the students perceive that some are high and some are low on these status characteristics. Moreover, everyone agrees that it is better to be high than low. Other status characteristics that can affect group behavior in the mathematics classroom are gender, race, and ethnicity. These rankings on status exist prior to the group interaction; and they have the power to affect expectations for competence at the task at hand. High status students are expected to be more competent than low status students. Whether the task is relevant to the particular differences in status or not, people use the status information to organize their expectations for competence on a new collective task. Within the group, the high status students actually talk more and as a result learn more. Thus expectations based on status become fulfilled in differential behavior in the group, behavior that confirms the initial difference in expectations for competence. Researchers in complex instruction have consistently found these status differences within small groups working on challenging collective tasks in the classroom (Cohen, in press). Careful observation of groups at work in any classroom will reveal this process of status generalization.

What can be done about status generalization? With a theoretical understanding of the nature of these problems, it has been possible to design interventions to modify differential expectations for competence. Unlike what many people think, these problems are not a consequence of low self esteem or low self concept of the low status student. Because the origin of this inequity lies in different expectations for competence held by and for high and low status students, it is necessary to treat the expectations of classmates as well as expectations that the low status students hold for themselves. Two of the interventions that have proven effective in elementary classrooms are the Multiple Ability Treatment and Assignment Competence to Low Status Students.

In the Multiple Ability Treatment, the teacher modifies expectations for competence by convincing students that many different competencies or abilities are necessary for a successful group product. Moreover, the teacher specifically says, "No one will be good at all these abilities. Everyone will be good at some of the abilities." In this way students develop a *mixed* set of expectations for competence for self and other rather than a uniformly high or uniformly low set of expectations.

In the other intervention, teachers observe low status students as they work together with others in the group. When the student actually does demonstrate competence, the teacher points this out to the group and reminds the group of the relevance of this competence to a successful outcome for their task. Thus the low status student is assigned a high level of competence on what is called a "specific status characteristic" that is directly relevant to the collective task. Cohen and Lotan found that the more frequently that teachers used these two treatments the higher were the participation rates of low status students (Cohen, & Lotan, 1995). (For a detailed discussion of the theory and research behind these two interventions, see Cohen, in press and Cohen & Lotan, in press; for a practical discussion addressed to teachers and staff developers, see Cohen, 1994). These treatments will only work with tasks that really do require multiple intellectual abilities -- where different students can make different kinds of contributions.

Conclusion

In summary, there are several insights that this sociological theory and research can supply to researchers, staff developers, and curriculum developers in mathematics education who are using small groups. I have emphasized the importance of a careful analysis of the nature of the task assigned to the groups. The process of talking and working together will be far more critical for true group tasks with ill-structured problems than for collaborative seatwork.

For those curricula where interaction is essential, there are tremendous implications for the organization of the classroom and for the teacher's role. Teachers must learn how to delegate authority, how to hold groups accountable, and how to teach students the skills for cooperation and the skills for mathematical discourse.. Roles and norms are indispensable in a system that involves delegation of authority.

This preparation of teachers cannot be done in a three-day workshop nor during "tired time" after school. What is particularly challenging for the mathematics educator is that all of this should take place *in addition to* the preparation in mathematical content and the careful study of new curricular materials. Not only should workshops or seminars continue for several weeks, but clearly, classroom follow-up is a necessity for this level of staff development.

Available curricula may have to be adapted so as to assist teachers in delegating authority and so as to permit the successful treatment of status problems. The creation of activity cards will assist groups to solve problems on their own. In addition, status problems cannot be successfully treated if tasks only require single mathematical skills and abilities. When the teacher tries to use the Multiple Ability Treatment, the students will realize that "Everyone will be good at some relevant abilities." is obviously untrue. Thus tasks must truly involve multiple skills in which different students can make different kinds of contributions. In addition, students should have multiple opportunities to understand abstract concepts.

Finally, if all students are to benefit from group discussion, then status problems must be treated directly. Assuming that the tasks have the requisite multiple ability character, it takes considerable time with individualized follow-up for teachers to learn how to modify expectations for competence. Neglect of status problems is ultimately demoralizing for the conscientious teacher, to say nothing of the students who feel they are doing all the work or the students who are left out of the discussion. If the mathematics educator and the teacher wish to set the stage for the co-construction of mathematical knowledge, then it must be a process to which all students have access.

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FIGURE 1

CONTRAST OF COLLABORATIVE SEATWORK

AND COMPLEX INSTRUCTION

COLLABORATIVE SEATWORK	COMPLEX INSTRUCTION
RIGHT ANSWER OR ALGORITHM	PROBLEM WITH AN ILL STRUCTURED SOLUTION
TASK CAN BE DONE ALONE	TRUE GROUP TASK
INDIVIDUAL PRODUCT	INDIVIDUAL PRODUCT
STUDENTS TOLD TO HELP EACH OTHER	STUDENTS TAUGHT HOW TO HELP EACH OTHER
NO GROUP PRODUCT	GROUP PRODUCT
ONE-WAY EXCHANGE	TWO-WAY EXCHANGE

RAINBOW LOGIC

OBJECTIVE

This is an exercise developed by the Family Math program to give the students practice in communicating their deductive thinking and spatial reasoning. Students must deduce through a series of questions the pattern of a 3x3 color grid. The grid is constructed using rules about the permissible ways in which squares may be placed. Within those rules the group must discuss and decide on the best questions to ask of the Grid Designer.

MATERIALS:

Colored paper squares for each player
4 each of each of 4 colors (more than needed for solution)
3x3 grids

PROCEDURE

For the first round, the teacher may be the Grid Designer. A group can be selected to demonstrate the exercise. The rest of the class can gather round to watch. After the first round, students should take turns being the Grid Designer in their separate groups. Group sizes can vary from 3 to 5. The Person who is Grid Designer can also play the role of observer.

The Grid Designer prepares a secret 3x3 color grid, using 3 squares of each color.

> Rule: All of the squares of the same color must be connected by at least one full side. See Figure A.3 for examples of permissible and impermissible grids.

The goal is for the players to be able to give the location of all colors on the grid after as few questions as possible. Therefore the group should discuss and decide before asking the gridkeeper a question. In the course of the discussion students should share the logic of their thinking. Why will this question get the maximum amount of useful information for solving the problem? During this discussion, there are two new behaviors that the students should learn:

DISCUSS AND DECIDE

GIVE REASONS FOR YOUR SUGGESTIONS

Rules for asking and answering questions

Players ask for the colors in a particular row or column (rows are horizontal. columns are vertical.)

The Grid Designer gives the colors, **but not necessarily in order.**

Each player should use a grid and colored paper squares to keep track of the clues. Squares may be put beside the row or column until exact places are determined.

NOTE: if this seems too easy for the class, try playing with a 4x4 grid with the same rules.

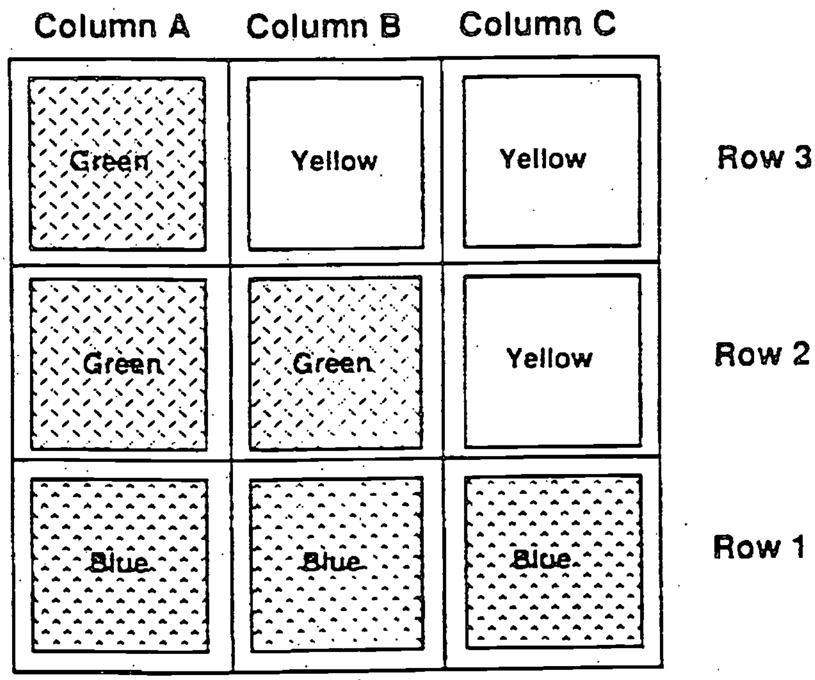
Discussion

The observer(Grid Designer) for a particular round should keep track of how often people gave reasons for their suggestions. The observer should also watch the character of the discussion to see if people really discussed before they came to a decision. Perhaps one person jumped in and asked the question of the Grid Designer before everyone in the group was heard from or before a controversy was actually resolved.

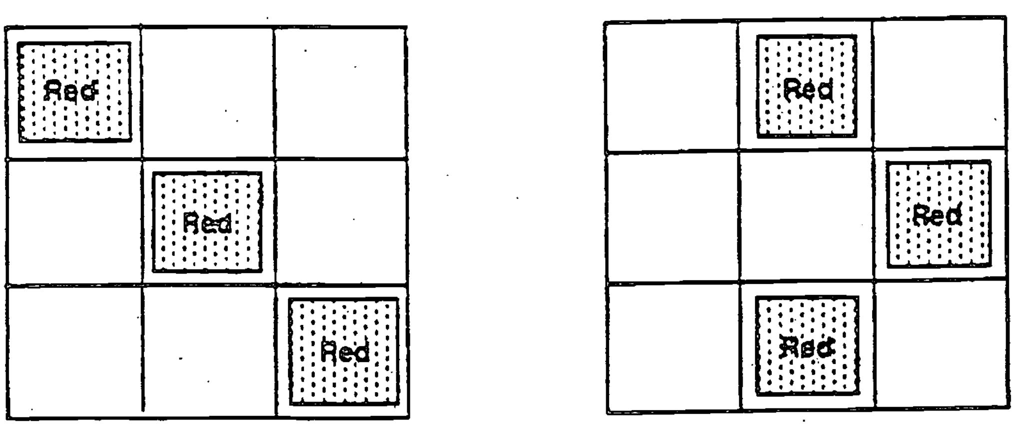
After most groups have had the chance to complete a few rounds of the excercise. the teacher should stop the action and have observers from each group report what they have seen. Then the class may discuss how to improve the process of discussion and the process of giving reasons. Let the class proceed to give everyone else a turn at being Grid Designer and Observer. After they have finished the final round, ask the observers to come up an form a panel, to discuss whether they heard improved discussion and giving of reasons in the group in the second part of the lesson. Alternatively, students could write about what they have learned concerning the three cooperative norms and how they fit into groupwork in their subject matter.

FIGURE A.3
RAINBOW LOGIC

Example of a Secret Grid



Patterns Like the Ones Below are Not Allowed



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