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ABSTRACT

Six- to eight-year-old children (N=42) who were identified by their teachers as within the average range of ability in mathematics were individually tested on three different mathematics tasks. On the flashcard task and the nonverbal task where children replicated the number of buttons placed under a box, the same 14 addition problems with sums up to 20 were used. The third task investigated children's understanding of the associativity of length where they had to determine if string segments of various length, number of cuts, and different spatial orientations were of equal length. These data were analyzed using Rasch statistics which places both the items and the children along a hierarchical scale of difficulty. The results indicated that within each of the tasks there existed a sequential construction of increasingly complex cognitive abilities which was measured by providing the correct answers and the strategy types used. Further, a comparison between the flashcard, nonverbal, and associativity of length tasks elicited a developmental relationship between the ability to generate more sophisticated strategies to solve mathematics problems and the evolution of operational structures as measured on the Piagetian associativity task. These findings were discussed relative to the dispute as to whether mathematical knowledge consists of the internal construction of relationships or the mapping of standard mathematical symbols onto a preexisting mental model of number and number transformation. Remedial implications of the findings follow. Contains 49 references. (Author)

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Children's Construction of the Operation of Addition

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Six-to eight-year-old children (N=42) who were identified by their teachers as within the average range of ability in mathematics were individually tested on three different mathematics tasks. On the flashcard task and the nonverbal task where children replicated the number of buttons placed under a box, the same fourteen addition problems with sums up to 20 were used. The third task investigated children's understanding of the associativity of length where they had to determine if string segments of various length, number of cuts, and different spatial orientations were of equal length. These data were analyzed using Rasch statistics which places both the items and the children along a hierarchical scale of difficulty. The results indicated that within each of the tasks there existed a sequential construction of increasingly complex cognitive abilities which was measured by providing the correct answers and the strategy types used. Further, a comparison between the flashcard, nonverbal, and associativity of length tasks elicited a developmental relationship between the ability to generate more sophisticated strategies to solve mathematics problems and the evolution of operational structures as measured on the Piagetian associativity task. These findings were discussed relative to the dispute as to whether mathematical knowledge consists of the internal construction of relationships or the mapping of standard mathematical symbols onto a preexisting mental model of number and number transformation. Remedial implications of the findings followed.

Children's Construction of the Operation of Addition

An examination of the development of cardinal principles of number and operational reasoning in preschool children has frequently led to results that are held to contradict Piaget's (1965, 1987) claim regarding the construction of logical-mathematical knowledge in young children. Specifically, it has been argued that very young children are able to hold number as invariant when the order of objects is changed (e.g., Gelman, 1972) and to transform object sets using operational reasoning (e.g., Gelman, 1972; Levine, Jordan, & Huttenlocher, 1992). This study will argue that the methodologies generating these conclusions lack sufficient examination of children's forms and structures of mental activity.

Piaget (1965, 1987) argued that mathematical operations are biologically based systems of mental activity that are "entirely built up from the coordinations of action-schemes and from ensuing coherent, deductive modes of reasoning . . . " through the self-regulated mechanism of equilibration (Sinclair, 1990, p. 27). Cultural practices serve as stimuli for children to generate mathematics problems (Saxe, 1988) in which the truth of one's actions are verified in a transforming reality (Sinclair, 1990). However, the expansion and regulation of logical mathematical activity does not have as their source linguistic communication and/or diligent perception. Rather, language and cultural practices serve as tools to guide children's thinking by encouraging reflections relative to what their forms and structures of mental activity can logically coordinate and differentiate (e.g., von Glasersfeld, 1990).

Operational reasoning (i.e., the logical relations of compensation, identity, and reversibility) is preceded by a period during which children reflect on their counting activity using one-to-one correspondence. During this stage of development, observable object features are attended to (affirmation), but children's ability to infer what is not seen (negation) is minimal, nor can they anticipate reversible actions. This limitation is due to children's inability to group the activity of their actions simultaneously within an enclosed, organized structure of mental activity. Thus, their field of attention consists of centrations on the whole over the parts (overgeneralization) or of fixations on individual parts over of the whole (lack of coordinated differentiation).

For example, Piaget, Coll, & Marti (1987) examined children's ability to conserve addition in a task where children are presented with two identical strings that are equally segmented into four colors such that each color represents a unit element of the whole string. The pieces of string are cut and placed in various spatial orientations to represent the same cognitive complexity inherent in the operational structures of addition (see Figure 1). Specifically, the elements are the invariant line segments of each color (i.e., unit interval), the parts the various subassemblies of elements or the subsets (i.e., unit intervals that combine to form subgroups), and the whole is the sum of all elements (i.e., set of all subsets). Children whose organizing activity is not yet coordinated within an enclosed, organized structure attend to the difference in number of cuts or segment length, but they fail to simultaneously attend to the effect of the cuts on the length of the various segments or vis versa. Thus, there is no additivity because the cuts do not serve to delimit length (i.e., create differentiated parts to be added together).

As children reflect on their counting when solving meaningful problems, their organizing activity expands, enabling them to assimilate the logic of the conserved cardinal number. As a cardinal number, "as many" refers to the union of equivalent units (elements) found in a set (part) (Piaget, 1965). Each member (element) of the set (part of the whole) can now be understood as included hierarchically in the number(s) that preceded it (e.g., 3 consists of the inclusion of 1 and 2) (Kamii, 1985). However, while the recoordination of schemes results in partial negation, there continues to be a lack of coordination between the parts to each other and the whole and the simultaneous ability to deduce more composite unit structures from the elements contained within the parts. Thus, while, children may solve mathematical problems, their thinking is limited to simple, successive, stepwise, progressions (Piaget, 1965; Chapman, 1988). In the fence task, children at this level perceive that the cuts both modify the relations of continuity and mark boundaries (i.e., unit parts) that delimit length. However, the whole is not conserved because children do not logically coordinate an increase in the elements of one fence part with a decrease in the others.

Children's continued reflection and evaluation upon their actions on objects, results in schemes recoordinating themselves to the degree necessary for the apprehension of number as a unit. Children now desire to seek out and reflect upon relationships between other parts in relation to the expanded whole that is created by the coordination of these parts. This cognitive activity marks the onset of <u>operational</u> logic. Numbers can be thought of as continuous points along a line and the operations of addition and subtraction (which evolve simultaneously) are reflected upon as an increase or decrease of a related whole (Chapman, 1988; Vergnaud, 1982). In the operation of addition, there is one level of abstraction which is additive (i.e., the inclusion of each unit in each succeeding unit, where the groups are compared successively). In the fence task, children conserve the whole by adding the parts as well as the elements contained within the parts despite their redistribution. Piaget, Coll, & Marti (1987) argued that all conserving addition is simultaneously associative because it conserves the sum of all parts. Specifically, differences between parts result only from the displacement of elements that are considered in the process of changes in distribution.

Contrary to Piaget's assertions, Gelman & Meck (1986) and Gelman, Meck, & Merkin (1986), argued that preschool children can enumerate sets of objects and understand that the final count of an object set represents the number of objects contained in a set regardless of the order of enumeration. Further, in a conservation task where the arrays to be compared were small in number and where training conditions were provided, preschool children conserved number despite differences in the length and density of the arrays (Gelman, 1972). Gelman (1972) attributed children's inability to conserve on traditional Piagetian tasks to the redirection of their attention to the action of lengthening and shortening the arrays performed by the experimenter. Such redirection distracted them from their initial focus on number. Specifically, when external reinforcement is used to train children to attend to the difference in the number of objects rather than to differences in length or density (Gelman, 1972), children attend only to number and not to length or density. Gelman concluded that preschool children understand that the perceptual dimensions are irrelevant and thus hold number as invariant. Gelman additionally argued that children understood the transformation as subtraction because they answered directed questions about how many mice were on the plate before and after the transformation.

There are two concerns that warrant discussion with regard to Gelman's conclusions. First, when children require external training and reinforcement to direct attention, then they are learning fragmented skills removed from the logic orders of their reflective activity (Kamii, 1985; Sinclair & Sinclair, 1986). Because these skills are isolated from the dynamic, biologically-based structures of the knower, the skills are static in nature (Cobb, Yackel, &

Wood, 1992) and cannot be generalized (Smedslund, 1961). Mathematical equations have limited meaning, purpose, and generalization if they are not supported by a mental framework that can coordinate (i.e., reflectively abstract) the relationships between the elements and the parts and the parts to the whole that are inherent in mathematical equations (Kamii, 1985; Steffe, 1988, 1992).

Second, although children provided the correct count when objects were taken away, there was no evidence that children were reflecting on the number change using operational structures or understanding number as invariant. In fact, when children four years of age were asked to predict a reverse count on object sequences that were successfully enumerated, a statistically significant proportion of children were unable to apprehend what a reverse count would yield (Baroody, 1993).

Gelman and her associates additionally proposed that young children are unable to solve conventional equations because they are not yet able to "map" the terminology of mathematical language (e.g., less, more, same) onto their existing mathematical structures (Gallistel, 1990; Gelman, 1972, Gelman & Gallistel, 1978; Gelman & Meck, 1992). Levine et al., (1992) concurred with Gelman's premise, stating that children's earliest experiences adding and subtracting are based on experiences in which object sets are combined and separated. With development, the use of physical referents to represent number quantities present in calculation is gradually replaced by an accompanying increase in the ability "to create representations for quantities referred to linguistically and/or by an increased reliance on memorization of number facts and schooled calculation algorithms" (p.102). These findings were later elaborated upon with the suggestion that the start point for simple calculation in preschool children involves the mapping of standard mathematics symbols onto a preexisting mental model of number and number transformation (Huttenlocher et al., 1994).

Support for the premise of Levine et al., (1992) was generated by comparing children's performance across three problem types: (1) nonverbal problems; (2) story problems; and (3) number-fact problems where simple story problems and number-fact problems were verbally presented. On the nonverbal problem, the children replicate the number of objects that were added to or taken away from a total amount hidden from view. According to Levine et al., the presentation and response modes are completely nonverbal. The authors contended that the improved performance on the nonverbal task, as compared to the verbal tasks, is due to the fact that, "the operation of adding and subtracting is provided by the physical act of combining or separating sets. These physical referents may make it easier for the child to represent the terms of the problem and the operation involved in the calculation than on verbal problems" (Levine et al., p. 99). Thus, children transform sets by adding or subtracting elements prior to solving simple word problems or number fact problems.

However, one cannot conclude that the presentation and response modes are completely nonverbal simply because a verbal justification was not requested. Further, one cannot imply that the children were <u>calculating</u> (i.e., transforming the sets using the logic of operational structures) because they replicated the total number of disks that were presented as two separate groups (e.g., Levine et al, 1992). To draw such a conclusion, it is necessary to examine the nature of the mental reflections children are generating in relation to their actions. Specifically, assigning a tag of one to the first disk of the second set rather than counting on from the last number in the previous set, suggests that children are taking one whole (mentally) then another whole and combining them to make an expanded, homogeneous whole. In this more complex whole, the previous wholes disappear in one sense, but continue to exist as a transformed whole (Kamii, 1985).

The purpose of this study is to examine the quality of children's thought structures (6-8 years of age) across different types of math tasks: (a) flashcard task with socially transmitted signs and symbols; (b) nonverbal task (Levine et al., 1992) with the modification that solutions are justified, and; (c) associativity of length task (Piaget, Coll, & Marti, 1987) that examines the emergence of the operative structures of addition. It is hypothesized that although children might successfully "solve" algorithms and/or replicate the number of disks on a mat that were presented as separate sets, they may be mentally engaging in a task using schemes that are not yet coordinated enough to reflect on the relationships inherent in operational thought. These less sophisticated strategies should be related to non-conserving strategies on the Piagetian task.

METHOD

Subjects

A total of 42 children across three age levels (6, 7, & 8) participated in the study. There were 14 children at each of the three age levels. The mean ages and standard deviations (in parentheses) for the three age groups were: 6.5 (.34); 7.4 (.27); and 8.5 (.23). The combined age mean for all three age groups was 7.4 (.28). The percent of students in kindergarten through third grade at each of the age levels were: (a) 50% kindergarten, 50% grade 1; (b) 71% grade 1, 29% grade 2; and (c) 50% grade 2, 50% grade 3.

All children were from a lower-to middle social-economic status in a suburban town in northern New Jersey. The numbers of boys/girls in each age group were: 6 girls, 8 boys for the 6 year age group and 7 girls and boys for both the middle and oldest age groups.

The children's classroom teachers judged them to have average mathematical abilities for their grade. The mean performance (standard scores) (SC) and standard deviations (SD) on the Wide Range Achievement Test (WRAT) for the middle and oldest age groups validated teacher judgement: 101.8 (7.3), and 101.7 (4.1) for the middle and oldest age groups respectively, in which all children score within one SD of the mean (expected mean SC=100; SD=15). The mean SC(SD) of the lowest age group was 108.9 (9.79). The larger variability of scores for the youngest age level is a result of three children having achieved more than one SD above the mean (118-131). These students were not excluded from the data analysis because the main interest is in performance relationships between tasks for all subjects combined rather than group differences. The inclusion of these three children provides important information toward this end.

Materials and Apparatus

Associativity of length task.

The children's construction of operational structures was examined in a task assessing the associativity of length (Piaget, Coll, Marti, 1987) in which they were presented with two identical strings that were equally segmented into four colors. The pieces of string were cut into various parts to represent the same type of cognitive complexity inherent in the operative structures of addition as previously detailed. The lengths of the segments were 5.5, 12.5, 9.0, and 7.5 centimeters. Each piece of the string segments was compared until the child agreed that the length was the same. As each segment piece was compared, the other segments were pushed aside so that the entire length of the string would be not seen together as one unit. The strings were then placed on a board with pins to make identical fences (2.5cm in length) for a cat to walk on (see Figure 1 for the spatial arrangement of the fences in this problem and the problems to follow). The experimenter then walked the cat on both fences and asked, "Did the cat walk the same length on both fences or was the walk on one fence longer than on the other fence?" The children then justified their answers.

For problem 2, one cut was made on each fence at different segments while the child watched. The cat then walked each fence and the question was asked again. In problem 3, the pieces of the fence used in problem 2 were placed in different spatial orientations. Again the cat walked the fence and the same question was asked. If the child included the space between the cuts as adding to the length of the fence in his or her explanation, the child was told to consider only the pieces of the fence. For problems 4-6, two new pieces of string were compared as previously described in problem 1 and three more fences were constructed. In the fourth problem, the two fences were presented without cuts and in the same spatial orientation. In the fifth problem, two cuts were made in the experimenter's fence and one cut made in the child's fence at a different segment. Finally, the fence segments were placed in different spatial orientations for the sixth problem. The authors of the task stated that the cuts deprived the children of the perceptual facilitation afforded by the continuity of the line segments. This difficulty further increases when both the elements and the parts in the total figure change orientation.

Because this task involves understandings of length equality, a preliminary task was administered to assess children's construction of this notion. Specifically, they were first shown two standard, unsharpened pencils (7 ½ inches) that were placed next to each other and asked, "Are the pencils the same length or is one pencil longer than the other" and "why?" One pencil was displaced relative to the other and the children were then asked the same question and to justify their choice. The pencil was then returned to its initial position and the child was again questioned about length equality or was asked, "Can anything be done to make them the same length again?" Each child was then asked to provide a justification for the response given.

Flashcard task

Fourteen addition number problems were written on 3x5 index cards in vertical form. Each numeral was approximately 1" in length and written with a black marker. There were seven number problems whose sums were less than 10 and seven number problems whose sums range from 11-20. The cards were randomly selected from a shuffled presentation with the provision that if there was repetition of either the augend or the addend across consecutive problems, the cards were put at the end of the pile and moved until there was no repetition in either augend or addend across consecutive problems.

Each child was told, "You will be shown some number problems. Some problems may be hard for you. Just do the best you can to give me an answer." The card was then held in front of the child until an answer was supplied or the child stated he or she does not know. When an answer was supplied, the child was asked, "How did you know that?" The experimenter noted anything the child did to assist with solving the number fact.

Nonverbal calculation task.

The materials for the nonverbal calculation task (Levine et al., 1992) consisted of two 12"x12" white cardboard mats with black horizontal lines to divide the mats in half, a set of 40 black buttons (3/4 of an inch in diameter), a box for the buttons, and a cover for the buttons. One side of the cover had an opening so the experimenter could easily put in or take out the buttons. The experimenter and child sat at a small table facing each other, each with a mat in front of himself/herself.

The addition problem of 2+3 was demonstrated by having the experimenter place two buttons on her mat in full view of the child and then pushing them under the cover. Three buttons were then placed on the horizontal line next to the cover and slid under the cover one by one. Next, the experimenter placed five buttons on the horizontal line on the child's mat and lifted the cover to show the two buttons on her mat, saying, "See, yours is just like mine." This demonstration item was presented again following the same procedure, except this time the child was asked to place the appropriate number of disks on the mat after the transformation had been made by the experimenter. In contrast to study design of Levine et al., (1992) in which no verbal response was required, the child was asked to justify what was laid out if he or she did not count verbally. If the child placed the wrong number of buttons on the mat, the response was corrected and the item was repeated.

The nonverbal problems were presented immediately following the demonstration problem. For each addition problem, the experimenter placed the set of disks comprising the augend on the mat and then pushed them under the cover. The experimenter then put the set of disks comprising the addend in a horizontal line next to the cover and slid them under the cover one by one. As in the demonstration procedure, the children then indicated how many disks were hiding under the cover by placing the appropriate number of disks on his or her mat. The children were asked to justify their solutions.

The same number facts that were used on the flash card task were used for the nonverbal task with changes in the order of presentation. The order of problem difficulty was determined in the same manner as the flashcard task. In addition, the order of the nonverbal and flashcard tasks was switched for every other child to control for any effects of memory. None of the children commented that they remembered a number problem from the previous task, although some remembered problems within tasks. The children sometimes used this information to help solve other problems in the flashcard task.

Scoring

Associativity of length task.

The associativity of length task was used to judge levels of operative thinking according to the theory of Piaget. There were three levels of behavior scored in this task. A child was judged to be at level IA if there was an absence of any quantitative length concept in the sense of additivity. For example, children at this level commented on the difference in the number of cuts between the fences but did not attend to how these cuts affected the equality of the elements within the parts or the equality of the parts to each other. Often these children commented on the shape of the fence, stating that one is longer because of its shape (e.g., "My fence is longer cause it's shaped like a triangle and is flat out").

Children at level IB perceived the cuts as affecting length, but were unable to apprehend the notion of compensation between parts where an increase in one part of the fence leads to a decrease in another part (e.g., "My fence is longer cause the white one is shorter than the green one," "Mine is longer cause you cut different colors on mine and each color is a different length than the other") in at least one of four problems where the strings were altered in some way.

Children scored at level IIA, had all problems correct. Thus, they were consistently able to simultaneously coordinate the parts with changes in the distribution of the elements such that the two were apprehended as necessarily equal in length (e.g., "If you keep cutting until a million pieces it would still be the same," "Cause well, if they were together and added up in a straight line they would be the same").

The pencil task, which was introduced prior to the associativity task, was judged only in terms of correctness of response as its purpose was to help to delineate children's understanding of the terms "same length" which was the language used in the associativity of length task. This task also served to validate children's ability to conserve on the associativity of length task.

Thus, nine different performance criteria were generated in relation to the associativity of length task. These criteria consisted of a stage variable and dichotomous correct/wrong variables for each of the specific problems tested. A reliability check was performed on fifteen of protocols by a student familiar with Piagetian theory. The obtained reliability was $\underline{r} = .93$. These performance criteria and the performance criteria for the flashcard and nonverbal tasks are found on Table 1 (see items 1-9 for the associativity of length task).

Insert Table 1 about here

Flashcard task.

Six strategies were coded for the type of answers provided based on scoring schemata identified in past research (Baroody, 1987; Carpenter & Moser, 1984; Geary, Brown, & Samaranayake, 1991; Siegler, 1986, 1987; Svenson & Sjoberg, 1983). The strategies were: (0)

don't know (DK) - no attempt to solve the problem; (1) counting fingers (CF) - fingers used to represent problem integers and then counting fingers to derive the total; (2) fingers (F) - using fingers to represent integers but not recounting the fingers prior to stating the sum; (3) verbal counting (C) - observation of audible counting or lip movement without the use of fingers; (4) decomposition (D) - decomposition of a problem into an easier problem or using a previous problem solution help derive the total; and (5) retrieval (R) - no counting observed. If children were not observed counting but stated that they counted (subvocalization) they were coded as having used the verbal counting strategy.

According to Siegler (1986, 1987) memory retrieval is the preferred strategy because it places fewer demands on the resources of the working memory and requires less time to execute. The other four strategies serve as backup strategies. However, because decomposition involves the ability to relate the elements into various subsets around an expanded whole, decomposition and retrieval strategies are considered as the preferred strategies for the purposes of this study. Also scored was whether the children stated the correct answer. These two components of the flash card task generated 28 performance criteria which are items 10-37 on Table 1.

Nonverbal calculation task.

Levine et al., (1992) coded children's strategies with respect to four of the flashcard strategies stated above (counting fingers, fingers, counting, and unobserved) and added the strategy of imitating the experimenter. These authors suggested that although children infrequently used overt counting with or without the use of fingers on this task, it is plausible that the children were frequently counting silently (Jordan et al., 1992; Levine et al., 1992). In this student sample, it was clearly evident that children were consistently counting the buttons (subvocally or vocally). Further, the manner in which the buttons were counted differed between children. Because the counting strategies appeared indicative of children's underlying cognitive activity, five different counting strategies were coded to assess task performance: 0) no attempt and/or consistent approach to the problem solution (NA); 1) interval count using fingers (IF) - counting two sets as one whole with the assistance of fingers; 2) interval count (I) - counting two sets as one whole without the assistance of fingers; 3) sequenced group count (G) - counting two separate groups from one using the terms first/then when combining groups; and 4) summing group count (A) - counting two separate groups from one while using additive terms that included "in all," "I added them, " or "plus." Two children used their fingers to assist them with their count in a small number of problems. Because they directed their counting activity on two separate groups and used additive terms, this strategy was coded as level 4. Because strategies G and A consist of the assimilation of the two sets as a homogeneous whole, they were considered the more complex strategies. As in the flash card task, there were performance criteria generated for correctness of an answer stated and for the type of strategy generated which are represented as variables 38-65 in Table 1.

It is recognized that Levine et al. (1992) had designed this task in an effort to assess calculation abilities that were independent of verbal labels. However, in the current research the labels were generated by the children, thus demonstrating a predilection to use these terms to justify their answers.

Procedures

Testing.

Children were individually tested by the first author for approximately forty minutes. The associativity of length task was administered first followed by either the flashcard task or the nonverbal task with the order of flashcard and nonverbal tasks changed with each child. Finally, the WRAT (second edition) test of mathematics was administered. The testing was administered during the latter part of the school year (late April into early May).

Statistical Analysis

To examine the pattern of relationships among all three tasks, Rasch analysis was used on the combined age group data. The Rasch model is an unidimensional analytical model that has as its foundation the assumption that performance of a given test item is determined by two factors only; the ability of the subject and the difficulty of the test item (Elliot, 1983; Hautamaki, 1989). In contrast to relational statistics such as factor analysis and correlations which regards all incorrect answers equally as errors, Rasch analysis treats "errors" in relation to the difficulty of test items and how well they match each person's ability. It follows that there is a greater possibility of getting an easier test item correct than a more difficult one. Further, a person who falls higher on the logits scale has a greater probability of getting items correct as compared to a person at the lower end of the logits scale. Therefore, the model consists of a "necessary precondition" where success on easier test items is a precursor to success on a more difficult item (Bond, 1995a, 1995b; Bond & Bunting, 1995; McNamara, 1996).

Thus, the treatment of item difficulty corresponds to a conception of cognitive development as having a hierarchical structure making the Rasch model appropriate to the analysis of the three tasks described in this study. Specifically, this model allows for the estimation of item difficulty (logits) within and among all three tasks as well as determination as to how well all of the items fit with each other. This will help determine whether one underlying cognitive ability is implicated or if substantially different abilities are tapped by the various tasks. Such an investigation will help to discriminate the underlying cognitive demands in the various strategies generated within tasks, their relationship to operative level thinking, and the consistency of student performance levels across tasks.

RESULTS

Total Item and Subject Pool

The analysis of all 65 items together for all 42 children produced a fit to the Rasch unidimensionality premise, with items spread along the logits (difficulty) scale from -3.59 logits (easiest items, items 2.1 & 38.1) to +4.11 (most difficult item, item 23.5), centered around 0.0 logits (with item 51.1 nearest the midpoint of the scale at +.02) as shown in Table 2.

(Insert Table 2 about here)

According to the conventional interpretation of t (acceptable from +2 to -2), the vast majority of test items fit the Rasch model. However, items 5, 7, 60, and 63 just exceeded the conventionally accepted boundary of t=+2 or -2, when p<.05 with infit t values of +2.1, +2.2, -2.4, and -2.1 respectively. Because of the exploratory nature of this investigation was based on a small subject sample, it is reasonable to include these items in the results below.

The spread of persons along the logit scale shown in Figure 2 revealed that the most successful person was an eight-year-old at +5.94 logits and the least successful person was a six-year-old at -3.86 logits.

(Insert Figure 2 about here)

However, Table 3 shows that there were four subjects with negative t values below 2.0: subjects 10 and 13 in the youngest age group with t values of -3.58 and -2.12; subject 28 in the middle age group with a t value of -5.04; and subject 33 in the oldest age level with an infit t of -2.23. These large negative t values indicate that these children performed in an "all-or-none" manner; they got the majority of easy items correct and the remainder of the items wrong. This type of performance is similar to that required by deterministic Guttman scaling rather than what is expected by a probabilistic item response (IRT) model. However, one of the youngest students (3) with an infit t value of +2.24 shows more variation from the pattern predicted by the Rasch model. This student did well on the Piagetian task problems but used a less mature strategy of counting fingers on the flaschcard task which differentiated her performance from the performance of others.

(Insert Table 3 about here)

Figure 2, where the person ability locations are indicated by each subject's age (viz 6, 7, 8), reveals a general trend for ability to increase with age. As children progressed in age there was a gradual increase in their ability to solve more difficult problems within tasks although there was a degree of overlap between the age groups. This overlap is demonstrated by the student spread along the logits scale for the youngest to oldest age groups respectively: -3.86 to +3.03 (for the 6 year olds); +.80 to +2.13 (for the 7 year olds); and +1.23 to +5.94 (for the 8 year olds). The large spread of students for the youngest age level is due to the three students whose performance was consistently advanced on all tasks administered. This superior performance (i.e., better than all the 7 year olds in the sample) was corroborated by their standard scores on the WRAT (123, 131, 118). When these three students are not considered, the spread of students along the logits scale for the same age level is -3.86 to +1.47. By far, the smallest variability in student performance was at the middle age level.

<u>Associativity of Length Task</u>

When comparing pencil lengths placed side by side, all but one student (i.e., 98%) identified the two pencils as having the same length. However, when one pencil was extended up from the other, only 33% of the children judged the pencils as equivalent in length. When the length between the fence pairs was compared, problems 1 & 4 (same spatial orientation with no cuts on Figure 1) had the greatest success rate where 86% and 88% of the students achieve success on each problem respectively. This was followed by the problem with one cut, same spatial position (48%), different number of cuts, same spatial position (45%), one cut, different spatial position (41%), and finally, the fewest number of students achieved success on the task with different spatial positions and different number of cuts (29%). These variations in item difficulty are depicted by the distribution of the Piagetian item locations along the logits scale in Figure 2. Specifically, the spread of items along the logits scale ranged from -3.59 (item 2) to

+3.01 (item 9). These statistics represent increasing levels of cognitive complexity within and between developmental stages are defined by Piaget.

Only those students who achieved success on all of the problems were judged to be at stage IIA (29%). Fifty-seven percent of the children achieved at level 1B, while 14% of the children achieved at level IA.

Flashcard Task

Strategy type.

Items 10-23 on Table 2 consists of strategy types for each of the 14 flashcard problems that produced a spread of items along the logits scale from -2.75 logits (item 14.1) to +4.11 (item 23.5). Within each number problem, the item difficulty for each strategy type also increased. Table 4 additionally shows that as the answer to the sums increased in quantity, fewer of the more sophisticated strategies were generated.

(Insert Table 4 about here)

This observation is demonstrative of the increasing cognitive demands inherent in strategy type as students progressed from using no attempt and/or inconsistent means to solve the problem to retrieving an answer without counting. Figure 2 serves to make clear the progressive relative difficulty of the six strategies on the flashcard task for each of the fourteen number problems. For example, the number problem 6+1 (CF), is located at -2.75 logits. In contrast, students found that using the CF strategy for the number problems over ten (e.g., 2+9 & 9+6) more difficult (item difficulties of -.38 & -.69 for the two problems respectively). Further, the use of the decomposition and retrieval strategies for number problems under 10 (e.g., 6+1 R at -1.04 logits) was often easier than the use of the less complex strategies (i.e., CF, F, & C) for number problems over 10.

Figure 2 also shows that as the problems increased in complexity, the initial item difficulty (logits) increased for each strategy as well. For example, for the number problems less than 10 the spread of items along the logits scale ranged from -2.75 logits (6+1 CF) to +1.85 logits (6+3 R). In contrast, the spread of items along the logits scale for number problems more than 10 ranged from -.69 logits (9+6 CF) to +4.11 logits (9+7 R). The extension of the difficulty range in logits for number problems less than 10 into the number problems more than ten shows that some children used the more sophisticated strategies for number problems less than 10 (D & R) while at the same time they used the less sophisticated strategies for the number problems more than 10 (CF, F, C).

Number correct.

An examination of item difficulty for these same number problems (items 24 to 37 in Table 2) shows that the item difficulty (logits) for number problems less than 10 ranged from -2.50 (3+1, 1+4, & 6+1) to -.68 (2+5) whereas the item logits for number problems more than 10 ranged from -1.36 (2+9) to .86 (9+7). Thus, the number problems more than 10 were more complex problems.

The item difficulties for the number correct start and end at points along the logits scale that are lower than the item difficulties for type of strategy used (see Figure 2). Thus, children who used the less complex strategies (CF, F, & C) could state correct answers.

Nonverbal Task

Strategy type.

The percentage of students using one of the five strategies on the nonverbal task for each of the 14 number problems is reported in Table 5. A small increase in the number of times students used the most complex strategies (G & A) was noted for number problems more than 10, but generally the spread of scores along the logits scale was relatively constant regardless of whether then sums are less/more than 10.

(Insert Table 5 about here)

Items 52-65 which consisted of the strategies for the nonverbal task for each of the 14 number problems are found in Table 2 and produced a spread of scores along the logits scale from -3.44 (3+1 I) to +2.76 logits (9+5 A). Within each of the strategy types, the item difficulty increased showing increased cognitive complexities inherent in the more advanced strategies. The spread of item scores along the logits scale (see Figure 2) for number problems less/more than ten showed only a small degree of difference in item difficulty (logits). Specifically, the item difficulty (logits) for number problems less than 10 ranged from -3.44 (3+1 I) to 2.41 (5+3 A) and the logits for number problems more than 10 ranged from -2.94 (9+7 IF) to +2.76 (2+9 A & 9+5 A). This is due to the fact that once children began to use the strategies for counting two separate groups from one with and without additive terms, they did so for both sets of number problems.

Number correct.

The number correct consists of items 38-51 on Table 2. The success of children on the buttons task generated a spread of item difficulties along the logits scale from -3.59 (3+1) to .55 (2+9, 9+5) (see Figure 2). The spread in item difficulty (logits) in number correct for number problems less than 10 ranged from -3.59 (3+1) to -.19 (2+5 & 5+3). The range of item difficulty for number problems more than 10 was from -.42 (5+7 & 5+8) to .55 (2+9 & 9+5) indicating that children found the number problems less than 10 easier to answer although there was some overlap in item difficulty.

A comparison of items 38-51 (number correct) to items 52 -65 (strategy type) on Table 2 indicates that the majority of the children were supplying correct answers to number problems before they were able to generate the strategy of counting the two separate groups from one to represent the whole (strategies G & A). Thus, children were providing correct answers to the problems prior to reflecting on the numerical amounts as being explicitly nested.

Between Tasks Analysis

Flashcard and nonverbal tasks.

When the spread of items along the logits difficulty scale for the flashcard and nonverbal tasks are compared in Figure 2, the flashcard task and the nonverbal task are of comparative difficulty in terms of getting the correct answer. However, the elementary counting strategies are much more readily applied in the nonverbal task (IF & F) than in the flashcard task (CF, F, & C). Within the buttons task, there is a large gap between counting the buttons as one group (IF & I) and counting the two separate groups from one to form a homogeneous whole (G & A). This gap suggests that while the nonverbal presentations of addition problems might lessen the difficulty of counting with and without fingers, the use of the operational strategies (i.e., G & A) is considerably more difficult and comparable to the difficulty of applying the strategies of D and R in the flashcard presentation. For example, for the number problem 9+7 the item difficulties for the R and D strategies were +4.11 and +2.71 respectively. In contrast, the item difficulty for the most difficult strategy used for the same number problem on the nonverbal task (A) was +2.50. (See Figure 2 for a chart of the total item and subject pool along the logits scale).

Flashcard, nonverbal, and associativity of length tasks.

An examination of the relationship between the use of additive operative structures on the associativity of length task and the performance on the flashcard and nonverbal tasks for both the type of strategy used and number of times the item was correct, reveals discrepancies in children's understanding of operational knowledge. Specifically, on both the flashcard and nonverbal tasks, children stated the correct answer to number problems that are often claimed to indicate understanding of the operation of addition prior to the emergence of operative structures as indicated on the associativity of length task. This contention is based on the difference in the spread of scores along the logits scale for both tasks. The range in item difficulty for number correct on the flashcard and nonverbal tasks respectively is -2.50 to +.86 and -3.59 to +.55. In contrast, the item difficulty for the first fence problem where the children conserved length was +1.92 and extended to +3.01. The strategies used to provide correct responses prior to conserving are mainly the lower level strategies on both the flashcard (CF, F, & C) and nonverbal (IF & I) tasks.

When performance on the nonverbal task and associativity of length task is compared in terms of item difficulty (see Figure 2), the first problem along the logits scale on the nonverbal tasks in which the strategy of A was used (1+4) had an item difficulty of +1.77 logits. This is at the approximate location for the problem of conserving equality in fences that were altered with one cut (i.e., +1.92 logits). Thus, those students who counted both groups from one using additive terms tended to conserve equality. A comparison of the performance between the item difficulty on the flashcard and the associativity of length tasks shows that the great majority of children who conserved length despite changes in appearance also solved number problems more than 10 using the most sophisticated strategies (D & R).

DISCUSSION

The results of children's performance in this study revealed that within each of the three tasks there existed a sequential construction of increasingly complex cognitive abilities that was measured by the generation of correct answers and the use of a variety of strategies. Further, a comparison between the flashcard, nonverbal, and the associativity of length tasks elicited a developmental relationship between the ability to use more sophisticated strategies to solve mathematical problems and the evolution of operational structures as measured on the Piagetian task. Such findings support the notion that mathematical principles consist of complex structures of organizing activity that evolves by its recoordinations of this activity (e.g., Kamii, 1985; Piaget, 1965, 1987; Steffe, 1988; von Glasersfeld, 1995). This dynamic systems approach to the evolution of logical-mathematical structures disputes the notion that the acquisition of mathematics skills involves the mapping of standard mathematics symbols onto a preexisting mental model of number and number transformation (Gelman, 1972; Huttenlocher et al., 1994).

For example, on the flashcard task, children's strategies were altered according to the complexity of problems solved. The great majority of children who consistently used the R strategy for number problems less than 10 did not use this strategy for number problems more than 10. The strategies of decomposition and retrieval for number problems more than 10 were generally used by those children who had conserved length when one of the fence configurations is altered, although counting was still used to solve some of the more complex problems.

Children had greater success solving number problems whose sums were less than 10 as compared to sums more than 10 even though they changed to the use of lower level strategies (e.g., counting fingers and fingers versus retrieval) for assistance in solving the more difficult problems. This performance pattern was most evident with the children at the youngest age level although it was observed to some degree for children at the middle age level as well. These results further exemplify the dynamic relationship between the number of variables to be manipulated in problems and the quality of strategies generated to derive solutions.

Many children at the youngest age level who got the number problems more than 10 incorrect stated that they used their fingers to count. However, when they ran out of fingers for one-to-one correspondence, the number of counts to follow became meaningless and/or ineffective tools were used (e.g., trying to bob head with each count). The children at the middle age level were better able to use tools such as fingers to help them count because they appeared to better understand that quantity (number of fingers representing a cardinal value) continued to exist without the object representation. As a result, they were able to use the same fingers twice to count on in an efficient manner.

The above performance patterns are consistent with the notion that young children cannot understand all of the relationships inherent in understanding cardinal value while engaged in a counting activity using one-to-one correspondence even if they are able to supply an appropriate label (Baroody, 1992a, b; Sinclair, Siegrist, & Sinclair, 1982). Such findings dispute the notion that the difficulty four year olds experienced generating referents to a cardinal designation in the flashcard task is due to the lack of realization by children that such intervening representations assist with problem solution (Levine et al., 1992). The knowledge of referents is not the stumbling block; the problem is the manner in which the tools are reflected upon. As discussed, this reflection is guided by the degree of coordination in mental structuring activity (action schemes).

What is left unanswered in the patterns of performance within and between the flashcard and associativity of length tasks, is why so many children who used the retrieval strategy to solve number problems less than 10 were unable to demonstrate operative thinking on the associativity of length task (i.e., decompose the parts in relation to the whole). Does this behavior indicate that although several children were not demonstrating operational thinking on the associativity of length task, they were able to coordinate the relations inherent in an additive equation? The fact that many of these children demonstrated difficulty representing cardinal value suggests that they were attending to each number in the equations as isolated elements. As such, there was minimal ability to reflect on the relationship between the hierarchical inclusion of each number within each of the subsets in a way that gave meaning to the equation as an operation (i.e., transforming sets and subsets into any of its related parts). Thus, it is possible that children were retrieving an answer as a result of explicitly taught skills while at the same time being unable to reflect upon the relationship of the parts to each other and to the whole. Examining the findings on the nonverbal task and how they relate to the associativity of length task can address these speculations.

When the strategy of counting two separate groups from one using additive terms in the nonverbal task is compared to children's ability to conserve length on the altered fence configurations, the item difficulties are very close to each other. Specifically, the first number problem along the logits scale in which children counted two groups from one using additive terms (1+4) has an item difficulty of +1.77 while the first altered fence pairs that children conserved (one cut, same spatial orientation) has an item difficulty of +1.92. Thus, those children who conserved length on at least one of the problems with altered fence configurations also tended to generate the most complex strategy (A) to solve problems on the nonverbal task. The strategy of counting two groups from one using the terms "first/then" was first observed for the problem 1+4 at a difficulty level along the logits scales of +1.09 and the problem that was most difficult (9+6) had a value of +1.94 on the logits scale. Because the fence problem with one cut that children conserved was at +1.92 logits, it is likely that the children using the G strategy were transitional in their ability to conserve. The range in item difficulty for children who counted both groups as one whole (IF & I) is significantly lower (-3.41 to -1.48) than the problems in which the children achieved success when the fences have been altered (+1.92 to +3.01).

These findings are consistent with the observation that children progress from writing numerals in sequential order without signs to represent the transformation of two groups into a whole to the writing the numerals as an equation (Kamii, 1985). Kamii (1985) argued that without the use of the signs and symbols that denote the relationship between the parts and the parts to the whole to represent mental actions on objects, there is no evidence to suggest that children understand the equation as an "operation." This argument could be generalized to the verbal use of the terms. The ability to transform the parts in relation to the whole, that is, to perform a calculation as an operation, involves the coordination of complex relationships between the elements within the parts and the parts to the whole that even some eight-year-old children in this data sample have not yet constructed.

An examination of the number of correct answers on the nonverbal task further exemplifies qualitative differences in how children reflected on their experiences on the nonverbal task. The item analysis shows that stating the correct answer to the number problems is easier than demonstrating the ability to generate strategies indicative of the construction of operative and transitional operative structures (i.e., A & G strategies respectively). If children are taught equations before their cognitive structures are coordinated enough to relate the elements of the equation to each other, then it appears that they provide answers to equations with minimal reflection of their own self-generated thought processes (Schifter, 1997). Instead, children learn the borrowed thoughts of the teacher through practice and repetition which significantly compromises the understanding of numbers and the equations that they are embedded in (Baroody, 1987; Carpenter, 1986; Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988) Such are the dynamics of specific skill instruction; it creates forms of knowledge that tend to be static in nature (Cobb, Yackel, & Wood, 1992) due to the separation of learning activity from children's structurations of activity. These findings are consistent with previous research findings where the great majority of children who have been taught a technique to solve equations in first grade are unable to represent the hierarchical relationship between the parts and the parts to whole when these thought processes were investigated (Kamii, 1985).

The children with more mature operative structures attended to the units and the relationships between those units and the whole. The relative difficulty and acquisition order of number combinations for these children appeared to be determined largely but the salience and complexity of the relationships underlying various number combinations (Baroody, 1989, 1992a, b; Baroody & Ginsburg, 1986; Kamii, 1985). Specifically, as quantity increases, the number of relationships to be constructed within and between the parts and parts to the whole also increases because there are more combinations to be considered at one time (Kamii, 1985).

Finally, although the WRAT was used as a tool to validate teacher judgement, the limitations of what this and similar types of tests can tell us about children's mathematical constructions warrants discussion. This test did discriminate three children at the youngest age level who consistently achieved above the "norm" of what is expected at their age on all tasks administered. What made these particular children excel on the WRAT was their ability to add two column additions without regrouping and for one child, success with a simple multiplication problem. However, the student at the 8 year-old level who solved the most difficult problems on three tasks combined as indicated by her position along the logits scale (+5.94), did not achieve the highest score on the WRAT as compared to other children in the oldest age group. Specifically, this child achieved a standard score of 98 whereas ten students achieved standard scores that ranged from 102-108. Perhaps the contradiction in test performance on the WRAT and how well children performed at the youngest and oldest age levels the other three tasks at each age level, is due to the fact that the complexity of children's thinking is not explored on these standardized tests (Baroody, 1987). Most likely, if the younger children's thought processes were probed, they would be unable to demonstrate a conceptual understanding of place value even though they "added" numbers in more than one place value column.

In summary, these data support the schema-based model of mathematics that argues that mathematical constructions consist of the internal coordination of structures of logical-mathematical activity that cannot be socially transmitted or "mapped onto" a preexisting mental model of number and number transformation. In fact, the learner comes to the learning situation with complex forms and structures of logical-mathematical activity that continually transform themselves as they seek expansion in meaningful learning activities that provoke conflict (Hatano, 1988). The pedagogical challenge thus becomes one of facilitating children's "reflective abstractions from, and progressive mathematization of, their initially situated activity" (Cobb, Yackel, & Wood, 1992, p. 23). As part of this process, the teacher intentionally seeks out students' multiple representations of concept, encourages discussion of these differences while respecting unique constructions, and decides how a lesson should proceed during instruction depending on what is occurring in that moment (Bauersfeld, 1995; Cobb, Wood, Yackel, & McNeal, 1992; Sherin, 1997; Sherin, Mendez, & Louis, 1997).

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Table 1
List of Items Included in the Rasch Analysis

Piagetian Tasks/Stage Indicators

1 **1.0** IA Absence of any quantitative length concept using additive structures **1.1** Perceived cuts as affection length but unable to make compensations when appearances differed **1.2** Conserved length despite appearances

Piagetian Tasks/Right or Wrong

2 **2.0** Unable to judge pencils as equal when side by side **2.1** Able to judge pencils as equal when side by side

3 **3.0** Unable to judge pencils as equal when not side by side **3.1** Able to judge pencils as equal when not side by side

4 **4.0** Unable to judge strings as equal when in same spatial orientation and no cuts **4.1** Able to judge strings as equal when in same spatial orientation and no cuts

5 **5.0** Unable to judge strings as equal when in same spatial orientation with one cut **5.1** Able to judge strings as equal in same spatial orientation with one cut

6 **6.0** Unable to judge strings as equal when in different spatial orientation with one cut **6.1** Able to judge strings as equal in different spatial orientation with one cut

7 **7.0** See 4.0 **7.1** See 4.1

8 **8.0** Unable to judge strings as equal when in same spatial orientation with two cuts **8.1** Able to judge strings as equal when in same spatial orientation with two cuts

9 **9.0** Unable to judge strings as equal when in different spatial orientation with two cuts **9.1** Able to judge strings as equal when in different spatial orientation with two cuts

Flash Card Task/Strategies

10 3+1 **10.0** No attempt to solve problem (DK) **10.1** Uses fingers to physically represent integers and then counts to arrive at the total (CF) **10.2** Uses fingers to represent integers but does not visibly count fingers prior to stating the sum (F) **10.3** Observation of audible count or lip movement (C) **10.4** Decomposes problem

into an easier problem and/or uses a previous number combination to arrive at the total (D) **10.5** Retrieves answer with no observed counting (R)

11 1+4 **11.0** See 10.0 **11.1** See 10.1 **11.2** See 10.3 **11.3** See 10.5

12 2+4 **12.0** See 10.0 **12.1** See 10.1 **12.2** See 10.2 **12.3** See 12.3 **12.4** See 12.4 **12.5** See 10.5

13 2+5 **13.0** See 10.0 **13.1** See 10.1 **13.2** See 10.2 **13.3** See 10.3 **13.4** See 10.5

14 6+1 **14.0** See 10.0 **14.1** See 10.1 **14.2** See 10.3 **14.3** See 10.5

15 5+3 **15.0** See 10.0 **15.1** See 10.1 **15.2** See 10.2 **15.3** See 10.3 **15.4** See 10.4 **15.5** See 10.5

16 6+3 **16.0** See 10.0 **16.1** See 10.1 **16.2** See 10.2 **16.3** See 10.3 **16.4** See 10.4 **16.5** See 10.5

17 2+9 **17.0** See 10.0 **17.1** See 10.1 **17.2** See 10.2 **17.3** See 10.3 **17.4** See 10.4 **17.5** See 10.5

18 4+8 **18.0** See 10.0 **18.1** See 10.1 **18.2** See 10.2 **18.3** See 10.3 **18.4** See 10.4 **18.5** See 10.5

19 5+7 **19.0** See 10.0 **19.1** See 10.1 **19.2** See 10.2 **19.3** See 10.3 **19.4** See 10.4 **19.5** See 10.5

20 5+8 **20.0** See 10.0 **20.1** See 10.1 **20.2** See 10.2 **20.3** See 10.3 **20.4** See 10.4 **20.5** See 10.5

21 9+5 **21.0** See 10.0 **21.1** See 10.1 **21.2** See 10.2 **21.3** See 10.3 **21.4** See 10.4 **21.5** See 10.5

22 9+6 **22.0** See 10.0 **22.1** See 10.1 **22.2** See 10.2 **22.3** See 10.3 **22.4** See 10.4 **22.5** See 10.5

23 9+7 **23.0** See 10.0 **23.1** See 10.1 **23.2** See 10.2 **23.3** See 10.3 **23.4** See 10.4 **23.5** See 10.5

Flash/Right or Wrong

24 3+1 **24.0** Unable to state correct answer **24.1** Able to state correct answer

25 1+4 **25.0** See 24.0 **25.1** See 24.1

26 2+4 **26.0** See 24.0 **26.1** See 24.1

27 2+5 **27.0** See 24.0 **27.1** See 24.1

28 6+1 **28.0** See 28.0 **28.1** See 24.1

29 5+3 **29.0** See 24.0 **29.1** See 24.1

30 6+3 **30.0** See 24.0 **30.1** See 24.0

31 2+9 **31.0** See 24.0 **31.1** See 24.1

32 4+8 **32.0** See 24.0 **32.1** See 24.1

33 5+7 **33.0** See 24.0 **33.1** See 24.1

34	5+8	**34.0** See 24.0	**34.1** See 24.1			
35	9+5	**35.0** See 24.0	**35.1** See 24.1			
36	9+6	**36.0** See 36.0	**36.1** See 36.1			
37	9+7	**37.0** See 37.0	**37.1** See 37.1			

Buttons/Right or Wrong

38	3+1	**38.0** Unable to state correct answer	**38.1** Able to state correct answer			
39	1+4	**39.0** See 38.0	**39.1** See 38.1			
40	2+4	**40.0** See 38.0	**40.1** See 38.1			
41	2+5	**41.0** See 38.0	**41.1** See 38.1			
42	6+1	**42.0** See 38.0	**42.1** See 38.1			
43	5+3	**43.0** See 38.0	**43.1** See 38.1			
44	6+3	**44.0** See 38.0	**44.1** See 38.1			
45	2+9	**45.0** See 38.0	**45.1** See 38.1			
46	4+8	**46.0** See 38.0	**46.1** See 38.1			
47	5+7	**47.0** See 38.0	**47.1** See 38.1			
48	5+8	**48.0** See 38.0	**48.1** See 38.1			
49	9+5	**49.0** See 38.0	**49.1** See 38.1			
50	9+6	**50.0** See 38.0	**50.1** See 38.1			
51	9+7	**51.0** See 38.0	**51.1** See 38.1			

Buttons/Strategy

52 3+1 **52.0** No consistent approach to the problem solution (NA) **52.1** Counts two sets as one whole with the assistance of fingers (IF) **52.2** Counts two separate groups from one without the assistance of fingers (I) **52.3** Counts two separate groups from one using the terms first/then when combining groups (G) **54.4** Counts two separate groups from one using additive terms (A)

53	1+4	**53.0** See 52.0	**53.1** See 52.2	**53.2** See 52.3	**53.3** See 52.4	
54	2+4	**54.0** See 52.0	**54.1** See 52.1	**54.2** See 52.2	**54.3** See 52.3	**54.4** See 52.4

55	2+5	**55.0** See 52.0	**55.1** See 52.1	**55.2** See 52.2	**55.3** See 52.3	**55.4** See 52.4
56	6+1	**56.0** See 52.0	**56.1** See 52.1	**56.2** See 52.2	**56.3** See 52.3	**54.4** See 52.4
57	5+3	**57.0** See 52.0	**57.1** See 52.2	**57.2** See 52.3	**57.3** See 52.4	
58	6+3	**58.0** See 52.0	**58.1** See 52.2	**58.2** See 52.3	**58.3** See 52.4	
59	2+9	**59.0** See 52.0	**59.1** See 52.1	**59.2** See 52.2	**59.3** See 52.3	**59.4** See 52.4
60	4+8	**60.0** See 52.0	**60.1** See 52.2	**60.2** See 52.3	**60.3** See 52.4	
61	5+7	**61.0** See 52.0	**61.1** See 52.2	**61.2** See 52.3	**61.3** See 52.4	
62	5+8	**62.0** See 52.0	**62.1** See 52.2	**62.2** See 52.3	**62.3** See 52.4	
63	9+5	**63.0** See 52.0	**63.1** See 52.1	**63.2** See 52.2	**63.3** See 52.3	**63.4** See 52.4
64	9+6	**64.0** See 52.0	**64.1** See 52.2	**64.2** See 52.3	**64.3** See 52.4	
65	9+7	**65.0** See 52.0	**65.1** See 52.1	**65.2** See 52.2	**65.3** See 52.3	**65.4** See 52.4

Table 2
Rasch Analysis for the Associativity of Length, Flashcard, and Nonverbal Tasks

Item	Diff. Est.	Error Est.	Infit t	Outfit t	Item	Diff. Est.	Error Est.	Infit t	Outfit t
1.1	-.56	.88	.9	2.2	25.1	-2.50	.91	-.6	.2
1.2	2.95	.73	.9	2.2	26.1	-1.36	.65	-.4	.4
2.1	-3.59	1.23	-.8	1.0	27.1	-.68	.54	-.9	-.5
3.1	2.77	.38	.3	6.7	28.1	-2.50	.91	-.6	-.2
4.1	-.68	.54	1.5	1.3	29.1	-.99	.58	.5	1.3
5.1	1.92	.36	2.1	6.1	30.1	-.99	.58	-.3	-.1
6.1	2.31	.37	.5	.4	31.1	-1.36	.65	-1.4	-.6
7.1	-.99	.58	2.2	1.0	32.1	.39	.42	-.7	-.1
8.1	2.05	.36	.1	0.0	33.1	.39	.42	.1	.4
9.1	3.01	.39	.8	0.0	34.1	.39	.42	-.3	-.2
10.1	-1.58	1.42	1.2	.8	35.1	.55	.40	-1.3	-.8
10.2	-.94	1.30	1.2	.8	36.1	.39	.42	-1.2	-.7
10.3	-.24	1.10	1.2	.8	37.1	.86	.39	-1.0	-.5
10.5	.18	1.01	1.2	.8	38.1	-3.59	1.23	-.8	1.0
11.1	-1.47	1.22	-1.3	-.2	39.1	-.99	.58	1.3	.6
11.2	-.93	1.04	-1.3	-.2	40.1	-2.50	.91	.2	2.0
11.3	-.46	.98	-1.3	-.2	41.1	-.19	.47	1.1	1.1
12.1	-.97	1.06	-.3	0.0	42.1	-.99	.58	1.2	1.8
12.2	-.16	.94	-.3	0.0	43.1	-.19	.47	1.8	1.5
12.3	.59	.77	-.3	0.0	44.1	-.68	.54	.5	1.2
12.4	1.51	.62	-.3	0.0	45.1	.55	.40	.4	.6
12.5	1.67	.61	-.3	0.0	46.1	.39	.42	.3	1.0
13.1	-.42	1.00	-1.0	-.1	47.1	-.42	.50	-1.1	-.4

13.2	.15	.89	-1.0	- .1
13.3	.67	.79	-1.0	- .1
13.4	1.62	.64	-1.0	- .1
14.1	-2.75	1.95	1.6	.4
14.2	-1.66	1.60	1.6	.4
14.3	-1.04	1.42	1.6	.4
15.1	-1.81	1.25	1.3	.5
15.2	- .09	.95	1.3	.5
15.3	.35	.85	1.3	.5
15.4	1.16	.66	1.3	.5
15.5	1.29	.67	1.3	.5
16.1	-1.06	1.03	.4	.4
16.2	.35	.79	.4	.4
16.3	.62	.76	.4	.4
16.4	1.76	.63	.4	.4
16.5	1.85	.61	.4	.4
17.1	- .38	1.05	1.4	1.1
17.2	- .08	1.01	1.4	1.1
17.3	.41	.88	1.4	1.1
17.4	1.59	.64	1.4	1.1
17.5	1.85	.61	1.4	1.1
18.1	- .27	.80	- .3	- .3
18.2	.67	.71	- .3	- .3
18.3	1.50	.63	- .3	- .3
18.4	2.85	.63	- .3	- .3
18.5	3.35	.66	- .3	- .3

48.1	- .42	.50	- .4	.4
49.1	.55	.40	.9	4.8
50.1	.21	.43	.2	.2
51.1	.02	.45	- .2	.3
52.2	-3.44	1.50	- .5	- .6
52.3	1.32	.64	- .5	- .6
52.4	1.97	.63	- .5	- .6
53.1	-3.41	1.53	-1.0	- .8
53.2	1.09	.66	-1.0	- .8
53.3	1.77	.62	-1.0	- .8
54.1	-2.84	1.69	.4	- .1
54.2	-2.00	1.47	.4	- .1
54.3	1.32	.63	.4	- .1
54.4	2.18	.62	.4	- .1
55.1	-2.84	1.69	- .7	- .7
55.2	-2.04	1.49	- .7	- .7
55.3	1.53	.63	- .7	- .7
55.4	2.06	.64	- .7	- .7
56.1	-2.84	1.69	-1.1	- .8
56.2	-2.02	1.47	-1.1	- .8
56.3	1.43	.63	-1.1	- .8
56.4	2.17	.63	-1.1	- .8
57.1	-2.84	1.31	-1.8	-1.3
57.2	1.31	.62	-1.8	-1.3
57.3	2.41	.64	-1.8	-1.3
58.1	-2.84	1.31	.7	.6

19.1	-.72	.88	.1	-.1
19.2	.61	.70	.1	-.1
19.3	1.82	.61	.1	-.1
19.4	2.84	.68	.1	-.1
19.5	4.09	.76	.1	-.1
20.1	-.75	.84	.2	-.3
20.2	.72	.67	.2	-.3
20.3	2.03	.63	.2	-.3
20.4	2.94	.68	.2	-.3
20.5	3.73	.74	.2	-.3
21.1	-.28	.81	0.0	0.0
21.2	.66	.69	0.0	0.0
21.3	1.72	.62	0.0	0.0
21.4	2.85	.66	0.0	0.0
21.5	3.77	.72	0.0	0.0
22.1	-.69	.88	1.1	.8
22.2	.67	.68	1.1	.8
22.3	1.51	.61	1.1	.8
22.4	2.72	.65	1.1	.8
22.5	3.36	.68	1.1	.8
23.1	-.34	.81	1.3	.7
23.2	.76	.68	1.3	.7
23.3	1.89	.62	1.3	.7
23.4	2.71	.65	1.3	.7
23.5	4.11	.80	1.3	.7
24.1	-2.50	.91	-.6	.2

58.2	1.18	.64	.7	.6
58.3	2.19	.62	.7	.6
59.1	-2.84	1.49	-2.0	-1.4
59.2	-2.06	.64	-2.0	-1.4
59.3	1.85	.63	-2.0	-1.4
59.4	2.76	.64	-2.0	-1.4
60.1	-2.84	1.31	-2.4	-1.7
60.2	1.52	.64	-2.4	-1.7
60.3	2.66	.64	-2.4	-1.7
61.1	-2.84	1.30	-1.8	-1.4
61.2	1.73	.64	-1.8	-1.4
61.3	2.64	.66	-1.8	-1.4
62.1	-2.84	1.31	-2.0	-1.4
62.2	1.52	.64	-2.0	-1.4
62.3	2.66	.64	-2.0	-1.4
63.1	-2.84	1.69	-2.1	-1.5
63.2	-2.04	1.47	-2.1	-1.5
63.3	1.73	.65	-2.1	-1.5
63.4	2.76	.66	-2.1	-1.5
64.1	-2.84	1.30	-2.0	-1.4
64.2	1.94	.65	-2.0	-1.4
64.3	2.61	.66	-2.0	-1.4
65.1	-2.94	1.59	-2.0	-1.4
65.2	-1.48	1.20	-2.0	-1.4
65.3	1.74	.63	-2.0	-1.4
65.4	2.50	.64	-2.0	-1.4

Table 3
Rasch Analysis for the Subject Estimates

Student	Diff. Est.	Error Est.	Infit t	Outfit t	Student	Diff. Est.	Error Est.	Infit t	Outfit t
1	-3.86	.37	.78	4.94	22	1.31	.20	- .08	- .27
2	.96	.20	1.92	1.22	23	1.63	.20	1.42	1.18
3	1.23	.20	2.24	.28	24	1.87	.20	1.01	- .57
4	0.00	.21	-1.82	- .83	25	1.47	.20	- .63	- .25
5	- .94	.23	0.00	-1.33	26	2.13	.21	- .99	.69
6	-1.60	.24	1.29	.23	27	1.95	.21	- .63	- .28
7	1.19	.20	.12	- .45	28	1.67	.20	-5.04	-1.89
8	3.03	.26	.11	1.08	29	2.08	.21	- .42	- .26
9	1.47	.20	- .89	1.09	30	5.94	1.00	.39	1.77
10	.84	.20	-3.58	-1.84	31	2.84	.25	1.37	.09
11	1.11	.20	1.06	- .27	32	2.84	.25	.27	.83
12	.84	.20	- .65	- .53	33	1.23	.20	-2.23	-1.00
13	- .99	.23	-2.12	-1.80	34	4.14	.41	.60	.22
14	2.72	.24	- .69	.32	35	3.34	.29	- .68	.73
15	1.03	.20	1.90	.12	36	2.61	.23	- .43	.29
16	1.23	.20	- .83	- .90	37	2.84	.25	-1.86	- .02
17	1.35	.20	1.06	.54	38	3.18	.27	.96	0.00
18	2.13	.21	- .50	- .65	39	3.42	.30	1.11	.36
19	1.23	.20	.93	- .40	40	3.42	.30	-1.70	.63
20	.80	.20	-1.47	- .86	41	3.42	.30	.43	1.42
21	2.13	.21	1.61	- .23	42	3.25	.28	- .37	2.73

Table 4
Percent of Students Using the Six Strategies and Achieving Success on the Flashcard Task for Each of the Number Problems

Strategy[a]	Percent of Students Using Strategies For Each Number Problem													
	3+1	1+4	2+4	2+5	6+1	5+3	6+3	2+9	4+8	5+7	5+8	9+5	9+6	9+7
0	5	5	7	10	2	5	7	10	12	10	10	12	10	12
1	2	2	5	5	2	7	10	2	12	14	16	12	14	14
2	5	-	10	10	-	5	5	7	19	26	29	24	19	26
3	7	7	21	23	5	17	29	26	31	24	21	26	29	19
4	-	-	5	-	-	5	2	7	9	17	12	14	11	19
5	81	85	52	52	91	61	47	48	17	9	12	12	17	10
	Percent of Students Achieving Success													
	95	95	91	86	95	88	88	90	74	74	74	71	74	67

[a]0-No attempt, 1-counting fingers, 2-fingers, 3-counting, 4-decomposition, 5-retrieval

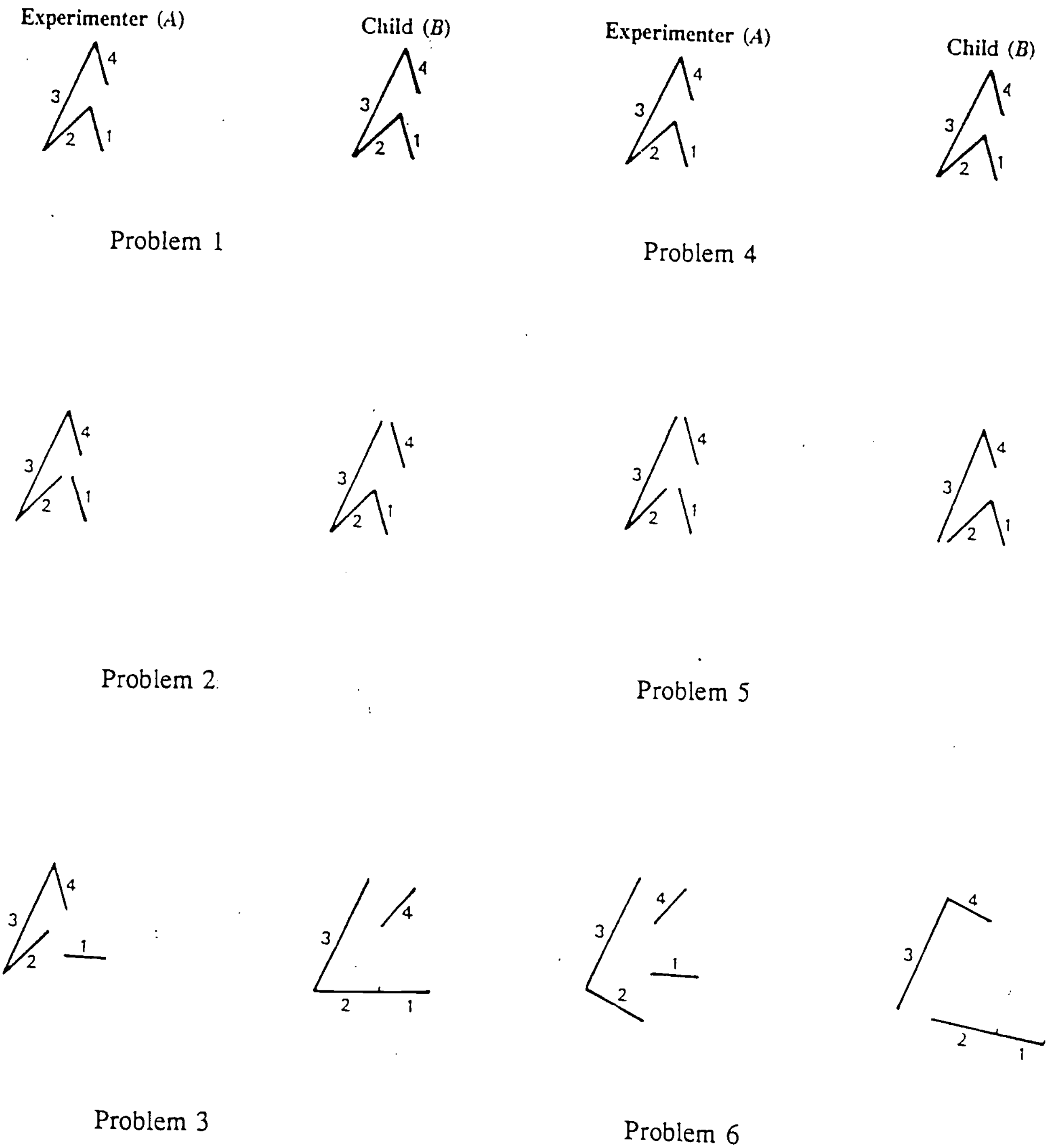
Table 5

Percent of Students Using the Five Strategies and Achieving Success on the Nonverbal Task for Each of the Number Problems

Strategy [a]	Percent of Students Using Strategies for Each Number Problem													
	3+1	1+4	2+4	2+5	6+1	5+3	6+3	2+9	4+8	5+7	5+8	9+5	9+6	9+7
0	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	-	-	2	2	2	-	-	2	-	-	-	2	-	5
2	36	31	33	38	36	36	33	45	41	45	41	43	50	41
3	17	17	22	15	19	26	24	22	26	22	26	24	17	19
4	45	50	41	43	41	36	41	29	31	31	31	29	31	33
	Percent of Student Achieving Success													
	98	88	95	81	88	81	86	71	74	83	83	71	76	79

[a]0-No attempt, 1-counting two sets as one whole with fingers, 2-counting two sets as one whole without fingers, 3-counting two separate groups from one without additive terms, 4-counting two separate groups from one with additive terms.

Figure 1.
Drawing Representations of the Fence Configurations for the Associativity of Length Task[a]



[a]Each number represents a different color segment.

Figure 2. Person and Item Levels for All Tasks

Item Est.	Subjects	Piagetian[a]	Flashcard-R	Flashcard Strategy[a]	Buttons-R	Buttons Strategy[a]
5.94 ↑	8					
	8	SP=string problem		9+7(R)		
4.0				5+7(R)		
3.5	888			9+5(R) 5+8(R)		
	8			4+8(R) 9+6(R)		
	88					
	6					
3.0		SP6				
	888			4+8(D) 5+7(D) 5+8(D) 9+5(D) 9+6(D) 9+7(D)		2+9(A) 9+5(A)
	6	pencil different				4+8(A) 5+7(A) 5+8(A) 9+6(A)
	8					9+7(A) 5+3(A)
		SP3				
	777					
						2+4(A) 2+5(A) 6+1(A) 6+3(A)
	8	SP5		5+8(C)		
2.0	77	SP2		9+7(C)		3+1(A) 2+9(G) 9+6(G)
				6+3(D) 6+3(R) 2+9(D) 5+7(C) 9+5(C)		1+4(A) 5+7(G) 9+5(G) 9+7(G)
				2+4(R) 2+5(R)		
	77					
	6 7			2+4(D) 2+9(R) 4+8(C) 9+6(C)		4+8(G) 5+8(G) 2+5(G)
	77			5+3(R)		6+1(G)
	6 77 8					3+1(G) 2+4(G) 5+3(G)
	66			5+3(D)		1+4(G) 6+3(G)
1.0	6 7					
			9+7			
	66			5+8(F) 9+7(F)		
	7			2+5(C) 4+8(F) 9+5(F) 9+6(F)		
			9+5	2+4(C) 5+7(F) 6+3(C)	2+9 9+5	
			4+8 5+7	5+3(C) 6+3(F) 2+9(C)	4+8	
					9+6	
			5+8 9+6	3+1(R) 2+5(CF)	9+7	
.0	6			2+4(F) 5+3(F) 2+9(F)	2+5 5+3	
				3+1(C) 4+8(CF) 9+5(CF)		
			2+5	1+4(R) 2+5(CF) 2+9(CF) 9+7(CF)	5+7 5+8	
		SP1		5+7(CF) 5+8(CF) 9+6(CF)	6+3	
	66					
-1.0		SP4	5+3 6+3	3+1(F) 1+4(C) 2+4(CF)	1+4 6+1	
			2+4 2+9	6+1(R) 6+3(CF)		
	6			1+4(CF)		9+7(I)
				3+1(CF) 6+1(C)		
				5+3(CF)		
-2.0						2+4(I) 2+5(I) 6+1(I) 2+9(I) 9+5(I)
			3+1 1+4			
			6+1		2+4	
				6+1(CF)		2+4(IF) 2+9(IF) 6+3(I) 4+8(I) 2+5(IF)
						5+3(I) 9+5(IF) 6+1(IF) 5+7 (I) 5+8(I)
-3.0						9+6(I) 9+7(IF)
	6	pencil same				3+1(I) 1+4(I)
					3+1	

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