This paper discusses the various aspects of mathematics teaching and learning at the elementary school level. The National Council of Teachers of Mathematics (NCTM) standards are reviewed by giving examples and a list of principles of learning in mathematics is offered. Principles include the following: 1) the need for pupils to perceive learning activities that are meaningful; 2) mathematics teachers needs to pay attention to the rate of presentation at which pupils may understand and attach meaning to what is taught; 3) the need for pupils to experience a variety of learning opportunities in the mathematics curriculum; 4) the need to experience individual and collaborative endeavors; 5) the need for teachers to minimize lecturing as a method of instruction; 6) the need for a work environment that is conducive to achieving in mathematics; 7) "mathematics for all" is important to stress; 8) achievement in three main categories of objectives in mathematics; 9) emphasis on a hands-on approach to learning; and 10) application of that which has been learned. (ASK)
Mathematics in the Elementary School

by

Marlow Ediger
MATHEMATICS IN THE ELEMENTARY SCHOOL

Teachers and administrators need to determine the best mathematics curriculum possible for each pupil. Additional people to assist in developing the mathematics curriculum include university professors in mathematics, parents, other lay persons, and interested personnel in mathematics education. The mathematics curriculum needs to be studied carefully and objectively to arrive at a consensus involving worthwhile objectives for pupil attainment.

Pupils need to become mathematics literate in a rapidly changing society. The National Council Teachers of Mathematics (NCTM) in 1989 came out with a comprehensive statement of objectives for pupils to achieve. Five broadly stated objectives are the following: Pupils need to

1. learn to value mathematics.
2. become confident in their ability to do mathematics.
3. become mathematical problem solvers.
4. learn to communicate mathematically.
5. learn to reason mathematically (NCTM, 1989, 23).

Each of the above named objectives is relevant for pupils in kindergarten through grade twelve, and beyond, throughout one's lifetime. To value mathematics means to have pupils feel it is important in school and in society in an ongoing way. The use of mathematics is practical in that in everyday situations individuals face problems involving number. There are items to purchase and pay for. There are checkbook balances to keep and maintain accurately. Monthly printouts come from the local bank for individual to verify if the accounts were kept accurately and match what the consumer has record of in his/her checkbook such as lists of canceled checks and money on hand balance. It is always good for the consumer to check the accuracy of his/her checkbook balance with that of the computerized bank statement.

Further utilitarian uses of number include credit card purchases with the accompanying receipts and the monthly statement itemizing purchases and payments made as issued by the credit card company. These are just a few ways that are practical in every day use that indicates the necessity for pupils to appreciate number.

The pupil may also value mathematics for its own sake. There is beauty inherent in seeing order in mathematics. No other academic discipline seemingly has this order which is so consistent. No wonder that Rene Descartes (b, 1596 and d, 1650) based his philosophy of idealism on mathematics. Descartes advocated using deductive reasoning in arriving at truth. His model stressed a person starting at a given point in knowledge where certainty was involved. Thus deductively, the individual presents clear and distinct ideas related to
the starting point. Each new idea is then different from the previous and at the same time is very clear, not vague nor subject to interpretation. (Ediger, 1995).

It is enjoyable to notice order in mathematics, such as in base ten. When counting by twos, the order of numbers in making a table are indeed unique:

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There certainly is order also when thinking of structural ideas in mathematics:

1. the commutative property of addition states that the order of two addends can be added in any sequence and the sum is the same. Thus, \(a + b = b + a\).

2. the associative property of addition states that three or more addends may be added in any order and the sum will be the same, such as \(a + b + c = c + b + a\). There are other arrangements of ordering the addends for addition and the answer or sum will be the same such as \(a + c + b = b + c + a\). There may be any number of addends, not just three.

3. the commutative and associative properties hold true for multiplication in the same way as was true of addition. Thus, \(a \times b = b \times a\) for the commutative property in multiplication. And, \(a \times b \times c = c \times b \times a\) for the associative property of multiplication. There may be other arrangements in considering the order of factors when multiplying such as \(b \times a \times c = c \times a \times b\). There also might be any number of factors to multiply, beyond that of three: \(a \times b \times c \times d = d \times c \times b \times a\). The symbol "\(\times\)" stands for "times," in multiplication.

3. the property of closure indicates that the answer to an addition problem containing whole numbers will also be a whole number. The whole numbers consist of the set: 0, 1, 2, 3, 4, 5, ... The three dots mean the numbers go on sequentially in an infinite manner.

Becoming confident in performing operations and doing problem solving is a must. Learners need to be successful in daily work in mathematics in order to become confident that they can and do achieve success. The mathematics teacher needs to adjust each pupil’s daily work to harmonize with his/her capacity and ability to attain success in goal attainment. The teacher may praise a pupil for doing well in mathematics so that increased confidence is in the offing. The praise needs to be honestly given to the learner who is truly doing better in achievement in mathematics. Praise might then be frequently provided to those who do improve in achieving objectives.
Problem solving in mathematics is always a practical and utilitarian goal. Problems need to be as realistic as possible. Lifelike problems identified by pupils for solving propel learners to put forth effort in learning. These problems are considered intrinsically worthwhile to be significant. There are perplexities here in that the involved learner is not certain of the direction to go which is necessary in offering solutions. Thus deliberation and reflection are needed. Thinking then centers upon clarifying the meaning of the problem as well as of possible alternatives for solution. The solution or hypothesis being considered is tentative and may be modified if evidence warrants. If the solution/hypothesis holds up under testing in a realistic situation, there might then be evidence that the correctness of the answer is no longer tentative. In mathematics, pupils deal with more exactness and accurateness as compared to other curriculum areas, such as social studies. In social studies, there is not the rightness of an answer as compared to mathematics, such as in the basic addition, subtraction, multiplication, and division facts. Thus, for example, in base ten, five plus five is always ten. There are no exceptions.

Being able to communicate well is important in all curriculum areas. Mathematics is no exception. In the following situations, pupils need to be able to communicate clearly and accurately in mathematics:
1. when working in committees and in large group instruction.
2. when discussing ideas with others.
3. when indicating work performed on paper.
4. when using the word processor to show mathematical operations and problem solving experiences.
5. when developing line, bar and circle graphs to show data and information.
6. when using multiple intelligences (Gardner, 1993) to show mathematical ideas. This includes using written work, art work, dramatizations, songs, interpersonal and intrapersonal activities, social studies content, and subject matter from science, information to indicate what has been learned in mathematics. Verbal intelligence, such reading, writing, and taking multiple choice tests, as well as other kinds of tests where reading alone is stressed, should not be used to the extent of crowding out those mentioned previously in multiple intelligences. Nor should verbal intelligence be minimized. Pupils then do possess multiple intelligences and these need to be honored and rewarded by the teacher. Possessing multiple intelligences is quite obvious as one observes professional athletes, musicians, dancers, and individuals in the business and academic worlds reveal their talents.

Sternberg (1997) states that when we expand the range of abilities we test for, we also expand the range of students we identify as smart. Among other curriculum areas, he stresses the following as
examples in mathematics: 
   Memory Remember the mathematical formula (Distance = Rate X Time).
   Analysis Solve a mathematical problem (using the D = RT formula).
   Creativity Create your own mathematical word problem using the D = RT formula.
   Practicality Show how to use the D= RT formula to estimate driving time from one city to another near you.

   Pupils then need to learn to communicate clearly and accurately in mathematics. It is a necessity to do so; otherwise misinformation will be presented. Also, the reasoning ability of pupils is important in mathematics. Pupils need to use logic when reasoning inductively or deductively. When reasoning inductively, pupils achieve sequentially from the specific to the general. Thus, with the commutative and associative properties of addition and multiplication, learners may generalize from a few cases where this property works to many situations. The cases/situations may be tested again and again in terms of correctness to notice the worth of the commutative as well as associative properties. With deductive reasoning, the mathematics teacher may present one or two examples in a meaningful way to pupils whereby the latter then makes application to specific situations of these properties. Pupils may be appraised at regular intervals if the commutative and associative properties of mathematics are understood and being applied.

   Good attitudes of pupils toward mathematics indicate that this academic discipline is valued or prized. Learners need to trust the self that with continued success in ongoing lessons and units in mathematics, they are becoming increasingly confident. Being a proficient problem solver is at the top of mathematics skills and abilities that need to be achieved. With all facets of learning, pupils need to be able to communicate with others in clear and distinct language using symbols in mathematics. Reasoning abilities also need to be cultivated continuously in ongoing lessons and units of study.

Principles of Learning in Mathematics

   The mathematics teacher has selected guidelines which need to be followed in order that pupils may achieve more optimally. These guidelines come from the psychology of learning as advocated by educational psychologists. First of all, pupils need to perceive learning activities that are meaningful. They need to understand that which is taught. Too frequently, rote learning is stressed in which pupils memorize content in ongoing lessons and units of study. Rote learning
is done without pupils realizing what is meant by the content committed to memory. Then too, what is memorized usually is placed in short term memory and forgotten. Whereas, what is understood has a good chance of becoming a part of long term memory. I recommend that teachers select important concepts and generalizations to teach that are taught in a manner whereby pupils attach meaning and understanding. For example, if pupils are to understand the concept of what a line is, they should see short and longer lines on the chalkboard as well as on monitors with software presentations. Next, pupils should look at illustrations and identify lines in individual pictures. Lines should be differentiated from line segments and rays. Pupils should also have chances to look at objects in the classroom and identify lines thereon. The teacher needs to observe that pupils do not become tired or become confused over what a line is, but rather keep attention placed upon the concept to be developed.

Second, the mathematics teacher needs to pay attention to the rate of presentation at which pupils may understand and attach meaning to what is taught. I have observed student teachers and cooperating teachers, whom I have supervised in the public schools, who are in a hurry to have pupils cover much content. The chances are in a hurried lesson that few pupils will grasp the meaning of what is taught. Rather, the teacher needs to observe pupils to pace learning experiences which assist pupils individually to understood that which is taught taught. Lessons may be paced too rapidly so that understanding of content by pupils is not possible, or the content may be paced too slowly whereby pupils loose attention and focus.

Third, pupils need to experience a variety of learning opportunities in the mathematics curriculum. There are more activities available for pupils than ever before. The teacher needs to select those which will guide pupils to achieve as optimally as possible. Which kinds of learning opportunities might then be available for pupils?

1. the basal textbook, carefully chosen, may provide rich experiences for pupils. This can be done if the teacher is creative and brings in novel, unique experiences along with using the basal mathematics textbook. Readiness for each learning activity needs to be provided pupils that precede teaching and learning from the basal. The readiness experiences provide sequential learnings for pupils. Concrete and semi concrete materials should be used as readiness experiences as well as during the time that pupils are engaged in using the basal.

2. videotapes and other audio-visual aids on the understanding level of pupils may provide excellent learning opportunities for pupils. For example, if pupils are studying the area of a square, there are good videotapes that provide interesting ways for pupils to determine the area of a square. Pupils may then observe how the person on in the video measures the length and width of the floor in the house to come up with the area in square feet. There are also films that convey this information
clearly and accurately to pupils. In one film, a pupil wishes to find out how many square inches there are on his desktop at home. The pace of the presentation is good in that pupils listening and observing may obtain the necessary information in an unhurried way in order to be able to measure to ascertain square inches of a square or rectangle. I like to use filmstrips in teaching. Here, the teacher may take as long as he/she wishes on a frame to teach a concept or generalization in mathematics. The teacher or the pupil determines when to turn to the next frame in sequence. I have observed student teachers and supervising teachers make their own slides to use in teaching. These slides then in content pertain to the stated objectives of the ongoing lesson or unit of study.

3. drawings may be made to illustrate concepts and generalizations to pupils. A paper marked in square inches may be an excellent way for pupils to learn how to determine the number of square inches in a given region. Fraction kits showing a pizza divided in halves, fourths, sixths, eighths, and tenths, among other divisions, may assist pupils to understand fractional values better. Pupils may make their own fractional kits to use in ongoing lessons. Fractional kits truly represent manipulative materials whereby learners may use a hands on approach in learning. Thus, the parts may be manipulated to show that two fourths equal one half; the two fourths may be placed over the half for meaningful learning.

4. software and technology may be used wisely in teaching mathematics. Drill and practice experiences provide opportunities for pupils to reflect upon what has not been mastered previously. Each program guides pupils through a set of experiences which assists in remedying what had not been learned previously. Tutorial software guides pupils to understand a new process such as adding fractions with unlike denominators. Sequence in the activities helps pupils to understand why the denominators need to be the same before adding can occur. Simulation emphasizes realistic problem solving activities for pupils. The problem is stated for the pupil but generally needs clarification. The learner, individually or in a committee, may then attempt to locate information to solve the problem. The solution is tentative and subject to testing. If necessary, the solution is modified or changed. Gaming stresses pupils playing a competitive game with two or thee sides involved. The game may also be for an individual pupil. Games can be very enjoyable for pupils as well as provide for needed learning experiences. Wholesome competition may be emphasized in that a team may win whereas there are also losers, but if pupils can enjoy and learn effectively, the competition can be quite healthy.

The point to be made from the above discussion is that pupils need to experience a variety of learning activities so that interest and meaning is involved while boredom is kept to a very minimum.

Fourth, pupils need to experience individual and collaborative
endeavors. Why? there are times when pupils need to be able to work effectively on individual tasks and activities. Then too, there are times whereby pupils need to work well with others in collaborative settings. Both working individually and with others in a positive way are important. In working with others, it is good to have pupils assist each other as needed as well as work politely with others. Learners should become caring individuals who are there to provide needed assistance. High academic achievement should be possible in either individual or group situations.

Esteem needs of individuals should be met whereby pupils are recognized for their talents and abilities. Being confident of the self helps pupils to reach toward higher goals in mathematics achievement. Multicultural education needs to be stressed in collaborative situations. Learners need to respect and accept those who differ according to race, religious beliefs, abilities, and past accomplishments in mathematics. Pupils who are mainstreamed need to be accepted and assisted as needed. There should be no discrimination among individuals due to differences possessed. Differences need to be prized, not cause stumbling blocks.

Fifth, teachers need to minimize lecture as a method of instruction. Explanations are a form of lecture but need to be used to clarify, enrich, and assist to make for meaning in learning. Explanations are short in length and are given in a precise manner to communicate in a rather brief period of time. How short in duration are explanations? It depends upon the amount of subject matter a pupil needs in order to continue working with the task at hand in mathematics. The teacher needs to avoid lengthy explanations whereby pupils lose interest in learning. Sometimes, the mathematics teacher will need to ask questions of the learner to clarify meanings so that the latter may proceed with the ongoing experience. Pupils should be actively engaged in learning and not be passive recipients of knowledge (Ediger, The Modern Elementary School, 1997).

Sixth, pupils need to work in an environment that is conducive to achieving in mathematics. A tense learning environment will not assist pupils to achieve as much as they can. Mathematics anxiety is a term given to pupils who fear mathematics and feel inferior in this curriculum area. A hostile learning environment whereby pupils are scolded for making mistakes is hardly conducive to appreciating and liking mathematics. A relaxed environment in which learners desire to achieve and attain is wanted. In this environment, pupils like to work with others in collaborative endeavors. Learners also like to work by themselves on individual tasks. Good, attractive bulletin board displays may also assist pupils to enjoy mathematics. These displays may be used to teach pupils or they may also be used as devices for pupils to learn on their very own. Wholesome attitudes toward mathematics as a curriculum area need to be in the offing on the student's part. Being
sensitive to pupil’s feeling is important. Ridiculing and humiliating pupils is a no no in teaching mathematics. Mathematics as a curriculum area should not be used to punish pupils for misdeeds.

Seventh, mathematics for all is important to stress. Mathematics is not just for the gifted and talented, but each and every pupil needs to achieve vital objectives in mathematics. To be numeracy literate is very important and each pupil needs to achieve as much as possible. Mathematics is a very useful curriculum area presently for pupils, as well as for the future at a work place.

Equity is important to stress whereby each gender receives good instruction to achieve vital objectives. Evaluation techniques and results indicate where pupils individually need more assistance. Remediation then may follow the diagnosis. Computer use, in particular, needs to stress fairness in time for technology use. Thus, boys and girls need to experience a fair share of time in computer use and develop proficiency in its use for each gender.

Eighth, pupils need to achieve three categories of objectives in mathematics. Knowledge objectives need to be carefully chosen and represent vital facts, concepts, and generalizations for pupils to achieve. The objectives need to reflect arithmetic, algebra, geometry, statistics, and graphing. Each objective is relevant and utilitarian in use. Skills objectives, as a second kind for pupils attainment, need to emphasize critical and creative thinking, as well as problem solving. The third kind to stress are attitudinal objectives. The attitudinal objectives should emphasize positive feelings pupils need, to achieve well in the mathematics curriculum. Quality attitudes are long term and difficult for pupils to develop, but achieving them assists pupils to do well in the knowledge and skills areas. Too frequently, bad attitudes hinder pupils from learning as much as they could. In daily lesson plans and entire units of study, the teacher needs to pay careful attention to what will be taught. Thus, three kinds of objectives--knowledge, skills, and attitudes---need adequate attention in curriculum development.

Ninth, a hands on approach in learning needs to be emphasized. Starting with kindergarten pupils for example, the teacher needs to have an adequate number of materials for pupils to use that identify a set of five objects. Each pupil needs to have five sticks to use in associating the concept of “five” with the actual number of sticks that are there. Pupils may use sticks, and other objects, to add, subtract, multiply, and divide. Objects may also be used for counting, forming geometrical designs, and finding the unknown in algebra.

Tenth, pupils need to apply that which has been learned. To make application in practical situations might well indicate that the pupil understands mathematical content studied. To apply what has been learned, pupils need to attach meaning to subject matter being used. Attaching meaning to content understood indicates that pupils have had and experienced prerequisite tasks such as adding,
subtracting, multiplying, and dividing. Thus, a hands on approach in learning has been used (Ediger, *Teaching Mathematics in the Elementary School*, 1997).

**Conclusion**

The mathematics teacher has numerous responsibilities in guiding pupils to achieve sequentially. Principles of learning need to be used to assist pupils to achieve as optimally as possible. Objectives need to be chosen which are vital and relevant. Knowledge, skills, and attitudinal objectives need to be in the offing for pupils to achieve. Rational balance among these kinds of objectives need to be stressed in ongoing lessons and units of study. Learning opportunities need to be varied so that each pupil's learning style is being accommodated. These learning opportunities are varied to provide for individual differences among learners so that each may achieve as much as possible. The mathematics teacher needs to possess a good repertoire of knowledge in mathematics, but there also needs to be skill on the teacher's part to have learners achieve stated objectives. Sequential experiences for pupils assist in attaining vital goals of mathematics instruction. In a relaxed environment, pupils should achieve as much as individual abilities permit.

**References**


Title: Mathematics in the Elementary School

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Publication Date: 7-22-98

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