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ABSTRACT

This document contains Wisconsin's Model Academic Standards for Mathematics. The Wisconsin Model Academic Standards for Mathematics are designed to be general guidelines which may be adopted or adapted by local school districts with cooperation and input from parents and other concerned citizens. They are important goals for all students from which individual schools can build a complete curriculum specific to the needs of the district and the students. Sample tasks and student work designed according to the standards are presented. Standards are presented for six areas: (1) Mathematical Processes; (2) Number Operations and Relationships; (3) Geometry; (4) Measurement; (5) Statistics and Probability; and (6) Algebraic Relationships. Contains a glossary. (ASK)

ED 420 538

WISCONSIN'S MODEL ACADEMIC STANDARDS FOR

Mathematics

Wisconsin's Model Academic Standards for Mathematics

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*Established by Governor Tommy G. Thompson, January 29, 1997

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A Letter From the Lieutenant Governor and the State Superintendent

To the Citizens of Wisconsin:

On behalf of the Governor's Council on Model Academic Standards, it is our pleasure to present Wisconsin's Model Academic Standards in the area of mathematics at grades four (4), eight (8), and twelve (12). Wisconsin has long been a model for other states in terms of educational quality. However, the world is rapidly becoming a more complex and challenging place. As a result, we must expect greater academic achievement from our children today if they are to be adequately prepared for the challenges of tomorrow. While Wisconsin's Model Academic Standards do demand more of our children, we are confident that they are equal to the task.

These standards are also significant because they herald the dramatically different way in which student achievement will be judged. In the past, achievement was determined by comparing a student's grades to those of his or her peers. In the future, mastery of subject matter will be objectively measured against these new standards at grades four, eight, and twelve. In this way we will know how well a student is learning, not how well that student is doing compared to others.

These model academic standards represent the work of writing teams made up of people from diverse backgrounds. Drafts were subjected to extensive public engagement in which hundreds of additional people offered input. The process of reaching consensus yielded a draft that enjoys very strong public support. Over 74 percent of respondents agreed or strongly agreed that the standards will prepare students for the future. Seventy percent felt they are sufficiently rigorous. Nearly 70 percent agreed or strongly agreed that they are clearly understandable and specific enough to guide local curricula and standards.

It must be stressed that these standards are not intended to limit local districts. Instead they are a model to be met or exceeded. The Council specifically encouraged local districts to implement standards that are more rigorous. However, districts must remember that assessment, including high school graduation exams based on standards, awaits every student in Wisconsin.

In closing, we want to commend the many members of the writing work groups. These teams, comprised of parents, teachers, business people, school board members, and administrators, gave freely of their time to produce the initial drafts. Finally, the citizens of Wisconsin must be thanked for devoting their time and effort to the development of the final draft of Wisconsin's Model Academic Standards.

Scott McCallum

Scott McCallum, Lt. Governor

John T. Benson

John T. Benson, State Superintendent

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Introduction

Defining the Academic Standards

What are academic standards? Academic standards specify what students should know and be able to do, what they might be asked to do to give evidence of standards, and how well they must perform. They include content, performance, and proficiency standards.

— Content standards refer to *what* students should know and be able to do.

— Performance standards tell *how* students will show that they are meeting a standard.

— Proficiency standards indicate *how well* students must perform.

Why are academic standards necessary? Standards serve as rigorous goals for teaching and learning. Setting high standards enables students, parents, educators, and citizens to know what students should have learned at a given point in time. The absence of standards has consequences similar to lack of goals in any pursuit. Without clear goals, students may be unmotivated and confused.

Contemporary society is placing immense academic demands on students. Clear statements about what students must know and be able to do are essential to ensure that our schools offer students the opportunity to acquire the knowledge and skills necessary for success.

Why are state-level academic standards important? Public education is a state responsibility. The state superintendent and legislature must ensure that all children have equal access to high quality educational programs. At a minimum, this requires clear statements of what all children in the state should know and be able to do as well as evidence that students are meeting these expectations. Furthermore, academic standards form a sound basis on which to establish the content of a statewide assessment system.

Why does Wisconsin need its own academic standards? Historically, the citizens of Wisconsin are very serious and thoughtful about education. They expect and receive very high performance from their schools. While educational needs may be similar among states, values differ. Standards should reflect the collective values of the citizens and be tailored to prepare young people for economic opportunities that exist in Wisconsin, the nation, and the world.

Developing the Academic Standards

How were Wisconsin's model academic standards developed? Citizens throughout the state developed the academic standards. The first phase involved educators, parents, board of education members, and business and industry people who produced preliminary content and performance standards in 12 subjects including English language arts, mathematics, science, social studies, visual arts, music, theatre, dance, family and consumer education, foreign language, health education, and physical education. These standards are benchmarked to the end of grades 4, 8, and 12.

The next step required public input aimed at getting information to revise and improve the preliminary standards. This effort included forums and focus groups held throughout the state. The state superintendent used extensive media exposure, including telecommunications through the Department of Public Instruction (DPI) home page, to ensure the widest possible awareness and participation in standards development.

Each subject had at least two drafts taken to the general public for their review. Based on this input, the standards were revised to reflect the values of Wisconsin's citizens.

In January 1997, Governor Thompson appointed the Governor's Council on Model Academic Standards. The Council augmented the existing Department of Public Instruction task forces with additional appointees by the Council, these newly configured task forces produced another draft of model academic standards for the subjects that are part of the state assessment system. These include English language arts, mathematics, reading, science, and social studies.

Once these draft standards were completed, public review became the focus. Using a series of statewide forums coupled with a wide mailing distribution and telecommunications access through both the Wisconsin Department of Public Instruction and the lieutenant governor's home page, Wisconsin citizens provided valuable feedback. As with previous drafts, all comments received serious consideration.

Who wrote the academic standards and what resources were used? Each subject area's academic standards were drafted by teams of educators, parents, board of education members, and business and industry people that were sub-groups of larger task forces. This work was done after reviewing national standards in the subject area, standards from other states, standards from local Wisconsin school districts, and standards like the nationwide New Standards Project.

After the creation of the Governor's Council on Model Academic Standards, four task forces representing English language arts (reading was folded into this group), mathematics, science, and social studies were appointed. Combining the existing DPI task force members with the Council's appointees further ensured that the many voices of Wisconsin's citizenry were represented through the parents, educators, school board members, and business and industry people sitting on those task forces. Documents reviewed included the national standards in the subject area, standards from other states, and standards from local Wisconsin schools. The two most frequently used resources were the first and second drafts of content and performance standards developed by the DPI and the Modern Red Schoolhouse standards developed by the Hudson Institute and Dr. Finley McQuade.

How was the public involved in the standards process? The DPI was involved in extensive public engagement activities to gather citizen input on the first two drafts of the academic standards. Over 19 focus group sessions, 17 community forums, and more than 450 presentations at conferences, conventions, and workshops were held. More than 500,000 paper copies of the standards tabloids were distributed across the state in addition to more than 4,000 citizen visits to the standards on the DPI web page. Input from these activities, along with more than 90 reviews by state and national organizations, provided the writers with feedback on Wisconsin's model academic standards.

Continuing the emphasis on public engagement started by the DPI with previous standards drafts, the Governor's Council on Model Academic Standards held nine community forums. In addition, more than 50,000 paper copies of the standards were distributed. Recipients included each public school building as well as all major education stakeholders and interest groups. Lending the prestige of their offices to the standards development, members of the Council met with editorial boards of media outlets throughout the state discussing the model academic standards.

Will academic standards be developed in areas other than the 12 areas listed above? Yes, currently the DPI has convened six task forces to develop academic standards in agriculture, business, environmental education, marketing, technology education, and information and technology literacy. Task force members include educators, parents, school board members, and representatives of business and industry. These academic standards will be completed by the start of the 1998-99 school year.

Using the Academic Standards

How will the Department of Public Instruction use the Wisconsin Model Academic Standards? Upon completing its work, the Governor's Council on Model Academic Standards submitted academic content and performance standards for English language arts, mathematics, science, and social studies to the governor. On January 13, 1998, Governor Thompson signed Executive Order 326, thus approving and issuing the model academic and performance standards developed by the Governor's Council. These approved standards will be used as the basis for state testing, especially as it relates to the Wisconsin Reading Comprehension Test, the Wisconsin Student Assessment System, and the planned High School Graduation Test.

Additionally, the DPI will use the Wisconsin Model Academic Standards as the basis for revision of its *Guides to Curriculum Planning* and as the foundation for professional development activities that it sponsors.

Must a district adopt the Wisconsin Model Academic Standards? Adopting the Wisconsin Model Academic Standards is voluntary, not mandatory. Districts, however, must have academic standards in place by August 1, 1998. At a minimum, districts are required to have standards in reading and writing, geography and history, mathematics, and science. Districts may adopt the model state standards, or standards from other sources, or develop their own standards.

How will local districts use the academic standards? Districts may use the academic standards as guides for developing local grade-by-grade level curriculum. Implementing standards may require some school districts to upgrade school and district curriculums. In some cases, this may result in significant changes in instructional methods and materials, local assessments, and professional development opportunities for the teaching and administrative staff.

What is the difference between academic standards and curriculum? Standards are statements about what students should know and be able to do, what they might be asked to do to give evidence of learning, and how well they should be expected to know or do it. Curriculum is the program devised by local school districts used to prepare students to meet standards. It consists of activities and lessons at each grade level, instructional materials, and various instructional techniques. In short, standards define what is to be learned at certain points in time, and from a broad perspective, what performances will be accepted as evidence that the learning has occurred. Curriculum specifies the details of the day-to-day schooling at the local level.

What is the link between statewide academic standards and statewide testing? Statewide academic standards in mathematics, English language arts, science, and social studies determine the scope of statewide testing. While these standards are much broader in content than any single Wisconsin Student Assessment System (WSAS) test, they do describe the range of knowledge and skills that may appear on the tests. If content does not appear in the academic standards, it will not be part of a WSAS test. The statewide standards clarify what must be studied to prepare for WSAS tests. If students have learned all of the material indicated by the standards in the assessed content areas, they should do very well on the state tests.

Relating the Academic Standards to All Students

Parents and educators of students with disabilities, with limited English proficiency (LEP), and with accelerated needs may ask why academic standards are important for their students. Academic standards serve as a valuable basis for establishing meaningful goals as part of each student's developmental progress and demonstration of proficiency. The clarity of academic standards provides meaningful, concrete goals for the achievement of students with exceptional education needs (EEN), LEP, and accelerated needs consistent with all other students.

Academic standards may serve as the foundation for individualized programming decisions for students with EEN, LEP, and accelerated needs. While the vast majority of students with EEN and LEP should be expected to work toward and achieve these standards, accommodations and modifications to help these students reach the achievement goals will need to be individually identified and implemented. For students with EEN, these decisions are made as part of their individualized education program (IEP) plans. Accelerated students may achieve well beyond the academic standards and move into advanced grade levels or into advanced coursework.

Clearly, these academic standards are for all students. As our state assessments are aligned with these standards and school districts adopt, adapt, or develop their own standards and multiple measures for determining proficiencies of students, greater accountability for the progress of all students can be assured. In Wisconsin this means all students reaching their full individual potential, every school being accountable, every parent a welcomed partner, every community supportive, and no excuses.

Applying the Academic Standards Across the Curriculum

When community members and employers consider what they want citizens and employees to know and be able to do, they often speak of broad areas of applied knowledge such as communication, thinking, problem solving, and decision making. These areas connect or go beyond the mastery of individual subject areas. As students apply their knowledge both within and across the various curricular areas, they develop the concepts and complex thinking of educated persons.

Community members need these skills to function as responsible citizens. Employers prize those employees who demonstrate these skills because they are people who can continue learning and connect what they have learned to the requirements of a job. College and university faculty recognize the need for these skills as the means of developing the level of understanding that separates the expert from the beginner.

Teachers in every class should expect and encourage the development of these shared applications, both to promote the learning of the subject content and to extend learning across the curriculum. These applications fall into five general categories:

1) Application of the Basics

2) Ability to Think
   - — Problem solving
   - — Informed decision making
   - — Systems thinking
   - — Critical, creative, and analytical thinking
   - — Imagining places, times, and situations different from one's own
   - — Developing and testing a hypothesis
   - — Transferring learning to new situations

3) Skill in Communication
   - — Constructing and defending an argument
   - — Working effectively in groups
   - — Communicating plans and processes for reaching goals
   - — Receiving and acting on instructions, plans, and models
   - — Communicating with a variety of tools and skills

4) Production of Quality Work
   - — Acquiring and using information
   - — Creating quality products and performances
   - — Revising products and performances
   - — Developing and pursuing positive goals

5) Connections with Community
   - — Recognizing and acting on responsibilities as a citizen
   - — Preparing for work and lifelong learning
   - — Contributing to the aesthetic and cultural life of the community
   - — Seeing oneself and one's community within the state, nation, and world
   - — Contributing and adapting to scientific and technological change

Overview of Mathematics

The Wisconsin Model Academic Standards for Mathematics have been developed by a team of Wisconsin citizens including classroom teachers, professional educators, parents, business persons, and school board members. The team used various resource documents in its deliberations, including the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM), *A Guide to Curriculum Planning in Mathematics* of the Wisconsin Department of Public Instruction (DPI), Modern Red Schoolhouse, and state standards and guidelines developed by other entities such as the New Standards Project, the National Institute for Educational Research in Japan, and the states of California, Colorado, Michigan, Oregon, and Virginia.

The Wisconsin Mathematics Standards are designed to be general guidelines which may be adopted or adapted by local school districts with cooperation and input from parents and other concerned citizens. They are not meant to be a full curriculum nor a prescription for instructional practice carried out from week-to-week in classrooms. They are important goals for ALL students from which individual schools can build a complete curriculum specific to their district's and children's needs.

Scope

The content of these standards reflects the shift in mathematical emphasis necessitated by technological advances in an information society. Some topics in mathematics (i.e., estimation, place value) have become more important and several new areas in mathematics (i.e., fractals, predictive statistics) have developed. The understanding of mathematical concepts has become imperative for each citizen as everyday functions become more mathematically complex and as low-skill jobs become nonexistent.

Mathematics instruction must, therefore, be made accessible, understandable, and meaningful for all students. Access includes:

- learning experiences that enable students to acquire and build knowledge and skills
- different instructor roles that use a variety of teaching techniques, adapting them as appropriate for different purposes of instruction and students' needs
- adaptive learning environments so that all students achieve success, and
- servicing all students, including populations previously sometimes underserved, i.e., EEN, LEP, etc.

Likewise, mathematics assessment must address the understanding of all students (as assessed with accommodations which match instructional methodologies) so that mathematics instruction can be evaluated and improved for all students.

Not all students in secondary school elect to pursue "college preparatory" courses. Therefore, the following content and performance standards do not reflect the content of those higher level courses. Rather, they reflect the content of a core mathematical experience that ALL students should encounter, regardless of what mathematics courses they choose.

These standards are intended as points of reference, not limitations. Some students will accomplish much more than these standards envision; yet the standards set the targets for what all students should be challenged, encouraged, and expected to achieve.

Goals and Instructional Practice

Classroom practice geared to the attainment of the Wisconsin standards should be aimed at creating a community of learners and scholars, a place where the teachers and students actively investigate and discuss mathematical ideas, using a wide variety of tools, materials, and technology. Classes should engage students in more high-level mathematical thought and emphasize conceptual understanding, more so than in the past.

Important goals for students are:

- to develop a deep conceptual understanding in order to make sense of mathematics (students need to know not only *how* to apply skills and knowledge, but also *when* to apply them and *why* they are being applied)
- to master specific knowledge necessary for its application to real problems, for the study of related subject matter, and for continued study in mathematics
- to learn and view mathematics as a way of thinking about and interpreting the world around them
- to recognize that mathematics is a creative part of human culture in much the same way as the fine arts

Connections

Mathematics should be viewed as a unified whole made up of connected, big ideas rather than as a disjointed collection of meaningless, abstract ideas and skills. Learning is easier when students see the connections between various concepts and procedures, and between the various branches of mathematics. Students should also be aware of the connections between, and applications of, mathematics and other disciplines, such as the sciences, art, music, business, medicine, and government.

Problem Solving

Mathematics is important because its concepts and procedures can be applied to the solution of problems of varying kinds and complexity. Solving problems challenges students to apply their conceptual understanding in a new or complex situation, to exercise their basic skills, and to see mathematics as a way of finding answers to many of the problems they encounter both within and outside the classroom. Students grow in their ability and persistence in problem solving through extensive classroom experience in posing, formulating, and solving problems at a variety of levels of difficulty and at every level in their mathematical development.

Reasoning

The ability to reason is such a vital part of mathematical behavior that it is safe to assert that mathematics cannot be done without it. At all levels, students should be able to provide a reason why they have chosen to apply a particular skill or concept, or why that skill works the way it does. Further, students should habitually check their results and conclusions for their reasonableness; that is, "does this make sense?" Proportional and spatial reasoning are specific kinds of reasoning that all students should have at their disposal. And, finally, it is important that all students should be able to apply the logical reasoning skills of induction and deduction to make, test, and evaluate mathematical conjectures, to justify steps in mathematical procedures, and determine whether conclusions are valid by analyzing an argument.

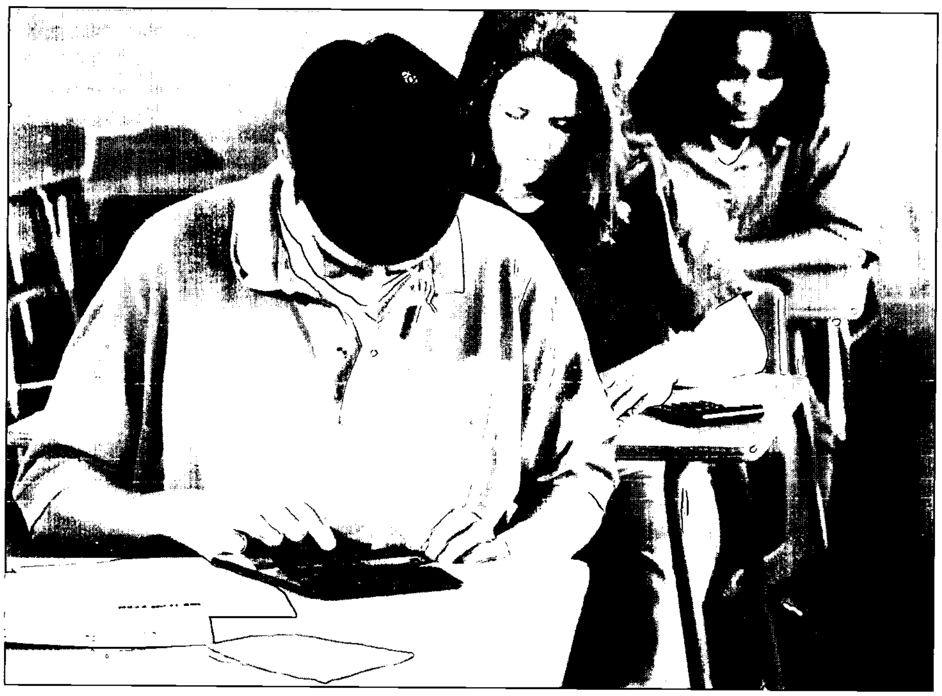
Communication

Whether working alone, or as part of a team, students must be able to communicate their thinking to others. Students must learn not only the signs, symbols, and specialized terms of mathematics, but also how to use this mathematical language in oral, symbolic, and written communication. These communication skills become even more relevant when students leave their classroom world for the world of work.

Technology

Calculators, computers, spreadsheets, graphing utilities, and other forms of electronic information technology are now standard tools for mathematical problem solving in science, engineering, business, medicine, government, and finance. Thus, the use of technology must be an integral part of teaching and learning mathematics. Such use should aim at enhancing conceptual understanding and problem-solving skills. However, the tools of technology are not a substitute for proficiency in basic computational skills.

In the text that follows, terms with an asterisk (*) are defined and/or exemplified in the Glossary of Terms following which appears on page 22 of this document.



A. MATHEMATICAL PROCESSES

CONTENT STANDARD

Students in Wisconsin will draw on a broad body of mathematical knowledge and apply a variety of mathematical skills and strategies, including reasoning, oral and written communication, and the use of appropriate technology, when solving mathematical, real-world* and nonroutine* problems.

Rationale: In order to participate fully as a citizen and a worker in our contemporary world, a person should be mathematically powerful. Mathematical power is the ability to explore, to conjecture, to reason logically, and to apply a wide repertoire of methods to solve problems. Because no one lives and works in isolation, it is also important to have the ability to communicate mathematical ideas clearly and effectively.

PERFORMANCE STANDARDS

BY THE END OF GRADE 4 STUDENTS WILL:

A.4.1 Use reasoning abilities to

- perceive patterns
- identify relationships
- formulate questions for further exploration
- justify strategies
- test reasonableness of results

A.4.2 Communicate mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models*

A.4.3 Connect mathematical learning with other subjects, personal experiences, current events, and personal interests

- see relationships between various kinds of problems and actual events
- use mathematics as a way to understand other areas of the curriculum (e.g., measurement in science, map skills in social studies)

A.4.4 Use appropriate mathematical vocabulary, symbols, and notation with understanding based on prior conceptual work

A.4.5 Explain solutions to problems clearly and logically in oral and written work and support solutions with evidence

*Terms with an asterisk are defined in the Glossary of Terms.

BY THE END OF GRADE 8 STUDENTS WILL:

A.8.1 Use reasoning abilities to

- evaluate information
- perceive patterns
- identify relationships
- formulate questions for further exploration
- evaluate strategies
- justify statements
- test reasonableness of results
- defend work

A.8.2 Communicate logical arguments clearly to show why a result makes sense

A.8.3 Analyze nonroutine* problems by modeling*, illustrating, guessing, simplifying, generalizing, shifting to another point of view, etc.

A.8.4 Develop effective oral and written presentations that include

- appropriate use of technology
- the conventions of mathematical discourse (e.g., symbols, definitions, labeled drawings)
- mathematical language
- clear organization of ideas and procedures
- understanding of purpose and audience

A.8.5 Explain mathematical concepts, procedures, and ideas to others who may not be familiar with them

A.8.6 Read and understand mathematical texts and other instructional materials and recognize mathematical ideas as they appear in other contexts

BY THE END OF GRADE 12 STUDENTS WILL:

A.12.1 Use reason and logic to

- evaluate information
- perceive patterns
- identify relationships
- formulate questions, pose problems, and make and test conjectures
- pursue ideas that lead to further understanding and deeper insight

A.12.2 Communicate logical arguments and clearly show

- why a result does or does not make sense
- why the reasoning is or is not valid
- an understanding of the difference between examples that support a conjecture and a proof of the conjecture

A.12.3 Analyze nonroutine* problems and arrive at solutions by various means, including models* and simulations, often starting with provisional conjectures and progressing, directly or indirectly, to a solution, justification, or counter-example

A.12.4 Develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

A.12.6 Read and understand

- mathematical texts and other instructional materials
- writing about mathematics (e.g., articles in journals)
- mathematical ideas as they are used in other contexts

B. NUMBER OPERATIONS AND RELATIONSHIPS

CONTENT STANDARD

Students in Wisconsin will use numbers effectively for various purposes, such as counting, measuring, estimating, and problem solving.

Rationale: People use numbers to quantify, describe, and label things in the world around them. It is important to know the many uses of numbers and various ways of representing them. Number sense is a matter of necessity, not only in one's occupation but also in the conduct of daily life, such as shopping, cooking, planning a budget, or analyzing information reported in the media. When computing, an educated person needs to know which operations (e.g., addition, multiplication), which procedures (e.g., mental techniques, algorithms*), or which technological aids (e.g., calculator, spreadsheet) are appropriate.

PERFORMANCE STANDARDS

BY THE END OF GRADE 4 STUDENTS WILL:

B.4.1 Represent and explain whole numbers*, decimals, and fractions with

- physical materials
- number lines and other pictorial models*
- verbal descriptions
- place-value concepts and notation
- symbolic renaming (e.g., 43 = 40+3 = 30+13)

B.4.2 Determine the number of things in a set by

- grouping and counting (e.g., by threes, fives, hundreds)
- combining and arranging (e.g., all possible coin combinations amounting to thirty cents)
- estimation, including rounding

B.4.3 Read, write, and order whole numbers*, simple fractions (e.g., halves, fourths, tenths, unit fractions*) and commonly-used decimals (monetary units)

B.4.4 Identify and represent equivalent fractions for halves, fourths, eighths, tenths, sixteenths

B.4.5 In problem-solving situations involving whole numbers, select and efficiently use appropriate computational procedures such as

- recalling the basic facts of addition, subtraction, multiplication, and division
- using mental math (e.g., 37 + 25, 40 x 7)
- estimation
- selecting and applying algorithms* for addition, subtraction, multiplication, and division
- using a calculator

B.4.6 Add and subtract fractions with like denominators

B.4.7 In problem-solving situations involving money, add and subtract decimals

*Terms with an asterisk are defined in the Glossary of Terms.

BY THE END OF GRADE 8 STUDENTS WILL:

B.8.1 Read, represent, and interpret various rational numbers* (whole numbers*, integers*, decimals, fractions, and percents) with verbal descriptions, geometric models*, and mathematical notation (e.g., expanded*, scientific*, exponential*)

B.8.2 Perform and explain operations on rational* numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

B.8.3 Generate and explain equivalencies among fractions, decimals, and percents

B.8.4 Express order relationships among rational numbers using appropriate symbols (>, <, ≥, ≤, ≠)

B.8.5 Apply proportional thinking in a variety of problem situations that include, but are not limited to

- ratios and proportions (e.g., rates, scale drawings*, similarity*)
- percents, including those greater than 100 and less than one (e.g., discounts, rate of increase or decrease, sales tax)

B.8.6 Model* and solve problems involving number-theory concepts such as

- prime* and composite numbers
- divisibility and remainders
- greatest common factors
- least common multiples

B.8.7 In problem-solving situations, select and use appropriate computational procedures with rational numbers such as

- calculating mentally
- estimating
- creating, using, and explaining algorithms* using technology (e.g., scientific calculators, spreadsheets)

BY THE END OF GRADE 12 STUDENTS WILL:

B.12.1 Use complex counting procedures such as union and intersection of sets and arrangements (permutations* and combinations*) to solve problems.

B.12.2 Compare real numbers using

- order relations (>, <) and transitivity*
- ordinal scales including logarithmic (e.g., Richter, pH rating)
- arithmetic differences
- ratios, proportions, percents, rates of change

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

B.12.4 In problem-solving situations involving the application of different number systems (natural, integers, rational*, real*) select and use appropriate

- computational procedures
- properties (e.g., commutativity*, associativity*, inverses*)
- modes of representation (e.g., rationals as repeating decimals, indicated roots as fractional exponents)

B.12.5 Create and critically evaluate numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social)

B.12.6 Routinely assess the acceptable limits of error when

- evaluating strategies
- testing the reasonableness of results
- using technology to carry out computations

C. GEOMETRY

CONTENT STANDARD

Students in Wisconsin will be able to use geometric concepts, relationships and procedures to interpret, represent, and solve problems.

Note: Familiar mathematical content dealing with measurement of geometric objects (e.g., length, area, volume) is presented in "D. Measurement."

Rationale: Geometry and its study of shapes and relationships is an effort to understand the nature and beauty of the world. While the need to understand our environment is still with us, the rapid advance of technology has created another need—to understand ideas communicated visually through electronic media. For these reasons, educated people in the 21st century need a well-developed sense of spatial order to visualize and model real world* problem situations.

PERFORMANCE STANDARDS

BY THE END OF GRADE 4 STUDENTS WILL:

C.4.1 Describe two-and three-dimensional figures (e.g., circles, polygons, trapezoids, prisms, spheres) by

- naming them
- comparing, sorting, and classifying them
- drawing and constructing physical models to specifications
- identifying their properties (e.g., number of sides or faces, two- or three-dimensionality, equal sides, number of right angles)
- predicting the results of combining or subdividing two-dimensional figures
- explaining how these figures are related to objects in the environment

C.4.2 Use physical materials and motion geometry (such as slides, flips, and turns) to identify properties and relationships, including but not limited to

- symmetry*
- congruence*
- similarity*

C.4.3 Identify and use relationships among figures, including but not limited to

- location (e.g., between, adjacent to, interior of)
- position (e.g., parallel, perpendicular)
- intersection (of two-dimensional figures)

C.4.4 Use simple two-dimensional coordinate systems to find locations on maps and to represent points and simple figures

*Terms with an asterisk are defined in the Glossary of Terms.

BY THE END OF GRADE 8 STUDENTS WILL:

C.8.1 Describe special and complex two- and three-dimensional figures (e.g., rhombus, polyhedron, cylinder) and their component parts (e.g., base, altitude, and slant height) by

- naming, defining, and giving examples
- comparing, sorting, and classifying them
- identifying and contrasting their properties (e.g., symmetrical*, isosceles, regular)
- drawing and constructing physical models to specifications
- explaining how these figures are related to objects in the environment

C.8.2 Identify and use relationships among the component parts of special and complex two- and three-dimensional figures (e.g., parallel sides, congruent* faces)

C.8.3 Identify three-dimensional shapes from two-dimensional perspectives and draw two-dimensional sketches of three-dimensional objects preserving their significant features

C.8.4 Perform transformations* on two-dimensional figures and describe and analyze the effects of the transformations on the figures

C.8.5 Locate objects using the rectangular coordinate system*

BY THE END OF GRADE 12 STUDENTS WILL:

C.12.1 Identify, describe, and analyze properties of figures, relationships among figures, and relationships among their parts by

- constructing physical models
- drawing precisely with paper and pencil, hand calculators, and computer software
- using appropriate transformations* (e.g., translations, rotations, reflections, enlargements)
- using reason and logic

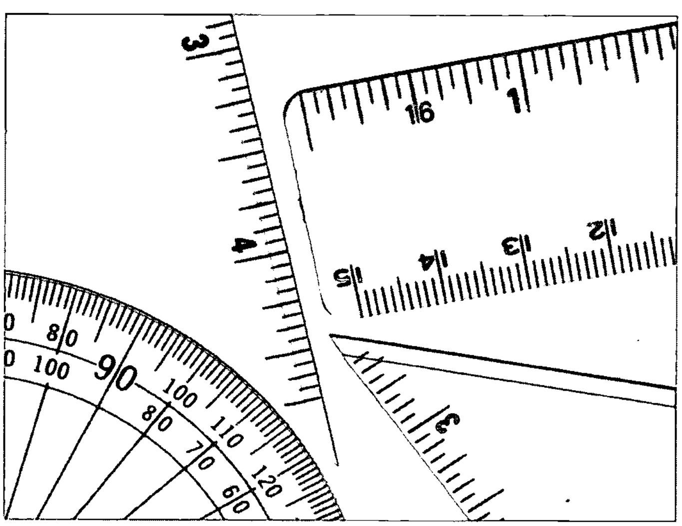
C.12.2 Use geometric models* to solve mathematical and real-world problems

C.12.3 Present convincing arguments by means of demonstration, informal proof, counter-examples, or any other logical means to show the truth of

- statements (e.g., "these two triangles are not congruent")
- generalizations (e.g., "the Pythagorean* theorem holds for all right triangles")

C.12.4 Use the two-dimensional rectangular coordinate system* and algebraic procedures to describe and characterize geometric properties and relationships such as slope*, intercepts*, parallelism, and perpendicularity

C.12.5 Identify and demonstrate an understanding of the three ratios used in right-triangle trigonometry (sine, cosine, tangent)



D. MEASUREMENT

CONTENT STANDARD

Students in Wisconsin will select and use appropriate tools (including technology) and techniques to measure things to a specified degree of accuracy. They will use measurements in problem-solving situations.

Rationale: Measurement is the foundation upon which much technological, scientific, economic, and social inquiry rests. Before things can be analyzed and subjected to scientific investigation or mathematical modeling*, they must first be quantified by appropriate measurement principles. Measurable attributes* include such diverse concepts as voting preferences, consumer price indices, speed and acceleration, length, monetary value, duration of an Olympic race, or probability of contracting a fatal disease.

PERFORMANCE STANDARDS

BY THE END OF GRADE 4 STUDENTS WILL:

D.4.1 Recognize and describe measurable attributes*, such as length, liquid capacity, time, weight (mass), temperature, volume, monetary value, and angle size, and identify the appropriate units to measure them

D.4.2 Demonstrate understanding of basic facts, principles, and techniques of measurement, including

- appropriate use of arbitrary* and standard units (metric and US Customary)
- appropriate use and conversion of units within a system (such as yards, feet, and inches; kilograms and grams; gallons, quarts, pints, and cups)
- judging the reasonableness of an obtained measurement as it relates to prior experience and familiar benchmarks

D.4.3 Read and interpret measuring instruments (e.g., rulers, clocks, thermometers)

D.4.4 Determine measurements directly* by using standard tools to these suggested degrees of accuracy

- length to the nearest half-inch or nearest centimeter
- weight (mass) to the nearest ounce or nearest 5 grams
- temperature to the nearest 5°
- time to the nearest minute
- monetary value to dollars and cents
- liquid capacity to the nearest fluid ounce

D.4.5 Determine measurements by using basic relationships (such as perimeter and area) and approximate measurements by using estimation techniques

*Terms with an asterisk are defined in the Glossary of Terms.

BY THE END OF GRADE 8 STUDENTS WILL:

D.8.1 Identify and describe attributes* in situations where they are not directly* or easily measurable (e.g., distance, area of an irregular figure, likelihood of occurrence)

D.8.2 Demonstrate understanding of basic measurement facts, principles, and techniques including the following

- approximate comparisons between metric and US Customary units (e.g., a liter and a quart are about the same; a kilometer is about six-tenths of a mile)
- knowledge that direct measurement* produces approximate, not exact, measures
- the use of smaller units to produce more precise measures

D.8.3 Determine measurement directly* using standard units (metric and US Customary) with these suggested degrees of accuracy

- lengths to the nearest mm or 1/16 of an inch
- weight (mass) to the nearest 0.1 g or 0.5 ounce
- liquid capacity to the nearest millileter
- angles to the nearest degree
- temperature to the nearest C° or F°
- elapsed time to the nearest second

D.8.4 Determine measurements indirectly* using

- estimation
- conversion of units within a system (e.g., quarts to cups, millimeters to centimeters)
- ratio and proportion (e.g., similarity*, scale drawings*)
- geometric formulas to derive lengths, areas, volumes of common figures (e.g., perimeter, circumference, surface area)
- the Pythagorean* relationship
- geometric relationships and properties for angle size (e.g., parallel lines and transversals; sum of angles of a triangle; vertical angles*)

BY THE END OF GRADE 12 STUDENTS WILL:

D.12.1 Identify, describe, and use derived attributes* (e.g., density, speed, acceleration, pressure) to represent and solve problem situations

D.12.2 Select and use tools with appropriate degree of precision to determine measurements directly* within specified degrees of accuracy and error (tolerance)

D.12.3 Determine measurements indirectly*, using

- estimation
- proportional reasoning, including those involving squaring and cubing (e.g., reasoning that areas of circles are proportional to the squares of their radii)
- techniques of algebra, geometry, and right triangle trigonometry
- formulas in applications (e.g., for compound interest, distance formula)
- geometric formulas to derive lengths, areas, or volumes of shapes and objects (e.g., cones, parallelograms, cylinders, pyramids)
- geometric relationships and properties of circles and polygons (e.g., size of central angles, area of a sector of a circle)
- conversion constants to relate measures in one system to another (e.g., meters to feet, dollars to Deutschmarks)

E. STATISTICS AND PROBABILITY

CONTENT STANDARD

Students in Wisconsin will use data collection and analysis, statistics and probability in problem-solving situations, employing technology where appropriate.

Rationale: Dramatic advances in technology have launched the world into the Information Age, when data are used to describe past events or predict future events. Whether in the business place or in the home, as producers or consumers of information, citizens need to be well versed in the concepts and procedures of data analysis in order to make informed decisions.

PERFORMANCE STANDARDS

BY THE END OF GRADE 4 STUDENTS WILL:

E.4.1 Work with data in the context of real-world situations by

- formulating questions that lead to data collection and analysis
- determining what data to collect and when and how to collect them
- collecting, organizing, and displaying data
- drawing reasonable conclusions based on data

E.4.2 Describe a set of data using

- high and low values, and range*
- most frequent value (mode*)
- middle value of a set of ordered data (median*)

E.4.3 In problem-solving situations, read, extract, and use information presented in graphs, tables, or charts.

E.4.4 Determine if the occurrence of future events are more, less, or equally likely, impossible, or certain

E.4.5 Predict outcomes of future events and test predictions using data from a variety of sources

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*Terms with an asterisk are defined in the Glossary of Terms.

BY THE END OF GRADE 8 STUDENTS WILL:

E.8.1 Work with data in the context of real-world situations by

- formulating questions that lead to data collection and analysis
- designing and conducting a statistical investigation
- using technology to generate displays, summary statistics*, and presentations

E.8.2 Organize and display data from statistical investigations using

- appropriate tables, graphs, and/or charts (e.g., circle, bar, or line for multiple sets of data)
- appropriate plots (e.g., line*, stem-and-leaf*, box*, scatter*)

E.8.3 Extract, interpret, and analyze information from organized and displayed data by using

- frequency and distribution, including mode* and range*
- central tendencies* of data (mean* and median*)
- indicators of dispersion (e.g., outliers*)

E.8.4 Use the results of data analysis to

- make predictions
- develop convincing arguments
- draw conclusions

E.8.5 Compare several sets of data to generate, test, and, as the data dictate, confirm or deny hypotheses

E.8.6 Evaluate presentations and statistical analyses from a variety of sources for

- credibility of the source
- techniques of collection, organization, and presentation of data
- missing or incorrect data
- inferences
- possible sources of bias

E.8.7 Determine the likelihood of occurrence of simple events by

- using a variety of strategies to identify possible outcomes (e.g., lists, tables, tree diagrams*)
- conducting an experiment
- designing and conducting simulations*
- applying theoretical notions of probability (e.g., that four equally likely events have a 25 percent chance of happening)

BY THE END OF GRADE 12 STUDENTS WILL:

E.12.1 Work with data in the context of real-world situations by

- formulating hypotheses that lead to collection and analysis of one- and two-variable data
- designing a data collection plan that considers random sampling, control groups, the role of assumptions, etc.
- conducting an investigation based on that plan
- using technology to generate displays, summary statistics*, and presentations

E.12.2 Organize and display data from statistical investigations using

- frequency distributions
- percentiles*, quartiles, deciles
- line of best fit* (estimated regression line)
- matrices

E.12.3 Interpret and analyze information from organized and displayed data when given

- measures of dispersion*, including standard deviation and variance
- measures of reliability
- measures of correlation*

E.12.4 Analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in journals, magazines, news media, advertising, etc.

E.12.5 Determine the likelihood of occurrence of complex events by

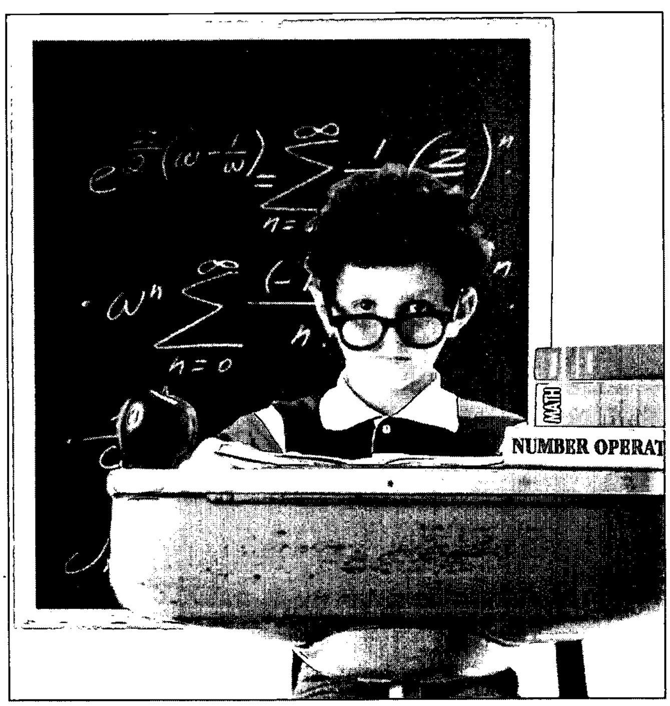
- using a variety of strategies (e.g., combinations*) to identify possible outcomes
- conducting an experiment
- designing and conducting simulations*
- applying theoretical probability

F. ALGEBRAIC RELATIONSHIPS

CONTENT STANDARD

Students in Wisconsin will discover, describe, and generalize simple and complex patterns and relationships. In the context of real-world problem situations, the student will use algebraic techniques to define and describe the problem to determine and justify appropriate solutions.

Rationale: Algebra is the language of mathematics. Much of the observable world can be characterized as having patterned regularity where a change in one quantity results in changes in other quantities. Through algebra and the use of variables* and functions*, mathematical models* can be built which are essential to personal, scientific, economic, social, medical, artistic, and civic fields of inquiry.



PERFORMANCE STANDARDS

BY THE END OF GRADE 4 STUDENTS WILL:

F.4.1 Use letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations (e.g., N + 0 = N is true for any number)

F.4.2 Use the vocabulary, symbols, and notation of algebra accurately (e.g., correct use of the symbol "=", effective use of the associative property of multiplication)

F.4.3 Work with simple linear patterns and relationships in a variety of ways, including

- recognizing and extending number patterns
- describing them verbally
- representing them with pictures, tables, charts, graphs
- recognizing that different models* can represent the same pattern or relationship
- using them to describe real-world phenomena

F.4.4 Recognize variability in simple functional* relationships by describing how a change in one quantity can produce a change in another (e.g., number of bicycles and the total number of wheels)

F.4.5 Use simple equations and inequalities in a variety of ways, including

- using them to represent problem situations
- solving them by different methods (e.g., use of manipulatives, guess and check strategies, recall of number facts)
- recording and describing solution strategies

F.4.6 Recognize and use generalized properties and relationships of arithmetic (e.g., commutativity* of addition, inverse relationship of multiplication and division)

*Terms with an asterisk are defined in the Glossary of Terms.

BY THE END OF GRADE 8 STUDENTS WILL:

F.8.1 Work with algebraic expressions in a variety of ways, including

- using appropriate symbolism, including exponents* and variables*
- evaluating expressions through numerical substitution
- generating equivalent expressions
- adding and subtracting expressions

F.8.2 Work with linear and nonlinear patterns* and relationships in a variety of ways, including

- representing them with tables, with graphs, and with algebraic expressions, equations, and inequalities
- describing and interpreting their graphical representations (e.g., slope*, rate of change, intercepts*)
- using them as models of real-world phenomena
- describing a real-world phenomenon that a given graph might represent

F.8.3 Recognize, describe, and analyze functional relationships* by generalizing a rule that characterizes the pattern of change among variables. These functional relationships include exponential growth and decay (e.g., cell division, depreciation)

F.8.4 Use linear equations and inequalities in a variety of ways, including

- writing them to represent problem situations and to express generalizations
- solving them by different methods (e.g., informally, graphically, with formal properties, with technology)
- writing and evaluating formulas (including solving for a specified variable)
- using them to record and describe solution strategies

F.8.5 Recognize and use generalized properties and relations, including

- additive and multiplicative property of equations and inequalities
- commutativity* and associativity* of addition and multiplication
- distributive* property
- inverses* and identities* for addition and multiplication
- transitive* property

BY THE END OF GRADE 12 STUDENTS WILL:

F.12.1 Analyze and generalize patterns of change (e.g., direct and inverse variation) and numerical sequences, and then represent them with algebraic expressions and equations

F.12.2 Use mathematical functions* (e.g., linear*, exponential*, quadratic*, power) in a variety of ways, including

- recognizing that a variety of mathematical and real-world phenomena can be modeled* by the same type of function
- translating different forms of representing them (e.g., tables, graphs, functional notation*, formulas)
- describing the relationships among variable quantities in a problem
- using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum*, minimum*)

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities

- numerically
- graphically, including use of appropriate technology
- symbolically, including use of the quadratic formula

F.12.4 Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities

Sample Proficiency Standards

The following general statements reflect ratings of student work created in response to a mathematically challenging task.

Advanced

Student work is distinguished in that it goes well beyond the criteria for Proficient in an insightful and creative approach to the task. It includes:

- evidence of reflection upon one's work
- multiple solutions and/or solution strategies
- effective presentation of ideas, using a variety of forms (pictorial, graphic, symbolic, algebraic, verbal)
- evidence of exploration, conjecturing, generalizing, validating and justifying with use of examples and counterexamples when appropriate.

Proficient

Student work completely addresses all aspects of the task. It includes:

- appropriate application of concepts, procedures, and structures although an occasional minor computational error may be present
- clear and complete explanations
- coherent use of mathematical words, symbols, or other visual representations that are appropriate to the task
- logical conclusions based upon known facts, properties and relationships.

Basic

Student work addresses most of the essential conditions of the task. It includes:

- some evidence of the application of appropriate knowledge and skills
- reasonably clear explanations (which may not be complete)
- some accurate conclusions (although reasoning may be faulty or incomplete)
- evidence of some minor misconceptions

Minimal

Student work addresses some of the essential conditions of the task. While it may include some positive elements, the work is characterized by:

- the presence of at least one major conceptual or procedural error
- unsatisfactory or missing communication
- a lack of detail/superficiality
- reasoning that is seriously flawed or completely missing

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Example: A task for fourth grade students.

SUMS-O

Mrs. Lobato and her students enjoy playing an addition facts game called SUMS-O. The game uses all of the 100 flash cards with the basic addition facts. (*Basic facts* add 2 one-digit numbers.) It is played like BINGO, except that players get to make their own game boards.

SUMS-O Rules

- A player writes 9 different numbers in the squares on the game board.
- The game leader picks a flash card and players with the sum of the 2 numbers on the flash card cross it off their game board.
- The winner must have crossed off all three numbers in a row, column, or diagonal.

SUMS-O Game Board

Chris and Toni made these game boards.

6	7	8
3	2	1
9	11	13

CHRIS

2	4	6
12	14	16
22	24	26

TONI

Sample Proficiency Standards

MATHEMATICS

B. Number Operations and Relationships

CONTENT STANDARD

Students in Wisconsin will use numbers effectively for various purposes, such as counting, measuring, estimating, and problem solving.

PERFORMANCE STANDARD

B.4.5 In problem solving situations involving whole numbers, select and efficiently use appropriate computational procedures such as recalling the basic facts of addition, subtraction, multiplication and division

MATHEMATICS

E. Statistics and Probability

CONTENT STANDARD

Students in Wisconsin will use data collection and analysis, statistics and probability in problem solving situations, employing technology where appropriate.

PERFORMANCE STANDARDS

E.4.4 Determine if the occurrence of future events is more, less, or equally likely, unlikely, impossible, or certain

E.4.5 Predict outcomes of future events and test predictions using data from a variety of sources

SAMPLE TASK

Successful completion of the task involves a thorough knowledge of the basic facts of addition, good number sense, and a realization that chances of winning are increased by: 1) choosing the sums that are more likely to occur in the game than other sums, and 2) placing those numbers in the optimal positions.

Advanced

Student work is distinguished in that it goes well beyond the criteria for Proficient in an insightful and creative approach to the task. It should include definite evidence of the recognition that the best chances of winning involve correct choice and placement of those numbers that will come up most often as sums. Written explanations should be clear and well organized.

Proficient

Student work completely addresses all aspects of the task. It should include correct use of computational procedures although an occasional minor error is allowed. Written or symbolic explanations should be easy to follow. Conclusions about the chances of winning of the two hypothetical students should be justified by examples, counter-examples, or citation of known mathematical properties.

Basic

Student work addresses most of the essential conditions of the task. Students may fail to address the chances of both hypothetical students. There might be some misconception about odd or even numbers. Respondent's own game board may be well filled out, but rationale may be sketchy, missing, or with evidence of some misconceptions.

Minimal

Student work addresses only some, or even none, of the essential conditions of the task. Written explanations, if any, may not fit numerical evidence presented. Student may criticize the choice of the hypothetical students, but then repeat the same choice in his/her own board. Rationale is completely faulty. Rules for filling out the game board are not followed.

EXPLANATION OF RATINGS FOR STUDENT WORK

Proficient

All aspects of the task are addressed. The explanation for Question 1 is clearly presented. Although the explanation for the student's own choice of a game board is non-verbal, it is well presented in such a way that one can easily understand why the various numbers were chosen. Inclusion of commutative pairs of addends (e.g., 9+6 and 6+9; 4+1 and 1+4) is important.

On the other hand, this response does not merit an Advanced Rating. The organization on page 2 is not completely systematic and the better choices of 12 and 13 were not even considered. The location of numbers on the student's own game board suggests that optimal placement (center square is best, four corners are next best) of sums that have the highest likelihood of being called was not considered.

1. Explain what you think Chris' and Toni's chances of winning SUMS-O might be and why.

Chris has a better chance of winning because on Toni's card, there are the numbers 22, 24 and 26. If a basic fact is a one-digit plus another one-digit 9+9 is the biggest problem, but it only equals 18. So 22, 26, and 24 would be impossible to get a basic fact of.

2. Write numbers in the game board that you would use to try to win SUMS-O.

11	6	7
~~6~~5	8	10
9	15	4

MY GAME BOARD

3. Explain why you think this will give you a very good chance of winning. Because

=15 possible for 15: 9+6, 8+7, 7+8, 6+9

=10 possible for 10: 9+1, 8+2, 7+3, 6+4, 5+5, 4+6, 3+7, 2+8, 1+9

=8 possible for 8: 8+0, 7+1, 6+2, 5+3, 4+4, 3+5, 2+6, 1+7, 0+8

=6 possible for 6: 0+6, 1+5, 2+4, 3+3, 4+2, 5+1, 6+0

=5 possible for 5: 5+0, 4+1, 3+2, 2+3, 1+4, 0+5

=4: 0+4, 1+3, 2+2, 3+1, 4+0 possible for 4

=11: 9+2, 8+3, 7+4, 6+5, 5+6, 4+7, 3+8, 2+9 Possible for 11

=7: 0+7, 1+6, 2+5, 3+4, 4+3, 5+2, 6+1, 7+0 possible for 7

9: 9+0, 8+1, 7+2, 6+3, 5+4, 4 5, 3+6, 2+7, 1+8, 0+9 possible for nine

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Basic

All aspects of the task are addressed but there is no recognition that Toni has some impossible numbers. The explanation for Question 3 is clear and understandable, but shows some misconception that chance of winning is based solely on a balance of odd and even numbers, and not on choice of sums that are more likely to occur than others.

1. Explain what you think Chris' and Toni's chances of winning SUMS-O might be and why.

Toni's' winning is probably ~~better~~ good because she/he has all even numbers and ~~it's~~ has a good chance to get doubles. Chris's is okay too because sometimes you pull out four a lot and most of his can get four in them.

2. Write numbers in the game board that you would use to try to win SUMS-O.

4	8	10
5	9	13
12	18	14

MY GAME BOARD

3. Explain why you think this will give you a very good chance of winning.

I think this will give me a good chance of winning because I have some odd some even which balances it off because sometimes you get odd numbers and sometimes you get even,

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Minimal

The explanation for Question 1 indicates a misinterpretation of the rules; the student believes the numbers on the board themselves must be added and the sum must also be on the board. The choice of numbers put on the student's own gameboard violate the rules of the game—repetitions and impossibles sums (0 and 1). However, the student **did try** and that is an important fact in his/her favor. It must also be noted that all of the indicated sums and differences written on the paper are correct with the one exception of 22 + 24 = 26.

Yeses and why..........

1. Explain what you think Chris' and Toni's chances of winning SUMS-O might be and why.

Yes, for chris because 7+1=8.... 9+2=11.... 11+2=13..... 6+1=7....

~~Yes, because that is~~ well ~~because~~ Yes, because he has good ones too like 2+4=6..12+4=16.... 22+24=26.... 6+6=12..........

2. Write numbers in the game board that you would use to try to win SUMS-O.

9	1	10
0	1	1
9	2	10

MY GAME BOARD

3. Explain why you think this will give you a very good chance of winning.

I got agood chance of winning because 9+1=10, 10-1=9, 9-0=9, and 1+1=2........

Glossary of Terms

Algorithm. An established step-by-step procedure used to achieve a desired result. For example, the addition algorithm for the sum of two two-digit numbers where carrying is required:

$$\begin{array}{r} 55 \\ +\ 27 \\ \hline 82 \end{array}$$

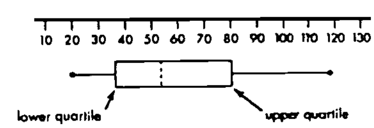
Arbitrary unit (of measure). A unit that is not part of the standardized metric or US Customary systems. For example, using one's own shoe size to measure the length of a door opening or saying that the area of an exhibition hall floor is "about the size of two football fields."

Associative property. When adding or multiplying three numbers, it doesn't matter if the first two or the last two numbers are added or multiplied first. For example,

$$\begin{array}{ccccc} 3 + 9 + 7 & = & (3 + 9) + 7 & = & 3 + (9 + 7) \\ & & 12 + 7 & = & 3 + 16 \\ & & 19 & = & 19 \end{array} \qquad \begin{array}{ccccc} 3 \times 9 \times 7 & = & (3 \times 9) \times 7 & = & 3 \times (9 \times 7) \\ & & 27 \times 7 & = & 3 \times 63 \\ & & 189 & = & 189 \end{array}$$

Attribute (measurable). An identifiable property of an object, set, or event that is subject to being measured. For example, some of the measurable attributes of a box are its length, weight, and capacity (how much it holds).

Box plot. A graphic method that shows the distribution of a set of data by using the median, quartiles, and the extremes of the data set. The box shows the middle 50 percent of the data; the longer the box, the greater the spread of the data.



Central tendencies. A number which in some way conveys the "center" or "middle" of a set of data. The most frequently used measures are the mean and the median.

Combinations. Subsets chosen from a larger set of objects in which the order of the items in the subset does not matter. For example, determining how many different committees of four persons could be chosen from a set of nine persons. (See also, Permutations)

Commutative property. Numbers can be added or multiplied in either order. For example, $15 + 9 = 9 + 15$; $3 \times 8 = 8 \times 3$.

Congruence. The relationship between two objects that have exactly the same size and shape.

Correlation. The amount of positive or negative relationship existing between two measures. For example, if the height and weight of a set of individuals were measured, it could be said that there is a positive correlation between height and weight if the data showed that larger weights tended to be paired with larger heights and smaller weights tended to be paired with smaller heights. The stronger those tendencies, the larger the measure of correlation.

Deciles. The 10th, 20th, 30th, ...90th percentile points. (See definition for Percentile)

Direct measurement. A process of obtaining the measurement of some entity by reading a measuring tool, such as a ruler for length, a scale for weight, or a protractor for angle size.

Dispersion. The scattering of the values of a frequency distribution (of data) from an average.

Distributive property. Property indicating a special way in which multiplication is applied to addition of two (or more) numbers. For example,
$5 \times 23 = 5 \times (20 + 3) = 5 \times 20 + 5 \times 3 = 100 + 15 = 115$.

Expanded notation. Showing place value by multiplying each digit in a number by the appropriate power of 10. For example, 523 = 5 x 100 + 2 x 10 + 3 x 1 or $5 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$.

Exponential function. A function that can be represented by an equation of the form $y = ab^x + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$ and $b > 0$ and $b \neq 1$.

Exponential notation (exponent). A symbolic way of showing how many times a number or variable is used as a factor. In the notation 5^3, the exponent 3 shows that 5 is a factor used three times; that is $5^3 = 5 \times 5 \times 5 = 125$.

Frequency distribution. An organized display of a set of data that shows how often each different piece of data occurs.

Function. A relationship between two sets of numbers or other mathematical objects where each member of the first set is paired with only one member of the second set. Functions can be used to understand how one quantity varies in relation to (is a function of) changes in the second quantity. For example, there is a functional relationship between the price per pound of a particular type of meat and the total amount paid for ten pounds of that type of meat.

Functional notation. A convenient way to show how a function takes a value and transforms it into a new value. A commonly used notation is called Euler notation and is written f (x) and read "f of x." For example, if f (x) = 3x + 5, this means that the function f will take any value from the domain of x and multiply it by 3 and then add 5 to make the new value for x (e.g., f (4) = 3*4 + 5 = 17).

Identity. For addition: The number 0; that is N + 0 = N for any number N. For multiplication: The number 1; that is, N x 1 = N for any number N.

Indirect measurement. A process where the measurement of some entity is not obtained by the direct reading of a measuring tool or by counting of units superimposed alongside or on that entity. For example, if the length and width of a rectangle are multiplied to find the area of that rectangle, then the area is an indirect measurement.

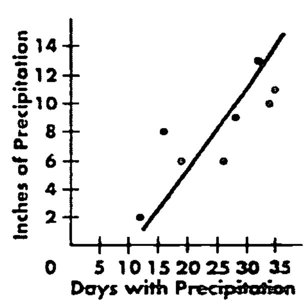
Integers. The set of numbers: {..., -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6,...}

Intercept. The points where a line drawn on a rectangular-coordinate-system graph intersect the vertical and horizontal axes.

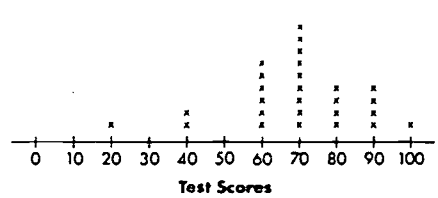
Inverse. For addition: For any number N, its inverse (also called opposite) is a number -N so that N + (-N) = 0 (e.g., the opposite of 5 is -5, the opposite of -3/4 is 3/4).

For multiplication: For any number N, its inverse (also called reciprocal) is a number N* so that N x (N*) = 1 (e.g., the reciprocal of 5 is 1/5; the reciprocal of -3/4 is -4/3).

Line of best fit. A straight line used as a best approximation of a summary of all the points in a scatter-plot* (See definition below). The position and slope of the line are determined by the amount of correlation* (See definition above) between the two paired variables involved in generating the scatter-plot. This line can be used to make predictions about the value of one of the paired variables if only the other value in the pair is known.



Line plot. A graphical display of a set of data where each separate piece of data is shown as a dot or mark above a number line.



Linear equation. An equation of the form $y = ax + b$, where a and b can be any real number. When the ordered pairs (x, y) that make the equation true for specific assigned values of a and b are graphed, the result is a straight line.

Matrix (pl.: matrices). A rectangular array of numbers, letters, or other entities arranged in rows and columns.

Maximum/minimum (of a graph). The highest/lowest point on a graph. A relative maximum/minimum is higher/lower than any other point in its immediate vicinity.

Mean. The arithmetic average of a set of numerical data.

Median. The middle value of an ordered set of numerical data. For example, the median value of the set {5, 8, 9, 10, 11, 11,13} is 10.

Mode. The most frequently occurring value in a set of data. For example, the mode of the set {13, 5, 9, 11, 11, 8, 10} is 11.

Model (mathematical). A [verb] and a noun. [Generate] a mathematical representation (e.g., number, graph, matrix, equation(s), geometric figure) for real-world or mathematical objects, properties, actions, or relationships.

(Non)-Linear functional relationship. (See definition of Function above.) Many functions can be represented by pairs of numbers. When the graph of those pairs results in points lying on a straight line, a function is said to be linear. When not on a line, the function is nonlinear.

Nonroutine problem. A problem is nonroutine if it cannot be solved simply by substituting specific data into a formula that the student knows, or by following step-by-step the solution of a type of problem with which the student is already familiar.

Outlier. For a set of numerical data, any value that is markedly smaller or larger than other values. For example, in the data set {3, 5, 4, 4, 6, 2, 25, 5, 6, 2} the value of 25 is an outlier.

Patterns. Recognizable regularities in situations such as in nature, shapes, events, sets of numbers. For example, spirals on a pineapple, snowflakes, geometric designs on quilts or wallpaper, the number sequence {0, 4, 8, 12, 16,...}.

Percentile. A value on a scale that indicates the percent of a distribution that is equal to it or below it. For example, a score at the 95th percentile is equal to or better than 95 percent of the scores.

Permutations. Possible arrangements of a set of objects in which the order of the arrangement makes a difference. For example, determining all the different ways five books can be arranged in order on a shelf.

Prime number. A whole number greater than 1 that can be divided exactly (i.e., with no remainder) only by itself and 1. The first few primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37....

Pythagorean theorem (relationship). In a right triangle, $c^2 = a^2 + b^2$, where c represents the length of the hypotenuse (the longest side of the triangle which is opposite the right angle), and a and b represent the lengths of the other two, shorter sides of the triangle.

Quadratic function. A function that can be represented by an equation of the form $y = ax^2 + bx + c$, where a, b, and c are arbitrary, but fixed, numbers and $a \neq 0$. The graph of this function is a parabola.

Quartiles. The 25th, 50th and 75th percentile points. (See definition of Percentile.)

Range (of a set of data). The numerical difference between the largest and smallest values in a set of data.

Rational number. A number that can be expressed as the ratio, or quotient, of two integers, a/b, provided $b \neq 0$. Rational numbers can be expressed as common fractions or decimals, such as 3/5 or 0.6. Finite decimals, repeating decimals, mixed numbers and whole numbers are all rational numbers. Nonrepeating decimals cannot be expressed in this way, and are said to be irrational.

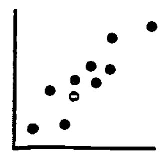
Real numbers. All the numbers which can be expressed as decimals.

Real-world problems. Quantitative and spatial problems that arise from a wide variety of human experiences, applications to careers. These do not have to be highly complex ones and can include such things as making change, figuring sale prices, or comparing payment plans.

Rectangular coordinate system. This system uses two (for a plane) or three (for space) mutually perpendicular lines (called coordinate axes) and their point of intersection (called the origin) as the frame of reference. Specific locations are described by ordered pairs or triples (called coordinates) that indicate distance from the origin along lines that are parallel to the coordinate axes.

Scaling (Scale drawing). The process of drawing a figure either enlarged or reduced in size from its original size. Usually the scale is given, as on a map 1 inch equals 10 miles.

Scatter plot. Also known as scattergram or scatter diagram. A two dimensional graph representing a set of bi-variate data. That is, for each element being graphed, there are two separate pieces of data. For example, the height and weight of a group of 10 teenagers would result in a scatter plot of 10 separate points on the graph.



Scientific notation. A short-hand way of writing very large or very small numbers. The notation consists of a decimal number between 1 and 10 multiplied by an integral power of 10. For example, $47{,}300 = 4.73 \times 10^4$; $0.000000021 = 2.1 \times 10^{-8}$.

Similarity. The relationship between two objects that have exactly the same shape but not necessarily the same size.

Simulation. Carrying out extensive data collection with a simple, safe, inexpensive, easy-to-duplicate event that has essentially the same characteristics as another event which is of actual interest to an investigator. For example, suppose one wanted to gather data about the actual order of birth of boys and girls in families with five children (e.g., BBGBG is one possibility). Rather than wait for five children to be born to a single family, or identifying families that already have five children, one could simulate births by repeatedly tossing a coin five times. Heads vs. tails has about the same chance of happening as a boy vs. a girl being born.

Slope. A measure of the steepness or incline of a straight line drawn on a rectangular-coordinate-system graph. The measure is obtained by the quotient "rise/run" (vertical change divided by horizontal change) between any two points on that line.

Stem-and-leaf plot. A way of showing the distribution of a set of data along a vertical axis. The plot at right shows the data 13, 19, 33, 26, 19, 22, 34, 16, 28, 34. The ten's digits of these data are the stems and the one's digits are the leaves.

1|3699
2|268
3|344
Key: 1|5 means 15

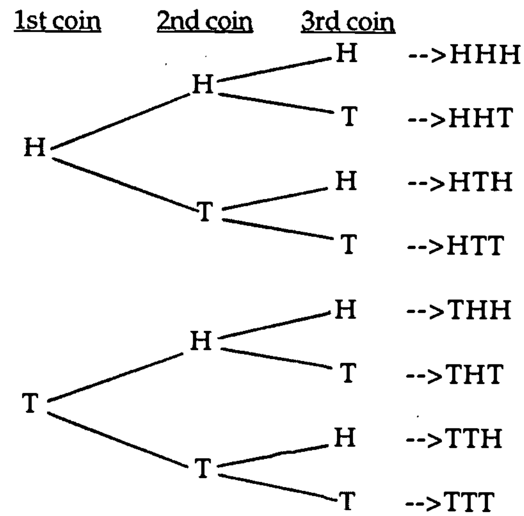
Summary statistics. A single number representation of the characteristics of a set of data. Usually given by measures of central tendency and measures of dispersion (spread).

Symmetry. A figure has symmetry if it has parts that correspond with each other in terms of size, form, and arrangement. For example, a figure with line (or mirror) symmetry has two halves which match each other perfectly if the figure is folded along its line of symmetry.

Transformation. A change in the size, shape, location or orientation of a figure.

Transitive property. For equality: If $a = b$ and $b = c$, then $a = c$.
For inequality: If $a > b$ and $b > c$, then $a > c$; or If $a < b$ and $b < c$, then $a < c$.

Tree diagram. A schematic way of showing the number of ways a compound event may occur. For example, the tree diagram at the right shows the eight possible ways the tossing of three coins could happen.

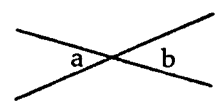


Unit fraction. A fraction with a numerator of 1, such as 1/4 or 1/7.

Variable. A quantity that may assume any one of a set of values. Usually represented in algebraic notation by the use of a letter. In the equation $y = 2x + 7$, both x and y are variables.

Variance. The value of the standard deviation squared.

Vertical angles. The pair of angles that are directly across from each other when two straight lines intersect. Angles a and b at the right are an example of vertical angles.



Whole numbers. The numbers: 0, 1, 2, 3, 4, 5,....

Appendix

The following people contributed to the development of these Wisconsin Academic Standards in Mathematics by serving as a reviewer and/or a member of a focus group. Their contributions are gratefully acknowledged.

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