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ABSTRACT

This report describes the theoretical background of an international comparison project on pupils' mathematical beliefs and outlines its realization. The first chapter briefly discusses problems with the underlying concepts of "belief" and "conception." The central concept, view of mathematics, is introduced in the second chapter. The meaning of beliefs for pupils' learning is also considered in detail and some research results presented. The third chapter addresses the issue of international comparison. The fourth chapter is devoted to the description of research design and plans for its realization. The last chapter provides an overview of the state of the research project, the pilot study in two stages, and the main study. The project is currently still in the first stage of the pilot study, i.e., the preliminary data has been gathered using a questionnaire from 10 countries including Australia, Estonia, Finland, Germany, Hungary, Italy, Norway, Russia, Sweden, and the United States with 200 pupils in each. Some preliminary results from the five country comparison are also discussed. Contains 77 references. (Author/NB)

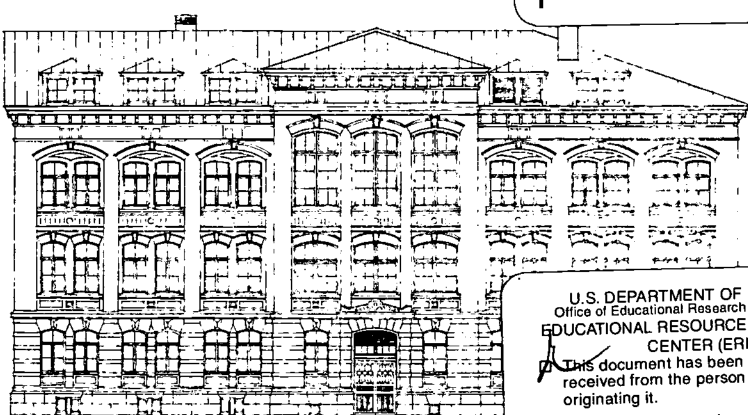
Research Report 152

Erkki Pehkonen

Pupils' View of Mathematics

Initial report for an international comparison project

Department of Teacher Education
University of Helsinki



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Erkki Pehkonen

Pupils' View of Mathematics: Initial report for an international comparison project. 47 p.

Summary

The report deals with the theoretical background of an international comparison project on pupils' mathematical beliefs, and outlines its realization. In the first chapter, problems with the underlying concepts "belief" and "conception" are discussed briefly. The central concept "view of mathematics" is introduced in the second chapter. In addition, the meaning of beliefs for pupils' learning is considered in detail, and some research results are given.

In the third chapter, the issue of international comparison is dealt with. The fourth chapter is devoted to the description of research design and plans for its realization. The last chapter gives an overview of the state of the research project: the pilot study in two stages, and the main study.

Today, the project is still in the first stage of the pilot study, i.e. the preliminary data has been gathered with a questionnaire from ten countries (Australia, Estonia, Finland, Germany, Hungary, Italy, Norway, Russia, Sweden, the USA), with N ≈ 200 pupils in each. At the end, there are some preliminary results from the five-country comparison.

Key words:
conceptions, mathematics, pupils, international comparison

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Erkki Pehkonen

Oppilaiden matematiikkakuva: Kansainvälisen vertailuprojektin alkuraportti. 47 s.

Tiivistelmä:

Raportti käsittelee oppilaiden matemaattisia uskomuksia selvittelevän kansainvälisen vertailuprojektin teoreettista taustaa ja hahmottelee sen toteutuksen. Ensimmäisessä luvussa keskustellaan lyhyesti pohjana oleviin käsitteisiin "uskomus" (belief) ja "käsitys" (conception) liittyvistä ongelmista. Keskeinen käsite "matematiikkakuva" otetaan käyttöön toisessa luvussa. Lisäksi tarkastellaan uskomusten merkitystä oppilaan oppimiselle yksityiskohtaisesti, ja annetaan joitakin tutkimustuloksia.

Kolmannessa luvussa käsitellään kansainvälisen vertailun tilannetta. Neljäs luku on omistettu tutkimussuunnitelman ja sen toteutuksen kuvailemiseen. Viimeinen luku antaa yleiskuvan tutkimusprojektin tilasta: kaksivaiheinen esitutkimus ja varsinainen tutkimus.

Tällä hetkellä projekti on vielä esitutkimuksen ensimmäisessä vaiheessa, ts. alustavia tietoja on kerätty kyselylomakkeella kymmenestä maasta (Australia, Eesti, Italia, Norja, Ruotsi, Saksa, Suomi, Unkari, USA, Venäjä), jokaisessa N ≈ 200 oppilasta. Lopuksi on annettu joitakin alustavia tuloksia viiden maan vertailusta.

Avainsanat:
käsitykset, matematiikka, oppilas, kansainvälinen vertailu

Preface

This project has its roots in the earlier research project "Open Tasks in Mathematics" (Pehkonen & Zimmermann 1990), sponsored by the Finnish Academy. In that project, one of the original ideas was to compare the situation in Finland and Germany. In the background study of the mentioned research project, we used a questionnaire on pupils' mathematical conceptions (for the questionnaire see e.g. Pehkonen 1992) which has played a central role in developing the new international comparative study.

As an answer to the often-posed question "What are the benefits of an international comparison in this field?", the main reason is the question of the transferability of research results obtained e.g. in the United States. How far is it possible to carry over the results directly, e.g. to the European situation? Or are pupils' conceptions culture-bound?

Hereby, I would like to thank all persons who have helped me in initiating this project, especially the pupils and teachers in those Finnish schools where the questionnaire was administered. Without them, there would be no results to be considered. To Professor Olaf Prinits, Dr. Klara Tompa, and Mr. Arne Engström, I want to express my gratitude for their interest in my research project, and for their willingness to administer the questionnaire in their own countries which gave me the idea for an international comparative study.

My thanks are due to my colleague, Dr. Maija Ahtee, who kindly read my draft manuscript and gave me valuable ideas to improve it. I am also thankful to our Head of Department, Professor Irina Koskinen, for her continuous interest in and support for my research work, and for accepting this initial report to be published in the Research Reports series.

Helsinki, May, 1995

Erkki Pehkonen

Contents

Introduction

The sixth original target field in the three-year research project "Open Tasks in Mathematics" (Pehkonen & Zimmermann 1989, 1990), sponsored by the Finnish Academy, was as follows: To clarify whether there are differences in the teaching of mathematics between Finland and Germany (especially Hamburg). From this idea of comparison, an international comparison project has developed, in the first stage (pilot study) of which pupils' conceptions have been charted with the same questionnaire, in many countries.

The starting point for the project was the following: When the author was realizing the mentioned research project, some mathematics educators from the neighboring countries visited the University of Helsinki and got acquainted with the project. They were interested in carrying out a similar inquiry in their own country. Thus, the situation was interesting and challenging for the researcher: There was an opportunity for a large international comparison in the case of pupils' conceptions about mathematics teaching, and the field seemed to be uncovered.

The very first stage. When the author visited Estonia in the fall 1989 (the Universities of Tallinn and Tartu), the Estonian colleagues in Tartu showed their interests in the pupils' and teachers' questionnaire about conceptions on mathematics teaching. They wanted to administer the same questionnaires in Estonia. Later on, Professor Olaf Prinits (University of Tartu) visited the University of Helsinki in April 1990, in order to get acquainted with the realization of the research project, "Open Tasks in Mathematics", in schools.

Dr. Klára Tompa (Centre for Evaluation Studies, National Institute of Public Education, Budapest) expressed interest in the research project during her stay as a stipendiate at the Department of Teacher Education (University of Helsinki) in the fall of 1990. She voluntereed to collect the Hungarian data for pupils' conceptions.

Arne Engström (University of Lund), who was at that time working on his doctoral thesis, visited Helsinki for three days in March 1991, in order to get acquainted with the research project in theory and its realization in schools. He was ready to carry out the same test (pupils' questionnaire) in Sweden.

With these initial contacts (1989–1991), the international comparison project was born. Since then the author has enlargened it. Any time, e.g. after conference presentations on the preliminary results of the comparison project, if somebody showed interest in the comparative results introduced, the author asked him/her to collect corresponding data in his/her own country. Today, there are comparable data from altogether ten different countries.

1. Theoretical background

Mathematics teaching in schools has been under change in almost all countries, and the process is continuing. In particular, our conception of school mathematics and its teaching has been changing (e.g. Zimmermann 1991a). Mathematics in school is no longer understood as a static system which pupils are supposed to adopt as such. Instead it is thought that pupils learn at their best when they are working actively on a new topic. This change has been supported, among others, by the national accounts of mathematics teaching (e.g. Cockcroft 1982, NCTM 1989, 1991).

Within research in school, the understanding of learning has concentrated, in the first place, on the following of cognitive academic achievement. Affective by-results, which are in connection with an individual's metacognitions, however, determine the quality of learning. But they are often left aside in studies. During the last decade, researchers around the world have paid more and more attention to mathematics learning from the viewpoint of metacognitions, especially in the form of pupils' and teachers' beliefs. Beliefs are situated in the "twilight zone" between the cognitive and affective domain. They have a component in each domain.

Behind the mentioned active understanding of learning, one finds the view of learning which is compatible with constructivism. In that view, it is essential that a learner is actively working, in order to be able to elaborate his knowledge structure (e.g. Davis & al. 1990; Ahtee & Pehkonen 1994). Thus, the meaning of pupils' own beliefs (subjective knowledge) concerning mathematics and its learning is emphazised as a regulating system of his knowledge structure. Since the teacher is the central influential factor as an organizer of learning environments, his beliefs are also essential. Therefore, teachers' and pupils' mathematical beliefs play a key role when trying to understand their mathematical behavior (Noddings 1990, 14).

1.1. Beliefs and belief systems

During this century, beliefs and belief systems were, to some extent, examined in the beginning of the century, mainly in social psychology (Thompson 1992). But briefly after that, behaviorism spread to the research in the psychological domains. Then the focus was on the observational parts of human behavior, and beliefs were nearly forgotten. New interest in beliefs and belief systems emerged mainly in the 1970s, through the developments in cognitive science. (Abelson 1979)

An individual continuously receives perceptions from the world around him. According to his experiences and perceptions, he makes conclusions about different phenomena and their nature. The individual's personal knowledge, i.e. his beliefs, are a compound of these conclusions. Furthermore, he compares these beliefs with his new experiences and with the beliefs of other individuals, and thus his beliefs are under continuous evaluation and change. When he adopts a new belief, this will automatically form a part of the larger structure of his personal knowledge, of his belief system, since beliefs never appear fully independently. Thus, the individual's belief system is a compound of his conscious or unconscious beliefs, hypotheses or expectations and their combinations. (Green 1971)

Different conceptions of beliefs

Although beliefs are popular as a topic of study, the theoretical concept of "belief" has not yet been dealt with thoroughly. The main difficulty has been the inability to distinguish beliefs from knowledge, and the question is still unclarified (e.g. Abelson 1979, Thompson 1992).

As an implication of this fuzziness in the definition of the concept, one means different matters with beliefs, depending on the discipline and the researchers who deal with them. For example, beliefs are considered equal to concepts, meanings, propositions, rules, preferences or mental images (Thompson 1992). In social psychology, for example, the impressions of and reactions to other people are typically divided into beliefs, expectations and attitudes. For them, beliefs are statements thought to be true, whether or not they are. Expectations are explicit or implicit predictions about people's future behaviors, and attitudes are emotional reactions to them. (Brophy & Evertson 1981; 8, 25)

There are many variations of the concepts "belief" and "belief system" used in studies in the field of mathematics education. As a consequence of the vague definition of the concept, researchers often have formulated their own definition for "belief", which might even be in contradiction with others. For example, Schoenfeld (1985, 44) states that in order to give a first rough impression "belief systems are one's mathematical world view". He later modifies his definition, interpreting beliefs as an individual's understandings and feelings that shape the way that the individual conceptualizes and engages in mathematical behavior (Schoenfeld 1992). Hart (1989, 44) – under the influence of Schoenfeld's (1985) and Silver's (1985) ideas – uses the word belief "to reflect certain types of judgments about a set of objects".

Some researchers think that beliefs are some kind of attitudes (e.g. Underhill 1988). Whereas Lester & al. (1989, 77) explain that "beliefs constitute the individual's subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements". On the other hand, Thompson (1992) understands beliefs as a subclass of conceptions. Yet another different explanation is given by Bassarear (1989) who sees attitudes and beliefs on the opposite poles of a bipolar dimension.

In Germany, researchers usually speak instead of beliefs (*Vorstellungen*[1]) and conceptions (*Auffassungen*) on "subjective theories" (e.g. Bauersfeld 1983, Tietze 1990, Jungwirth 1994), and the central term to be

[1] For the concept "belief", one may find several different translations into German. For example, the following translations were found easily in the International Review of Mathematical Education (ZDM-journal): *Einschätzung, Einstellung, Meinung, Sichtweise, Überzeugung, Vorstellung* (in alphabetical order).

used there is "a subjective experience domain" (Bauersfeld 1983). Recently, Törner & Grigutsch (1994) administered a questionnaire among first year mathematics students at the University of Duisburg, and named their research object "mathematical world view", according to Schoenfeld (1985). This concept was elaborated in the further research (Grigutsch & al. 1995), and theoretically based on the theory of attitudes. These are both in near relationship to the concept of mathematics-related belief systems.

There are, however, also some exceptions in Germany. In his study, Zimmermann (1991b) uses the word *Vorstellung* (belief), and discusses its meaning from the viewpoint of German-language literature. However, since he wants to emphasize the cognitive aspect of his research object, he chooses the concept: "a view of mathematics".

Beliefs on the dimension affective–cognitive

An important question on beliefs is how they are situated on the dimension "affective – cognitive". If we were to stress the connections between beliefs and knowledge, we would see beliefs mainly as representatives of the cognitive structure of the personality. Whereas to see beliefs as attitudes, i.e. as a rather fixed form of reactions toward a certain object, means that we locate beliefs in the affective part of the personality.

In research, there are representatives for both viewpoints. Some researchers consider beliefs as a real part of cognitive processing. Most researchers acknowledge that beliefs contain some affective elements, since the birth of beliefs happens in the social environment in which we live. (McLeod 1989) From the six definitions of belief given above, those of Underhill (1988), and Lester & al. (1989), for example, stress the affective component, whereas the definitions of Bassarear (1989), and Thompson (1992) are more on the cognitive side.

In his study, Saari (1983) tried to structure the central concepts of the affective domain. He grouped them using three categories: feelings, belief systems and optional behavior. Belief systems may be seen to be developed from simple perceptual beliefs or authority beliefs – via new beliefs, expectations, conceptions, opinions and convictions – to a general conception of life. In structuring the concepts of the affective domain, Saari understands attitude, for example, as a component-structured concept. It has a component on each of the three dimensions: feelings, belief systems and optional behavior. Thus, according to him, beliefs form one component of attitude.

Such a viewpoint that attitude has a component structure seems to be rather common in psychology today. One may find the following definition in the dictionary of psychology (Statt 1990, 11): Attitude is "a stable, long-lasting, learned predisposition to respond to certain things in a certain way. The concept has a cognitive (belief) aspect, an affective (feeling) aspect, and a conative (action) aspect." In many definitions on attitudes within research on mathematics education, one may notice the same threefold structure (e.g. Hart 1989, Taylor 1991).

Our definitions

Here, we understand beliefs as one's stable subjective knowledge (which also includes his feelings) of a certain object or concern to which tenable grounds may not always be found in objective considerations. The reasons why a belief is adopted are defined by the individual self – usually unconsciously. The adoption of a belief may be based on some generally known facts (and beliefs) and on logical conclusions made from them. But each time, the individual makes his own choice of the facts (and beliefs) to be used as reasons, and his own evaluation on the acceptability of the belief in question. Thus, a belief, in addition to knowledge, also always contains an affective dimension. This dimension influences the role and meaning of each belief in the individual's belief structure.

In one's belief system, beliefs are usually held with a different degree of conviction (Abelson 1979). For example, Kaplan (1991) refers to the concepts "deep belief" and "surface belief" which could be understood as unconscious beliefs and conscious beliefs. One interpretation here could be that unconscious beliefs are basic beliefs, and conscious beliefs are conceptions. As a matter of fact, in accordance with Saari (1983), we explain here conceptions as conscious beliefs, i.e. we understand conceptions as a subset of beliefs. Thus for us, conceptions are higher order beliefs which are based on such reasoning processes for which the premises are conscious. Therefore, there seems to be a basis for conceptions, at least they are justified and accepted by the person himself.

One variation of conceptions are views. They are very near conceptions, but they are more spontaneous, and the affective component is more emphasized in them. Conceptions are more considered than views, and the cognitive component will be more stressed in them.

That pupils often have misconceptions is well-known among teachers. For example, Perso (1993) carried out research on pupils' misconceptions in algebra, and explained them as some kind of beliefs. Thus for us, misconceptions form a subclass of beliefs. A variation of misconceptions is formed by misinterpretations. MacGregor & Stacey (1993) have investigated pupils' misinterpretations in algebra, when studying the connections between language and mathematics.

Since beliefs lie on the border region of the affective and cognitive domain, i.e. they belong at least partly to both domains, it looks sensible to make the following separation: In unconscious beliefs (basic beliefs), the affective component dominates, whereas in conscious beliefs (conceptions), the cognitive component is more emphasized.

The individual compares his beliefs with new experiences and with the beliefs held by other individuals, and therefore, his beliefs are under continuous evaluation and change. Thus his beliefs will develope in social settings with other persons. When an individual adopts a new belief, this will be organized to form a part of the large structure of his personal knowledge, of his belief system, since beliefs do not appear in fully independence. Some beliefs depend on the other, for the individual, more important beliefs. Thus, they form a belief system which might be in connection with his other belief systems. Such a structure is a compound of the individual's conscious and unconscious beliefs, hypotheses and expectations and their combinations. (Green 1971; see also Rokeach 1968)

1.2. Distinctions between beliefs and knowledge

The method used to distinguish and to compare beliefs and knowledge is to consider the properties of the structures compound by each. However, it should be noticed that not all researchers are taking the distinguishing problem so seriously. Some researchers have argued that it is not important to distinguish between knowledge and beliefs, but rather to find out how belief/knowledge systems influence teachers' and pupils' behavior in mathematics classes (Thompson 1992). On the other hand, some researchers think that the distinction between these two seems to be more or less an academic philosophical problem (e.g. Audi 1988).

What is knowledge?

The concept of knowledge will be discussed very briefly here, stressing its near connections with beliefs. Those interested in a broader discussion may refer to the literature, e.g. Goldin (1990), Steffe (1990), or Fennema & Franke (1992).

According to the classical definition "knowledge is a well reasoned true belief", all knowledge (also scientific) is based on beliefs. A premise for knowledge is that all the beliefs which form its basis are logically true and justified, in the sense that also all other facts in the phenomen world speak for them. Thus, beliefs are individuals' subjective knowedge which expressed as sentences might be (or might not be) logically true. Knowledge always has this property. (Lester & al. 1989)

Skemp (1979) tried to clear up the problem of distinguishing between knowledge and beliefs, as follows:

> "Knowledge is the name we give to conceptual structures built from and tested against our own experiences of actuality. Beliefs are what we have accepted as facts for other reasons. These are frequently used in combination as the basis for the functioning of a director system."

But he does not stress the objectivity of knowledge which we usually think to be one of the hallmarks of knowledge. Actually, he let his readers, in this quote, to think also of the subjective dimension of knowledge, and thus leaves the situation a bit fuzzy. Also, Thompson (1992) states that distinctions between knowledge and beliefs are fuzzy, because of their close connections.

Thus, we behold as the most important characteristic of knowledge to be its objectivity. Knowledge should be free of emotional factors, and its truth should be considered as such, without subjective affective coloring. Another important characteristic of knowledge is its collectivity. Knowledge should be accessable for everybody (in principle), in order to be checked for its truth.

But conceptions which will be considered as knowledge may change with time. And one reason for the difficulty to distinguish between beliefs and knowledge could be this relativity of knowledge, in the sense that knowledge might be historically changing. Our conception of knowledge is changing all the time, also within mathematics. For example, in the 1700s one generally accepted piece of knowledge among the mathematicians was that all infinite series, with the limit zero of the general term, are convergent. This conception was rejected as knowledge when the well-known counter-example, $\Sigma(1/n)$, was found at the end of that century, and, in consequence, the theory of infinite series was developed. Another example is the very general misconception nowadays that boys are on the average more talented in mathematics than girls. Although research, however, has not given any support to this misconception, it is nevertheless still very strong.

Properties of belief systems

In order to solve the problem of distinguishing between knowledge and beliefs, some structural differences between belief systems and knowledge systems have been noticed. For example, Rokeach (1968) organized beliefs along a dimension of centrality to the individual. The beliefs that are most central are those on which the individual has a complete consensus; beliefs about which there is some disagreement would be less central.

Whereas, Green (1971) introduces three dimensions which are characteristic for belief systems: quasi-logicalness, psychological centrality and cluster structure. These dimensions of Green, which are strictly different ones, will be discussed here more closely.

Quasi-logicalness. Knowledge systems are usually formed logically from premises and from conclusions deduced from them. Whereas, the relationships between beliefs within a belief system cannot be said to be logical, since beliefs are arranged according to how the believer himself sees their connections. In other words, each person has in his belief system a structure which can be called quasi-logical, with some primary beliefs and derivative beliefs. This quasi-logical order is unique for each person, and it reflects the thinking and valueing of the person in question.

Also, Abelson (1979) pointed to this lack of logic in belief systems: Within a belief system, beliefs are not necessarily held in consensus with other beliefs. Therefore, one could have beliefs which contradict other beliefs held by the same person at the same time. Furthermore, the believer is usually aware that others may have different beliefs. Whereas, one important feature of knowledge systems is that it cannot contain contradictions.

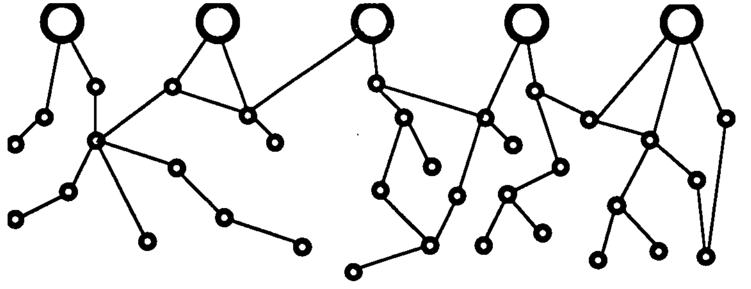

Figure 1.1. The quasilogical structure of beliefs may be described also as a net.

Figure 1.1 shows a schematic example of the quasi-logical relationships in one's belief system. Big circles represent his primary beliefs, and smaller ones his derivate beliefs. An example, in the case of a teacher's mathematical beliefs, is given e.g. in Pehkonen (1994b).

On the theme of the meaning of computers and calculators for mathematics teaching, one may easily generate an example of a set of teachers' beliefs where one belief is a primary one, and the others are derivate beliefs. For example, a teacher may hold the following belief: "Technology helps remarkably in mathematics teaching". From this primary belief, he may have conducted some derivate beliefs, such as: "A teacher should allow pupils to use pocket calculators in classrooms whenever suitable", "Pupils should have time for computer exercises", "The school should invest money in a computer class".

Psychological centrality. Some beliefs are more important for an individual than others. The first ones could be said to be psychologically more central, and the others are peripheral in the individual's belief system. Figure 1.2 shows this relationship schematically, for examples see e.g. Pehkonen (1994b).

Thus, beliefs have their own psychological strength, i.e. the degree of conviction with which they are held. The degree of conviction may vary from belief to belief. The most central beliefs are held most strongly, whereas the peripheral ones may be changed more easily; compare this with Rokeach's concept of centrality (Rokeach 1968).

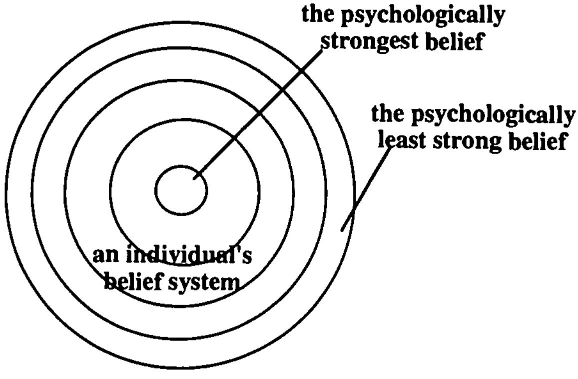

Figure 1.2. The structure of a belief system according to Green (1971).

The dimension of psychological centrality is lacking in knowledge systems. When speaking of one's beliefs, it is possible to say that "I am 60 % sure that we will some day find an intellectual life form another than human beings in the stars". Whereas one cannot say that somebody knows a topic strongly.

Cluster structure. Beliefs are held in clusters which are not necessarily in connection, or only in loose connection with each other. Or as Green (1971) has said: "Nobody holds a belief in total independence of all other beliefs. Beliefs always occur in sets or groups." This cluster structure enables the individual even to hold conflicting beliefs within his own belief system (cf. quasi-logicalness). The clustering property may help to explain some inconsistencies found in an individual's belief system.

Let us take an example from research done by Hasemann (1987, 29–30). He described a girl (Yvonne) interviewed by him on her fraction skills. He noticed that she "added fractions by using the rule "numerator plus numerator, denominator plus denominator", but she did the diagrammetical solutions to the problems correctly". Both algorithms seem to belong in her belief system to totally different clusters, since she accepted two different answers for the same task – and "she believed that they both were correct". This contradicting situation did not disturb her. The disturbance occurred when the interviewer took a realistic situation (a cake in oven), and she grasped that, in reality, two different solutions could not exist.

In addition to the cluster structure, Abelson (1979) pointed out that belief systems rely heavily on evaluative and affective components. A belief system typically has extensive categories of concepts which are grouped into "good" and "bad". As a typical example, those who support so-called "green values", also usually believe that nuclear power is bad, materialism and waste are bad, natural alternative energy sources are good, re-cycling is good. Knowledge systems lack such evaluations.

2. On pupils' mathematics-related beliefs

During the last decade, much research on pupils' beliefs has been carried out. Among others, Underhill (1988) has published a review of them. Here, we will restrict our considerations to pupils' mathematics-related beliefs. For teachers' beliefs, one should look at Pehkonen (1994b).

Instead of speaking of pupils' mathematics-related beliefs, we often abbreviate our language by referring to "pupils' mathematical beliefs". But one should note that it is only a label for a great variety of beliefs. The research results of Martha Frank (1985) form an example of pupils' mathematical beliefs. She condensed her results on pupils' beliefs about mathematics in junior high school, as follows (Frank 1988, 33):

1. Mathematics is computation.
2. Mathematics problems should be quickly solvable in just a few steps.
3. The goal of doing mathematics is to obtain "right answers".
4. The role of the mathematics student is to receive mathematical knowledge and to demonstrate that it has been received.
5. The role of the mathematics teacher is to transmit mathema-

tical knowledge and to verify that students have received this knowledge.

The mathematical beliefs given by Frank are very general ones and possessed by many pupils. According to my experiences and perceptions as well as my discussions with colleagues from different countries, these beliefs also seem to fit in the teaching situation of many European countries.

2.1. A pupil's view of mathematics

Pupils' mathematical beliefs are often divided into subgroups, e.g. beliefs on the nature of mathematics, beliefs on mathematics learning and teaching, and beliefs on oneself as a learner of mathematics (e.g. Underhill 1988). Such a division is actually artificial, in the sense that many beliefs belong to more than one of the four groups. For example, pupils' beliefs on the nature of school mathematics influence their conceptions of how mathematics will be learned (and how it should be taught). Nevertheless, such a grouping of beliefs may help us to structure the situation.

Now we are ready to explain our research object: "View of Mathematics". Its leading idea originates from Törner & Grigutsch paper (1994), but also has some components from the work of Zimmermann (1991b). As a matter of fact, it is merely a variation of the concept "mathematical world view" which originated from the book of Schoenfeld (1985), and has been used recently by Törner and Grigutsch (1994).

A pupil's view of mathematics is a wide spectrum of beliefs (and conceptions) which contains, among others, four main components:
(1) beliefs about mathematics,
(2) beliefs about oneself as a learner and as a user of mathematics,
(3) beliefs about mathematics teaching, and
(4) beliefs about mathematics learning.

As stated earlier, a pupil's beliefs form a structure which was called, especially in American literature, his belief system. Here we will call such a belief system his view of mathematics, since we consider such a name as more informative.

The four main groups of beliefs given in the definition, in their turn, may be split into smaller pieces. In the following, we will give some examples from each main group, in order to enlighten the variety of aspects within a pupil's view of mathematics.

For example, the following beliefs all pertain to the first main component of the view of mathematics: (1a) beliefs about mathematics as a school subject, (1b) beliefs about the birth of mathematical knowledge, (1c) beliefs about mathematics as a university discipline, (1d) .... (The dots here mean that the list could be continued.) In the second main group of a pupil's view of mathematics, there are: (2a) beliefs concerning his self confidence, (2b) beliefs regarding how successful he beholds himself as a problem solver, (2c) .... The third component contains, among others: (3a) beliefs about the nature of teaching mathematics, (3b) beliefs about how teaching should be organized, (3c) beliefs about what the role of the teacher is, (3d) beliefs about what the degree of autonomy given to pupils is, (3e) ... The fourth main group of mathematical beliefs consists of: (4a) beliefs about the nature of learning mathematics, (4b) beliefs about how learning should be organized, (4c) beliefs about what the role of the learner is, (4d) beliefs about what the degree of autonomy expected from pupils is, (4e) beliefs about who sets the criteria for correctness, (4f) ...

And these smaller parts (atoms) of one main component might be split again into still smaller pieces (quarks), perhaps without an end. For example, within the atomic belief group (1a) "beliefs about mathematics as a school subject", one may find the following quark beliefs: (1aa) beliefs about the nature of school mathematics, (1ab) beliefs about mathematical contents, (1ac) beliefs about mathematics textbooks, (1ad) beliefs about the nature of mathematical tasks, (1ae) beliefs about tests in mathematics, (1af) ... Thus, we have a glimse of the huge variety of beliefs which together form a pupil's view of mathematics.

The differentiation of this component structure in a pupil's view of mathematics could still continue. It is important to notice that these components are not separate, but rather that different aspects in the view of mathematics influence each other, and thus form clusters.

There is an on-going research project in Helsinki concerning the dynamics of the components within a pupil's view of mathematics from which some preliminary papers have been published (Malmivuori 1994, 1995). Also Törner & Grigutsch (1994) pointed out these dynamics: There is a large number of beliefs concerning interrelationships between these components.

2.2. The meaning of mathematical beliefs

During the last years, pupils' thinking processes have been studied intensively. About ten years ago, researchers noticed that pupils' beliefs seemed to form a key to the understanding of their behavior, also in mathematics (Wittrock 1986). The central role of beliefs for the successful learning of mathematics has been pointed out again and again by several mathematics educators (e.g. Schoenfeld 1985, Silver 1985, Frank 1988, Garofalo 1989, Baroody & Ginsburg 1990, Borasi 1990, Schoenfeld 1992). In this connection, the following points are given as an explanation for these effects: Beliefs may have a powerful impact on how children will learn and use mathematics, and therefore, they may also form an obstacle for the effective learning of mathematics. Pupils who have rigid and negative beliefs of mathematics and its learning easily become passive learners, who emphasize more remembering than understanding in learning.

Beliefs and learning seem to form a circle: Pupils' experiences in mathematics learning influence and form their beliefs. On the other hand, beliefs have a consequence on how pupils will behave in mathematical learning situations, and therefore, how they are able to learn mathematics. (Spangler 1992) Thus, pupils' beliefs revealed through research reflect teaching practices in the classroom. The way mathematics are taught in the classroom will little by little form the pupils' view of mathematics.

The latter is easily understood when we remember that an individual's mathematical beliefs, his view of mathematics, form a regulating system for his knowledge structure. Within this frame, the individual may act and think. On the other hand, this frame broadly influences his mathematical performance. Let's take an example: There is a pupil who understands mathematics merely as calculations. His understanding is often resulting from a one-sided, calculations emphasizing teaching at the primary level. Then tasks which demand thinking and where mere calculation do not lead to an answer might be for him difficult or even impossible.

Mathematical beliefs as a regulating system

In her dissertation, Martha Frank (1985) introduced a schematic picture of some factors affecting pupils' problem solving behavior. Since most of the factors act via pupils' belief systems, we have organized the components in the scheme in another manner (Figure 2.1). This scheme, in fact, shows the regulating character of a pupil's view of mathematics (his mathematical belief system).

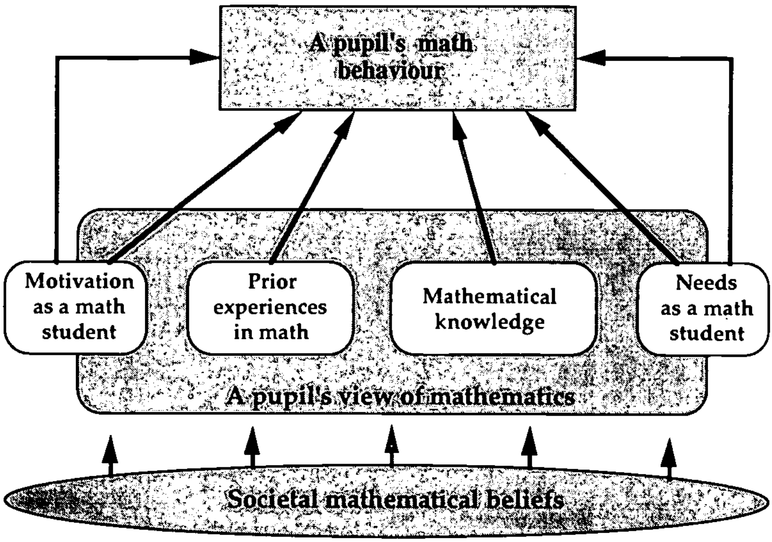

Figure 2.1. Factors affecting pupils' mathematical behavior.

Beliefs play a central role as a background factor for pupils' thinking and acting. A pupil's mathematical beliefs act as a filter which influences almost all his thoughts and actions concerning mathematics. A pupil's prior experiences of mathematics affect fully at the level of his beliefs – usually unconsciously. When he uses his mathematical knowledge, his beliefs are also highly involved.

In contrast to this, a pupil's motivation and needs as a learner of mathematics are not always connected with his mathematical beliefs. Additionally, there are many societal mathematical beliefs, perhaps myths, e.g. thatmathematics is merely calculation (for more myths see Frank 1990 or Paulos 1992), which also influence a pupil's mathematical behavior via his belief system.

The scheme of Figure 2.1 shows such a situation in which a pupil's mathematical performance is influenced by several factors which affect through a system or a net of his own beliefs. However, this is only part of the truth, in fact, the situation is much more complex. Pupils act within a very complex net of influences – Underhill (1990) speaks about a web of beliefs. For example, their mathematics teacher, classmates, friends, parents, relatives and teachers of other subjects all have their own views of mathematics and its teaching and learning (Figure 2.2). These beliefs affect more or less learners' beliefs, and usually in a contradicting way.

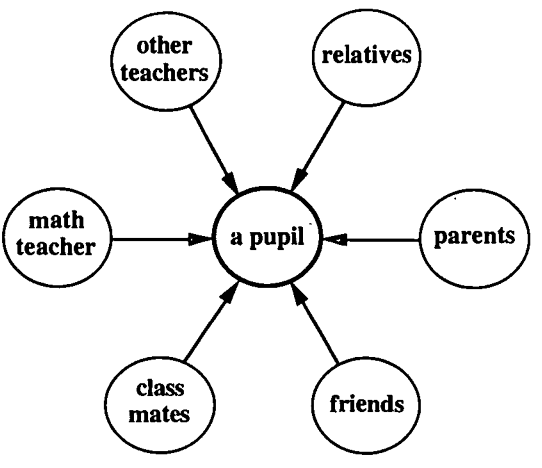

Figure 2.2. There is a variety of persons in a pupil's environment whose beliefs influence him.

If we were to condense an answer to the question "Why do research on mathematical beliefs?", we might stress at least two points: The knowledge of pupils' conceptions provides an opportunity for teachers, (1) to better understand pupils' thinking and actions, and (2) to support pupils' learning. Both these points will help teachers to better organize their teaching to correspond with pupils' view of mathematics.

Mathematical beliefs as an indicator

There is another practical meaning of beliefs: A view of mathematics (mathematical beliefs) may form a practical indicator in a situation which one is not otherwise able to observe[2]. Since the view of mathematics transmitted through beliefs, expressed by an individual, gives a good estimation of his experiences within mathematics learning/ teaching, we will have a method to indirectly evaluate the instruction he has received / has given: In the case of a teacher, the view of mathematics may act as an indicator

(1) for teachers' university studies,
(2) for teachers' professional view,
(3) for teachers' in-service training.

In the case of pupils and students, the view of mathematics could function as an indicator

(4) for students' experience teaching (in schools and universities).

Generally, one may consider the view of mathematics as an indicator

(5) of the functioning of the whole school system.

Mathematics teaching forms a part of the general education provided by the school, which will be realized within a societal context. In the research done, one may see connections with the change processes within the society, connections which have arisen outside the framework of mathematics education (Pehkonen & Törner 1994). Thus, the view of mathematics also has a role as an indicator

(6) for social sensitivity.

Mathematical beliefs as an inertia force

If we aim to develop mathematics teaching in schools, we are compelled to take into account teachers's beliefs (their view of mathematics)

[2] The idea of using beliefs as an indicator is accredited to Prof. Günter Törner (University of Duisburg).

– and also pupils' beliefs. Usually, the question is of experienced teachers' rigid attitudes and their steady teaching styles, which will act as an inertia force for change. Thus, the problem is how to help teachers to develop and enlargen their own pedagogical knowledge. Also, pupils' view of mathematics might be improper for the development, and the conditions of change should be considered. Therefore, beliefs have a central position when trying to change teaching.

If a teacher thinks that mathematics learning happens at its best by doing calculation tasks, his teaching will concentrate on doing as many tasks as possible. This phenomena was already observed more than ten years ago: Teachers' different teaching philosophies (belief systems) will lead to different teaching practices in classrooms (Lerman 1983; also Ernest 1991).

2.3. Empirical research on pupils' mathematical beliefs

About thirty years ago, there was a sharply delineated distinction between the cognitive and affective domains. Through the research work done in the 1980s, our view of the situation became clearer, and the boundaries between these two domains became increasingly blurred. Today, we have a fairly extensive literature on pupils' beliefs, and a moderate but growing literature on teachers' beliefs (Schoenfeld 1992).

Over the last decade, many studies on pupils' beliefs have been undertaken (e.g. Frank 1985, Schoenfeld 1989, Stodolsky & al. 1991, Zimmermann 1991b, Pehkonen 1992, Törner & Grigutsch 1994). In addition, there is a review of research results on pupils' beliefs compiled by Underhill (1988). In the reviews of McLeod (1989, 1992) about affect in mathematics education, one may find still more information on the research results in this field.

Some research on pupils' beliefs

In the research on pupils' mathematical beliefs, different methods (questionnaires and interviews) have been used. Also, the size of test groups varied from some tens to some thousands. In the following, we will discuss some research realized during the last decade, with the point of emphasis on the research design.

The test subjects of Frank (1985) were 27 mathematically-talented junior high school pupils. They participated in Purdue University's two-week intensive program on problem-solving with computers. Of them, 15 pupils were observed daily. Four of them were individually interviewed, each pupil at least four hours. In the interviews, the pupils were questioned about their classroom experiences in mathematics and were encouraged to discuss their beliefs about mathematics. Most of the interview time was spent with the pupils using the "think-aloud" technique to solve problems. Frank condensed her results into the five pupils' beliefs given in the beginning of chapter 2.

Schoenfeld (1989) tested 230 mathematics pupils in high school (grades 10–12) with a questionnaire, containing 70 closed and 11 open questions. The data suggest that the pupils were motivated and they worked hard with mathematics, but problem-solving seemed to be only rhetoric for them. Their answers revealed the following beliefs about mathematics: Any problem that cannot be solved within 12 minutes will be impossible. Mathematics is best learned by memorization.

Stodolsky & al. (1991) compared pupils' attitudes and conceptions about learning mathematics and social studies by interviewing 60 fifth-graders. Among many other results, they reached a view of mathematics through the eyes of fifth graders. In the interview, pupils were asked to define mathematics. The majority of pupils described it in terms of the basic arithmetic operations and dealing with numbers. In addition to basic operations, about 30 percent of fifth graders mentioned fractions and desimals. A smaller number of responses defined mathematics as measuring, doing problems, geometry, counting, and telling time.

In Hamburg, Zimmermann (1991b) used the same questionnaire as in Pehkonen (1992), but he gathered responses from a larger group of pupils. There were more than 2600 pupils in grades 6–9 from all school forms and levels, mixed together, and responses from their 85 teachers. He was interested, in the first place, to determine the position of problem-orientation in teachers' and pupils' conceptions on mathematics teaching. One of his ultimate aims was to relate pupils' conceptions to their teachers' responses to corresponding statements. Using cluster analysis, he found six pupil groups and five teacher groups, and noticed a certain correspondence between two pairs of them: Problem Orientation and Schema Orientation.

Pehkonen (1992) gathered the data from Finnish seventh-graders (N=514) using a questionnaire with 32 closed and three open questions. He found that the pupils have a pragmatic, calculation-centered, and task-oriented view of mathematics. But additionally, they emphasized the process more than the product, and the importance of rigorous working procedures.

Törner & Grigutsch (1994) administered a questionnaire with 50 statements among first year students in mathematics and chemistry (N=106) at the University of Duisburg. Their main result was as follows: The concept "mathematical world view" seems to be a complex structure which essentially affects doing mathematics. The different aspects of the "world view" influence each other, and form clusters. They extracted four such clusters, and with the aid of them, compounded a prototype of an ideal "mathematical world view" for doing mathematics.

Reorganization of some well-known research results

Pupils' mathematical beliefs have been a central research topic for years, especially in the Unites States. For the American research, it seems to be typical to use the interview methodology, and therefore, to obtain some detailed information which may be generalized. In the following, the main results of three researchers (Schoenfeld, Frank, Garofalo), which one may find in the teacher-read literature, are briefly given, and then compared with each other and situated into the framework discussed here: View of Mathematics (Figure 2.2).

In his book, Schoenfeld (1985, 43) stated three typical pupils' beliefs on mathematics and its learning:

S1. Formal mathematics has little or nothing to do with real thinking or problem solving.

S2. Mathematics problems are always solved in less than 10 minutes, if they are solved at all.

S3. Only geniuses are capable of discovering or creating mathematics.

In her dissertation on mathematical beliefs and problem-solving, Martha Frank (1985) extracted the following five pupils' beliefs which also were published in her later paper Frank (1988, 33):

F1. Mathematics is computation.

F2. Mathematics problems should be quickly solvable in just a few steps.

F3. The goal of doing mathematics is to obtain "right answers".

F4. The role of the mathematics student is to receive mathematical knowledge and to demonstrate that it has been received.

F5. The role of the mathematics teacher is to transmit mathematical knowledge and to verify that students have received this knowledge.

The third researcher whose results will be given here is Garofalo (1989, 502–503). He delivered some examples of pupils' mathematical beliefs which are common on the lower secondary level:

G1. Almost all mathematics problems can be solved by the direct application of the facts, rules, formulas, and procedures shown by the teacher or given in the textbook.

G2. Mathematics textbook exercises can be solved only by the methods presented in the textbook; moreover, such exercises must be solved by the methods presented in the section of the textbook in which they appear.

G3. Only the mathematics to be tested is important and worth knowing.

G4. Mathematics is created only by very prodigious and creative people; other people just try to learn what is handed down.

In the following, these results are put into the framework of the View of Mathematics (Figure 2.3), in order to give an overview on these results. Some of the beliefs were not so easy to classify within the main components. In fact, some beliefs belong to several groups at the same time. But the most important observation is clear: Nine beliefs of twelve, i.e. 75 % of all, pertain to the first main component (beliefs about mathematics).

The distribution of beliefs within the first main component is not uniform. Altogether seven beliefs of twelve lie in the first subgroup (1a: school subject). Further on the next level, three of them pertain to the first quark belief group (1aa: nature of school mathematics). The beliefs S1, F1, and F3 describe mathematics as computational procedures which have little to do with real thinking, and the goal of which is to get the "right answer".

beliefs on mathematics	beliefs on oneself	beliefs on teaching	beliefs on learning
school subject S1, S2 F1, F2, F3 G1, G2 birth of math S3 G4 uni discipline	self confidence success in problem-solving	nature of teaching organization of teaching role of teacher F5 degree of autonomy	nature of learning G3 organization of learning role of learner F4 degree of autonomy criteria for correctness

Figure 2.3. The beliefs obtained by research set into the view of mathematics (the numbers refer to the beliefs given above).

In the third quark belief group (1ac: mathematics textbooks), there is belief G2 which explains the structure of textbooks: Pupils have found that the mathematics needed in textbook exercises are usually presented just before. The fourth quark belief group (1ad: nature of mathematical tasks) contains the beliefs S2, F2, and G1. The first two are, as a matter of fact, the same and refer to the time needed for solving a problem (usually very short). The third (G1) gives support to the beliefs of the first quark group (1aa).

Beliefs S3 and G4 belong to the second atomic group of beliefs (1b: birth of mathematical knowledge), in delivering an image of how mathematics is discovered or created. These beliefs also pertain to the second main component (beliefs about oneself as a learner of mathematics), since they reflect learners' views of their possibilities to do mathematics (self-confidence).

Three last beliefs lie in the third and fourth main component (beliefs about teaching and learning mathematics). Two of them reflect pupils' and teachers' roles in the classroom. Pupils consider themselves as passive receivers of knowledge (F4). A teacher is expected to use the time in the classroom for explaining the content of the textbook (F5). The third one (G3) explains the meaning of assessment (tests) for pupils' learning.

2.4. On methodologies of investigating beliefs

In mathematics education research, there seems to be some fixed schools concerning the methodology used in research. Some researchers use questionnaires with "hard" statistics. They are satisfied with their methods, and often consider information gathered with "soft" methods (e.g. interviews) not to be research at all. The other ones claim that the only right way to do research in education is to use interviews and observations. They often do not value the research done with hard statistics, since "an average person" does not exist. And therefore, these two ultimate groups of researchers may have difficulties to understand each other.

Here the well-known western dualism seems to come to life again: You have only two alternatives, which are exclusive, and you are compelled to choose your side. Why could we not here replace the old two-value logic with e.g. the fuzzy logic which corresponds better with our reality? According to our view, we need both kinds of methodologies. If we are going to have high level information, we will need the both methodologies at the same time.

Problems of investigating experience

The following considerations are mostly developed from the paper of Perttula (1995): The object and the basic structure of a subjective experience is always the same, independent of the discipline from which its content was derived in the first place. Actually, research methods in educational problems could be divided into two categories, according to how the research is realized: natural science methods and human science methods. In natural science methods, the phenomenon is described with variables, and the variables in turn, into numerical data, in order to deal with the data statistically. Then the starting point for the evaluation of the reliability of the research is that the researcher uses research methods enabling the use of measuring and statistical methods of handling with the data. In human science methods, the consideration of the reliability of research goes back to the correspondence between the basic structure of the phenomenon and the research method.

According to Perttula (1995, 40), the researcher, before starting the research work, should philosophically analyze his conception of man's basic nature. The analyses will then reveal, what the researcher's meaning is as a constructor of knowledge during the research process. Therefore, the researcher should beforehand have a view of what kind of phenomenon he is going to investigate. In the qualitative research process, the researcher is necessarily a part of the relationship of meanings in his research object. But this is partly true also in the case of quantitative methods.

In the qualitative research process, the main components are as follows: (1) the experience of another person, (2) his way to express his experience, (3) the researcher's experience of the experience and its expression of another person, (4) the researcher's way to express his experience on the experience of another person. Lincoln & Guba (1985) state that the reliability of the qualitative methods is a compound of its truth value, applicability, permanence and neutrality. In measuring reliability, the central starting point is the researcher's ability to catch the phenomenon as such, as it has been experienced by the test person. It is worth noting that the researcher is no superhuman who may see "pure phenomena" without involving relationships to his own meanings.

Beliefs as a subject of investigation

Using a questionnaire methodology, researchers usually remain on the surface level of beliefs. One may only reach conscious beliefs, i.e. the conceptions of test persons. In addition, the belief system of the researcher strongly limits the results of his inquiry. In compounding the questionnaire, he decides the framework within which test persons should stay. Furthermore, the researcher obtains only those parts of the conceptions which test persons deem as appropriate to respond to the statements, and which they are willing to share.

With interviews and observations, one may try to go deeper, and also to elicite out also unconscious beliefs (basic beliefs) which lie behind the explicated conceptions. Since a structured interview often remains almost on the same level as a good questionnaire, it might be a good idea to realize the interviews with phenomenographic methods (Lincoln & Guba 1985). The so-called theme interview methodology, in which some main questions are shown to the test persons beforehand and form the core of the discussion, could be useful. During the interview, more questions may be asked if it is felt that "all answers" to the main question have not yet been elicited. The narrative mode of interviews will encourage the test persons to reflect on their experiences and feelings associated with them. (For a realization of the method, see the paper Pehkonen & Törner 1994.)

From the methods used, it follows that the data analysis is inductive (not deductive), since then it is more probable that new phenomena may be identified from the data. The inductive data analysis differs, however, from the conventional semantic analysis in the following point: Instead of beforehand-defined variables, the categories used will be formed during the data processing. That is, the rules for a classification will be compounded during the classification, not beforehand. Thus, the inductive data analysis is not conducted through a beforehand given theory, but the theory, the so-called grounded theory, will be formed from the data.

Classroom observations offer a powerful opportunity to see the teaching/learning situation from inside. If the observations are made for a longer period, a skilful observer will see or guess, within his own conceptions, the conceptions and basic beliefs which determine a teacher's and his pupils' behavior in the classroom. Classroom observations, compounded with other methods of data gathering (e.g. questionnaires and/or interviews), will give important triangulation for the reliability of the results. Another useful method which is compatible with the constructivist understanding of learning is the use of free writing, e.g. in the form of open-ended questions.

In research of beliefs, a questionnaire methodology has its disadvantages. In the first stage of investigating a new field, it is possible to chart the problems with a questionnaire. But high level information could be obtained by using interpretative methods, such as interviews and observations. However, a disadvantage of these methods is the central role of the interpreter. The results obtained are more or less dependent from the viewpoints of the interpreter.

3. International comparisons

International comparative studies pertain to the area of comparative education. Within this field, Bereday (1964) classified international comparisons as "the so-called simultaneous comparisons". They have their own method of treatment which consists of four components: "1) description, collection of pedagogical facts, 2) interpretation, the analysis of the facts applying the methods of different social sciences, 3) juxtaposition, the preliminary comparison of facts, 4) comparison, the final fusion of the facts with similarly-assembled data from other countries for the purpose of comparison" (ibid. xi).

Benefits of comparative studies

All the important conditions for views of mathematics may be seen a priori as nationally defined. Thus, university schooling, curricular organizations, special kinds of school form and organization will provide the limits for transfering results obtained from one country to another country. Therefore, it is clear that an international comparison offers an opportunity to clarify the influence of these national factors. For example, comprehensive school using inner differentiation is characteristic for the Finnish schooling system, whereas in Germany, they are realizing mainly a parallel school system using outer differentiation. There are also differences between Finland and Germany in the amount of teacher training both in subject studies and in pedagogical studies. Therefore, comparative international analyses are to be seen as a central research object. The juxtaposition of international situations may just reveal hidden determinants.

Comparative studies generally pertain primarily to basic research, which usually does not stress an applicable point of view. But in this case, through a comparative study, we may be able to see our own system from the outside, which could help us to better determine its weaknesses and strengths. In the results of a comparative study, we might notice that pupils' conceptions in some countries are desirable, and then begin to think how to develop similar conceptions in our own country.

Another benefit is the question of the transferability of the research results obtained. Is it possible, for example, to carry over the results obtained in the United States directly to the European situation? That is something we have been believing in, and as a consequence we have used the results in our own country without questioning them. Or is it true that pupils' conceptions are culture-bound? If this is generally valid, it might have interesting consequences on the interpretation of research results. Perhaps it is not possible to directly generalize the results from the United States to all industrial countries. This possibility was explored in the paper published by Pehkonen and Lepmann (1993), in the case of teachers' beliefs. To ensure the situation, we need systematic comparative studies of pupils' conceptions.

International comparison of pupils' beliefs

During the last decade, many studies on pupils' beliefs have been published – especially in the United States, see the literature in Pehkonen (1992). However, the question of the international comparison of pupils' mathematical beliefs still seems to be an almost unexplored field. The main question here is: "Are there essential differences in conceptions of mathematics teaching in different countries?" We know that mathematics can be understood as a universal discipline. So, the question arises whether pupils' conceptions on mathematics and on mathematics teaching and learning are also universal, or whether they are, perhaps, culture-bound.

However, before this present project (Graumann & Pehkonen 1993, Pehkonen 1993, 1994a, Pehkonen & Tompa 1994) almost no research into variations between pupils' beliefs on an international scale seems to have been done. Only in the Second International Mathematics Study (Kifer & Robitaille 1989) were pupils' responses to some questions on the affective domain dealt with in a background questionnaire. The study indicates that there are large differences between countries on measures of mathematical beliefs and attitudes.

Today, there seems to be a growing interest in the international comparison of pupils' conceptions (e.g. Berry & Sahlberg 1994a, 1994b).

4. Research design and its realization plans

In the background studies of the earlier research project "Open Tasks in Mathematics" (cf. Pehkonen & Zimmermann 1989, 1990), pupils' conceptions of mathematics teaching, as well as their experiences and wishes regarding mathematics teaching, were clarified with the aid of a questionnaire and interviews. From these background studies, a large independent international research project has gradually evolved which has also been of interests to foreign colleagues.

4.1. The purpose of the research

The theoretical framework for the study is the constructivist understanding of learning (Davis & al. 1990; Ahtee & Pehkonen 1994), according to which learning happens in a learner's mind, and at least some parts of the new knowledge structure is constructed by the learner himself. Therefore, it is most important for the teacher to know what his pupils think about the subject to be learnt and what kind of previous knowledge (preconceptions or misconceptions) they have. The way, the learner acts and thinks during the learning process is regulated by his belief system (Figure 2.1). Hence, the knowledge of pupils' conceptions about mathematics teaching (his view of mathematics) forms a necessary basis for the teacher's decisions in organizing classroom teaching.

According to the cognitive view of learning, a pupil's own activity is essential. Therefore, when trying to develop teaching and learning, the ways pupils think ought to be found out, and pupils' expectations and wishes should be taken into account. Schoenfeld (1987, 29) emphasizes the same point of view when saying: "When you prepare for a class, the key issue is how pupils will interpret what you show them rather than how much of it they will absorb". And the answer to the question of how your pupils are going to interpret your teaching depends on their beliefs.

Research problems

The purpose of this research project is to clarify pupils' views of mathematics. But the main question lies in the comparison of pupils' conceptions: Are there essential differences in pupils' views of mathematics in different countries?

The objectives of the research can be extracted from its purpose, and categorized into three target fields: To clarify (A) pupils' views of mathematics, (B) interrelationships between the components of pupils' views of mathematics, and (C) whether there are any differences in these points in different countries.

4.2. Practical realization

The study was planned to consist of both quantitative data (questionnaire) and qualitative data (interviews, observations), in order to ensure the triangulation of the data. The questionnaire used was intended to determine the pupils' conceptions about mathematics and its learning/teaching with structured questions and open questions. The questionnaire information will provides a first rough approximation of pupils' conceptions. In the second stage, this view is deepened with interviews and classroom observations.

As a background factor, the conceptions of their teachers is sketched with the aid of a questionnaire and follow-up interviews.

Pupils to be tested

In the first place, the test subjects are seventh-graders of the comprehensive school, i.e. about 13 year-old pupils. According to educational psychology (e.g. Good & Brophy 1977), children at this age level are in the transition phase from the (Piagetian) stage of concrete operations to the stage of formal operations. They have already developed enough critically to consider and to perceive the world outside, e.g. the way mathematics is taught to them. Another reason is that mathematics teaching during the first six school years (primary level) in most countries emphasizes numbers and basic calculations using them (arithmetics). The systematic teaching of more symbolic mathematics (algebra) begins during the seventh grade.

Later on when the project provides some results, the scope of the test subjects may, perhaps be extended to older pupils, e.g. to see the development of pupils' conceptions through the school age, and to university students. Another possibility for extension is to different school forms. Both these possibilities also form interesting objects for international comparison.

Measurements

Questionnaire. In the first stage of the project, the questionnaire method is used. The results of the questionnaire should provide a first approximation of pupils' mathematical conceptions. Its main purpose is to survey the field of the international comparison and to reveal possible interesting problem areas for further investigation.

The questionnaire used for this stage of the pilot study was developed for the earlier research project "Open Tasks in Mathematics". Its purpose was to clarify pupils' views of mathematics teaching, and their experiences and wishes about mathematics teaching with 32 structured questions and a few of open questions; see the questionnaire in Pehkonen (1992).

Interviews and observations. In the second stage of the project, the information received through the questionnaire will be specified and elaborated with the aid of intensive interviews, so-called theme interviews and classroom observations. However, only a small group (4–5 pupils) from each class are interviewed to prevent the study from expanding too much. Classroom observations will be loosely structured, and there should be a sequence of lessons to be observed. All the interviews will be videotaped, if possible.

The examination of mathematical classroom activities and the learning atmosphere in classrooms could also be of interest. This kind of information would enrich the picture of pupils' mathematical conceptions, obtained through questionnaire and interviews. A possibility could be to concentrate on some target pupils, and to determine their mathematical belief system through intensive interviews.

Methodology

In the first stage of the pilot study which was to act as a survey, data was gathered through a questionnaire. When considering the results of the structured questions, the statistics are mainly on the level of percentage tables. With the aid of cluster analysis, one may try to sketch the conceptions (and basic beliefs) behind the explicated views. The responses to the open questions are classified according to a certain category system. The description of results are on the level of percentage distributions, and contain a large variety of pupils' responses.

In the second stage of the pilot study, the theme interview methodology will be used. With the aid of the intensive interviews (theme interviews), an attempt will be made to clarifythe conceptions held by the pupils. The mathematical conceptions of the teachers teaching these pupils will offer a possibility to explain the results, to some extent.

In a sense, the questionnaire and interview results check each other, and thus the reliability of pupils' responses. For responses to open questions, the permanence of classification will be checked with parallel classification. The comparability of classification in different countries will be controlled allowing each classifier to work through the common set of papers (e.g. in English).

5. The state of research

In the research project, we are now in the first stage of the pilot study, which has almost been realized. Thus, we are ready to begin with the second stage (interviews). The main study may begin fully when the results of the pilot study have been elaborated, and the details in the design of the study fixed.

5.1. The pilot study

In the pilot study of the international comparison, one may separate at least two stages: The first stage is working with the questionnaire, and the second one, dealing with pilot interviews.

The first stage. In the first stage of the pilot study running in 1989–94, the main indicator was the questionnaire used earlier. The survey consisted of collecting data from about 200 seventh-graders in each country. The questionnaire was planned to determine pupils' conceptions about teaching mathematics.

The questionnaire has been administered in the following ten countries (the name of the local coordinator is given in brackets): Australia (Dr. Kevan Swinson, Queensland University of Technology), Estonia (Dr. Lea Lepmann, University of Tartu), Finland (Dr. Erkki Pehkonen, University of Helsinki), Germany (Nordrhein-Westphalen; Prof. Günter Graumann, University of Bielefeld), Hungary (Dr. Klara Tompa, Institute of Public Education, Budapest), Italy (Prof. Fulvia Furinghetti, University of Genova), Norway (Dr. Gard Brekke, Telemark College of Teacher Education), Russia (Prof. Ildar Safuanov, University of Tatarstan), Sweden (Arne Engström, University of Lund), and the USA (Georgia; Prof. Tom Cooney, University of Georgia). In addition, there are still some more responses coming, at least from England, Japan, Canada and Portugal.

The representatives of the first four pilot study countries (Estonia, Finland, Hungary and Sweden) decided to write a common report of the results obtained through the questionnaire. This paper is still under elaboration (Pehkonen & al. 1994). In this pilot report, the main question will be, as follows: What kind of answers can we provide to the research problems (in chapter 4) with the aid of the questionnaire data? From the preliminary results of this very first stage, some papers have been published in conference proceedings (Graumann & Pehkonen 1993, Pehkonen 1993, Lepmann 1994) and in journals (Pehkonen 1994a, Pehkonen & Tompa 1994), and some are still under elaboration (Pehkonen & Tompa 1995). Furthermore, some minor not published papers also exist (e.g. Heikkinen 1994).

The second stage. The second stage of the pilot study will deal with the development of interviews and observations for an international comparison. One of the recent questions in the research project has been: How may one implement a comparison of interview results in two different countries using different languages?

Plans for the main study

In the pilot study, the data was collected by some colleagues who showed interest in the study. Now for the main study, we aim to build a European network of belief researchers within which the problems can be dealt with on a proper scale.

According to the preliminary plans and connections with belief research in different parts of Europe, cooperating researchers will participate from at least ten European countries. The details of the design for the study have purposefully been left open, in order to be worked out together as a group. According to propositions presented by the researchers who have accepted the invitation, there are several possible approaches to be thought over: For example, one cooperating researcher proposed the use of mainly qualitative methods, such as intensive interviews, and the considerations of teachers' impacts on pupils' conceptions.

When considering the possible benefits from pursuing this collaborative European approach, one may point out that with the proposed project, we can gather together European researchers interested in pupils' beliefs, and do such research which will compete with that done in the United States. The field of international comparison in teachers' and pupils' beliefs is a vast area where it is today possible to do much pioneer research.

5.2. On some preliminary results

The first observation from the questionnaire results obtained and coded in the computer is that the distribution of pupils' responses in structured items is almost similar in each country (Pehkonen 1993, 1994a). The overall view of the conception profiles shows strong similarities. However, there are seventeen items where the means of some countries differed statistically very significantly from each other.

The five-country comparison

In the published paper Pehkonen (1994a), responses to the questionnaire in five countries (Finland, Estonia, Hungary, Sweden, the USA) were dealt with. In particular, the following seventeen items, with the largest differences (of all 32 items) between the countries, were discussed.

The leading title in the questionnaire (cf. Pehkonen 1992) for all statements was as follows: Good mathematics teaching includes ...

(1) doing calculations mentally
(3) doing computations with paper and pencil
(5) the idea that everything ought to be expressed always as exactly as possible
(6) drawing figures (e.g. triangles)
(7) the idea that one ought to get always the right answer very quickly
(8) strict discipline
(9) doing word problems
(10) the idea that there is always some procedure which one ought to exactly follow in order to get the result
(11) the idea that all students understand
(12) the idea that much will be learned by memorizing rules
(14) the use of calculators
(16) he idea that everything will always be reasoned exactly
(18) the idea that there will be as much repetition as possible
(25) the idea that games can be used to help students learn mathematics
(27) the idea that students are led to solve problems on their own without help from the teacher
(28) the constructing of different concrete objects (e.g. a box or a prism) and working with them
(32) the idea that the teacher always tells the students exactly what they ought to do

In order to compare the differences, these statements were categorized into five groups:

Basic Calculations (items 1, 3, 6, 9, 14),
Mathematical Rigor (items) 5, 10,16,
Mechanistic Learning (items 7, 12, 18),
Pupil-Centeredness (items 11, 25, 27, 28),
Teacher-Directedness (items 8, 32).

In the following figure (Figure 5.1), a score for each country has been computed as an average of the item averages, to enable us to compare the countries on these dimensions.

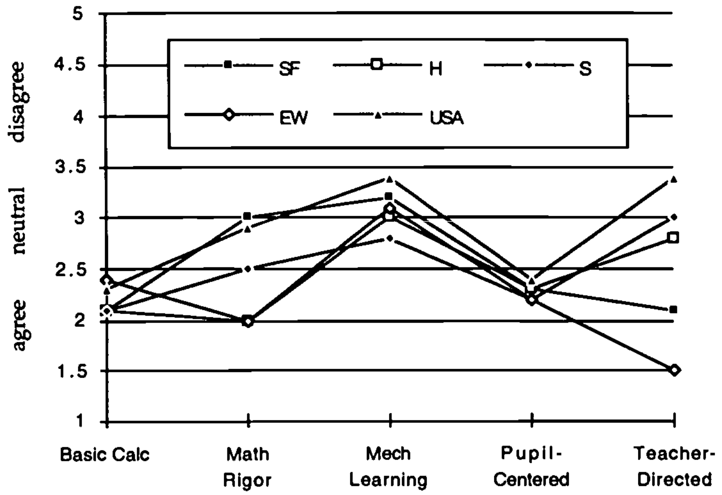

Figure 5.1. The comparison of the countries on the five dimensions (SF= Finland, H= Hungary, S= Sweden, EW= Estonia, USA).

Figure 5.1 shows that the largest differences in the pupils' conceptions in different countries emerge in the case of Teacher-Directedness. The Estonian pupils felt that mathematics is teacher-directed, whereas the American pupils tended to disagree. Another large difference can be found in the case of Mathematical Rigor. Both the Finnish and American pupils took a more neutral attitude, whereas the Estonians and Hungarians agreed that mathematics teaching involves rigor. On the other dimensions, the "averages" of the countries were similar to each other.

End note

When interpreting the results of the survey, the readers' conceptions of mathematics teaching and learning are very strongly involved. For example, the responses to statement 20 (only the talented can solve most of the tasks) might give us the picture that mathematics teaching has been successful since the pupils disagreed with the statement. We might think that they have learned to struggle with mathematics and not to think of it as being impossible. But there could be another contradictory interpretation of this same fact: Mathematics teaching could have failed because it deals with problems too simple and easy, since the majority of pupils consider mathematics to be accessible to everyone.

In the first evaluation, the mathematics teaching (experienced by pupils) in Estonia and Hungary seems to be more formalistic than in Finland, Sweden and the US (Georgia). In Finland and Estonia, pupils consider teacher-directedness a more integral part of mathematics teaching than in Hungary, Sweden and the US (Georgia). But the main finding in this research is that differences between countries are significantly larger than within the country, e.g. those between girls and boys (cf. Pehkonen 1992).

Thus, it seems, as a first approximation, that pupils' conceptions regarding mathematics teaching are culture-bound. On the other hand, remembering the similarities in the overall view of the conception profiles, we could conclude that there is also some part in pupils' mathematical conceptions which is universal. Therefore, the next question to investigate will naturally be: Which components in pupils' view of mathematics are universal and which are culture-bound?

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