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ABSTRACT

This report describes a workshop on developing a set of three introductory mathematics courses using new pedagogical techniques and computer technology. The courses provide underprepared students with multiple entry points into the study of mathematics and serve as a gateway for students' continued study of mathematics. These workshop courses in quantitative reasoning, statistics, and calculus abandon the traditional lecture approach in favor of an interactive format in which students work collaboratively on activities designed to help them explore and discover mathematical ideas for themselves. This document contains an executive summary detailing the project overview, purpose, background and origins, project description, evaluation/project results, and conclusions. (ASK)

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ED 418 839

Workshop Mathematics: Using New Pedagogy and Computers in Introductory Mathematics and Statistics Courses

GRANTEE ORGANIZATION:

Dickinson College
Department of Mathematics and Computer Science
Carlisle, PA 17013

GRANT NUMBER:

P116B11132

PROJECT DATES:

Starting Date:	Oct. 1, 1991
Ending Date:	Sept. 30, 1994
Numberof Months:	36

PROJECT DIRECTOR:

Professor Nancy Baxter Hastings
Department of Mathematics and Computer Science
Dickinson College
Carlisle, PA 17013
Telephone: (717) 245-1626

FIPSE Program Officer: Preston Forbes

GRANT AWARD:

Year 1	$102,399.00
Year 2	$113,313.46
Year 3	$ 70,844.49
Total	$286,556.95

5Cl01191

WORKSHOP MATHEMATICS: Using New Pedagogy and Computers in Introductory Mathematics and Statistics Courses

We have developed a distinctive set of three introductory mathematics courses which uses new pedagogical techniques and computer technology. They provide underprepared students with multiple entry points into the study of mathematics and serve as gateway for students' continued study of mathematics. These workshop courses in Quantitative Reasoning, Statistics, and Calculus with Review abandon the traditional lecture approach in favor of an interactive format in which students work collaboratively on activities designed to help them explore and discover mathematical ideas for themselves. Having developed a complete set of curricular materials for each course and undertaken extensive assessment activities, we are initiating a comprehensive dissemination program.

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Primary products of the project, all to be published by Springer-Verlag Publishers in 1995 or 1996:

Workshop Statistics: Discovery with Data
Workshop Calculus with Review I & II
Quantitative Reasoning: An Interactive Approach for the Liberal Arts

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WORKSHOP MATHEMATICS:
Using New Pedagogy and Computers in Introductory Mathematics and Statistics Courses

Dickinson College, Carlisle PA 17013-2896

Project Director: Nancy Baxter Hastings (717)245-1667

A. Project Overview

In the Workshop Mathematics project we have addressed many concerns of undergraduate mathematics education: (1) the shortage of mathematics courses which aim to teach fundamental mathematical concepts to entering college students, (2) the failure of many introductory mathematics courses to be accessible or appealing to many entering college students (particularly women and minorities), (3) the concern that many instructors of introductory mathematics courses do not take advantage of recent research findings in mathematics and science education, and (4) the inability of many introductory mathematics courses to make effective use of new computer technology.

We have developed a distinctive set of three introductory mathematics courses which uses new pedagogical techniques and computer technology to provide students with a conceptual understanding of mathematical concepts and the ability to apply mathematical ideas and techniques in other disciplines and in their everyday lives. These courses, which provide underprepared students with multiple entry points into the study of mathematics and serve as gateway for students' continued study of mathematics, are Workshop on Quantitative Reasoning, Workshop Statistics, and Workshop Calculus with Review I & II.

These courses abandon the traditional lecture approach in favor of an interactive workshop format in which students work collaboratively on activities carefully designed to help them explore and discover mathematical ideas for themselves. The application of mathematics to real problems helps students understand important mathematical concepts, motivates their learning, and prepares them for further study in mathematics, science, and social science. The courses use a variety of software tools, including a spreadsheets, statistical analysis package, computer algebra system, mathematical programming language, and microcomputer-based laboratory systems.

The primary products of this project are curricular materials in the form of a *Student Activity Guide* for each course. These guides contain the activities which lead students to discover mathematical concepts in collaboration with their peers. We have designed these guides for flexible use, so that instructors at various institutions can easily adapt the materials for use with different types of students, lengths of class meetings, and the computer hardware and software tools.

We have undertaken an extensive effort of assessing and disseminating the program. The reactions both of students in the courses and colleagues who have attended our presentations have been very favorable. We expect many institutions to adopt similar introductory courses and to incorporate many of this project's features into existing courses. Toward this end we have received continued funding from the National Science Foundation for assessing the Workshop Calculus project and from Springer-Verlag Publishers, who will publish the *Student Activity Guides* for all of the Workshop courses.

B. Purpose

Our project set out to address the problems of introductory mathematics courses enumerated in the first paragraph above. We have concentrated most on empowering underprepared students to become actively engaged in the study of mathematics. We do this primarily through the development of activities which ask students to use the computer and to work collaboratively as they investigate mathematical phenomena and discover mathematical concepts. These activities lead students to acquire: (1) greater confidence in their ability to learn mathematics, (2) an appreciation that mathematics has direct relevance in their everyday lives as well as in most fields of academic inquiry, (3) stronger ability to apply mathematical ideas in follow-up courses in other disciplines, and (4) desire to undertake further study of mathematics itself.

Our niche, within the many fine mathematics projects which have proliferated in recent years, is our focus on mathematically-at-risk students. Most curricular projects for introductory mathematics, particularly the calculus reform projects, have concentrated on students entering the traditional calculus sequence. We have directed our efforts at students who are less mathematically prepared and may not be inclined to take such a course as they enter college.

C. Background and Origins

Our work on this Workshop Mathematics project began prior to receiving funding from FIPSE in 1991. In addition to our general concern for the national problems in mathematics education listed above, several campus-wide concerns at Dickinson College led us to begin our project. One concern was that nearly half of the students who needed to take calculus for use in other disciplines were not prepared mathematically to enter the department's traditional calculus sequence. Another concern expressed by colleagues in disciplines such as physics and economics was that their students were not comfortable applying their knowledge of statistics and calculus to problems in their fields. A third concern was the perception of faculty throughout the College that many students lacked fundamental skills of quantitative reasoning that all liberally educated citizens should possess.

We enjoyed a great deal of support from Dickinson and other funding agencies as we undertook this project. We have especially benefited from the support of Priscilla Laws, who has shared her experiences and advice based on her renowned FIPSE-funded Workshop Physics program. Critical to the success of the Workshop Mathematics project was a grant from the National Science Foundation to equip two classroom/laboratories with microcomputers.

D. Project Description

As we mentioned above, the primary outcome of our project is a set of curricular materials for teaching a unique set of introductory mathematics courses for underprepared students. The most distinctive features of these materials are that they (1) support a learner-centered pedagogical approach in which student activities of exploration and discovery replace instructor lectures, (2) emphasize students' intuitive and conceptual understanding of fundamental mathematical ideas rather than rote applications of techniques, (3) pose open-ended real-world applications for students to investigate, (4) encourage students to work collaboratively with their peers, (5) promote student development of a variety of communication skills, and (6) use technology as a tool for helping students to achieve each of these goals.

Recognizing that faculty at other institutions face different types of students, computer facilities, and class schedules, we have made a special effort to make these materials flexible enough to be adopted in a variety of settings. Since this workshop approach asks instructors to change their pedagogical approach dramatically, we are preparing extensive instructors' notes to accompany the materials.

E. Evaluation/Project Results

We have been extremely pleased by the results of the Workshop Mathematics project to date. We feel that our work has had a significant impact on Dickinson's academic program, that students enjoy the courses and learn a great deal from them, and that we have generated interest and enthusiasm for the project in colleagues around the world.

Dickinson has taught all of its Quantitative Reasoning, Statistics, and Calculus with Review courses with the workshop format since the project's inception. The Workshop Mathematics materials have also been tested by faculty members at other colleges and universities. Through presentations, minicourses, and workshops, we have compiled a list of more than 300 individuals to whom we have sent information and sample materials about the program. The publication of the Workshop Mathematics series by Springer-Verlag Publishers will greatly stimulate our dissemination efforts, as will our selection as a Project Kaleidoscope *Program That Works.*

While the reactions to Workshop Mathematics of colleagues from Dickinson and elsewhere have been gratifying, they do not address the key question of student learning gains and attitude changes from these courses. We have collected much data on these issues with very satisfying results. We have also received much positive feedback and constructive criticism from two nationally recognized experts in educational assessment.

We enthusiastically continue our work on this project. Most notably, Nancy Baxter Hastings has received a two-year grant from the National Science Foundation to undertake an extensive assessment program for Workshop Calculus with Review. This effort involves: (1) identifying a list of "community norms," key concepts that students taking an integrated pre-calculus/calculus course are expected to know, (2) designing and implementing internal and external assessment tools to evaluate student learning gains and attitudes with a particular emphasis on the impact of the use of technology, and (3) tracking student retention rates, continuation rates and performance in subsequent mathematics classes and classes in other disciplines that have a calculus prerequisite.

F. Summary and Conclusions

We are very pleased with the results of the Workshop Mathematics program to date, and we feel that we have learned a great deal about instituting such a fundamental pedagogical change in one's curriculum and about developing curricular materials to support such a change. One lesson is that institutional and departmental support are crucial, another is the importance of early and regular student feedback when developing and teaching these courses. With regard to dissemination, we believe strongly that potential adapters need to see the workshop approach in action to provide them with the understanding and insight necessary to transport it to their own institutions.

In conclusion, we greatly appreciate FIPSE's support of Workshop Mathematics. We believe that we have initiated a very important and rewarding program, and we look forward to its continued development, assessment, and dissemination.

WORKSHOP MATHEMATICS:
Using New Pedagogy and Computers in Introductory Mathematics and Statistics Courses

FIPSE Final Report, December 1994

A. Project Overview

In this Workshop Mathematics project we have addressed many concerns of undergraduate mathematics education: (1) the shortage of mathematics courses which aim to teach fundamental mathematical concepts to entering college students, (2) the failure of many introductory mathematics courses to be accessible or appealing to many entering college students (particularly women and minorities), (3) the concern that many instructors of introductory mathematics courses do not take advantage of recent research findings in mathematics and science education, and (4) the inability of many introductory mathematics courses to make effective use of new computer technology.

We have developed (and continue to develop) a distinctive set of three introductory mathematics courses: Workshop on Quantitative Reasoning, Workshop Statistics, and Workshop Calculus with Review I & II. These courses use new pedagogical techniques and computer technology to provide students with a conceptual understanding of mathematical concepts and the ability to apply mathematical ideas and techniques in other disciplines and in their everyday lives. They provide underprepared students with multiple entry points into the study of mathematics and serve as gateway for students' continued study of mathematics.

Our courses abandon the traditional lecture approach in favor of an interactive workshop format in which students work collaboratively on carefully designed activities to help them explore and discover mathematical ideas for themselves. The application of mathematics to real problems helps students understand important mathematical concepts, motivates their learning, and prepares them for further study in mathematics, science, and social science. The courses use a variety of software tools, including a spreadsheet, statistical analysis package, computer algebra system, mathematical programming language, and microcomputer-based laboratory systems.

The primary products of this project are curricular materials in the form of a *Student Activity Guide* for each course. These guides contain the activities which lead students to discover mathematical concepts in collaboration with their peers. We have designed these guides for flexible use, so that instructors at other institutions can easily adapt the materials for use with different types of students, lengths of class meetings, and computer hardware and software tools.

During period FIPSE funded this project, each course has been taught a number of times by a variety of faculty members at Dickinson College. Several instructors at other institutions have also tested the materials. The *Student Activity Guides* have been revised (and continue to undergo revision) based on feedback from these instructors and the outcomes of our extensive assessment activities. Springer-Verlag Publishers will publish the *Student Activity Guides* for all of the Workshop courses in 1995 or 1996.

We are now poised to undertake a comprehensive, national dissemination program. The reactions of both students in the courses and colleagues who have attended our presentations have been very favorable. We expect many institutions to adopt similar introductory courses and to incorporate many of this project's features, such as the workshop pedagogical approach and the effective use of computer technology, into existing courses. Toward this end we have received funding from the National Science Foundation to continue assessing the Workshop Calculus project and from Springer-Verlag Publishers to begin dissemination activities. We have submitted a pre-proposal to FIPSE requesting funding to mount an extensive dissemination program.

B. Purpose

Our project set out to address the problems of introductory mathematics courses enumerated in the first paragraph above. We have concentrated most on the problem of empowering underprepared students to become actively engaged in the study of mathematics. We do this primarily through the development of activities which ask students to use the computer and to work collaboratively as they investigate mathematical phenomena and discover mathematical concepts. These activities lead students to acquire (1) greater confidence in their ability to learn mathematics, (2) an appreciation that mathematics has direct relevance in their everyday lives as well as in most fields of academic inquiry, (3) a stronger ability to apply mathematical ideas in follow-up courses in other disciplines, and (4) the desire to undertake further study of mathematics itself.

Our niche, within the many fine mathematics projects that have proliferated in recent years, is our focus on mathematically-at-risk students. Most curricular projects for introductory mathematics, particularly the Calculus Reform projects, have concentrated on students entering the traditional calculus sequence. We have directed our efforts at students who are mathematically underprepared and are not inclined to take a mathematics course when they enter college. For example, the audience for our Workshop on Quantitative Reasoning course is primarily humanities majors who would otherwise not take a college mathematics course. Social science majors who need to learn statistical ideas and methods for use in their discipline take our Workshop Statistics course. Students who enroll in our Workshop Calculus with Review sequence are those who need to be able to use calculus in other disciplines, but who score very poorly on the Calculus Readiness Exam of the Mathematical Association of America. We have concentrated our efforts on these underprepared students.

C. Background and Origins

Our work on this Workshop Mathematics project began prior to receiving the funding by FIPSE in 1991. In addition to our general concern for the national problems in mathematics education listed above, several campus-wide concerns at Dickinson College came together and led us to begin the project. We realize, however, that these issues are in not unique to Dickinson.

One local concern was that nearly half the students who needed to take calculus for use in other disciplines were not prepared to enter the department's traditional calculus sequence. Dickinson's Department of Mathematics and Computer Science had long grappled with the question of what kind of course to offer these students. One solution was to offer these students a two-semester sequence in which much of the first semester was devoted to pre-calculus; faculty were dissatisfied with this approach because most students did not learn the precalculus any better than they had in high school. Another option was to offer a two-semester course in applied calculus which did not cover the material as rigorously as the traditional calculus sequence; this course did not address the problem of the students' weaknesses with precalculus and it did not prepare them to continue their study of higher-level mathematics. The Workshop Calculus with Review sequence grew in part from the desire to integrate precalculus skills with the calculus ideas for which they are needed and to provide a gateway for underprepared students to subsequent mathematics courses.

Another concern expressed by our colleagues in disciplines such as physics and economics was that students were not comfortable with applying their knowledge of statistics and calculus to problems in their fields. Some of our colleagues felt that the mathematics courses were taught too abstractly for students to recognize the connections to applications. With this concern in mind, we have incorporated an emphasis on genuine and engaging applications from a variety of disciplines into all of the courses in the Workshop Mathematics program

A third concern at Dickinson was the perception of faculty throughout the College that our students lacked fundamental skills of quantitative reasoning that all liberally educated citizens should possess. Faculty from a number of disciplines spent parts of two summers discussing this problem, trying to identify specifically what those fundamental skills of quantitative reasoning are, and laying the foundation for what has become the Workshop on Quantitative Reasoning course.

We enjoyed a great deal of support from Dickinson and other funding agencies as we undertook this project. A residential liberal arts college of about 1900 students, Dickinson has distinguished itself in the past decade by its national leadership in redesigning science courses to be taught in an investigative, hands-on, student-oriented learning environment. We have especially benefited from the support of Priscilla Laws, who has shared her experiences and advice based on her renowned FIPSE-funded Workshop Physics program. We have also appreciated the support of a very collegial faculty who have offered many helpful suggestions for our work developing these courses. A grant to Dickinson from the Knight Foundation enabled us to begin curricular work prior to the FIPSE funding, and Dickinson's Presidential Discretionary Fund supported summer study groups which brought faculty together to discuss issues of quantitative reasoning. Critical to the success of the Workshop Mathematics project was a grant from the National Science Foundation to equip two classroom/laboratories with microcomputers.

The College has also adjusted some pragmatic policies to help us in the implementation of Workshop Mathematics. All of Dickinson's courses in Quantitative Reasoning, Statistics, and Calculus with Review have been taught with the workshop materials since 1991; we have not had to make do with a few pilot sections. The College has set student enrollments in the Workshop courses at a maximum of 24 students, enabling us to use the computer-equipped classrooms with two students per machine. The College has also approved Workshop Calculus with Review to meet for five hours per week rather than the usual three, with faculty teaching this course receiving 1.33 teaching credits. The College has also provided support in the form of undergraduate teaching assistants to assist the instructors of these course and to staff the computer classrooms during evening hours.

D. Project Description

As we mentioned above, the primary outcome of our project is a set of curricular materials for teaching a unique set of introductory mathematics courses for underprepared students. The most distinctive features of these materials are that they:

- support a learner-centered pedagogical approach in which student activities of exploration and discovery replace instructor lectures,
- emphasize students' intuitive and conceptual understanding of fundamental mathematical ideas rather than rote applications of techniques,
- pose open-ended real-world applications for students to investigate,
- encourage students to work collaboratively with their peers,
- promote student development of a variety of communication skills such as reading, discussing, and writing, and
- use technology as a tool for helping students to achieve each of these goals.

These materials have been used in Dickinson classrooms for the past three years and have undergone substantial revisions based on feedback from instructors and students alike. We have also benefited from holding two conferences for local and national advisory group members. These conferences have kept us informed about programs directed by leading educators in a variety of disciplines and provided a means for these educators to share ideas for the Workshop Mathematics program with us. The end result is that the current versions of the *Student Activity Guides* (which continue to be revised) are much improved over those from the first year of the project.

Recognizing that faculty at other institutions face different types of students, computer facilities, and class schedules, we have made a special effort to make these materials flexible enough to be adopted in a variety of settings. Since this workshop approach asks instructors to change their pedagogical approach dramatically, we have also concentrated on preparing extensive instructor's notes to accompany the materials.

We provide below brief descriptions and detailed outlines for each of these courses:

WORKSHOP ON QUANTITATIVE REASONING

COURSE DESCRIPTION

This course aims to help students develop their skills of interpreting and assessing quantitative arguments. An alternative to remedial courses in college algebra, it emphasizes critical thinking with regard to quantitative information rather than numerical computations or symbolic manipulations. The intended audience consists of students with little confidence in their mathematical abilities who need to acquire basic quantitative skills. Fundamental ideas covered in the course include understanding scales and magnitudes of numbers, interpreting graphs and tables, appreciating randomness, and drawing conclusions from data. Course material is presented in the context of practical and important applications in an effort to motivate students to develop their quantitative skills. Examples of these applications are estimating the revenue that would be generated by a gasoline tax in the U.S., projecting college tuition charges that today's students will pay for their children's education, and interpreting the results of AIDS tests.

COURSE OUTLINE

Unit I. Working with numbers

- "order of magnitude" estimations
- rates and percentages, % change, % error, projections
- index numbers, constant vs. current dollars
- scales of measurement, measurement validity, uncertainty, and accuracy
- exponential and linear growth, logarithmic scales

Unit II. Interpreting graphs and tables

- averages, statistical distributions
- dotplots, boxplots, scatterplots
- statistical tendencies, association, correlation vs. causation
- bar graphs, segmented bar graphs, Simpson's paradox
- faulty and misleading presentations
- randomized response technique, capture/recapture method of estimation

Unit III. Understanding surveys and experiments

- surveys, biased samples, effects of interviewer, wording, social fibbing
- random samples, sampling variability, margin-of-error
- anecdotal evidence, observational studies, comparative experiments
- statistical vs. practical significance
- logic, deductive and inductive reasoning, scientific method

Unit IV. Appreciating randomness

- randomness, probability, odds, simulations
- frequentist and subjectivist interpretations
- basic rules of probability, independent events
- decision making under uncertainty, expected values
- minimax criterion, prisoners' dilemma

WORKSHOP STATISTICS

COURSE DESCRIPTION

This course helps students to learn fundamental concepts and techniques involved with the statistical analysis of data. It concentrates on major ideas such as distribution, variability, association, sampling, confidence, and significance. By asking students to analyze real data, both taken from available sources and generated by the students themselves, the course emphasizes that data analysis is a process requiring thoughtfulness and judgment on the part of the analyst. Understanding, interpreting, and communicating the results of statistical analyses are stressed at the expense of performing rote numerical computations. The flexible nature of the curricular materials allows them to be used with most statistics software packages and even with a suitable graphics calculator. They can be used as the sole instructional materials for the course or in conjunction with a textbook or other supplementary materials.

COURSE OUTLINE

Unit I. Exploring Data: Distributions

- dotplot, histogram, stemplot, boxplot, symmetry, skewness, mean, median, standard deviation, inter-quartile range, empirical rule, resistance, outliers

Unit II. Exploring Data: Relationships

- association, scatterplot, correlation, causation, contingency tables, marginal and conditional distributions, bar graphs and segmented bar graphs, Simpson's paradox

Unit III. Exploring Data: Models

- normal distributions, standardization, least squares line, fitted values and residuals, influential observations, transformations

Unit IV. Inference from Data: Background

- convenience sampling, random sampling, experimental design, population vs. sample, parameter vs. statistic, sampling variability, Central Limit Theorem

Unit V. Inference from Data: Principles

- motivation and interpretation of confidence intervals and significance tests, margin-of-error, nonsampling errors, p-values, practical vs. statistical significance

Unit VI. Inference from Data: Applications

- one-sample t-test, paired t-test, two-sample t-test, two-sample z-test, relative risk, chi-square test

WORKSHOP CALCULUS WITH REVIEW I & II

COURSE DESCRIPTION

This two course sequence integrates a review of basic pre-calculus concepts with the study of concepts encountered in a traditional first semester calculus course – functions, limits, derivatives, integrals and an introduction to integration techniques. The course is designed for students who are not prepared to enter Calculus I, but who need to develop mathematical skills for further study in the social sciences, natural sciences or mathematics. The course feeds into calculus II. Essential elements of Workshop Calculus include the emphasis on applications to enhance student motivation and the use of the computer to help explore mathematical ideas. Software tools include a computer algebra system, the mathematical programming language ISETL, and Micro-computer Based Laboratory (MBL) Tools.

COURSE OUTLINE

Preface: Calculus With Review

- Why are you taking this course?
- Continuous motion or jerks?

Unit One: Functional Relationships

- Defining and representing functions: relating position and time, describing a process for finding positions and times, using the function concept to buy pizza
- Creating distance functions: creating functions using the motion detector
- Analyzing functions: using MacMotion to analyze the behavior of the tangent line, thinking about the general situation, creating one-to-one functions

Unit Two: Functions

- Constructing functions: getting started with ISETL, representing functions in ISETL
- Combining functions
- Reflecting functions

Unit Three: Function Classes

- Polynomial and rational functions: examining some polynomial and rational functions, becoming familiar with your CAS, using your CAS to analyze the behavior of polynomial and rational functions
- Trigonometric functions: converting between radians and degrees, evaluating trigonometric functions, graphing the basic trigonometric functions, stretching, shrinking and shifting sinusoidal functions
- Exponential and logarithmic functions: expressing situations in terms of exponential functions, comparing exponential growth functions, graphing and evaluating logarithmic functions
- Fitting a curve to a discrete function: modeling data

Unit Four: Limits

- Defining "limit": representing sequences of numbers, using ISETL to conjecture values of limits, examining situations where the limit does not exist
- Examining the connection between limits and continuity: considering general situations where a function is discontinuous, using limits to determine if a function is continuous at a point, calculating limits of continuous functions
- Considering other situations: calculating limits as x approaches a hole, calculating limits as x approaches infinity, using your CAS to evaluate limits

Unit Five: Derivatives – A Numerical Approach

- Examining rates of change: calculating some average rates of change and interpreting the results, discovering a value for the rate of change
- Defining "derivative": finding the slope of the tangent line, developing a definition, evaluating derivatives using the definition, representing derivatives by expressions
- Thinking about the derivative of a function as a function: examining situations where the derivative does not exist, comparing the graphs of linear functions and their derivatives, investigating graph of derivatives of nonlinear functions, gleaning information about the graph of a function from its derivative

Unit Six: Derivatives – A Calculus Approach

- Learning the differentiation rules: examining the Power Rule, applying the Constant Times a Function Rule, proving the Sum and Difference Rules, investigating the Product Rule, utilizing the Quotient Rule, employing the Extended Power Rule, evaluating derivatives using your CAS
- Sketching curves: comparing functions with the same derivative, using the First Derivative Test, determining the concavity of a function, calculating higher order derivatives, analyzing the shape of a graph, applying the Second Derivative Test

- Differentiating trigonometric functions: finding the derivative of $Sin(t)$ and $Cos(t)$, using rules to find derivatives of trigonometric expressions
- Differentiating exponential and logarithmic functions: finding the derivative of e^x and $e^{f(x)}$, finding the derivative of $\ln(x)$ and $\ln(f(x))$

Unit Seven: Definite Integrals – A Numerical Approach

- Approximating areas: calculating some areas, describing some possible ways, applying a rectangular approach
- Developing mathematical notation: considering an arbitrary situation, passing to the limit, calculating Riemann sums, modeling areas
- Applying the Riemann sum approach to other situations: approximating distance traveled, finding a formula for distance when velocity is varying
- Representing the area between two curves by a definite integral
- Examining functions whose graphs dip below the axis: examining the value of the Riemann sum for a function that is sometimes negative, interpreting some definite integrals, generalizing your observations

Unit Eight: Definite Integrals – A Calculus Approach

- Accumulating amounts: representing accumulated earnings by sums and integrals, using partial Riemann sums to approximate an accumulation function
- Calculating antiderivatives: recalling some differentiation rules, examining relationships among antiderivatives, discovering some useful rules for finding antiderivatives, finding antiderivatives of linear combinations, finding specific antiderivatives
- Fundamental Theorem of Calculus: comparing the graphs of accumulation functions and antiderivatives, using your CAS to compare the graphs of accumulation functions and antiderivatives, testing Part II of the FTC, applying Part II of the FTC

Unit Nine: Integration Techniques

- Using substitution: equating areas, evaluating some definite integrals using substitution, finding antiderivatives using substitution, using substitution, population project: tracking the human race
- Using integration by parts: trying integration by parts, using integration by parts, sound project: sounding off
- Using the integration tables: trying the tables, fish bowl project: finding the right water level
- Approximating solutions: fitting a curve and then using a model, using the Trapezoidal Rule on a data set, using the Trapezoidal Rule on a function without a

simple antiderivative, debt versus debt project: estimating the national debt, temperature project: finding an average temperature

Unit Ten: Volumes

- Finding volumes of solids of revolution using cylinders
- Finding volumes of solids of revolution using washers
- Examining brandy bottles and fish bowls

Appendices

- Using a computer
- Using MacMotion and MathLogger
- Using ISETL
- Using your CAS
- Integration tables

E. Evaluation/Project Results

We have been extremely pleased by the results of the Workshop Mathematics project to date. We feel that our work has had a significant impact on Dickinson's academic program, that students enjoy the courses and learn a great deal from them, and that we have generated interest and enthusiasm for the project in colleagues around the world.

With regard to campus-wide impact, Dickinson has taught all of its Quantitative Reasoning, Statistics, and Calculus with Review courses with the workshop format since the project's inception in 1991. By the end of this academic year (1994-95), this will have involved three sections of Workshop on Quantitative Reasoning taught by two different instructors (one of whom from the Political Science Department) to about 75 students, 19 sections of Workshop Statistics taught by six different instructors to about 450 students, and 15 sections of Workshop Calculus with Review taught by seven different instructors to about 300 students.

The campus-wide impact of Workshop Mathematics does not end there, however. While Dickinson's Workshop Physics has led the College's efforts to reform all of its science offerings, Workshop Mathematics has also served as a model for revolutionary curricular changes in introductory Chemistry. The Workshop Mathematics program has also been featured in college publications and in presentations for alumni and advisory groups as a model of Dickinson's curricular innovation at its best.

The Workshop Mathematics materials have also been tested with courses by faculty members at other colleges and universities. Barr von Oehsen has used the Workshop Statistics materials in courses at Shippensburg University and at Piedmont College; lacking computer facilities he modified the materials for use with graphing calculators. Joan Garfield of the University of Minnesota used portions of the Workshop Statistics materials in a summer workshop for elementary school teachers. Carol Harrison of Susquehanna University and Michael Kantor of Guilford College are using the Workshop Calculus with Review materials. Kevin Callahan of the California State University at Hayward has used excerpts from both the Workshop on Quantitative Reasoning and Workshop Calculus with review materials.

Many other faculty members from around the world have expressed interest in using Workshop Mathematics materials in the future. We have compiled a database of more than 300 individuals to whom we have sent information and sample materials about the program. These individuals have learned about our project through our extensive dissemination activities. These activities have included:

- presentations at meetings such as the International Conference on Technology in Collegiate Mathematics (ICTCM) and the International Conference on Teaching Statistics (ICOTS) and at regional and national meetings of organizations such as the Mathematical Association of America (MAA) and the American Statistical Association (ASA),
- workshops and minicourses at the ICTCM and MAA meetings, and
- invited presentations and workshops at institutions such as the University of New Hampshire and Villanova University.

To assist these dissemination activities, we have also prepared a brochure which we mailed to individuals who express an interest in our work. This brochure contains an overview of the Workshop Mathematics program, quotes from faculty, teaching assistants, and students, assessment results, and course descriptions, outlines, and sample activities. The brochure also includes a postcard which individuals can return to request that we send them lengthy sample booklets or complete texts of the Workshop Mathematics materials. The response to these brochures has indicated considerable interest in the program.

The publication of the complete Workshop Mathematics series by Springer-Verlag Publishers will greatly stimulate our dissemination efforts. Springer-Verlag expects to publish Workshop Statistics in the spring of 1995, Workshop Calculus with Review in the winter of 1995-96, and Workshop on Quantitative Reasoning in the spring of 1996.

Another boost to our dissemination plan and evidence of the high regard in which Workshop Mathematics is held was our selection in November of 1994 as a Project Kaleidoscope *Program That Works*. Jeanne Narum, Director of Project Kaleidoscope, wrote that our program "was one of ten selected from a pool of 35 nominations because it exemplifies the PKAL vision of what works in undergraduate science and mathematics: it reflects creative and effective ways of integrating teaching and research in the sciences, it exhibits an institutional environment that supports hands-on, collaborative learning; and it contributes to the national effort to reform undergraduate mathematics and science education."

While the reactions to Workshop Mathematics of colleagues from Dickinson and elsewhere have been gratifying, they do not address the key question of student learning gains and attitude changes from these courses. We have collected much data on these issues, with very satisfying results.

Example 1. *Students develop an understanding of fundamental ideas.* On the first day of class in the fall of 1992 and 1993, Workshop Calculus students were asked to write a short paragraph describing what a function is, without giving an example. Although they had all studied functions in high school, many wrote gibberish, some left the question blank, and only a couple of students (out of a total of 130) were able to describe a function as a process. After completing the activities in the first unit of their Activity Guide (where they do tasks designed to help them understand what functions are, without being given an actual definition for "function"), nearly 80% gave correct, insightful answers to the original question.

Example 2. *Students can apply the concepts they study.* During the spring semester of 1994, 23 students in one of the Workshop Calculus sections were asked the following question which was taken from an examination developed by the U.S. Naval Academy to assess the effectiveness of using the Harvard Calculus materials at their institution.: At which of the labeled points on the graph given below in Diagram 1 is $f(x)$ the greatest?

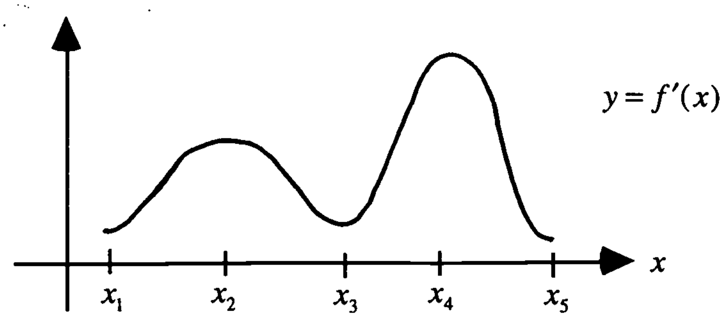
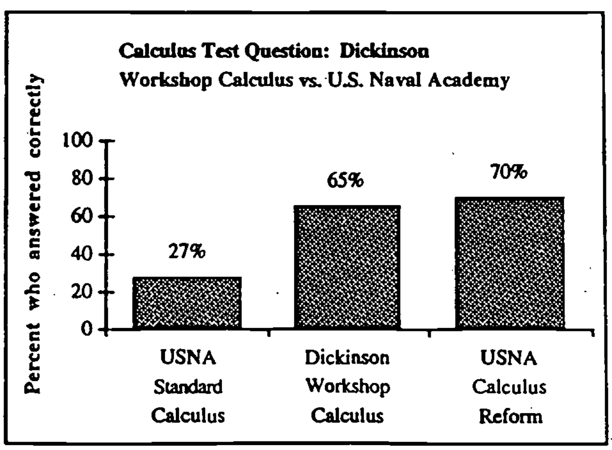


Diagram 1

Note that since f' is always positive, the graph of f is always increasing, so $f(x)$ takes its greatest value at the right endpoint. Out of the 23 students, 15 answered the question correctly with thorough explanations for a success rate of 65%. Prior to responding to the question students had completed tasks in their Activity Guides designed to help them understand the relationship between the graph of a function and its derivative. They had been given graphs of f', and asked to identify critical points of f; they had not, however, ever examined a function whose derivative was strictly positive.

Of the two groups of Naval Academy students, one of which had gone through a traditional lecture-based course and the other through a reform calculus course, 27% of the students in the traditional course answered correctly, while 70% of those in the reform course did[1]. Although Dickinson is less selective than the Naval Academy and the students in Workshop Calculus placed in the bottom 50% of those students at Dickinson who took the MAA Calculus Readiness placement exam, the Workshop Calculus students did almost as well as those in the reform course at the USNA. The following chart displays the outcomes.



[1] Penn, Howard Lewis. "Comparisons of Test Scores in Calculus I at the Naval Academy." Focus on Calculus. Spring, 1994.

On the final exam for Calculus with Review II in the spring of 1994, Dickinson students were asked three additional questions from the examination developed by the Naval Academy.

Question #1. The first question showed five graphs and asked the students to determine which graph had $f' > 0$ and $f'' < 0$. 81% of the students in the traditional lecture sections at the Academy responded to the question correctly, as did 93% of the students in the Academy's reform calculus sections. Similarly, 93% (25 out of 27) of the students in Dickinson Workshop Calculus course answered the question correctly.

Question #2. Question 2 showed the graph of a function h and asked at which point is $h'(x)$ the greatest. 82% of the students in the traditional lecture sections at the Academy responded to the question correctly, while 93% of the students in the Academy's reform calculus sections and 89% (24 out of 27) of the students in Dickinson Workshop Calculus course answered question #2 correctly.

Question #3. The last comparative question showed a graph of a function which was essentially made up of two line segments--one from (-1,0) to (0,-1) and the other from (0,-1) to (3,2). The students were asked to approximate the integral of the function from -1 to 3. 39% of the students in the traditional lecture sections at the Academy responded to the question correctly, as did 52% of the students in the Academy's reform calculus sections. 59% (16 out of 27) of the students in Dickinson Workshop Calculus course answered question #3 correctly.

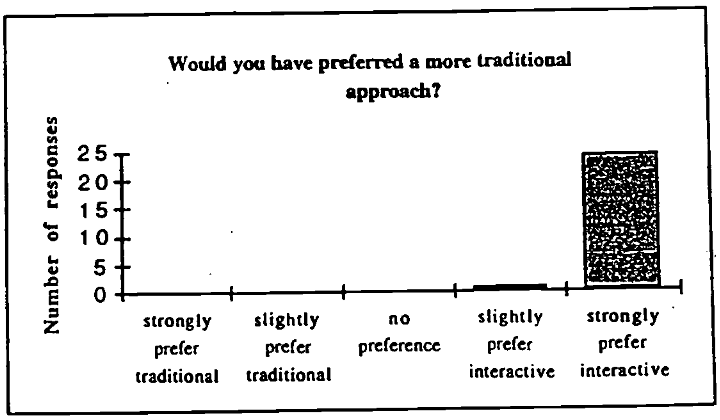
Obviously, we were very encouraged by these results. The table given below summarizes the percentage of the students in the USNA standard calculus, the USNA reform calculus and Dickinson's Workshop Calculus classes, who correctly answered the three questions.

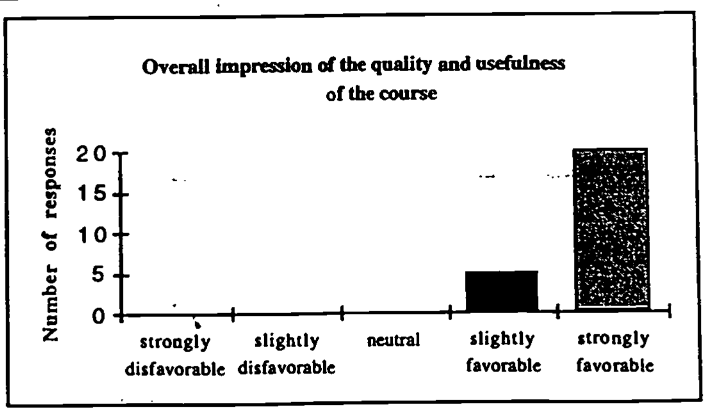
#	USNA regular	USNA reform	Dickinson workshop
1	81%	93%	93%
2	82%	93%	89%
3	39%	52%	59%

Figure 1. Percentage of students answering questions 1, 2 and 3 correctly.

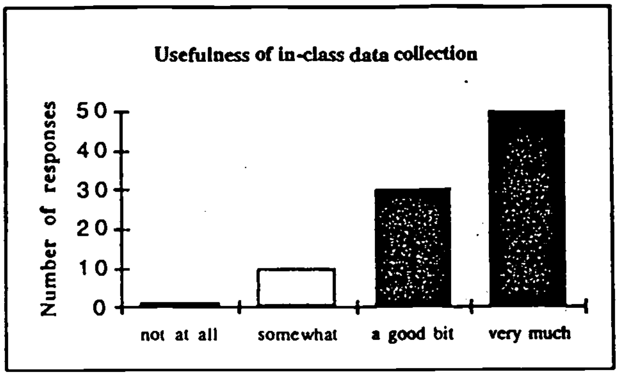
Since the Workshop Mathematics courses are "gateway" courses -- that is, since they are intended to instill confidence and to inspire students who do not think of themselves as being "good at math," to continue their study of mathematics -- we have done extensive assessment of student attitudes. This assessment data have been collected from student responses on: (1) comment sheets distributed at the end on each unit, where students are asked to write about what they have learned and to give feedback with regards to the learning environment and the instructional materials, (2) attitudinal questionnaires distributed at the end of the courses, where students rate the effectiveness of various activities, what they feel they have gained from the course, and how they feel about mathematics, and (3) all-college evaluation forms distributed at the end of the courses, where students describe changes they feel should be made and what they feel should be kept. As the following examples attest, the student response to the Workshop Mathematics courses has been very positive.

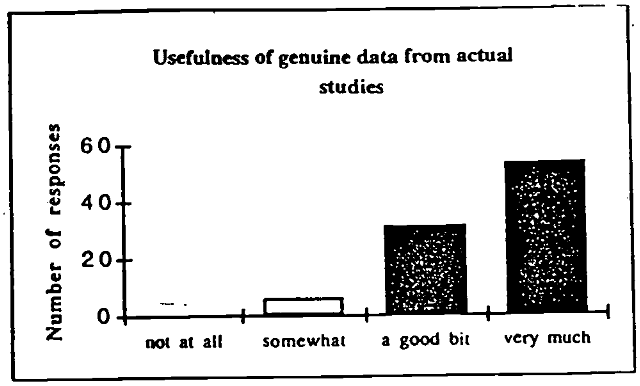
Example 3. *Students prefer the Workshop approach and they find the courses useful.* For example, the 25 students in the Quantitative Reasoning course in the spring of 1993, when asked about their reactions to the course, responded as follows:



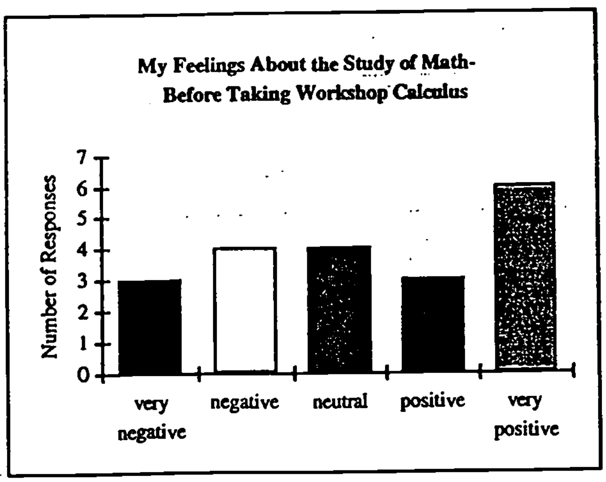


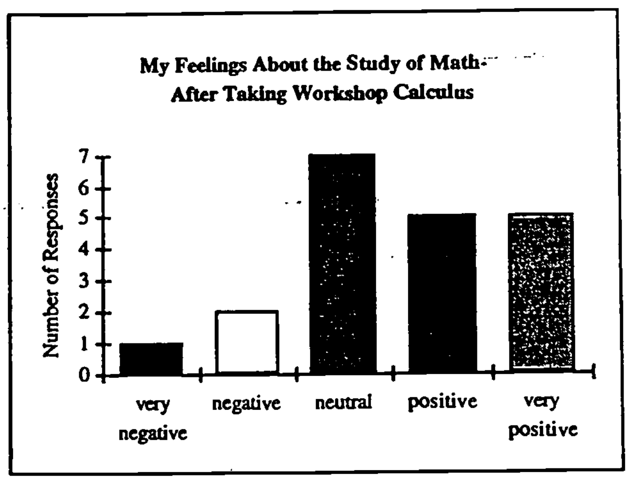
Example 4. *Students value the emphasis on real world applications and genuine data.* For example, the 90 students who took Workshop Statistics in the fall of 1991, spring 92 and fall 1992 responded as follows when asked about these issues.

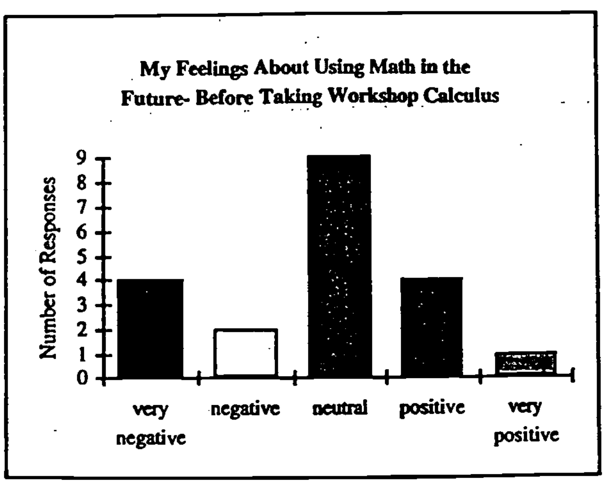


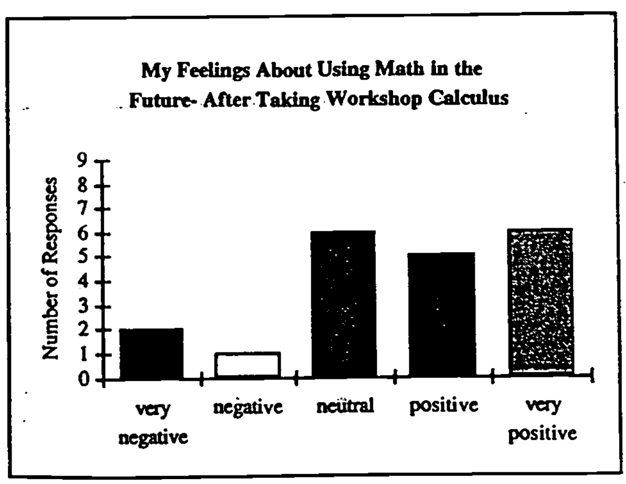


Example 5. *Students feel more positive about studying and using mathematics after taking Workshop Calculus.* Students are placed into Calculus with Review I & II based on their performance on the MAA Calculus Readiness Exam; it is recommended that those who score less than nine out of 20 take the workshop course. For the most part, these students have not fared well in high school mathematics courses. The following graphs describe 20 students attitudes about mathematics before and after completing the first semester of Workshop Calculus in the fall of 1993. Notice the shift to the right.









In addition to our own assessment activities, we invited two nationally recognized experts in educational assessment and evaluation to visit Dickinson to evaluate our program. Joan Garfield, statistics education assessment specialist from the University of Minnesota, evaluated the Workshop Statistics course and David Smith, Co-Director of Duke University's Project CALC, commented on the Workshop Calculus with Review sequence. These evaluators spent several days on campus, attended class meetings, and interviewed faculty, teaching assistants, and students.

Both evaluators applauded our efforts on these courses and provided very helpful suggestions which have guided our further refinement of the materials. They also helped us to identify distinctive features of our project in the context of other acclaimed curricular reform work in introductory mathematics and statistics. Garfield wrote in her report that "the workshop materials are unique in that they provide a complete structure for teachers to use that incorporates data gathering activities, use of the computer, and collaborative learning....This is an excellent alternative to traditional introductory statistics courses....I believe that students who complete this course will have developed positive attitudes about statistics, about the use of statistics in the world around them, as well as confidence to use statistics in other courses." Smith commented that he was "very pleased with the way workshop calculus is working out" and that upon observing a Workshop Calculus class he "could see clearly that students were coming to grips with the intended issues, progressing at different rates, but all making real progress." He added that "the results you have achieved -- getting significant numbers of students to go on in mathematics, from a population that had no intention of doing so, and thereby keeping them in the math/science pipeline longer -- speak for themselves."

Even though the original FIPSE funding for Workshop Mathematics has expired, we have enthusiastically continued our work on the project and made plans for more development, assessment, and dissemination. Springer-Verlag has awarded us a grant to assist with the development of materials for publication.

Nancy Baxter Hastings has received a two-year grant from the National Science Foundation to undertake an extensive assessment program for Workshop calculus with review. This effort involves: (1) identifying a list of "community norms," key concepts that students taking an integrated pre-calculus/calculus course are expected to know, (2) designing and implementing internal and external assessment tools to evaluate student learning gains and attitudes with a particular emphasis on the impact of the use of technology, and (3) tracking student retention rates, continuation rates and performance in subsequent mathematics classes and classes in other disciplines that have a calculus prerequisite.

We also intend to apply for funding from the National Science Foundation to hold two-week summer workshops for college faculty and from FIPSE to mount a comprehensive dissemination project for Workshop Mathematics.

F. Summary and Conclusions

While we are very pleased with the results of the Workshop Mathematics program to date, we recognize that the project is an ongoing one. We also feel that we have learned a great deal about instituting such a fundamental pedagogical change in one's curriculum and about developing curricular materials to support such a change.

One lesson is that institutional and departmental support are crucial. We have relied on institutional support in terms of funding classroom space, furniture, and equipment. Our institution has also supported this project by recognizing our curriculum development work as legitimate scholarly activity. Departmental support has been equally valuable in that we decided to teach all sections of these courses using the workshop approach and almost every member of our department has taught at least one of these courses. We strongly recommend that potential adopters of our project obtain institutional and departmental support rather than embark on a solitary crusade for this pedagogical approach.

Another moral that we have drawn is the importance of early and regular student feedback when developing and teaching these courses. We have relied heavily on student feedback as the courses progress. We have also found that it is quite important for the instructor to be forthright with students about the responsibility that the workshop courses place on them regarding their own learning.

With regard to dissemination, we believe strongly that potential adopters need to see the workshop approach in action to provide them with the understanding and insight necessary to transport it to their own institutions. While individuals certainly garner a taste of the workshop experience by reading our materials, hearing our presentations, and especially participating in our minicourses, they need to observe the courses first-hand in the presence of actual students to grasp fully the spirit of the workshop courses. Comments by numerous faculty members who have visited our courses attest to this conviction.

In conclusion, we greatly appreciate FIPSE's support of Workshop Mathematics. We believe that we have initiated a very important and rewarding program, and we look forward to its continued development, assessment, and dissemination.