D. Wood and J. Erskine (1976) and B. Thompson (1989) provided bibliographies of roughly 130 applications of canonical correlation analysis, but the features of such reports have not been widely studied. This report examines the features of recent canonical reports, including substantive inquiries, but also measurement applications examining multivariate validity and multivariate reliability. One particular area of interest focuses on interpretation of functions as against structure coefficients. Little appears to have changed since the publication of the Wood and Erskine study. The current review of the literature yields similar results about the confusing and somewhat arbitrary use of canonical terminology. Several analyses are highlighted that illustrate why students have so much trouble understanding canonical results. In addition to using confusing terminology, many authors failed to provide all the information needed to evaluate their conclusions. Recommendations for reporting canonical results include evaluating both the squared canonical correlation coefficients and statistical significance test results to decide which canonical functions to interpret. Both the canonical function coefficients and the canonical structure coefficients should be interpreted for noteworthy functions. One should usually not try to interpret the redundancy coefficients. One must, however, examine the communality coefficients for the variables that do not contribute to the overall canonical correlation solution, and one should evaluate the generalizability of the results through statistical or empirical means. Measurement applications are outlined.

(Contains 1 table and 51 references.) (SLD)
Features of Published Analyses of Canonical Results

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Abstract

Wood and Erskine (1976) and Thompson (1989) provided bibliographies of roughly 130 applications of canonical correlation analysis, but the features of such reports have not been widely studied. The present study examines the features of recent canonical reports, including substantive inquiries, but also measurement applications examining multivariate validity and multivariate reliability. One particular area of interest focuses on interpretation of function as against structure coefficients.
Features of Published Analyses
of Canonical Results

Hinkle, Wiersma, and Jurs (1979, p. 415) noted some twenty years ago that it was "increasingly important for behavioral scientists to understand multivariate procedures even if they do not use them in their own research." Similarly, Grimm and Yarnold (1995) recently noted that, "In the last 20 years, the use of multivariate statistics has become commonplace. Indeed, it is difficult to find empirically based articles that do not use one or another multivariate analysis" (p. vii). Thus, Emmons, Stallings, and Layne (1990) conducted an empirical study of 16 years of research reports in three journals, and found that the multivariate characteristic of the social science research environment with its many confounding or intervening variables has been addressed through the trend toward increased use of multivariate analysis of variance and covariance, multiple regression, and multiple correlation. (p. 14)

There were, and continue to be, good reasons for this trend. First, multivariate analyses limit the inflation of the Type I "experimentwise" error rates which can occur when a researcher conducts multiple univariate analyses. Second, multivariate methods honor 'real life' complexities "in which most outcomes have multiple causes, and in which most causes have multiple effects" (Thompson, 1986, p. 9). The general linear model (GLM)
allows a researcher to investigate relationships between potential causes (independent variables) and observed effects (dependent variables).

The GLM "produces an equation that minimizes the mean differences of independent variables as they are related to a dependent variable" (Vidal, 1997). The most general case of the parametric GLM is a structural model in structural equation modeling, which subsumes canonical correlation analysis as a special case (Bagozzi, Fornell, & Larcker, 1981; Dawson, 1998; Fan, 1996, 1997). Canonical correlation analysis, in turn, subsumes all other parametric multivariate analyses and regression as special cases (Baggaley, 1981; Thompson, 1991a), while regression subsumes all the univariate methods as special cases (Cohen, 1968). As Knapp (1978) noted, "Virtually all of the commonly encountered parametric tests of significance can be treated as special cases of canonical correlation analysis" (p. 410). Therefore, canonical correlation analysis is the logical choice for examination, if one wishes to understand classical parametric multivariate (and univariate for that matter) analysis procedures.

Canonical correlation analysis can be thought of (in somewhat simplistic terms) as a bivariate correlation between two sets of synthetic or latent variables (Thompson, in press-a). The principle aim of canonical correlation analysis is to find a
linear combination of the variables in one set that correlates maximally with the linear combination of variables in the second set. (It is possible to conduct canonical correlation with more than two sets of variables, however, for clarity sake let us focus on the bivariate analog). In order to accomplish this maximal correlation, CCA computes weights called canonical functions (analogous to beta weights in multiple regression). In fact, as Thompson explained,

These weights are all analogous, but are given different names in different analyses (e.g., beta weights in regression, pattern coefficients in factor analysis, discriminant function coefficients in discriminant analysis, and canonical function coefficients in canonical correlation analysis), mainly to obfuscate the commonalities of parametric methods, and to confuse graduate students.

(Thompson, 1995, p. 87)

In a similar effort to confuse the graduate students, the analogous systems of these weights are arbitrarily given different names (e.g., "equation," "factor," "function," "rule"), and so too the analogous synthetic/latent variables derived by applying the weights to measured/observed variables are arbitrarily given different names (e.g., "Yhat," "factor scores," "discriminant function scores," or "canonical function scores").

Table 1 summarizes the panoply of confusing jargon (one is
reminded of one of the old Bob Newhart television shows, where
one character regularly notes, "Hi, I'm Darrell, and this is my
other brother, Darrell").

The number of canonical functions that can be computed is
equal to the number of variables in the smaller of the two
variable sets (Thompson, 1991b). The canonical function
coefficients are then applied to an individual's set of measured
or observed scores (which have been converted to a standard
metric, i.e., $z$ scores) producing a "synthetic" variable score
(Thompson, in press-a). The synthetic variable score is the
focus of all statistical analysis, and is an estimate of the
latent construct actually of interest in all statistical
analyses.

Canonical correlation analysis is a rich analytic tool for
examining multiple dimensions of the synthetic variable
relationships. Thus, Wood and Erskine (1976) and Thompson (1989)
provided bibliographies of roughly 130 applications of canonical
correlation analysis. The researcher is able to examine the
relationships between the measured variables (within a set) and
the synthetic variable scores within a given function through two
avenues, examining the standardized function coefficients and the
structure coefficients.

As noted previously, the standardized canonical function
coefficient is the analog of the beta weight in regression, and
since many regression researchers erroneously interpret their
results by consulting only the beta weights (Thompson, 1997; Thompson & Borrello, 1985), one might expect many canonical researchers to make this same mistake. However, "if the variables within each set are moderately intercorrelated, the possibility of interpreting the canonical variates by inspection of the appropriate regression weights [function coefficients] is practically nil" (Meredith, 1964, p. 55). Structure coefficients must also be consulted since they allow a researcher to interpret the canonical variates even when the variables are intercorrelated.

A structure coefficient is the "bivariate product-moment correlation between the scores on an observed or measured variable and scores on a synthetic or latent variable" (Thompson, in press-a) for the given variable set. Structure coefficients inform the researcher about the contribution of each measured variable to the construction of the function. Squared structure coefficients represent the proportion of variance shared by a variable and the variable's canonical composite. Inspecting the relative contributions of the variables allows the researcher to interpret and understand the latent/synthetic scores on the given function.

Through inspection of both the standardized function coefficients and the structure coefficients for a given function, one is able to identify those variables which (a) contribute nothing to the understanding of the relationship between the
variable sets (both have near-zero structure coefficients and near-zero standardized function coefficients); (b) are arbitrarily denied credit for their predictive contributions (have a near-zero function coefficient and a large structure coefficient, i.e. approaching -1 or +1); (c) are demonstrating suppression effects (standardized function coefficients with a large absolute value and near-zero structure coefficients); and (d) are perfectly uncorrelated (in this unusual case, both the function and structure coefficients are identical--see Thompson, 1984). Clearly one can miss a great deal of the information provided in the canonical correlation analysis if one fails to examine both structure and standardized function coefficients in the results.

As mentioned earlier, canonical correlation analysis is a rich analytic tool for examining multiple dimensions of the synthetic variable relationships. In addition to the standardized function coefficients and the structure coefficients, three other coefficients are produced and beg mentioning: canonical communality coefficients; canonical adequacy coefficient, and canonical redundancy coefficient.

The canonical communality coefficients are "equal to the sum of the squared structure coefficients for a given variable across the canonical functions" (Thompson, in press-a). In this manner one is able to examine how much each variable (within a set)
contributes to the overall understanding of the variable set relationships (the overall canonical solution).

The canonical adequacy coefficient is equal to the mean of the squared structure coefficients for one variable set on one function. The canonical adequacy coefficient indicates how well, on the average, a given function reproduces the variance in the original measured variables (Thompson, 1984).

The redundancy coefficient is the product of the canonical adequacy coefficient multiplied by the squared canonical correlation. The redundancy coefficient is only useful when one is attempting to establish concurrent validity between identical sets of variables, for example when one is expecting g functions, in which case, the redundancy coefficient will (hopefully) equal 1.0 (Thompson, 1984). However, redundancy coefficients are not multivariate statistics, and are not optimized as part of the analysis, and thus usually have very limited utility (Cramer & Nicewander, 1979; Thompson, in press-a).

Many introductory statistics students find canonical correlation analysis "confusing." Thompson (1980) points out, "the neophyte student of canonical correlation analysis may be overwhelmed by the myriad coefficients which the procedure produces" (p. 16). Certainly the list of coefficients covered thus far is indicative of this possibility. Efforts to learn the meanings of each of these coefficients and the proper
interpretations of the combinations of coefficients can indeed be a daunting task.

A second source of is that researchers utilizing canonical correlation analysis do not consistently utilize the same terminology in referring to the coefficients and frequently interchange words and meanings. Wood and Erskine (1976) elucidate:

One researcher's canonical loading becomes another's canonical weight; canonical dimension to one is a canonical variate to another; and canonical correlation is the relationship between data sets for one, but only the relationship between variates for another. (p. 864)

The present study examined features of recent canonical reports, including substantive studies, but also measurement applications examining multivariate validity and multivariate reliability. A search of the database PsycINFO was conducted to identify articles published from 1988 to February, 1998.

Portrait of Contemporary Canonical Practices

Little has changed in the 20 years since Wood and Erskine's (1976) commentary on the confusing and somewhat arbitrary use of canonical terminology. The current review of literature reporting canonical correlation analysis yielded similar results. For example, structure coefficients were called by many terms, e.g., "correlation loadings" (Strack, 1994), "canonical loadings"
(Van de Geer, 1993), and "canonical correlates" (Retzlaff & Bromley, 1991). Other authors reported correlations, but did not specify what was being correlated.

Sadly, many authors report only function coefficients or only structure coefficients. One cannot assume that function and structure coefficients are the same or produce similar interpretations of the data. Thompson (1991b) pointed out, "the structure and function coefficients for a variable set will be equal only if the variables in a set are all exactly uncorrelated with each other." Therefore, reporting and interpreting both the function and structure coefficients is necessary in studies which reveal correlations among variables, for a dissenting opinion, see Harris, 1989.

The difficulties in the published literature are demonstrated through several examples of reported analyses of canonical results. The first example is a study by Roszkowski, Spreat, and Waldman (1983). This article is a good example of why students have difficulty understanding canonical results. The authors did not use tables for reporting their results. This increases the difficulty in examining conclusions, and requires considerable conscientious effort to sort through the reported results. The authors also chose to utilize the terms "canonical components" and "loadings" to refer to the structure coefficients. "Loadings" is a term that has been interchanged
with meanings and other terms that it has lost specificity. The word 'loadings' is therefore neither descriptive nor helpful in understanding the analysis, and some journal editorial policies therefore now explicitly proscribe the use of this term (cf. Thompson, 1994). Similar difficulties in terminology were noted in many articles (e.g., Adams, Lawrence, & Cook, 1979; Brush & Schoenfeldt, 1979; Fuqua, Seaworth, & Newman, 1987; Jelinek & Morf, 1995; Reid & Anderson, 1992; Strack, 1994; Tomasco, 1980).

In addition to confusing terminology, many authors failed to provide all the information necessary to evaluate their conclusions. Some authors chose to be overly selective by presenting only partial results, omitting those coefficients that fell below a specified criterion (e.g., Brush & Schoenfeldt, 1978; Gerbing & Tuley, 1991), or reporting ranges of results (e.g., Roszkowski, Spreat, & Waldman, 1983). This slipshod style of reporting prevents the reader from fully evaluating the reported conclusions. (A more cynical reviewer might conclude this was the intended purpose.) As noted previously, information on both standardized function coefficients and structure coefficients across value ranges are necessary to evaluate the potential influences of suppressor effects, to distinguish those variables which may arbitrarily not be getting predictive credit, to identify useless variables, and to identify perfectly uncorrelated variables. When coefficients are absent from the
reported results, the reader is unable to search for these potentially interesting anomalies.

Not all of the articles were exercises in frustration. Four articles stood apart from the rest by reporting and interpreting both function and structure coefficients (McIntosh, Mulkins, Pardue-Vaughn, Barnes, & Gridley, 1992; McLean, Kaufman & Reynolds, 1988; Reynolds, Stanton, McLean, & Kaufman, 1989; Sexton, McLean, Boyd, Thompson, & McCormick, 1988). These four articles were refreshing. They were clear, concise, easily inspected for verification of reported results, and provided complete information on standardized function coefficients, and structure coefficients. Amid the muddled, incomplete efforts of their peers, these articles stood out as shining examples of how canonical articles should be presented in the literature.

**Recommended Reporting Practices**

Clearly, the beginning statistics student has good reason to be confused, not withstanding the four exceptions to the unfortunate rule. This confusion may be resolved, at least in part, by employing a set of guidelines suggested by Thompson (1991b). These guidelines offer substantive and thoughtful suggestions for reporting and interpreting canonical results and are offered in five sequential steps.

The first step is to evaluate both the squared canonical correlation coefficients and statistical significance test
results to decide which canonical functions to interpret. Statistical significance tests are, of course, tied to sample size (Cohen, 1994; Thompson, in press-b), and most researchers know in advance of running their data whether the sample size was large enough for adequate examination. There is a problem, however, with most statistical packages in evaluating all of the canonical function coefficients. Many packages (e.g., SPSS, SAS) do not test each separate function. Rather, combinations of functions are reported with only one of the set reflecting the statistical significance of a single function. Additionally, when conducting statistical tests, the researcher must pay attention to the distribution of the data to evaluate the multivariate normality of the data.

Second, interpret both the canonical function coefficients and the canonical structure coefficients on the noteworthy functions (Thompson, in press-a). As mentioned earlier, it is vital to examine both the function coefficients and the structure coefficients in order to accurately interpret the results. Failure to inspect the canonical structure coefficients can lead to erroneous conclusions about the relationships of the variables.

Third, (usually) do not try to interpret the redundancy coefficients (Thompson, 1991b, in press-a). As mentioned earlier, redundancy coefficients are useful when one is
attempting to establish concurrent validity between identical sets of variables, for example when one is expecting $g$ functions, in which case, the redundancy coefficient will equal 1.0. This use of redundancy coefficients is appropriate, but is not a multivariate procedure. Any other use of the redundancy coefficient is discouraged as one should not attempt to apply a univariate statistic to a multivariate interpretation.

*Fourth*, one must examine the communality coefficients for those variables which do not contribute to the overall canonical correlation solution. This information may be very helpful in determining those measured variables which are useless and may possibly be omitted from the overall analysis (Thompson, 1984).

*Finally*, evaluate the generalizability of the results through statistical or preferably empirical means. A single study does not establish fact. Science requires replication and the extension of findings from a single study to understand relationships among variables. Statistical significance tests do not evaluate generalizability or replicability. Procedures such as bootstrap, and jack-knife are appropriate techniques for evaluating generalizability, and though once tedious and time-consuming tasks, can now be more easily accomplished through computer programs. Thompson (1995) demonstrates the use of bootstrap in a canonical correlation analysis. Of course, true "external" replications are more serious tests of result
replicability.

Chant and Dalgleish (1992) offer a SAS macro procedure for performing a jackknife analysis on canonical correlation and structure coefficients in a discriminant analysis in an effort to measure the standard error. Dalgleish and Chant (1995) offer a SAS macro procedure for performing a bootstrap analysis for the same coefficients.

Measurement Applications

Multivariate Reliability and Validity

Reliability estimates of multivariate data are best calculated utilizing procedures that take into consideration the potential intercorrelations of the variables. Classical reliability theory does not consider this potentiality. Classical reliability theory is defined as the ratio of the true variance to total variance. Yarnold (1984) attempted to extend this classical theory to cover multivariate procedures. Unfortunately, Yarnold's solution merely averaged the univariate reliabilities, thus failing to account for any intercorrelations among the variables.

Redundancy analysis appeared to be the next logical extension of the classical reliability definition to the multivariate case (Levin, 1993). However, redundancy analysis is not truly multivariate and is not sensitive to the intercorrelations of the variables being predicted (Cramer & Nicewander, 1979).
Rae (1991) suggested that canonical correlation analysis provides a measure of multivariate reliability that honors the reality of the data, including potential intercorrelations of variables. The multivariate reliability index, or canonical reliability coefficient, is the average of the squared canonical correlations between the observed scores and the latent true scores. (This is based on the earlier work of Conger and Lipshitz (1973), who invoked average squared Mahalonobis distances in calculating a canonical reliability coefficient.) Rae (1991) points out that when the measures comprising a variable set are perfectly uncorrelated, the results of the canonical reliability coefficient calculations will be identical to those using Yarnold's solution.

Redundancy analysis, so far panned in this paper, can be a valuable procedure, when evaluating multivariate validity. As mentioned earlier, redundancy coefficients are useful when one is attempting to establish concurrent validity between identical sets of variables, for example when one is expecting g functions, in which case, the redundancy coefficient will equal 1.0.

Sexton, McLean, Boyd, Thompson, and McCormick (1988) effectively utilized canonical correlation analysis to investigate the criterion-related validity of the Battelle Developmental Inventory (BDI) against the Bayley Scales of Infant Development. The concurrent validity of the BDI was supported
through large redundancy coefficients, thus indicating the scales are tapping essentially the same constructs.

**Additional Measurement Concerns**

Concern regarding the differential influence of sampling error on function and structure coefficients prompted Thompson (1991a) to conduct a Monte Carlo study. The results indicated that both sets of coefficients are influenced by sampling error and generally to about the same degree. The use of the Wherry correction as an effective correction for sampling error was demonstrated by Thompson (1990), who noted that sampling error is less of a concern when researchers maintain a 10:1 ratio of variables to subjects.

Liang, Krus, and Webb (1995) offered a K-fold crossvalidation procedure for canonical analysis to investigate and ultimately reduce the sample-specific variance. They also noted that their method reduces the demands of the variables to subjects ratio.

**Conclusion**

Canonical correlation analysis is a rich analytic tool for examining multivariate questions. It has the reputation among introductory statistics students as being confusing. The present paper outlined some of the reasons for this confusion and potential solutions, through a review of recent published analyses. Features of measurement applications of canonical
correlation analysis were also reviewed.
References


Thompson, B. (in press-b). If statistical significance tests are broken/misused, what practices should supplement or replace them?. *Theory & Psychology*. [Invited address presented at the 1997 annual meeting of the American Psychological Association, Chicago.]


Educational and Psychological Measurement, 44, 49-59.
Table 1
The Confusing Language of Statistics
(Intentionally Designed to Confuse the Graduate Students)

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Standardized Weights*</th>
<th>Weight System</th>
<th>Synthetic/Latent Variable(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Regression</td>
<td>β</td>
<td>&quot;equation&quot;</td>
<td>Yhat (Y)</td>
</tr>
<tr>
<td>Factor Analysis</td>
<td>pattern coefficients</td>
<td>&quot;factor&quot;</td>
<td>factor scores</td>
</tr>
<tr>
<td>Descriptive Discriminant</td>
<td>standardized function coefficients</td>
<td>&quot;function&quot; -or- &quot;rule&quot;</td>
<td>discriminant function scores</td>
</tr>
<tr>
<td>Analysis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canonical Correlation</td>
<td>standardized function coefficients</td>
<td>&quot;function&quot;</td>
<td>canonical function scores</td>
</tr>
</tbody>
</table>

* Of course, the term, "standardized weight", is an obvious oxymoron. A given weight is a constant applied to all the scores of all the cases/people on the observed/manifest/measured variable, and therefore cannot be standardized. Instead, the weighting constant is applied to the measured variable in its standardized form, i.e., we should say "weight for the standardized measured variables" rather than "standardized weight".
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