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ABSTRACT

A study examined and explored the importance of language in successful math problem-solving. The participants of the study were 24 fourth-grade students in a public elementary school in an urban area. Students were instructed in solving math word problems with a variety of strategies ranging from task specific, procedural methods to teacher-directed explicit strategies. A comparison was made between student performance using task-specific instruction and teacher-directed explicit strategy instruction with special attention paid to the interpretation of language in word problems to determine if there would be a significant difference in levels of performance. The hypothesis that there would not be was rejected. (Contains 44 references; sample math computations, word problem pre- and posttests, and a math story chart form are appended.) (Author/CR)

Strategic Learning: The Implications of Language in Successful Math Problem-Solving

by

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Submitted in Partial Fulfillment for The Master of Arts Degree

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ABSTRACT

The purpose of this study is to examine and explore the importance of language in successful Math problem-solving. The participants of this study were 4th grade students in a public elementary school in an urban area. Students were instructed in solving Math word problems with a variety of strategies ranging from task specific, procedural methods to teacher-directed explict strategies. A comparison was made between student performance using task-specific instruction and teacher-directed explicit strategy instruction with special attention paid to the interpretation of language in word problems to determine if there would be a significant difference in levels of performance. The hypothesis that there would not be was rejected.

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I would like to acknowledge the assistance and guidance of three individuals whose classes proved invaluable: Dr Richard Walter, Dr. Phyllis Kavett, and Dr Albert Mazurkiewicz.

DEDICATION

I would like to dedicate this undertaking to my family: My mother-in-law, whose courage and tenacity inspired me to keep working until I reached my goal, my son, who patiently allowed me to work, and always offered to help, and especially to my husband, without whose help and support, this would never have been possible.

LIST OF TABLES PAGE NO.

TABLE OF CONTENTS

The National Council of Teachers of Math Professional Standards for Teaching Mathemathics(1989)designated successful Math problem-solving to be primary objective for effective learning. The emphasis for elementary mathematics instruction has necessarily shifted from the task-specific(procedural)realm to the conceptual. The degree of scucess students experience in solving problems is dependent upon the choices they make when faced with real problem-solving situations. With this increased attention paid to successful problem-solving, language plays a crucial role. In order for students to choose correctly when selecting solutions, they must have a clear understanding of what they have read.

Research points to literature as a means of motivating and clarifying mathematical concepts. It can often be used as a springboard to introduce a Math concept by exciting children to utilize creativity that connects personal experience to Mathematical thinking.(Whitin,1992). Good literature provides a means by which students can write parallel stories that model math concepts(Lewis, Long, Mackay, 1993), or through the use of metaphorical language(Whitin and Whitin, 1997)to teach math concepts. Student authorship allows the opportunity for students to share their ideas and explain their reasoning in a way that is personal and original. If Math understanding and successful Math-problem-solving are to grow, an atmosphere must be provided that nurtures speculation and experimentation. The cognitive structures students build are expanded, reorganized and strengthened as students interact with each other in groups, sharing their thoughts and strategies. As a students connects new knowledge to previously held beliefs and reorganizes his thinking to accomodate new strategies or solutions, an illumination or clarification will take place which is far more meaningful than a mere confirmation of a correct answer.

Strategy instruction, therefore must also extened beyond the task-specific to include an

array of strategies that can be modeled to arm students against the sea of problems they face. Polya's 4 step approach(1957)to problem-solving provides a framework for organizing information in order to make a sensible plan encompassing orientation, organization, execution and verification(Garofalo and Lester, 1989)Susan Goldman(1989)outlined three instructional models that are commonly used to enable students to employ problem-solving strategies: The direct instruction model-which is task-specific and utilizes a step-by-step procedural method to ensure mastery, the self-instruction model which combines modeling by the teacher along with verbal prompts, and the mediated-assisted performance model which encourages students to initiate strategies which are altered or expanded by the teacher. Which of these might prove better in problem-solving or whether they would in fact, prove to be equally valuable has yet to be studied.

Hypothesis

To provide evidence on these potential differences on the effect of strategy instructions, the following study was undertaken. It was hypothesized that there would be no significant difference in Mathematical problem-solving=achievement when students are taught by task-specific instructional models or by language-based explicit strategy instruction models.

Procedures

This study was conducted within a 4th grade elementary school classroom in Elizabeth, New Jersey. Participating students ranged in age from 8 to 11 years. There were 13 girls and 11 boys of varying mathematical ability.

This study was conducted in four phases. First, the students completed a Math computataion test to determine their ability to solve 4th grade level problems in addition, substraction, multiplication and division. The addition and substraction problems required renaming of ones, tens, hundreds and thousands. Multiplication problems included both basic facts and multiplication of two digits by one digit, and division problems required only knowledge of the basic facts of division. All of these computational procedures had been presented and reviewed prior to the computation test in the course of the school year from September until December. The numbers used in this computation test were used again in the subsequent word problem test.

The computation test was followed, a week later, by the second phase of the study, a word problem pre-test consisting of 10 problems utilizing the same numbers that had been used in the computation test. The computation method required to solve the problems also remained the same. In other words, if addition had been required for problems on the computation test, those same problems were used in the word problem pre-test to solve problems which required an addition algorithm.

The third phase of the study emphasized explicit strategy instruction which included the following:

1. A four step problem-solving plan consisting of a "Math Story Chart"(based on Polya's 4 step method,(1957).
2. Rewriting of the language of word problem information to be more consistent with the students' knowledge base.
3. Rewriting of word problems to include simpler number facts.
4. Reinforcement of part-whole relationshps in work problems.

Four-Step problem-solving plan-This strategy instruction included daily word problem in-

struction which presented the use of a 4-step problem-solving plan: Tell-Show-Solve-Answer and Look Back(based on Polya's problem-solving method). Students completed "Math Story Charts" in which they identified problem-solving steps:

Tell-(the information given within the problem).

Show-(the equation which demonstrates the algorithm to be used).

Solve-(the computational component of the problem).

Answer and Look Back-(check of the reasonableness of the answer).

This strategy was used with teacher-made materials(story charts)and teacher modeling , on a daily basis(20 minutes per day)over a period of three weeks. During the completion of daily word problem "story charts," time was allocated for discussion of the language of the problem. Students who had difficulty identifying the Tell step part of the chart often had difficulty with the language of the story.

Rewriting the language of word problems-During the daily 20 minute instructional strategy time, when a problem arose with story language, students rewrote their own story problems in language more familiar to them. This strategy included changing the information of the problem whithout changing the conditions or the algorithm chosen for completion of the problem.

Rewriting word problems with simpler number facts-When, during class instruction, students exhibited a lack of understanding of how to mathematically complete the problem, but understood the language and conditions of the problem, extra time was taken to rewrite a new problem containing simpler number facts.

Reinforcement of Part-Whole Relationships in word problems-Wherever possible, rein-

forcement of part-whole relationships was emphazied. When choosing the algorithm with which to solve problems, students were prompted with questions about the relationships among numbers.

The final phase of this study included a post-test consisting of 10 word problems. An analysis of the significance of the difference between the computation and word problem pre-tests, and the difference between the word problem pre-tests and post-tests means was conducted.

Results

Table I illustrates a difference between the level of student achievenment on computational tests and word problem tests. Using statistical tests to determine the significance of the difference in performance between the two types of tasks, a t of 5.92 reveals substantially better performance on computation tests than on word problem tests that utilize the same numbers.

Table I

Means, Standard Deviations and t of Samples' computation and word problem Pre-Test.

Sample	Mean	Standard Deviation	t
Computation Test	81.67	17.86	5.92
Pre-Test(word problem)	47.5	21.92	

According to Table I, students performed significantly better on tasks that were not

embedded in language. Word problem pre-tests scores reflected errors made both in procedure as well as choice of appropriate algorithms.

Table II

Means, Standard Deviations, and t of Samples'

Word Problem Pre-tests' and Post-tests.

Sample	Mean	Standard Deviation	t
Pre-test 10 word problems	47.08	21.77	-1.26
Post-test 10 word problems	55.42	24.13	

Table II illustrates the findings with respect to level of performanace demonstrated on the Post-test which was preceded by a three week period of explicit strategy instruction. Applying statistical tests to determine the difference, a t of -1.26 was computed, indicating that no significant gain was made between the two tests.

Conclusions

The findings illustrated by Table I point out a significant difference exists in terms of student achievement based upon the type of mathematical task students are required to undertake. These findings illustrate the difficulty students have in successfully computing problems that are embedded in language. There is a far greater level of achievement demonstrated on purely computational tasks, than on those that require both computation and interpretation of language. An examination of the error patterns in the word problem pre-test revealed that in most cases students chose incorrectly when determining the

appropriate algorithm to be utilized in solving problems, but that occasionally students made procedural mistakes or computational errors on problems that they did not make on the computation test. It may be reasonable, therefore, to conclude that for some students the additional burden of interpreting language, and choosing appropriate algorithms may have resulted in a breakdown in their ability to calculate problems correctly.

In the word problem post-test, students were encouraged to use the teacher-made Math 'story charts,' designed to organize the information given in the problems into 4 basic components(based on Polya's 4 steps of problem-solving). This method seemed to help students to reduce the language of the word problem into its most salient features. All but one student chose to use this 4 step chart 'study aid' when completing their Post-tests. When use of this device was teacher-directed during the strategy instruction period of this study, students appeared to enjoy using it. It appeared to be viewed less as an additional task, than as a means of clarifying and organizing the information given problems. Occasionally, strategies were combined to allow students to access information in the most clear, concise way. For example, while using the Math 'story charts,' students also changed the language of story problems to resemble conditions or experiences more pertinent to them, or simplified the use of numbers in story problems to make calculation of answers easier to access. In addition, wherever possible the relationships among numbers in word problems was examined and discussed.

All of these methods were utilized together or in isolation during the twenty minute strategy instructions, in conjuction with open discussion of plans of solution. The open, non-threatening co-operative approach to problem-solving was intentional. Students

had exhibited an anxiety and reluctance to solve story problems independentlty, but appeared more willing to tackle these problems when they were presented as a class project.

The purpose of the study, however, was to model the use of strategy interventions that students could choose from to successfully solve problems on their own. The results, reveal that no significant gains were made in levels of performance from Pre-tests to Post-test means.

In this study, however, an attempt was made to examine and explore the implications of language on successful problem-solving. The following conclusions might be drawn from observations of students' performance in class: When there is an effort made to create an atmosphere of cooperation and team work among teacher and students to solve problems in a non-threatening, open setting, problem-solving approaches could be utilized to introduce and understand Math content. In other words, Mathematical concepts could be presented and investigated by using Math problem-solving.

When real-life problems and problematical situations are utilized within the classroom to formulate problems that peek student interest, problem-solving can become a means through which to guide students to the discovery of solutions.

When a variety of problem-solving techniques are modeled, and students are allowed to choose from these methods, explain their reasoning and reorganize or reconstruct their strategy or plan, apply it to a problematical situation, explain the reasoning behind it, weigh it against the plans or strategies of others, discuss it, refine it or discard it and begin again, the learning process builds.

When students develop the ability to look back at a solved problem, and verify the answer by checking it against the conditions of the original problem, they are using life-

time skills. These are strategies that will serve students well throughout their lives, and the ability to plan, develop, and utilize a strategy successfully can only enhance confidence in one's problem-solving capability.

Time invested in discussions of the use of strategies, possible solutions or even faulty reasoning is time well spent. Calling attention to the use of language in problems allows students to focus on strategies for solution rather than resort to random guessing.

Strategic Learning: The Implications of Language in Successful Math Problem-Solving: Related Research

To achieve greater understanding of mathematical concepts, problem-solving must be presented as both the means by which math understanding is enhanced as well as reason for learning math. Problem solving, within the classroom can be presented either as a cooperative, communal objective or as an independent task.

To better understand the complexities of successful problem-solving, it may be best to examine each of its component parts. For the purpose of this study problem-solving will be discussed in terms of language, that is the semantic structure of problems and its relation to successful representation of the problem to be solved, processing, or the way in which students use representations to integrate information and choose methods or plans of solution, and strategies both instructional heuristics and student-generated ideas.

Regardless of the method or approach utilized by teachers, reading comprehension is a critical component of successful math problem-solving. Because of the nature of problem-solving, language plays a crucial role. Research suggests that children's literature can provide a bridge from language to mathematical concepts. Stories which present mathematical ideas that generate exploration and extension of concepts especially at primary and intermediate levels are an ideal means of connecting students' experiences to mathematical thought. Whitin and Gary(1994)have conducted studies on the use of children's literature and story writing as methods of introducing math concepts and initiating student problem-solving responses. When students create their own stories which mirror the conditions of other stories they have heard or read, they are learning to formulate their own ideas as representations for solving problems.

Additionally, research by Silver and Cai,(1996)indicates that encouraging students to pose their own math problems enables them to continue to generate problems of a more

complex nature. Their study revealed a relationship between successful problem-solving and higher levels of problem-posing(more linguistically complex problems-containing conditional or relational features). It also suggests that students of low-performing levels are often capable of sound problem-posing.

Lewis and Mayer(1987) identified the two component processes of successful math problem-solving to be problem comprehension and problem solution. Comprehension requires a translation of each part or sentence of the problem into a mental representation followed by integration across all of the sentences of the problem to complete execution of the problems' structure. Solution includes the planning and execution of the problem. The research of Lewis and Mayer considers the "miscomprehensions" of studens to be due to incorrect representations of problems. Students exhibit a preference for the use of consistent language in their own problems. That is, they will present information in language consistent with the conditions or actions in a given problem. When students are required to solve problems which are presented with inconsistent language(language not compatible with actions in the problem), they are more likely to encounter difficulties.

Lewis(1989)suggests that most errors on word problems are due to "misrepresentations" of the problem's structure rather than incorrect computation. Problems can be categorized, according to Lewis(1989)as change, combine or compare problem types, and each has a distinct semantic structure. Compare problems, which contain relational elements to variables within the problem are often problematic because of the more complicated structure of the representation. Lewis(1989)implemented a diagramming method that enabled college students to check a picture of known and unknown variables against

the conditions in the problem, and suggested that some form of explicit representation may be beneficial to younger students as well.

Riley and Greeno(1988)also presented evidence of levels of knowledge and ability to represent information appropriately to solve word problems. In an analysis of developmental differences of kindergarten through third grade students' ability to process and con- conceptualize change, combine, and compare problems, results revealed that kindergarten and first grade students demonstrated difficulty in solving compare problems possibly due to the linguistic ambiguity of the compare problem language.

Carpenter, Hiebert, Moser(1983)warned that premature instruction on written representations may cause students to view scuccessful problem-solving as the selection of the correct operation to impose on the numbers in a given problem. Their research revealed that students attended more closely to the semantic structure of problems when they were using direct modeling or counting-on methods in order to recreate the conditions of the problem. Their study indicated, however,that after several months of instruction in writing number sentences(by 2nd or 3rd grade)students had made the transition to using written representations correctly.

In a subsequent 1988 study, Carpenter, Moser, and Bebout further suggest the relationship between the number sentences(symbolic representations)that first and second graders wrote and the semantic structure of word problems. This study revealed that students who successfully solved problems with the use of manipulatives(counters), appeared to use this knowledge to solve problems with larger numbers by writing number sentences that modeled the action in the problem. However, Carpenter et.al. suggest that students exhibited stages of development in their attempts to solve word problems. At the

most basic stage, students successfully solved addition and subtraction problems with the use of manipulatives and matching counting strategies. At the subsequent stage(direct modeling stage)students successfully solved problems they modeled directly from the problem actions. At the more advanced stage(second grade students)were able to transform problem language to solve problems with different stategies. At this stage, the use of open number sentences i.e. a + [ ] = b can be used to represent the action of the problem. These symbolic representations are used to build on the counting strategies previously learned. Carpenter's research suggests that the use of open sentences as symbolic representations can provide a way to build on the students' prior knowledge.

Hazelwood, Stouffer and Warshauser demonstrated the use of algebraic equations with second graders who, using Polya's four step mehtod(Polya,1957)

1. Understand the problem.
2. Make a plan.
3. Carry out the plan.
4. Check the answer.

translated "story problems" into number sentences. Each of the "characters" in the story represented part of the equation chosen for solution. By using this method, students were utilizing the language of math and applying meaning to it. The number sentences produced by students in Step 2(make a plan)provide a symbolic representation of the actions of the problem.

Davis(1985)suggests that in order to solve a problem, one must represent the problem situation, as well as the procedures and heuristics required for solution. After problem

representations are mapped, processed, and solved, they should then be checked for appropriateness: This last step(what Polya's four step approach refers to as "check the answer" or "look back") is a step, according to Davis that is often overlooked by students.

Research by Venezky and Breger(1988)further indicates that grade school students did not demonstrate the same level of ability to self-monitor as did college students. Their research suggests that as students progress from novice problem-solver to expert, they exhibit levels of ability from linguistic compentency(knowledge of the general semantic structure of the situation described in the problem)to domain-specific(knowledge of Math Skills, both procedural and computational)to strategic knowledge(ability to select an appropriate plan, and to monitor and evaluate the solution). Venezky and Breger's research suggests the need for more practice in grade school classrooms of the metacognitive tasks of self-monitoring and evaluation.

Students appear to demonstrate sound thinking strategies and spontaneous self-monitoring when they perform mathematical calculations and conceptualize mathematical situations outside of school,(Resnick,1987). When the doing of math is goal-oriented i.e. combining enough coins to buy an ice cream cone, students do not suffer the same doubt and confusion about solution plans. It would follow that school activities that resemble real-life situations will be more likely to allow students to utilize out-of-school math thinking.

Recent research by Wood and Sellars(1996) appears to support the view that students perform better and score higher on standardized tests as a result of problem-centered instruction. Their research is based on comparison of problem-centered 2nd and 3rd grade classroom programs for one or two years to similar classrooms who received textbook

instruction in 2nd and 3rd grade. Problem-centered classroom instruction included activities in which students worked in pairs to solve problems, followed by class discussions. The development of instructional activities was based on the children's growing conceptions of mathematical algorithms and mathematical concepts. The extension activities and problems grew directly out of the ideas generated by students.

The success of this program is closely allied to the teacher's understanding of what children's thinking is telling them about how their math knowledge is growing. Teachers who participated in this study attended a week long in-service summer program designed to help them understand the nature of reform in mathematics education.

Students appear to have a knowledge of different types of problems and their methods of solutions often give us incite into not only how they will solve those problems, but their level of conceptualization(Peled and Fresher, 1988). Drawings often reveal whether students have accurate images of the conditions of a problem. Translating problem conditions into pictorial representations may bridge the gap between mental representations and physical reality(vanEssen and Hamaker,1989). Research indicates that for students who have a working knowledge of math algorithms, a pictoral representation will help to clarify the conditions present in the problem situation. A study conducted by vanEssen and Hamaker revealed that this method proved beneficial for intermediate level students(fifth grade)who used drawings to facilitate the solution of both practice problems and transfer problems(those which utilized algorithms in varying contexts). The experience of using self-generated drawings by 1st and 2nd grade students did not prove as successful. The implication being that drawings are useful strategies to help students recall problem con-

ditions by creating a concrete representation of the conditions, but they do not assist students who have domain-specific(addition or subtraction)difficiencies. For these students a visual representation will not lead to solution. This condition in itself, however, provides a means of analyzing student weakness in specific skills. Drawings, therefore, can be a tool for students to faciliatate problem solution as well as a tool for teachers to identify difficulties in particular areas of mathematics.

When students make systematic errors(Cox,1979)or those that show evidence of a consistent misuse of specific algoritmic computations, visual representations may not be sufficient as a means of remediation. In fact, some research suggests that the most talented of math students are either poor visualizers or non visualizers Presmeg(1986). Reliance on concrete, visual representations may be beneficial as a remedial strategy, but research indicates the need to assist students who rely too heavily on concrete imagery to derive meaning from text to make the transition to more flexible, abstract propositions in order to find problem solutions(Campbell, Collis, and Watson, 1995). The more abstract the the students' image, the greater their use of logical abstract reasoning appears to be in obtaining answers. This theory supports the use of both visual and abstract strategy instruction methods.

There is much to be learned from the choices students make, whether visual representations or abstract propositions that lead to correct problem-solving DeCorte and Verschaffel,(1981)analyzed student decision making by categorizing students' response to unfamiliar problems as either a semantic approach or a "thinking aloud" protocol. In the semantic approach, students rearranged the conditions of problems to correspond with familiar schema. Thus, the use of addition by students who are asked to solve substraction

problems i.e. x-7=5. to find the missing number, the student adds 7+5, because he is more comfortable using a schema that utilizes addition. In the "thinking aloud" protocol, a student reasons that the missing must be greater than the other two numbers and thereby uses a trial-and-error approach until locating the correct answer.

Research into eye fixations of students as they solve word problems(Hegarty,Mayer, and Green, 1992)suggests that students who construct mental models as they read word problems that contained inconsistent language required extra time, because of the additional processing necessary to gain meaning. Lower acheiving students, however, who used a direct translation approach(Kintsch and Greeno, 1985) did not require extra time for rereading because they appeared to rely heavily on the semantic structure of the problem in order to translate word by word, losing the qualitative aspects of the problem.

It seems essential, then, that teachers be cognizant of the fact that student errors are as revealing and informative as student progress. The teacher must interpret the choices students make, and monitor their decision making during problem-solving situations to develop strategies that will benefit all levels of math knowledge within the classroom. This redefines the teacher's role to be more facilitator than teacher whose job is to guide the thinking taking place, rather than control it. Lesson-planning is based on problem-solving rather than mastery of procedural skills. Methodology is flexible and subject to change based on the ideas generaated by students(Maher, 1988).

Research into teachers' content knowledge of students' choices in problem-solving (Carpenter, Fennema, Peterson, Carey, 1988)indicates that teachers' knowledge of their

students' ability to solve problems correlated with student acheivements, but that the same was not true of their knowledge of problem difficulty or predicated strategies, Carpenter et. al's. research points out a need for additional instruction in identification of the process that students use in the solution of specific problems.

Programs that encourage teachers to examine their own attitudes about the subject matter they teach, and share their views about strategy choices and curriculum development, (Simon and Schifter, 1991)have proved beneficial to teachers and affected the choices and decisions they made about instruction.

Schoenfield(1979)demonstrated the use of explicit instruction in problem-solving heuristics with college students, and suggests the need to explore the possibility that particular strategies are best applied to particular problems.

A later study with 4th grade students demonstrated the success of explicit instruction in the teaching of part-whole relationships as an instructional strategy to solve multiplication problems(Huinker, 1989). Students were instructed with verbal prompts that demonstrated how to analyze a problem's structure by determining the part-whole relationships. This gave students an opportunity to experience the meaning of an operation(multiplication)before representing it symbolically.

A 1996 study conducted by Anthony suggests the opposite. Two case studies revealed that two students present in the same classroom, attending to the same instructional strategies based on a constructivist philosophy of building math knowledge, had developed distinctly different learning styles. One student remained passive in his approach to problem-solving, relying on teacher-prompted assistance for each problem, regardless of the similar structure of problems presented. The other student adopted an active learner's

mode, independently researching problem-solutions and seeking resources other than the classroom teacher to solve problems. This research implies that development of math knowledge and constructs is unique to each individual.

It would seem that if math understanding and success in problem-solving is to grow, teachers must provide an atmosphere which nurtures speculation and experimentation. The cognitive structures students build are expanded, reorganized and strengthened as students interact with each other in groups, sharing their thoughts and strategies. As a student connects new knowledge to previously held beliefs and reorganizes his thinking to accomodate new strategies or solutions, an illumination or clarification will take place which is far more meaningful than a mere confirmation of a correct answer.

A teacher in this environment must remain ever mindful of variation within each classroom of the kinds and degrees of math understanding that exists. She must be sensitive to the fact that understanding of math concepts is not at some point in time attained by students and certainly never acquired simultaneously by an group as an entire unit. Instead, it is built steadily on a day-to-day basis; constantly being reinforced and supported by interaction, observation, structuring, reorganization, and discovery of information by teacher and fellow classmates.

The degree of success students experience in solving problems is dependent upon the choices they make when faced with a problem-solving situation. If problem-solving is routinely linked to a concept which is being introduced or explored in class, then students are denied the opportunity to choose a strategy. If for instance, in the course of learning the procedural steps of long division, students are asked to solve nothing but problems

which utilize division, then these exercises serve only as additional drill. However, if open ended, non-routine questions are presented to students on a regular basis for exploration and discussions, the possibility of increasing the students repertoire of strategies for future problem solutions is increased as interaction with class members reveals a variety of strategies can yield the same answer.

In this type of environment, experimentation is encouraged. Failure to arrive at a correct answer is only a temporary setback. A wrong answer derived from a sound strategy is parised. The method used to attain a correct response becomes as important as the answer itself. This openess encourages students to consider a number of possible pathways to a solution before they select a plan. In subsequent discussion, as students verbalize their reasoning and exchange ideas, they gain more insight into strategies that will be at their disposal for their next problem-solving situation. If students are to truly benefit from this type of open-classroom climate, the teacher must take on the responsibility of keeping discussion focused and on course! One way to accomplish this is to guide students' thinking by asking the right questions. Implicit in this is the understanding that the teacher has carefully selected the problems that she presents to her students, that she is aware of the prior knowledge required to reach a solution, and that has considered the possible strategies that students might select. By asking leading questions, teachers can determine if students have gained understanding from the question being asked, if they have a reasonable plan for solving the problem, and if they can verbalize their reasoning. In other words teachers' questions must clarify the meaning of the question itself, initiate a plan for solution and extend thinking beyond this problem.

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APPENDIX

Math - Computation

1. $125 + 138$	2. $150 - 65$	3. 15×6	4. $32 \div 4 =$ ___	5. $1,981 - 1,962$
6. $227 + 389$	7. $112 - 98$	8. 23×2	9. $30 \div 6 =$ ___	10. $\$55.86 + 5.35$
11. $108 + 97 + 328$	12. $\$94.25 - 15.50$	13. $12 \times 4 =$ ___	14. $33 \div 3 =$ ___	15. $15,423 - 12,800$
16. 16×8	17. $24 \div 2 =$ ___	18. $146 + 142 + 165$	19. $1,200 - 237$	20. $1,027 + 1,215$

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WORD PROBLEM PRE-TEST

1: RICK IS TRYING TO GET IN SHAPE FOR FOOTBALL SEASON. HE DID 125 PUSH-UPS ON SATURDAY AND 138 PUSH-UPS ON SUNDAY. WHAT WAS THE TOTAL NUMBER OF PUSH-UPS HE DID THAT WEEKEND?

2: KURT PLANS TO DO 150 PUSH-UPS. HE HAS ALREADY DONE 65. HOW MANY MORE DOES HE NEED TO DO?

3: SANDY RIDES HER BIKE FOR 15 HOURS EACH WEEK. HOW MANY HOURS DOES SHE RIDE IN 6 WEEKS?

4: A GROUP OF 32 BOYS WANTED TO PLAY KICKBALL. IF THEY MAKE 4 TEAMS, HOW MANY BOYS WOULD BE ON EACH TEAM?

5: BRENDA'S CLASS COLLECTED ALUMINUM CANS. THE FIRST WEEK THEY COLLECTED 227, AND THE SECOND WEEK THEY COLLECTED 389. HOW MANY CANS DID THEY COLLECT IN THESE TWO WEEKS?

6: THE ANDERSON FAMILY LIVED IN CALIFORNIA FROM 1962 TO 1981. HOW MANY YEARS DID THEY LIVE IN CALIFORNIA?

7: THERE ARE 23 BOXES IN THE ART ROOM. EACH BOX CONTAINS

TWO JARS OF PAINT. HOW MANY JARS OF PAINT ARE IN THE ART ROOM?

8: KATIE BABY-SAT A TOTAL OF 30 HOURS IN THE LAST 6 WEEKS. HOW MANY HOURS DID SHE BABY-SIT EACH WEEK?

9: JASON WEIGHED 98 POUNDS IN JANUARY AND 112 POUNDS IN SEPTEMBER. HOW MANY POUNDS DID HE GAIN?

10: LISA SAVED $55.86. SHE PUT $5.35 MORE INTO HER SAVINGS HOW MUCH HAD SHE SAVED THEN?

WORD PROBLEM POST-TEST

1. EVAN COLLECTS STAMPS. HE HAS 108 GERMAN, 97 FRENCH, AND 328 UNITED STATES STAMPS. HOW MANY DOES HE HAVE IN ALL?

2. JOANN DID BABYSITTING DURING THE SUMMER. SHE EARNED $94.25. SHE HAS SPENT $15.50, AND PLANS TO PUT THE REST OF HER SAVINGS IN THE BANK. HOW MUCH WILL SHE PUT IN THE BANK?

3. AT CAMP MOHAVE, THERE ARE 108 CABINS. EACH CABIN HAS BEDS FOR 8 GIRLS. HOW MANY GIRLS CAN SLEEP AT CAMP MOHAVE?

4. MRS. SIMS BOUGHT 24 YARDS OF MATERIAL TO MAKE DRESSES. SHE NEEDS 2 YARDS OF MATERIAL TO MAKE EACH DRESS. HOW MANY DRESSES CAN SHE MAKE?

5. LYN BOWLED 146, 142, AND 165. WHAT WAS HER TOTAL SCORE FOR THE THREE GAMES?

6. A GROUP OF MOUNTAIN CLIMBERS WANT TO CLIMB TO THE TOP OF MOUNT STEEPLE, WHICH IS 15,423 FEET ABOVE SEA LEVEL. THEY HAVE CLIMBED 12,800 FEET. HOW MANY MORE FEET DO THEY HAVE TO CLIMB?

7. AN AUDITORIUM HAS 20 ROWS OF SEATS. THERE ARE 15 SEATS IN EACH ROW. HOW MANY SEATS ARE IN THE AUDITORIUM?

8. LAN HAD 27 TENNIS BALLS. SHE PUT 6 BALLS IN EACH BOX. HOW MANY BOXES CAN SHE FILL? HOW MANY BALLS LEFT OVER?

9. SKIMMER'S POND WAS STOCKED WITH 1,200 TROUT. THE FIRST DAY, 237 TROUT WERE CAUGHT. HOW MANY TROUT WERE LEFT?

10. SHELLY INVITED 18 GIRLS AND 14 BOYS TO HER PIZZA PARTY. HOW MANY KIDS DID SHE INVITE IN ALL?

MATH STORY CHARTS

1. TELL:

SHOW:

SOLVE:

ANSWER AND
LOOK BACK:

2. TELL:

SHOW:

SOLVE:

ANSWER AND
LOOK BACK:

3. TELL:

SHOW:

SOLVE:

ANSWER AND
LOOK BACK:

4. TELL:

SHOW:

SOLVE:

ANSWER AND
LOOK BACK:

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