DOCUMENT RESUME

ED 416 100 SE 061 156

AUTHOR	Hinerman, Sandra Dillon
TITLE	Graphing Calculators in the Calculus Classroom.
PUB DATE	1997-12-00
NOTE	63p.; Masters Thesis, Salem-Teikyo University.
PUB TYPE	Dissertations/Theses - Masters Theses (042)
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	*Calculus; *Educational Technology; *Graphing Calculators; High School Students; High Schools; Mathematics Education; Teaching Methods

ABSTRACT

This study compared the test scores of AP Calculus students. Two methods were used to work the calculus problems: the traditional pencil and paper method and the graphing calculator method. Four researcher-constructed assessments on various calculus topics were administered over a six-week period to two sections of high school AP Calculus students. Each assessment consisted of two sections: a non-calculator section and a calculator-active section. A t-test for dependent samples when the means are related was performed on each of the four assessments at the 0.05 level of significance. The t-test showed that there was a statistically significant difference between the two sections of students' test scores on the first two assessments. Two sections of the students' test scores showed no statistically significant difference on the last two assessments. Contains 43 references. (Author/NB)

GRAPHING CALCULATORS IN THE CALCULUS CLASSROOM

A Thesis

Presented to

The Faculty of the Master of Arts Degree Program

Salem-Teikyo University

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts in Education

by

Sandra Dillon Hinerman

December 1997

Salem-Teikyo University
Salem, West Virginia

This thesis submitted by Sandra Kay Dillon Hinerman has been approved meeting the research requirements for the Master of Arts Degree.

12.11.97 — Date — Thesis Committee Chair
(Gaby van der Giessen, Ph. D.)

12/12/97 — Date
(Robin Hensel, Ed. D.)

12/11/97 — Date
(Prof. Gary S. McAllister)

ACKNOWLEDGMENTS

There were many people who played a part in my fulfilling the requirements to complete this thesis. I want to thank the following so very much, because without their help, this time-consuming project could have never been completed. The words "thank you" can never be enough.

Dr. Gaby van der Giessen;

Dr. Robin Hensel;

Dr. Gary S. McAllister;

My AP Calculus students;

Elizabeth Dean, for the many references, and as always emotional support;

Debbie Herrick, for the technical support, and being so interested in my progress;

Linda Hornbrook, my children's care-giver, for always being so accommodating about my hours of overtime;

Steve Ross, my master's degree buddy, for driving me to class, listening to me talk, and for all those breakfasts;

Dick and Shirley Hinerman, my in-laws, for emotional support and the many days and evenings of taking care of my children and husband;

Virgil Dillon, my father, for praise, emotional support, and proof-reading my thesis;

Wilma Dillon, my mother, for praise, emotional support, cleaning my house, and taking care of my children;

Alicia Hinerman and Alec Hinerman, my children ages 7 and 3, for understanding and being patient when Mommy had to do her homework;

Richard Hinerman, my husband, for praise, emotional support, so much patience and most of all your love.

TABLE OF CONTENTS

ABSTRACT

This study compared the test scores of AP Calculus students in which two methods were used to work the calculus problems. The two methods compared were the traditional pencil and paper method and the graphing calculator method. Four researcher-constructed assessments on various calculus topics were administered over a six-week period to two sections of high-school AP Calculus students. Each assessment consisted of two sections: a non-calculator section and a calculator-active section. A t-test for dependent samples when the means are related was performed on each of the four assessments, at the 0.05 level of significance. The t-test showed that there was a statistically significant difference between the two sections of the students' test scores on the first two assessments. On the last two assessments, the two sections of the students' test scores showed no statistically significant difference.

CHAPTER ONE

Introduction

In 1989, The National Council of Teachers of Mathematics, (NCTM), proposed a change in school mathematics with the publication of the Curriculum and Evaluation Standards (1989). The belief that technology has resulted in a change in the discipline of mathematics prompted the underlying assumption that graphing calculators be available to all students at all times (NCTM, 1989).

Because of this assumption and the rapid change in technology, many educators became proponents of using the graphing calculator to enhance the teaching of mathematics (Demana & Waits, 1992; Mercer, 1995; Vonder Embse & Engebretsen, 1996). Standardized tests, such as the American College Testing (ACT) and the Scholastic Aptitude Test (SAT), permitted calculator use for students beginning in March 1994 (Lechay, 1996; The College Board, 1992). The philosophy of teaching calculus changed with the implementation of calculus reform and this new teaching tool, the graphing calculator (Douglas, 1995; Finney, et. al., 1994). Consequently, in 1995, graphing calculators were required for the Advanced Placement (AP) Calculus Exam (The College Board, 1996). According to Waits and Demana, the incorporation of graphing calculators into calculus is a "natural evolution" (Waits & Demana, 1993, p. 1).

Research Questions

1. What is the opinion of educators on the use of graphing calculators in the classroom?

2. How has the use of graphing calculators influenced standardized tests such as the ACT and SAT?
3. Does the use of graphing calculators improve student understanding of calculus concepts?

Hypothesis

There is no difference in test scores of AP Calculus students when test questions have been worked using two different methods: the traditional pencil and paper method and the graphing calculator method.

Limitations

1. This study focuses on students in rural high schools.
2. Class sizes are limited because of small enrollments in the schools.
3. The groups were selected because of their attendance at a particular high school.
4. One group is much smaller than the other because that high school has fewer students.
5. The findings of this study can only be applied to students in a similar academic situation.

Definitions of Terms

1. Graphing calculator: This is a calculator with built-in function-graphing utilities. It has a standard computer processor, a display screen and are fully programmable. They are hand-held and cost about eighty dollars (Demana & Waits, 1992).
2. Technology in the classroom: This includes graphing and/or scientific

calculators, and computers and any math applicable software (Waits & Demana, 1992).

3. The Standards: Established by the National Council of Teachers of Mathematics, these statements are used to judge the quality of a mathematics curriculum or methods of evaluation. They are statements about what is valued in the teaching of mathematics (NCTM, 1989).
4. Advanced Placement (AP) Calculus: Calculus classes taught in high schools that are comparable to calculus courses in colleges and universities. A standardized examination is given to students at the end of the year. If competencies have been illustrated on this exam, the student can receive college credit (The College Board, 1996).

Assumptions

1. The students in this study are similar to other rural students enrolled in other AP Calculus classes.
2. The mathematical background of the students in this study is similar to the mathematical background of other AP Calculus students.
3. The students in this study have studied the objectives required by The College Board (1996).

Importance of the Study

This study is important because of the revision of the mathematics curriculum established NCTM (1989), calculus reform (Douglas, 1995), and the rapid growth of technology (Day, 1996; Demana & Waits, 1992).

CHAPTER TWO

Introduction

A call for reform in school mathematics suggests that new goals are needed. A shift from the industrial age to the information age is here. This shift has resulted in both the concepts of mathematics that students need to know and how they must master these concepts if they are to be productive citizens in the next century (NCTM, 1989).

The use of technology has dramatically changed the nature of science, business, industry and government. New societal goals for the information age include an opportunity for all students to become mathematically literate, informed electorates and lifelong learners (NCTM, 1989).

In order to enable students to become mathematically literate, The National Council of Teachers of Mathematics, (NCTM), proposed in the Curriculum and Evaluation Standards (NCTM, 1989) the following five educational goals:

1. learning to value mathematics,
2. becoming confident in their ability to do mathematics,
3. becoming mathematical problem solvers,
4. learning to communicate mathematically, and
5. learning to reason mathematically (NCTM, 1989, p. 6)

The outcome of the above goals is that students will become mathematically literate. Mathematical literacy, as defined in The Standards (1989), is an individual's ability to explore, conjecture, reason logically and use a variety of mathematical methods to effectively solve problems. As a result of becoming mathematically literate, the student's mathematical power should be enhanced.

As a result of these five goals, the Commission on Standards for School Mathematics was established by the Board of Directors of the NCTM to "create a set of standards to guide the revision of the school mathematics curriculum and it's associated evaluation" (NCTM, 1989, p. 1). The Commission embedded three features of mathematics into The Standards. The first feature is that "knowing" mathematics is "doing mathematics." The second feature is that some aspects of doing mathematics have changed in the last decade. Third, technology has resulted in changes in the discipline of mathematics itself (NCTM, 1989, p. 7).

The Commission believes that calculators with graphing capability should be available to students at all times. Because of this underlying assumption, the Commission advocates the multiple-representation approach to study a function as one of it's Standards. This includes the following: tabulator, symbolic and graphical representation. This feature along with the availability of graphing calculators has resulted in many Calculus reform projects (Dunham & Dick, 1994; NCTM, 1989).

Graphing calculators have been used to enhance the learning of mathematics since 1985, when Casio marketed the first of these calculators. According to Demana and Waits of The Ohio State University, this began a "revolution in delivering powerful computer graphing to millions of mathematics students" (Waits & Demana, 1996, p. 714). Like larger computers, graphing calculators have standard computer processors, display screens, and built-in software. They are fully programmable and contain built-in function- and parametric-graphing utilities. Statistical functionality with a graphics interface and powerful matrix-arithmetic capabilities are also standard features. Because of the screen design, the user can see, at the same time, the input and the solution of the function (Demana & Waits, 1992).

These hand-held calculators, costing about eighty dollars, are user-friendly and can be used at home or at school. An overhead-graphing calculator-device, typically called a viewscreen, allows students to see what is on the teacher's calculator. This teaching device costs less than three hundred dollars and is a "must" during classroom demonstrations if teaching with a graphing calculator is to be effective. (Demana & Waits, 1992, p.181).

For teachers who want to do demonstrations for their students, Texas Instruments lends out free classroom sets of all their calculators, plus an overhead viewscreen. These can be used in classrooms where students have few calculators, or during teacher workshops where teachers teach other teachers. The calculators will arrive overnight and may be kept for two weeks. Return packaging and labels are included so sending the calculators back is no burden to the classroom teacher (Kelemanic, 1994).

Although the Curriculum and Evaluation Standards (NCTM, 1989) recommends that all students should use a computer and graphing calculator on a regular basis, it is simply not practical for schools to acquire enough computers and software for all students to use on a daily basis. Also, this technology is rarely available to students at home. Therefore, the graphing calculator appears to be the obvious choice to adequately meet the needs of math students (Waits & Demana, 1992).

In addition to the above, Texas Instruments has recently introduced an inexpensive hand-held computer, the TI-92. It has all the capabilities of a graphing calculator with the addition of built-in computer symbolic algebra and computer interactive-geometry software. It truly represents a computer for all students (Waits, & Demana, 1996). This hand-held computer, however, is not permitted to be used during the Advanced Placement Calculus exam (The College Board, 1996).

Calculator Use on Standardized Tests

The use of graphing calculators on other standardized tests began when it was obvious that their use was becoming common in the classroom. The College Board (1992) has allowed graphing calculators to be used on the SAT since March 1994. The ACT first permitted the use of these calculators in the 1996-97 testing cycle (Lechay, 1996).

The College Board concluded an extensive field trial of the SAT in March 1992. In the field test, students who took the SAT were surveyed about their use and familiarity with graphing calculators. According to the survey, 94 percent of all surveyed students indicated they owned or had regular access to a calculator. Eighty-seven percent used calculators on classroom math tests. Only two percent of the students found that calculators were not helpful on the SAT (The College Board, 1992).

According to the College Board, no question on the SAT requires a calculator to determine the answer; however their field studies show that students using a calculator did do slightly better than students who did not. The College Board concluded that students who take a program of college-preparatory mathematics where a calculator has been used in class are more likely to do better on the SAT than students who do not have this background experience (The College Board, 1992).

These responses were similar across ethnic, racial, and gender groups, as well as for students from urban, rural and suburban schools. Results from this field test suggest that scores did improve slightly for students when permitted to use a calculator. All groups of students - males and females of different ethnic and racial backgrounds - were positively affected by the use of calculators. In one analysis, though, women benefited more than men (The College Board, 1992).

During the 1992 field trials, students' scores increased 10 to 20 points. The College Board credited this increase to the overwhelming use of calculators in the classrooms. The fact that students already felt comfortable with their calculators was another explanation. The field trial data suggested that the more the students used these calculators in class, the higher their performance on the SAT. There was no indication, however, that using a calculator allowed a student to finish the test more quickly (The College Board, 1992).

Since the ACT recently permitted the use of calculators, no correlation between the use of calculators and scores has been established. ACT researchers, however, have determined that the current scoring scale can continue to be used. The same mathematical skills and knowledge will still be required. As on previous tests, all math questions on the ACT can be answered without a calculator. On some questions, a calculator will be helpful; on others though, the answer can be found just as quickly, or more efficiently, without the calculator's help (Lechay, 1996).

Graphing Calculators in the Mathematics Classroom

Technology, such as graphing calculators, can be used to promote students' understanding of math concepts and problem-solving techniques. According to Vonder Embse and Engebretsen (1996), students using technology can model real-world phenomena and actually see relationships found in the model. Modeling problems that virtually all students have seen in real-life is an example of the potential that technology possesses.

According to Demana and Waits, "graphing calculators empower students by giving them the power of visualization to do mathematics" (Demana & Waits, 1992, p. 95). Students can now use a geometric representation to investigate traditional algebra topics. An example is the problem:

$$x^2 + 5x + 4 = (x + 4)(x + 1)$$

The question to the student is whether this answer is correct. To check answers, students compare the graphs of the problem and the answer in the same viewing window on the graphing calculator. The students may not be interested in the properties of the produced graphs rather they should be interested in whether or not the problem and the answer produce the same graph. If a student's answer is incorrect, two different graphs will become visible. This process is not to replace algebraic-manipulative skills, but is a method to check algebraic work. Other positive results follow out of the natural curiosity of students. The ease of obtaining graphs encourages students to associate certain shapes with certain algebraic expressions. Most importantly, graphing calculators can make the study of mathematics enjoyable (Demana & Waits, 1992).

Mercer's (1995) article presents a graphing unit for Algebra I students. His lessons progress from the traditional teaching of lines, to quadratics, to slope of a line. He advocates the use of the graphing calculator in his class since students enjoy seeing interesting and unusual curves. In his classroom, calculators are exchanged to share exciting discoveries. With a graphing calculator, students can quickly see a pattern in their graphs.

Mercer (1995) stresses that it is important for teachers to realize the impact these calculators can have on students. Not only do they change the method in which material is presented but now the types of questions used on quizzes and tests may be different. Traditional questions would ask students to graph a given equation with pencil and paper. Now teachers can ask for an equation, given a graph. Furthermore, connecting concepts is made easier. After teaching equations of line on a graphing calculator, a teacher can easily progress to teaching families of parabolas.

A graphing calculator can add an active laboratory experience to a math class. According to Day, (1996), the following four scenarios exemplify technology striving to realize the goals of NCTM's Standards (1989):

1. Students can explore, conjecture about and verify relationships about data sets. On their calculators, students can organize data into tables and, using a scatterplot, display graphical representations.

2. The graphing calculator is an effective tool for comparing and analyzing the effects of changes in a mathematical model. A mathematical model can be generated on the calculator, then changed by the student to analyze resulting differences in the graph or problem.

3. Technology is used to simulate complex events using a probability

model based on random outcomes. Students can simulate various outcomes of a probability problem, then analyze the data.

4. Apparent solutions to equations can be graphically or numerically generated, then zoomed in on to assure accuracy. A graphing calculator can graph equations that represent a situation. Graphically, students can find points of intersection. Fixed-point iteration can also be applied to a numerical approximation routine (Day, 1996).

The question arises as to whether the use of the graphing calculator is really beneficial in mathematical education. Conference proceedings – particularly the annual International Conference on Technology in Collegiate Mathematics - are most abundant with research and the results regarding the impact of this technology on teaching and learning mathematics. Most studies compare two groups of students where the content, instruction and testing are identical except that one group utilizes the graphing calculator. Most proponents of the graphing-calculator usually have a different set of curricular and instructional goals that are made possible by the addition of this instructional tool. In the studies mentioned below, experimental groups used different instructional materials than the control group. (Dunham & Dick, 1994).

According to research, students who use graphing calculators:

1. are better at finding an algebraic representation of a graph (Shoaf-Grubbs, 1992; Rich, 1991; Ruthven, 1990);
2. can better read and interpret graphical information (Boers-van Oosterum, 1990);
3. are better able to relate graphs to their equations (Rich, 1991; Ruthven, 1990);

4. have better understandings of the connections among graphical, numerical, and algebraic representations (Beckmann, 1989; Browning, 1989; Hart, 1992);
5. have more flexible approaches to problem solving (Boers-van Oosterum, 1990);
6. have more concentration on the mathematics of the problems than on the algebraic manipulation (Rizzuti, 1992);
7. do believe that calculators improve their ability to solve problems (McClendon, 1992).

According to Dick, (1992), calculators can lead to better problem solving in the following three ways:

1. Less time is spent on algebraic manipulation, therefore, allowing more time for instruction.
2. Calculators supply more problem solving tools.
3. When freed from computation, students can focus more on setting up the problem and analyzing the solution.

Dunham and Dick, (1994), indicate that graphing calculators have the potential to dramatically affect the teaching and learning of mathematics, specifically in the areas of functions and graphs. A more interactive and exploratory learning environment can result in changes in the roles of teachers and students.

Opponents of the Graphing Calculator

Not all educators are enthusiastic about the implementation of the graphing calculator into the mathematics classroom.

Day (1996) is fearful of technology being the focus of learning. He fears that students, concentrating on technology to produce answers, may not see the underlying mathematics. Students must learn to use the calculator to facilitate their understanding of math concepts. The risk may arise where students only value the correct answer.

Askey worries about the loss of basic skills. (Addington & Roitman, 1996). Cuoco (1995) believes that using technology to support the learning of mathematics is one of the reform movements that threatens to make mathematics more of an elitist discipline than it is now. According to Cuoco, people in ten years may claim to be good in math, but not the type of mathematics used by scientists and mathematicians. He claims the math in which students are skilled will be fundamentally different and useless in the next century.

To obtain a relative maximum by graphing a function and zooming in on a " 'hump' " is Dubinsky's (1995, p.146) fear in the calculus class. The power, beauty and real analysis of calculus will be lost. He advocates that no graphing tool can show various features of a graph where different scales must be used. His example is that 4×10^{-1} and 4×10^{-4} cannot be shown at the same time on a graphing calculator screen. His suggestion is to use concepts from calculus to arrive at a solution and use graphing capabilities only to help.

According to the results of the 1996 AP Calculus AB exam, students who had limited access to calculators throughout the school year, scored somewhat higher than students who had access to graphing calculators each day. The mean score on the free response, which is entirely calculator-active, was 18.6 for unlimited use, compared to 19.61 for students with limited access (Educational Testing Service, 1996).

Andrews (1995) maintains that this technological aspect of calculus is producing students with pitiful skills in arithmetic, algebra and trigonometry. He is especially adamant about the B^- or C^+ students becoming button pushers.

According to Goetz and Kahan (1995), the graphing calculator can enhance the teaching of Analysis but teachers must not rely fully on technology. Students must appreciate the calculator as a powerful tool but not depend on it entirely, and they should realize the calculator component can foster a deeper understanding of the concepts of calculus.

Graphing Calculators in the Calculus Class

In 1986, Douglas (1995) invited twenty-five teachers to Tulane University to examine the teaching of calculus. This workshop, funded by the Sloan Foundation, is often viewed as the beginning of the current calculus reform movement. The purpose of this meeting was to examine the forces that were changing calculus, including content, pedagogy, and the rapid advance of technology.

From the standpoint of content, the general sentiment of the conference concluded that students were not understanding the concepts. The agreement was courses covered too much, too fast. Therefore, the new focus would be to teach fewer topics, but more deeply. Students need to "learn to write, 'speak', and do mathematics." (Schoenfeld, 1995, p. 3).

A variety of pedagogical strategies have been adopted to make reform work more efficiently. Students in some classes are now required to work on extended projects. Lab reports or written work are required for many courses. This provides the instructor a new window on student understanding. According to Schoenfeld (1995), this emphasis on writing in mathematics is a positive outcome of reform.

Nine years later, Douglas deems the movement a success for two reasons: "technology and a growing emphasis on undergraduate education" (Douglas, 1995, p. 2). According to Douglas, technology has made an impact in two ways: improvements in the calculators and computer software, and the methods mathematicians are using with their own research.

According to Waits and Demana, the incorporation of graphing calculators into calculus is a "natural evolution" (Waits & Demana, 1993, p. 1). They believe the content of calculus is changing and less time should be spent on pencil and paper problems while more time should be spent on applications, problem solving, and concept development. Symbolic and visual mathematics, generated by computer, changed the teaching and learning of calculus. Teaching methods are moving toward an investigative, exploratory approach. Because graphing calculators are small, user-friendly, and powerful, they are implemental for all students. This implementation cannot take place without teacher training.

According to Ferrini-Mundy and Lauten (1994), teachers of calculus should be sensitive to students' existing understandings, yet they should emphasize the true concepts of calculus. In their study, they found students entering calculus who have taken appropriate recommended classes have a slight advantage over students who have not taken recommended classes. But after a short time in the calculus class this background advantage disappears. Their study also found that calculus teachers face challenges in teaching the concept of the limit, the Mean Value Theorem, and analyzing graphs of functions.

The function is the central organizing concept in the study of calculus. A number of studies have found that students prefer functions expressed as formulas and are reluctant to study a function using a graph or a table as part of their understanding of a function. (Ferrini-Mundy & Lauten, 1994). Students are especially uneasy when studying piece-wise functions. A graphing calculator can easily graph any function, including a piece-wise function, and give the student a graphical representation of the function. The student then can analyze the graph for not only discontinuities but visualize the limit at a particular point. The graphical representation offers the student and entire class abundant opportunities for explorations and discussion.

Visual thinking can be extremely useful for the calculus student to learn the connections between functions and their graphs. As a visual activity, used with the Mean Value Theorem, students graph a function and a secant line on their calculator, and then visually estimate the point at which the tangent line is parallel to the secant line (Hinerman & Reinhart, 1994). Students can then check their answers by solving the problem analytically. Many researchers believe students who solve problems visually have a deeper understanding than students who solve problems using only the analytic method (Ferrini-Mundy & Lauten, 1994).

Ferrini-Mundy and Lauten (1994) allege the limit is another problem for calculus students. The concept image of a limit differs slightly in calculus from their previous intuitive concepts learned in Precalculus. By entering a function into the calculator, a student can analyze a limit not only graphically but tabularly (Hinerman & Zeppuhar, 1996).

Ferrini-Mundy and Lauten, (1994) advocate the use of calculators in the calculus classroom. Complicated derivatives and obscure techniques of integration are giving way to technology and the conceptual core and understanding of calculus.

According to Kenelly (1996), of Clemson University, there is a need for more people to understand and use calculus concepts. Because of this, he has required every student since 1987 in each of his classes to have a graphing calculator.

Kenelly (1996) favors a graphing calculator over a computer because it is more personal. His experience is that computer classroom requirements attract more male students than female; however, calculator users report positive experiences regardless of gender. His views on the requirement of the graphing calculator are two-fold: first, routine arithmetic calculations are properly de-emphasized, and second the AP study moved to the technology age. Kenelly argues that the use of a graphing calculator is a major improvement that has significantly changed school mathematics.

The most significant change in Kenelly's (1996) classes is the study of calculus topics. Before calculators, a student's end product was the graph. Now Kenelly begins with the graph and discusses all the processes and underlying mathematical principles represented by the function and leading up to the graph.

His classroom has more discussion than lecture, fewer numerical manipulations, and more cooperative learning and team presentations. His students have a clearer understanding of the relationship between calculus concepts and the real world because of the modeling done by graphs and other features on the calculator. Students in his classes have no problem recognizing the national deficit as the derivative of the national debt. His engineering students use their graphing calculator and computer-linking ability to produce graphs and data for submitted reports. Calculators are used daily and are central to all courses he teaches. Kenelly said, "The instruction is not just modified, it is fundamentally changed!" (Kenelly, 1996, p. 25).

The graphing calculator can be used in AP Calculus to enhance the teaching of the derivative (Ferrini-Mundy & Lauten, 1994; Goetz & Kahan, 1995). Numerical derivatives can be found on the calculator for functions. The answer to a derivative problem can be checked on the calculator by graphing the derivative of the original function and the answer found by the student. If correct, one graph will appear on the screen (Goetz & Kahan, 1995). In addition, a teacher may use a calculator to present graphs of the first and second derivatives of a function on the calculator overhead, one at a time. Students can then interpret each derivative and using this information, find the graph of the original function (Ferrini-Mundy & Lauten, 1994).

According to Miller (1995), the use of the graphing calculator will develop spatial visualization that she advocates is a necessary skill to master certain concepts of calculus. This spacial skill is needed when finding volumes and surface areas of solids of revolutions (Ferrini-Mundy & Lauten, 1994). Miller also emphasizes that when calculus is taught using the "Rule of Three" (Miller, 1995, p. 547), three methods of problem solving - graphically, numerically, and algebraically - all students' individual styles of learning are addressed.

Graphing Calculators on the AP Calculus Exam

Because The Advanced Placement Development Committee in Mathematics recognizes that powerful calculators capable of enhancing the instruction of calculus are becoming more available, the Advanced Placement Calculus exam required the use of graphing calculators beginning with the test of spring 1995 (The College Board, 1996). Discussion of the graphing calculator requirement began in 1990. Computers were considered, but this requirement "would have restricted teacher innovations more than it would have expanded them" (Kenelly, 1995, p. 25). The graphing calculator announcement came in 1992 and professional development needs were significant. It was estimated that at least 10,000 AP Calculus teachers would need training on a graphing calculator (Kenelly, 1995).

In response to the announcement that the AP exam would require the use of the graphing calculator, the Technology Intensive Calculus for Advanced Placement group (TICAP) was formed. During the summers of 1992, 1993, and 1994 over 200 teachers were trained. These teachers then became instructors in the College Board's national program of teacher workshops to address AP teacher needs. TICAP was funded at the $1.5 million dollar level by the National Science foundation, the College Board, Educational Testing Service, Texas Instruments and Hewlett Packard. "The graphing calculator requirement may appear to have been slow in being delivered, but since it arrived, it's influence has been substantial" (Kenelly, 1995, p 25).

The Advanced Placement Calculus Exam now consists of two multiple-choice sections: Part A, consisting of 25 questions, forbids the use of a graphing calculator; Part B, consisting of 15 questions, permits the use of a graphing calculator. The free response section, consisting of six questions, *requires* the use of the graphing calculator (The College Board, 1996).

When designing the test, the Advanced Placement Development Committee considered two issues. The first is an equity issue. The committee acknowledges it cannot develop examinations that are fair to all students if the spread in capabilities of the technology is too wide, but it can develop examinations appropriate to any given level of technology. The committee develops exams based on the assumption that all students use calculators to:

1. produce the graph of a function within an arbitrary viewing window,
2. find the zeros of a function,
3. compute the derivative of a function numerically, and
4. compute definite integrals numerically. (The College Board, 1996).

Teachers are provided a list of graphing calculators that are acceptable for use on the examination. The capabilities listed above can either be built into the approved calculator or programmed by the student into the calculator before the examination. Students are also permitted to have stored into their calculators any other programs they want.

The second issue considered by the Advanced Placement Development Committee is teacher development. The AP Development committee acknowledges that this change places a huge burden on AP Calculus teachers. Teachers must learn to incorporate this new technology into their courses. The committee considers the use of the graphing calculator to be an integral part of the course and recommends its use on a regular basis so that students will feel comfortable with the calculator's capabilities (The College Board, 1996).

Waits and Demana (1993) began training teachers for graphing calculators in the 1980's with their C^2PC project. C^2PC, The Calculator and Computer Pre Calculus Project, trains teachers in a one-week, summer institute.

The philosophy shared by Waits and Demana (1993) regarding the use of the graphing calculator to enhance the instruction and learning of mathematics is summarized by three points:

1. Do analytically (pencil and paper), then SUPPORT numerically and graphically (with a graphing calculator).

2. Do numerically and graphically (with a graphing calculator), then CONFIRM analytically (with paper and pencil).

3. Do numerically and graphically, because other methods are IMPRACTICAL or IMPOSSIBLE! (Waits & Demana, 1993, p. 2).

Because of their experience with C^2PC, Waits and Demana founded The Calculator and Computer Enhanced Calculus (C^3E). The C^3E project incorporates the C^2PC philosophy and calculus concepts to result in five major reoccurring themes:

1. Use graphing calculators to visually support analytical calculus methods (paper and pencil manipulations).

2. Use graphing calculators as <u>tools</u> to actually <u>do</u> calculus "manipulations", thus making the need for paper and pencil calculus manipulations less important!

3. Simulate problem situations and solve them using graphing calculator visualization.

4. Illustrate mathematical ideas and applications in concrete geometric settings. Explore, investigate, make and test mathematical conjectures.

5. Solve easily stated and understandable problems that calculus students cannot solve with paper and pencil analytic methods, and problems that have no analytic solutions (Waits, 1992, pp. 4-13).

The goal of both C^2PC and C^3E, is to use the graphing calculator to enhance the teaching and learning of high school mathematics. Demana and Waits also founded The Algebra with Computer and Calculator Enhancement (AC^2E) Institute and Connecting Mathematics and Science for physics teachers. After attending these workshops, teachers have the opportunity to attend several follow-up weekend conferences held regionally throughout the United States (Slomer, 1993).

Because these institutes became so popular and so many teachers were becoming graphing calculator literate, a national support group was formed: Teachers Teaching with Technology (T^3). It comprises all the technology-enhanced math groups founded by Waits and Demana. Regional and national conferences are held throughout the year to continue updating the training for teachers (Slomer, 1993).

The textbook, Calculus: Graphical, Numerical, Algebraic, (Finney, Ross, et. al., 1994) stresses multiple representation to explore problem situations. According to the authors, the philosophy of teaching calculus is to first find an algebraic representation of the problem, then find a complete graph of this algebraic representation. The concept of a function, which is central to the study of calculus, is taught using three representations. A function is represented numerically with a table of numerical pairs; graphically; and symbolically using algebraic representation. By using three different approaches, different learning styles of students are considered and accommodated. According to the authors, the text builds "a richer understanding of calculus" (Finny, Ross, et.al., 1994, p. vii).

Overall, educators view the graphing calculator in the calculus mathematics class as a positive influence (Demana, 1992; Ferrini-Mundy & Lauten, 1994; Kenelly, 1996; Waits, 1992). Applications using fewer pencil and paper manipulations are emphasized (Demana & Waits, 1992; Kenelly, 1996). Students gain a better understanding of concepts due to the graphing visualization attained by the calculator (Ferrini-Mundy & Lauten, 1994; Miller, 1995).

Day (1996) stresses, however, that technology must be used appropriately. Technology should not be used merely to produce answers, but to generate other information for problem extension. Students develop as problem solvers as mathematical explorations are enhanced with calculators. When students generate data, teachers help them see the underlying mathematical principles. Technology can be stimulating because it is a hands-on, manipulative process, and students then can easily move to a more theoretical point of view. An environment can be established in "which students can more clearly recognize the power and usefulness of mathematics to describe and confirm assumptions and conjectures. . . . Technology becomes a valuable tool for learning" (Day, 1996, p. 89).

CHAPTER THREE

Research Question

Due to the recommendation of having technology in the classroom by the National Council of Teachers of Mathematics (1989), educators advocating it's use (Ferrini-Mundy & Lauten, 1994; Vonder Embse & Engebretsen, 1996; Waits & Demana, 1992), and The College Board (1996) changing the Advanced Placement (AP) Calculus Exam to require a graphing calculator on parts of the test, the results of this research attempted to determined if there is a difference in students' scores on relevant calculus tests when working calculus problems with the traditional pencil and paper method and working calculus problems with a graphing calculator.

Subjects

Subjects were enrolled in the AP Calculus classes of two rural high schools in Wetzel County, West Virginia during the 1996-1997 school year. One class was at Magnolia High School in New Martinsville, West Virginia; the other class was at Paden City High School in Paden City, West Virginia.

All subjects were seniors and had completed all necessary math requirements to enroll in AP Calculus. All students took the AP Calculus exam in May 1997.

Design

This study focused on methods AP Calculus students use to work calculus problems. The AP Calculus Course Description Booklet (The College Board, 1996) guided the curriculum. During the study, the students learned various applications of the definite integral. All students learned both the pencil and paper method and the graphing calculator method of working calculus problems. This was a result of the AP exam requiring that students know both methods.

Four tests that required both the pencil and paper method and the graphing calculator method were administered to the students. This study compared the group's scores on the paper and pencil sections to the scores on the graphing calculator section.

Procedures

A teacher-made pre-test was given at the beginning of the unit of the study. The pre-test determined if the students knew how to work definite integrals using both the traditional method and a graphing calculator. The AP Calculus teacher then taught, over a six-week period, The College Board's (1996) objectives of applications of the definite integral. This included: area between two curves, finding the volume of solids of revolutions using washers, discs, and shells, and the relationship between position, velocity, and acceleration functions.

These objectives were taught using both the traditional lecture method and the graphing calculator view screen with an overhead.

Instrumentation

Teacher-made tests were administered throughout the unit. Each test consisted of two parts, a calculator-active section and a section where calculators are not permitted. Questions on each part were similar in nature.

The applications of the definite integral objectives were also tested on the semester exam. The teacher-made semester exam also consisted of two sections, a calculator part and a non-calculator part. Again, questions on each section were similar in nature.

The AP Calculus teacher graded all of the above exams.

On each of the exams, a t-score was calculated and a comparison was made as to students' success on each part.

CHAPTER FOUR

Results

Data Analysis

The purpose of this study was to investigate the two methods that AP Calculus students use to work calculus problems. The methods investigated were solving problems with the aid of a graphing calculator and solving problems using the traditional pencil and paper method.

Hypothesis

There is no difference in the percent of correct answers a student will achieve on a calculus test when working calculus problems using the traditional pencil and paper method and working calculus problems using the graphing calculator.

Data

Four exams were given to AP Calculus students over a six-week period. Seventeen students took the first three exams. The fourth exam, a semester exam, was administered to thirteen students. It was the policy of the school administration that semester exams be optional.

The AP Calculus Course Description Booklet (The College Board, 1996) guided both the exams and the curriculum. The researcher wrote all four exams. The calculator-active and non-calculator sections on each exam asked similar types of questions relevant to that particular exam. Because the College Board's curriculum was used, the researcher assumed that student achievement in these AP Calculus classes is normally distributed.

The first exam was a teacher-made pre-test on integrals. This included both definite and indefinite integrals. The test consisted of two parts: a calculator-active section and a non-calculator section. The possible raw score for each section was 100%.

The mean for the calculator-active section was 98.059 and the mean for the non-calculator active section was 88.000. A *t*-test for dependent samples when the means are related was used to analyze the data. A calculated *t*-value of 3.403 exceeded the critical *t*-value of 2.120. At the 0.05 level, the *t*-test revealed a significant difference, so the null hypothesis was rejected. (Table 1).

Table 1

T-Scores

Pre-test on Integrals

Test Type	N	Test Means	Calculated *t*	Critical *t*
Calculator	17	98.059	3.403	2.120
Non-Calculator	17	88.000		

The second test was a teacher-made performance assessment and consisted of questions regarding finding area under a curve and finding the volume of solids of revolution using discs and/or washers. It also consisted of two parts: a calculator-active section and a non-calculator section. The possible raw score for both sections was 100%.

The mean for the calculator-active section was 87.647 and the mean for the non-calculator active section was 75.765. A *t*-test for dependent samples when the means are related was used to analyze the data. A calculated *t*-value of 2.909 exceeded the critical *t*-value of 2.120. At the 0.05 level, the *t*-test revealed a significant difference, so the null hypothesis was rejected. (Table 2).

Table 2

T-Scores

Test on Area and Volume

Test Type	**N**	**Test Means**	**Calculated *t***	**Critical *t***
Calculator	17	87.647	2.909	2.120
Non-Calculator	17	75.765		

The third exam was a teacher-made performance assessment consisting of questions regarding finding the volume of solids of revolution using the shell method, and position, velocity, and acceleration problems. The test consisted of two parts: a calculator section and a non-calculator section. The possible raw score for each section was 100%.

The mean for the calculator-active section was 77.941 and the mean for the non-calculator active section was 81.353. A *t*-test for dependent samples when the means are related was used to analyze the data. A calculated *t*-value of 0.479 did not exceed the critical *t*-value of 2.120. At the 0.05 level, the *t*-test did not reveal a significant difference, so the null hypothesis was accepted. (Table 3).

Table 3

T- Scores

Test on Shells and Velocity

Test Type	N	Test Means	Calculated *t*	Critical *t*
Calculator	17	77.941	0.479	2.120
Non-Calculator	17	81.353		

The fourth exam was an optional semester examination consisting of multiple-choice questions on topics that were covered in class during the second semester. These multiple-choice questions were taken from the following sources: Calculus AB (Brook, Smith, & Worku, 1989), The Entire 1985 AP Calculus AB Examination and Key (The College Board, 1985), and 1993 AP Calculus AB: Free-Response Scoring Guide with Multiple-Choice Section (The College Board, 1993). The test consisted of two parts: a calculator section and a non-calculator section. The possible raw score for each section was 100%.

The mean for the calculator-active section was 89.231 and the mean for the non-calculator active section was 78.462. A *t*-test for dependent samples when the means are related was used to analyze the data. A calculated *t*-value of 1.620 did not exceed the critical *t*-value of 2.179. At the 0.05 level, the *t*-test did not reveal a significant difference, so the null hypothesis was accepted. (Table 4).

Table 4

T-Scores

Semester Exam

Test Type	N	Test Means	Calculated *t*	Critical *t*
Calculator	13	89.231	1.620	2.179
Non-Calculator	13	78.462		

Findings and Interpretations

An analysis of the four *t*-tests revealed there was a significant difference on two of the teacher-made assessments but not a significant difference on the other two teacher-made assessments. The *t*-test showed a significant difference on the integral test and the test on area and volume, suggesting that the graphing calculator did play a role in the students' scores.

No significant difference was found on the test on shells and velocity nor the semester exam, indicating that the graphing calculator made no difference in the students' scores.

CHAPTER 5

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

The National Council of Teachers of Mathematics proposed a change in school mathematics in 1989 with the Curriculum and Evaluation Standards (1989). The belief that technology has resulted in a change in the discipline of mathematics prompted the underlying assumption that graphing calculators be available to all students at all times (NCTM, 1989). Research shows that many educators advocate this new tool in the classroom. A graphing calculator can add an active laboratory experience (Day, 1996). Students using this technology can model real-world phenomena and actually see relationships found in the model (Vonder Embse & Engebretsen, 1996). According to Demana and Waits, "graphing calculators empower students by giving them the power of visualization to do mathematics" (Demana & Waits, 1992, p. 95). Calculators can lead to better problem solving by students since less time is spent on algebraic manipulation, therefore, allowing more time for instruction. (Dick, 1992).

Some educators are hesitant about the use of this new tool in the classroom. Day (1996) is fearful of technology becoming the focus of learning. Andrews (1995) maintains that this technological aspect of mathematics is producing students with pitiful skills in arithmetic and algebra. The graphing calculator can enhance the teaching of mathematics, but teachers must not rely fully on technology (Goetz & Kahan, 1995).

The use of graphing calculators on standardized tests such as the SAT and the ACT began when it was obvious that their use was becoming common in the classroom (Lechay, 1996; The College Board, 1992). Research found that when using a calculator on the SAT, students' scores increased 10 to 20 points. The College Board (1992) credited this increase to the overwhelming use of calculators in the classrooms. Since the ACT began permitting the use of calculators in the 1996-97 testing cycle, no correlation between the use of calculators and scores has been established. On some questions of the ACT, a calculator will be helpful; on others, though, the answer can be found just a quickly, or more efficiently, without the calculator's help (Lechay, 1996).

According to Waits and Demana, the incorporation of graphing calculators into calculus is a "natural evolution" (Waits & Demana, 1993, p.1). They believe the content of calculus is changing and less time should be spent on pencil and paper problems while more time should be spent on application, problem solving, and concept development. Many researchers believe calculus students who solve problems visually with a graphing calculator have a deeper understanding than calculus students who solve problems using the analytical method (Ferrini-Mundy & Lauten, 1994).

According to Miller (1995), the use of the graphing calculator will develop spatial visualization that she advocates is a necessary skill to master the calculus concepts of volumes and surface areas of areas of solids of revolution. Miller emphasizes that when calculus is taught using the "Rule of Three" (Miller, 1995, p.547), three methods of problem solving – graphically, numerically, and algebraically – all students' individual styles of learning are addressed.

Conclusions

This study was conducted in two AP Calculus classrooms in Wetzel County, West Virginia during the 1996-1997 school year. The students were enrolled at two high schools: Magnolia High School in New Martinsville, and Paden City High School in Paden City. All subjects were seniors who took the AP Calculus exam in May 1997. The purpose of this study was to determine if there was any difference in students' test scores when working calculus problems using traditional paper and pencil or when working calculus problems using a graphing calculator.

Four teacher-made calculus assessments were administered over a six-week period. Each exam had two sections: a calculator-active section and a non-calculator section. Students were required to work both sections of the exams.

A t-test, at the 0.05 level, revealed no statistically significant difference between the calculator and non-calculator sections on two of the four exams. One of these exams assessed the students' knowledge on volume using shells and rectilinear motion. The other exam was a semester exam.

A t-test, at the 0.05 level, did, however, suggest a statistically significant difference between the calculator and non-calculator sections on the other two exams. These exams assessed the students' knowledge on basic integral problems, area, and volume using washers.

It is noted that the calculator made a statistically significant difference in the scores on the first two exams the teacher administered. On the first test, students merely had to type in data on the calculator section and the calculator immediately revealed an answer. The pencil and paper section required a great amount of synthesis for the students since they had to actually work the entire problems themselves. This could therefore, reflect the literature's notion of a calculator solving easy manipulations to leave more time for applications, problem solving, and concept development (Waits & Demana, 1993).

The second exam, the test on area and volume, required a great deal of problem solving and analysis on both sections. Most students did better on the calculator-active section. The students were still required to understand the concepts of area and volume, but after setting the problem up, the calculator could do the integration for them. These results agree with Waits (1992) who stresses using graphing calculators as tools to do calculus manipulations, and making pencil and paper calculus manipulations less important.

By the third and fourth exams, there was no significant difference noted between the calculator sections and the non-calculator section. The third exam, written very much like the second exam, showed no significant difference at the 0.05 level. Many more students scored a perfect score on the non-calculator active section and missed many more points on the calculator-active section. Perhaps, the calculator became the focus of the test questions. According to Day (1996), students may concentrate so much on the technology to produce answers, the underlying mathematics may be lost.

The fourth exam, a multiple-choice semester exam, also showed no significant difference at the 0.05 level. Raw scores were similar for both sections. The format of this exam was very much like that of the multiple-choice section of the AP exam. Since the students had recently heavily reviewed for and taken the AP exam only a few weeks earlier, one might hope that the students were quite confident in working problems both with a calculator and without.

Results from this study, therefore, indicate that there may be times when there appears to be a difference in students' test scores when working calculus problems with a calculator and working calculus problems with pencil and paper. Educators are using the calculator in the classroom (Dunham & Dick, 1994; Ferrini-Mundy & Lauten, 1994; Hinerman & Reinhart, 1994; Waits & Demana, 1993). It has become necessary for students to use the calculator on the AP Calculus exam (The College Board, 1996). The National Council of Teachers of Mathematics has advocated it's use (NCTM, 1989), and much research has recently been published on the calculator in the classroom (Dunham & Dick, 1994).

Recommendations

The following recommendations have arisen from the results of this study:

1. Allow the students to become familiar with the calculator on an almost every-day experience.
2. Stress to the students that the calculator cannot problem-solve or synthesize for them. They still must understand the concepts.
3. Use the calculator in the classroom more to do basic calculus manipulations and save more time to do problem solving and concept development.

Recommendations for Follow-Up Studies

1. A study might be conducted to determine any significant difference in AP Calculus scores of students who used their calculators daily and those who had limited access.

2. A study might be conducted to determine any significant difference in males and females who use the graphing calculator for visual comprehension of calculus concepts.
3. A study might be conducted to determine any significant difference in Calculus students' understanding of Calculus concepts and on whether they used the graphing calculator or used only pencil and paper methods to learn in previous math courses.

APPENDIX A
INTEGRAL PRE-TEST
NON-CALCULATOR SECTION

AP CALCULUS TEST ON INTEGRATION NAME ______________
5 points each - 70 total

Evaluate the indefinite integrals. You may leave any <u>TWO</u> problems blank.

1. $\int x^3 \, dx$ ____________ 2. $\int x\sqrt{1 - x^2} \, dx$ ______________

3, $\int \sin (x/2) \, dx$ _______________ 4. $\int \cos^3 4x \sin 4x \, dx$ _____________

5. $\int \frac{x^3 + 1}{x^2} \, dx$ _____________ 6. $\int 5x (1 + x^2)^4 \, dx$ _____________

7. $\int \sqrt{\cot x} \csc^2 x \, dx$ _____________ 8, $\int \sec 2x \tan 2x \, dx$ _____________

9. $\int \frac{1}{2\sqrt{x}} \, dx$ ____________ 10. $\int x^{(1/3)} + x^{(2/5)} \, dx$ ___________

Evaluate the definite integral.

11. $\int_{1}^{2} 1/x^2 \, dx$ ____________

12. $\int_{0}^{1} x\sqrt{1 - x^2} \, dx$ ________________
see #2

13. $\int_{0}^{8} |x - 3| \, dx$ ______________

14. Find the average value of $f(x) = 2x^2 + 3$ on [0,2] ____________________

15. Use the Trapezoid Rule to approximate $\int_{2}^{3} \frac{1}{(x - 1)^2} \, dx$ with n = 4. ____________

APPENDIX B
INTEGRAL PRE-TEST
CALCULATOR SECTION

AP CALCULUS INTEGRATION TEST NAME ____________________

CALCULATORS MAY BE USED.

1. $\int_0^e \frac{(\ln x)^2}{x} \, dx$ ____________________

2. $\int_{-2}^4 x^4 + 3x^2 - 10x - 9 \, dx$ ____________________

3. $\int_0^7 x\sqrt{3x^2 + 9} \, dx$ ____________________

APPENDIX C
AREA AND VOLUME TEST
NON-CALCULATOR SECTION

AP CALCULUS TEST 7-2 NAME ______________________
(Sections 7-1 and 7-2)
THIS SECTION IS *NON*-CALCULATOR ACTIVE!

Find the area under the curve bounded by the given functions. Show work on this test.

1. $y = x^2$ $y = 2x + 3$ ______________________

2. $y = x - 1$ $x = 3 - y^2$ ______________________

3. Use the washer method to find the volume of the solid when the two functions are revolved around the given axis.

$y = 4x$ $y = 4x^2$ around the y-axis ______________________

APPENDIX D
AREA AND VOLUME TEST
CALCULATOR SECTION

NAME ______________________

THIS SECTION OF THE TEST IS ***CALCULATOR ACTIVE***!

Find the area of the region bounded by the given functions.

4. $y = 3^x$ $y = 2x + 1$ ______________________

5. $y = \frac{1}{1 + x^2}$ $y = x^2 / 2$ ______________________

Use the washer method to find the volume of the solid when the two functions are revolved around the given axis.

6. $x^2 = y - 2$ $2y - x - 2 = 0$ $x = 0$ $x = 1$ around $y = 5$ ______________

7. $y = x^2 + 1$ $y = x + 3$ around the x-axis ____________________

APPENDIX E
TEST ON APPLICATIONS OF THE INTEGRAL
NON-CALCULATOR SECTION

AP CALCULUS APPLICATIONS OF THE DEFINITE INTEGRAL

NAME ____________________

NO CALCULATOR IS PERMITTED ON THIS PORTION OF THE TEST

Use the shell method to find the volume of the resulting solid when the given function(s) is(are) revolved around the given axis.

1. $y = 1/x$ $x = 1$ $x = 2$ $y = 0$ around the y-axis

____

2. $x = 2y - y^2$ $x = 0$ around the x-axis

____

3. $y = e^x$ $x = 0$ $x = 1$ around the y-axis

____

A particle is moving along the x-axis such that the position at given time t is $s(t) = (2/3)t^3 - (11/2)t^2 + 12t$. If $0<t<5$, find the velocity of the function at the time the acceleration is equal to zero.

____

APPENDIX F
TEST ON APPLICATIONS OF THE INTEGRAL
CALCULATOR SECTION

AP CALCULUS   APPLICATIONS OF THE DEFINITE INTEGRAL

NAME ____________________

CALCULATORS MAY BE USED ON THIS PORTION OF THE TEST

Use the shell method to find the volume of the resulting when the given function(s) is(are) revolved around the given axis.

1. $y = x^3 + 1$ $2y = 2 - x$ $x = 0$ $x = 1$ around the y-axis

____

2. $y = \sec x \tan x$ $x = 0$ $x = 1$ around the y-axis ____________________

3. Use the best method to find the volume - disc or shell.

$y = 4x$ $y = x^2$ around the x-axis ____________________

4. A particle is moving along the x-axis with the following conditions: $a(t) = 6t - 12$ with $v(0) = 7$ and $s(0) = -3$. Find when the velocity of the particle is 0.

____

5. Using the above information, find the position of the particle when the velocity equals 0.

APPENDIX G
SEMESTER EXAM
CALCULATOR ACTIVE

AP CALCULUS SEMESTER EXAM NAME ____________

THIS PORTION IS CALCULATOR ACTIVE.

Circle the correct answer.

1. At how many points do the graphs of $y = 4^x$ and $y = x^4$ have the same slope?

A. 0 B. 1 C. 2 D. 3 E. 4

2. The region enclosed by the x-axis, the line $x = 3$ and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

A. 3π B. $2\sqrt{3}\pi$ C. $9/2\ \pi$ D. 9π $\frac{36\sqrt{3}\pi}{5}$

3. The area of the region enclosed by the graphs of $y = 3 - 2e^{-x}$ and $y = e^x$ is

A. 2.079 B. 1.079 C. 0.987 D. 0.920 E. 0.079

4. A particle moves along a line so that at time t, where $0 \leq t \leq$, its position is given by $s(t) = -4\cos t - (t^2/2) + 10$. What is the velocity of the particle when its acceleration is zero?

A. -5.19 B. 0.74 C. 1.32 D. 2.55 E. 8.13

5. A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = t^4 - 2t^3 - t^2 + 2$. At the instant when the acceleration becomes zero,

the velocity of the particle is approximately

A. -4.15 B. -.594 C. .152 D. .178 E. 1.985

6. $\int_{e}^{e^2} \frac{(\ln x)^2}{x}\, dx =$

A. 0.096 B. 1.00 C. 2.718 D. 2.333 E. 3

7. The average value of $f(x) = \frac{\sin x}{x^2 + x + 1}$ on the interval $0 \leqslant x \leqslant 2$ is

A. .0618 B. 0.219 C. 0.439 D. 0.877 E. none of these

8. The function f given by $f(x) = x^3 + 12x - 24$ is

A. increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
B. decreasing for $x < 0$, increasing for $x > 0$
C. increasing for all x
D. decreasing for all x
E. decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

9. $\int_{-2}^{-1} x^{-2}\, dx$

A. -(1/2) B. 7/16 C. 7/4 D. -(7/4) E. 1/2

10. If $f(x) = x^3 - x$, then

A. $\sqrt{3}/3 = x$ is a local maximum of f
B. $\sqrt{3}/3 = x$ is a local minimum of f
C. $\sqrt{3} = x$ is a local maximum of f
D. $\sqrt{3} = x$ is a local minimum of f
E. $-\sqrt{3} = x$ is a local minimum of f

11. The average value of $f(x) = \sqrt{9 - x}$ on the interval $0 \leq x \leq 9$ is

A. 6   B. 2   C. 1 1/2   D. -6   E. –2

APPENDIX H
SEMESTER EXAM
NON-CALCULATOR ACTIVE

THIS PORTION IS NON-CALCULATOR ACTIVE!

CIRCLE THE CORRECT ANSWER.

1. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

A. 0 B. 2 C. 4 D. 8 E. 12

2. $\frac{d}{dx}(2^x) =$

A. 2^{x-1} B. $(2^{x-1})x$ C. $(2^x)\ln 2$ D. $(2^{x-1})\ln 2$ E. $\frac{2x}{\ln 2}$

3. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

A. $1/x$ B. $1/x^2$ C. $\frac{\ln x - 1}{x^2}$ D. $\frac{1 - \ln x}{x^2}$ E. $\frac{1 + \ln x}{x^2}$

4. $\int_1^2 \frac{x^2 - 1}{x + 1}\,dx =$

A. 0.5 B. 1 C. 2 D. 2.5 E. ln 3

5. $\int \sec^2 x \, dx =$

A. $\tan x + C$ B. $\csc^2 x + C$ C. $\cos^2 x + C$
D. $\frac{\sec^3 x}{3} + C$ E. $2 \sec^2 x \tan x + C$

6. $\int \frac{x \, dx}{\sqrt{3x^2 + 5}} =$

A. $1/9\,(3x^2 + 5)^{3/2} + C$ B. $1/4(3x^2 + 5)^{3/2} + C$
C. $1/12(3x^2 + 5)^{1/2} + C$ D. $1/3\,(3x^2 + 5)^{1/2} + C$
E. $3/2(3x^2 + 5)^{1/2} + C$

7. $\lim_{n \to \infty} \frac{4n^2}{n^2 + 10{,}000n} =$

A. 0 B. 1/2,500 C. 1 D. 4 E. nonexistent

8. $\lim_{x \to \infty} \frac{1 - \cos x}{2 \sin^2 x} =$

A. 0 B. 1/8 C. 1/4 D. 1 E. nonexistent

9. Let R be the region in the first quadrant enclosed by the graph of $y = (x + 1)^{1/3}$, the line $x = 7$, the x-axis, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by:

A. $\pi \int_0^7 (x + 1)^{2/3} \, dx$ B. $2\pi \int_0^7 x(x + 1)^{1/3} \, dx$
C. $\pi \int_0^2 (x + 1)^{2/3} \, dx$ D. $2\pi \int_0^2 x(x + 1)^{1/3} \, dx$
E. $\pi \int_0^7 (y^3 - 1)^2 \, dy$

10. The volume of revolution formed by rotating the region bounded by $y = x^3$, $y = x$, $x = 0$ and $x = 1$ about the x-axis is represented by

A. $\pi\int_0^1 (x^3 - x)^2 \, dx$

B. $\pi\int_0^1 (x^6 - x^2) \, dx$

C. $\pi\int_0^1 (x^2 - x^6) \, dx$

D. $2\pi\int_0^1 (x^6 - x^2) \, dx$

E. $2\pi\int_0^1 (x^2 - x^6) \, dx$

REFERENCES

Addington, S. & Roitman, J. (1996). Who is Dick Askey and why is he so upset about the Standards? Mathematics Teacher, 89, 626 - 627.

Andrews, G. E. (1995). The irrelevance of calculus reform: rumination of a sage-on-the-stage. (UME Trends Vol. 6, No. 6). Lafayette, IN: Purdue University, Joint Policy Board for Mathematics.

Beckman, C. E. (1992). An approach to teaching a college course in trigonometry using applications and a graphing calculator (Doctoral dissertation, Illinois State University, 1992). Dissertation Abstracts International, 52, 2850A.

Boers-van Oosterum, M. A. M. (1990). Understanding of variables and their uses acquired by students in traditional and computer-intensive algebra (Doctoral dissertation, University of Maryland College Park, 1990). Dissertation Abstracts International, 52, 1538A.

Brook, D. E., Smith, D. M., Worku T. (1990). Calculus AB. USA: Research and Education Association.

Browning, C. A. (1989). Characterizing levels of understanding of functions and their graphs (Doctoral dissertation, Ohio State University, 1989). Dissertation Abstracts International 49, 2957A.

Cuoco, A. (1995). Some worries about mathematics education. Mathematics Teacher, 88, 186 - 187.

Day, R. (1996). Classroom technology: tool for, or focus of, learning? Mathematics Teacher, 89, 134 - 137.

Demana, F. (1992). Graphs and Continuity. Unpublished manuscript, The Ohio State University at Columbus.

Demana, F. & Waits, B. (1992). A computer for all students. Mathematics Teacher, 85, 94 - 95.

Dick, T. (1992). Super calculators: Implications for calculus curriculum, instruction and assessment. In J. T. Fey (Ed.) Calculators in Mathematics Education, 1992 Yearbook of the National Council of Teachers of Mathematics (pp. 145 - 57). Reston, VA: The Council.

Douglas, R. (1995). The first decade of calculus reform. (UME Trends Vol. 6. No. 6). Lafayette, IN: Purdue University, Joint Policy Board for Mathematics.

Dubinsky, E. (1995). Is calculus obsolete? Mathematics Teacher, 88, 146 - 148.

Educational Testing Service. (1996). [AP Calculus AB test results.] Unpublished raw data.

Ferrini-Mundy, J. & Lauten, D. (1994). Learning about calculus learning. Mathematics Teacher, 87, 115 - 120.

Finney, R. L., Thomas, G. B., Demana, F. & Waits, B. K. (1994). Calculus graphical, numerical, algebraic. Reading, MS: Addison-Wesley.

Goetz, A. & Kahan, J. (1995). Surprising results using calculators for derivatives. Mathematics Teacher, 88, 30 - 33.

Hart, D. K. (1992). Building concept images: supercalculators and students' use of multiple representations in calculus (Doctoral Dissertation, Oregon State University, 1992). Dissertation Abstracts International, 52, 4254A

Hinerman, S. & Reinhart, J. (1994, January). Topics from Pre Calculus and Calculus using the TI-82. Presentation at the annual meeting of the West Virginia Council of Teachers of Mathematics, Canaan Valley, WV.

Hinerman, S. & Zeppuhar, B. (1996, March). Using the TI-82 in Pre Calculus and to find the "average" student. Presentation at the annual meeting of the West Virginia Council of Teachers of Mathematics, Flatwoods, WV.

Kelemanik, G. (1994). A potpourri. Mathematics Teacher, 87, 566 - 569.

Kenelly, J. (1995). Advanced placement: both a leader and a follower in calculus reform. (UME Trends, Vol. 6, No. 6). Lafayette, IN: Purdue University, Joint Policy Board for Mathematics.

Kenelly, J. (1996). Technology in Mathematics Instruction. Calculus The Dynamics of Change. The Mathematical Association of America.

Lechay, D. (Ed.). (1996, Autumn). Activity. (Available from the American College Testing, 2201 North Dodge Street, P.O. Box 168, Iowa City, IA 52243-0168)

McClendon, M. A. (1992). The development of a graphics calculator study guide for calculus students (Doctoral dissertation, University of Oregon, 1992). Dissertation Abstracts International 52, 2450A.

Mercer, J. (1995). Teaching graphing concepts with graphing calculators. Mathematics Teacher, 88, 268 - 273.

Miller, N. (1995). Women in consortium calculus. Mathematics Teacher, 88, 546 - 548.

National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National Council of Teacher of Mathematics.

Rich, B. (1991). The effect of the use of graphing calculators on the learning of function concepts in precalculus mathematics (Doctoral dissertation, University of Iowa, 1991). Dissertation Abstracts International 52, 835A.

Rizzuti, J. M. (1992). Students' conceptualizations of mathematical functions: the effects of a pedagogical approach involving multiple representations (Doctoral dissertation, Cornell University, 1992). Dissertation Abstracts International 52, 3549A.

Ruthven, K. (1990). The influence of graphic calculator use on translation from graphic to symbolic form. Educational Studies in Mathematics

21, pp. 431-50.

Schoenfeld, A. (1995). A brief biography of calculus reform. (UME Trends Vol. 6. No. 6). Lafayette, IN: Purdue University, Policy Board for Mathematics.

Shoaf-Grubbs, M. M. (1992). The effect of the graphics calculator on female students' cognitive levels and visual thinking. In Lewis Lum (Ed.), Proceedings of the Fourth International Conference on Techology in Collegiate Mathematics (pp. 394-98). Reading MS: Addison - Wesley.

Slomer, D. (Ed.). (1993, Fall). Connections. (Available from Ohio C^3E, Department of Mathematics, The Ohio State University, 231 W. 18th Avenue, Columbus, OH 43210).

The College Board. (1996). Advanced placement course description mathematics, May 1997. USA: College Entrance Examination Board and Educational Testing Service.

The College Board. (1993). 1993 AP Calculus AB: free-response scoring guide with multiple-choice section. USA: College Entrance Examination Board.

The College Board. (1992). Q and A for calculator policy. College Entrance Examination Board.

The College Board. (1985). The entire 1985 AP Calculus AB examination and key. USA: College Entrance Examination Board.

Vonder Embse, C. & Engebretsen, A. (1996). A mathematical look at a free throw using technology. Mathematics Teacher, 89, 774 - 778.

Waits, B. K. (1992). The power of visualization in calculus. Unpublished manuscript, The Ohio State University at Columbus.

Waits, B. & Demana, F. (1992). A case against computer symbolic manipulation in school mathematics today. Mathematics Teacher, 85, 180 - 183.

Waits, B. & Demana, F. (1996). A computer for all students - revisited. Mathematics Teacher, 89, 712 - 714.

Waits, B. K. & Demana, F. (1993). Graphing claculator intensive calculus: a first step in calculus reform for all students. Unpublished manuscript, The Ohio State University at Columbus.

SE061156

ERIC

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
Educational Resources Information Center (ERIC)

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: Graphing Calculators in the Calculus Classroom	
Author(s): Sandra Dillon Hinerman	
Corporate Source: SALEM-TEIKYO UNIVERSITY BENEDUM LIBRARY SALEM, WV 26426-0520	Publication Date: Dec 1997

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic/optical media, and sold through the ERIC Document Reproduction Service (EDRS) or other ERIC vendors. Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following two options and sign at the bottom of the page.

[✓] **Check here For Level 1 Release:** Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical) *and* paper copy.

The sample sticker shown below will be affixed to all Level 1 documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY Sandra Dillon Hinerman TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Level 1

The sample sticker shown below will be affixed to all Level 2 documents

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY ______ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

Level 2

[ ] **Check here For Level 2 Release:** Permitting reproduction in microfiche (4" x 6" film) or other ERIC archival media (e.g., electronic or optical), but *not* in paper copy.

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but neither box is checked, documents will be processed at Level 1.

"I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic/optical media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Sign here→ please

Signature: Sandra Dillon Hinerman	Printed Name/Position/Title: Sandra Dillon Hinerman	
Organization/Address: 109 Beechwood Dr. New Martinsville, WV 26155	Telephone: 304-455-1541	FAX:
	E-Mail Address: rshine@ovnet.com	Date: 1/10/98

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, *or*, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
1100 West Street, 2d Floor
Laurel, Maryland 20707-3598

Telephone: 301-497-4080
Toll Free: 800-799-3742
FAX: 301-953-0263
e-mail: ericfac@inet.ed.gov
WWW: http://ericfac.piccard.csc.com

(Rev. 6/96)