Philosophical analysis indicates that underlying much of the Western scientific world view is the metaphysical presupposition of duality, the claim being made that the world is made sense of in terms of either/or and in terms of polarities (e.g., light versus dark). By way of contrast, no concept is more important in Asian philosophical and religious thought than that of nonduality. The basic ideas of quantum physics are not so much difficult as that they are strange. In some situations, electrons that are usually referred to as 'particles' may exhibit 'wave-like' behavior. Both matter and radiation can be viewed as having a dual (wave-particle) nature. In an empirical study of student thinking, the powerful heuristic metaphor of the map is used to construct graphic representations of United Kingdom Advanced level students' understanding of quantum physics. The nature of students' understanding is represented by their construction of groupings of ideas in a personal psychological space with underlying dimensions providing a co-ordinate system for their perceptions. The relationships between students' conceptions of quantum phenomena at the level of the population group are investigated using a structured questionnaire and multivariate analytical techniques (Multidimensional Scaling, Cluster Analysis, and Factor Analysis). A novel quantitative methodology is used to probe students' qualitative implicit understanding. Findings confirm the primacy of dualism in student thinking. Contains 51 references. (Author)
Dualistic Thinking Underlying Students' Understanding of Quantum Physics

by

Azam Mashhadi and Brian Woolnough
Abstract

What is important in the examination of people's mindscape is not what they articulately know or say they believe...What matters is something deeper: the feel of the world around us, the sense of reality, the taste that spontaneously discriminates between knowledge and fantasy. Roszak (1972: xxiv)

Philosophical analysis indicates that underlying much of the Western scientific world view is the metaphysical presupposition of duality. The claim being made that the world is made sense of in terms of either/or, in terms of polarities (e.g. light versus dark). By way of contrast no concept is more important in Asian philosophical and religious thought than that of nonduality (Loy, 1988).

The basic ideas of Quantum Physics are not necessarily difficult as that they are strange. In some situations, electrons that are usually referred to as 'particles' may exhibit 'wave-like' behaviour. Both matter and radiation can be viewed as having a dual (wave-particle) nature. In an empirical study of student thinking the powerful heuristic metaphor of the map is used to construct graphic representations of UK Advanced level students' understanding of Quantum Physics. The nature of students' understanding being represented by their construction of groupings of ideas in a personal psychological space, with underlying dimensions providing a co-ordinate system for their perceptions. The relationships between students' conceptions (at the level of the population group) of quantum phenomena are investigated using a structured questionnaire, and multivariate analytical techniques (Multidimensional Scaling, Cluster Analysis, and Factor Analysis). A novel quantitative methodology is used to probe students' qualitative implicit understanding. The findings confirm the primacy of dualism in students' thinking.

Category of presentation: Paper
Strand: Education
Key words: dualistic thinking, quantum physics, cognitive mapping

Azam MASHHADI's doctoral thesis, at the University of Oxford, addressed the question of What is the nature of the understanding of the concept of 'wave-particle duality' among Advanced level Physics students? Following degrees in Physics and Astrophysics (University of London) and Astronomy (University of Sussex) he taught for several years at a college in London (UK) before completing a MSc in Educational Research Methodology (Oxford). His research interests include student learning, teacher education, the use of IT in education, research methodology, and philosophy of science education.

Brian E. WOOLNOUGH is a lecturer in science education at the University of Oxford, and Vice-Master of St Cross College, Oxford. He has been editor of Physics Education (1986-1990), and is currently series editor for the Open University Press series on Developing Science and Technology Education. He has published many articles on science education; his recent books include Effective Science Teaching (OUP, 1994), and Practical Science (OUP, 1991).

1 All correspondence should be addressed to this author.
But first the notion that man has a body distinct from his soul is to be expunged...
If the doors of perception were cleansed, everything would appear to man as it is, infinite. For man has closed himself up, till he sees all things thro' narrow chinks of his cavern.
William Blake (1792), The Marriage of Heaven and Hell

1 Introduction
Shaw and Gaines (1992: 6) point out that people have tended to conceptualise the world in terms of restricted sorts that are then dichotomised is a phenomenon identified in antiquity (Lloyd, 1966), and is common across many cultures (Maybury-Lewis and Almagor, 1989). The implication is that 'dualistic thinking' underlies how individuals make sense of the world.

2 Dualistic thinking
Dualistic thinking is thinking which differentiates that-which-is-thought-about into two opposed categories: being and nonbeing, life and death and so on. The argument being that dualistic categories are part of a conceptual grid which is normally but unconsciously superimposed upon our immediate experience:

The world appears to be experienced dualistically: as a collection of discrete objects (one of them being me) causally interacting in space and time. Typically in everyday and school science language the speaker stands outside the 'object', regarding it as an object of consideration. The dualistic subject-object style of thinking may well carry with it an overwhelming bias about the way the individual comes to develop a world view (Loy 1988: 25).

Polarity or dualism is often used for categorisation (e.g. light versus dark or rational versus intuitive). As Helen Haste (1993: 44) expresses it:

The effect is that once symbols, metaphors or images have been attached to one pole, by implication their negative becomes attached to the other: things of the body become other than things of the mind.

The philosopher of religion David Loy (1988: 18) also points out:

The problem with such thinking is that, although distinctions are usually made in order to choose one or the other, we cannot take one without the other since they are interdependent: in affirming one half of the duality we maintain the other as well.

In a paper on Nondual Thinking Loy (1986) argues that much of Asian philosophy constitutes a radical critique of thinking as it is considered to usually occur:

Another nonduality, the nondifference of subject and object, is a crucial - perhaps the crucial - concept for several of those Eastern systems which criticize reasoning/conceptualizing - particularly Mahayana Buddhism, Advaita Vedanta, and Taoism.
Loy (1986: 294)

The general nature of reasoning is thought to move between assertion and negation, i.e. between 'it is' and 'it is not'. There is usually a distinction between subject and object, an experiencing self that is distinct from what is experienced, whether it is sense-object,
physical action, or mental event (Loy, 1988: 25). This dualistic view is almost in diametric opposition to a worldview based on the nonduality of seer and seen. The Western dualistic or commonsense 'relative' world consists of a collection of discrete objects, interacting causally in space and time. Metaphors form the basis for taken for granted assumptions about the world. There is a primacy of a metaphor of dualism in Western culture. Bowes (1986: 58) points out that:

Our view of how the world is, is intimately related to the question of how we think (logic), what knowledge means, how we know, and how we make sure of its truth (epistemology), how we talk about it, our conceptual scheme, so that what there is, is truly represented (philosophy), also to the questions of what knowledge is for, what we do with it, and last but not least what we try to know.

The scientific worldview has its own metaphysical presuppositions. Presuppositions that originated in ancient Greece and involved ways of perceiving the world that were articulated by Plato and in particular Aristotle. The Aristotelian worldview developed into the modern scientific worldview. John Barrow (1988: 24) in *The World Within the World* lists a number of presuppositions about the nature of reality that scientists usually take for granted:

1. There exists an external world which is external to our minds, and which is the unique source of all our sensations.
2. This external world is ultimately rational. 'A' and 'not A' cannot be true simultaneously.
3. The world can be analysed locally without destroying its essential structure.
4. The elementary entities do not possess what we call freewill.
5. The separation of events from our perception of them is a harmless simplification.
6. Nature possesses regularities, and these are predictable in some sense.
7. Space and time exist.
8. The world can be described by mathematics.
9. These presuppositions hold in an identical fashion everywhere and everywhen.

Barrow (1988: 24)

Greek logic, as formulated by Aristotle, forms the basis of the Western intellectual tradition and has so-called 'laws of thought' (the Law of Identity, the Law of Non-Contradiction and the Law of Excluded Middle). According to the Law of Excluded Middle a thing must be either A or not-A, there is no middle possibility. The Law of Non-Contradiction essentially states that A cannot be not-A, when not-A represents a category that has been created precisely so that A may be excluded from it (Bowes, 1986: 64). In theory this 'law' is an analytical procedure for creating mutually exclusive categories, opposites cannot belong together. This either/or logic is assumed unconsciously in the world of everyday or commonsense experience. These logical principles are offered as laws of thought, not as laws of thought in the mechanistic world view but laws of thought, period.

These laws are normally taken for granted, and it is assumed that these axioms define the way people think. The question arises as to whether or not these laws are necessarily true in all situations? Doubts arise once the meanings of 'is' and 'not' in Aristotle's laws are considered further (Macrone, 1995). Such words can be used in a number of ways, giving rise to semantic muddles. For example, consider a statement such as 'a daffodil is either yellow or it is not'. The simplicity of the statement is undermined by disagreement between individuals on how yellow a daffodil has to be to be 'yellow', and even perhaps disagreement on what 'yellow' means. Qualities, or predicates, are often subjective. In
mathematics it cannot be proven that an infinite number is either even or odd. In quantum physics it cannot be stated that 'light is either a wave or not-a-wave'.

It was not until Kant that Western philosophy became fully aware of the role of mind in sense-perception. The mind does not just receive perceptions, but interprets and synthesises perceptions into the phenomenal world of experience. Perception involves conception. Contemporary philosophy has shifted from Kant's Aristotelian categories to language as the means by which the organisation of the phenomenal world occurs. The individual makes sense of the world through both language and through the process of active interaction with other individuals. The philosopher John Searle comments:

I am not saying that language creates reality. Far from it. Rather, I am saying that what counts as reality ... is a matter of the categories that we impose on the world; and those categories are for the most part linguistic. And furthermore: when we experience the world we experience it through linguistic categories that help to shape the experiences themselves. The world doesn't come to us already sliced up into objects and experiences: what counts as an object is already a function of our system of representation, and how we perceive the world in our experience is influenced by that system of representation.
Magee (1978: 184)

Any language has to have some kind of grammatical structure. This structure incorporates tacit assumptions that direct individuals to direct attention to attend to particular aspects of what goes on in and around us and to ignore others. Benjamin Lee Whorf suggests that through the linguistic systems in our minds we project our grammar onto the world:

Every language binds the thoughts of its speakers by the involuntary patterns of its grammar...Languages differ not only in how they build their sentences but in how they break down nature into the elements to put into those statements... For example, English terms, like "sky", "hill", "swamp", persuade us to regard some elusive aspects of nature's endless variety as a distinct thing, almost like a table or chair. Thus English and similar tongues lead us to think of the universe as a collection of detached objects of different sizes...Thus as goes our segmentation of the face of nature, so goes our physics of the cosmos....
Benjamin Lee Whorf (1950: 153)

Similarly Hilgartner and diRienzi (1995: 475) argue that this results in a presupposition of a fundamental dualism:

...we project the structure of our grammar onto the cosmos. And in so doing, we presuppose a fundamental dualism, with the cosmos divided into two parts: an immaterial or mystical side (verb-like, and suggested by terms such as "soul" or "spirit" or "mind"), contrasted against a material or physical or "real" side (noun-like, and suggested by terms such as "body" or "the physical" or "matter").

Putnam (1969), and Finkelstein (1969) both consider the question of whether traditional two-valued (Aristotelian) logic is a result of environmental conditioning, and is applicable only to a very large, but still limited range of macroscopic experience. Winch (1970) and Foucault (1970) both situate rationality in cultural practices and criticise the traditional assumption of an abstract and universal logic as a form of covert imperialism. Reichenbach (1951: 189) points out the full significance of departing from the Newton-Euclidean corpuscular model of nature, and the abandonment of the idea of corporeal substance:
With the corporeal substance goes the two-valued character of our language, and even the fundamentals of logic are shown to be the product of an adaptation to the simple environment into which human beings are born.

Riegel (1973) argues that Piagetian formal operational thought is linked to Aristotelian logic, whereas a post-formal operational thought would be linked to the pre-Socratic Greek philosophy and to the dialectical philosophies of Hegel and Marx. Campbell and Bickhard (1986: 109) present the argument that formal operational thinking restricts the thinker to an inadequate world view that has to be replaced by a dialectical world view. To appreciate quantum theory fully needs perhaps a post-formal world view or mode of thought.

3 Quantum reality

Physicist: ... and so we conclude an electron is a particle.
Philosopher: But you also claim an electron is a wave.
Physicist: Yes, it's also a wave.
Philosopher: But surely, not if it's a particle.
Physicist: We say it's both wave and particle.
Philosopher: But that's a contradiction, obviously.
Physicist: Are you then saying it's neither wave nor particle?
Philosopher: No. I'm asking what you mean by "it".

Reality is quantum mechanical. The quantum theory is probably the most successful theory in the history of science, yielding descriptions for all the fundamental forces of nature except gravity and accounts for phenomena ranging from starlight to the periodic table. It has also been responsible for technologies spanning nuclear reactors to lasers. However as the physicist Richard Feynman (1965: 129) in The Character of Physical Law famously remarked:

...after people read the paper a lot of people understood the theory of relativity in some way or other, certainly more than twelve. On the other hand, I think I can safely say that no one understands quantum mechanics.

Two immediate implications of the comment is that, firstly, there is something fundamentally different about quantum physics, and secondly it raises the question of what is meant by 'understanding'. The theory of relativity was in many ways a continuation of 'classical physics' - it is quantum physics that represents a new conceptual revolution (Selleri, 1990). In less than a century physics has abandoned a world view consisting of concepts that were mechanistic, deterministic and largely absolute, and espoused a world view comprising concepts that are relative, frequently non-deterministic and stochastic in nature (Lahti, 1990). Quantum theory has two characteristic features that distinguish it from classical physics. Firstly, quantisation, i.e. physical quantities are not allowed to take a continuous set of values. Secondly, it is not possible to predict the outcome of an individual measurement.

The basic ideas of quantum physics are not necessarily difficult as that they are strange. In some situations, electrons that are usually referred to as 'particles' may exhibit 'wave-like' behaviour. Electromagnetic radiation, known classically as a wave phenomena, is explained in terms of particles called photons. Both matter and radiation can be viewed as having a dual (wave-particle) nature. What are electrons really like? Are they like particles or waves? Are they like both particles and waves, or like neither? These questions
illustrate the psychological difficulties with which students are confronted when trying to incorporate the concepts of quantum physics into their over-all conceptual framework.

At present in England and Wales upper secondary school students (ages 16-18) wishing to read for a physical science degree at university will follow the two year Advanced Level Physics course. The quantum physics section of the syllabi for the various examining boards will typically not include the Heisenberg Uncertainty Principle, the Schrödinger wave equation, and there is no explicit mention of introducing students to conceptions of the 'nature of science'. At the heart of quantum physics at both A-level and generally lies the concept of 'wave-particle duality'. The concept of wave-particle duality also involves the idea of a paradigm change, from classical physics to quantum physics. The conceptual challenge of coming to terms with quantum physics was commented on by Einstein:

We know that light has certain characteristics which we designate for short, respectively, as undulatory and corpuscular. It has no meaning to say, it is a wave and it is a corpuscle. Up to now we just have no reasonable theory which explains all its characteristics. However there is no contradiction, any more that it signifies a contradiction that a man feels and has weight.

Albert Einstein (In Stachel, 1986:363)

What if the sphere of experience of students shifts to the conceptually strange domain of quantum physics? In the process of constructing a conceptual map of students' understanding of quantum physics this study is also investigating if dualistic thinking still lies underneath students' conceptions of quantum physics.

4 What is meant by 'understanding'?
This study is concerned with investigating students' understanding of quantum physics. McCubbin (1984: 67) expresses the twin problems that the acceptance of the importance of understanding gives rise to:

The case for promoting understanding as an explicit educational objective is a difficult one to deny. It is also a peculiarly difficult one to make, in practice, because of the twin problems of defining and assessing understanding.

Studies of student understanding in science tend to tacitly assume some meaning for the term. Understanding is generally accepted to be an active process in which meaning is constructed, with new information being interpreted with regard to currently activated knowledge (Bransford, 1979). As Carey (1986: 1123) expresses it:

To understand some new piece of information is to relate it to a mentally represented schema, to integrate it with already existing knowledge.

A concept is understood, ultimately, through its relations with other concepts (Sowa, 1983). A new concept therefore cannot be explicitly understood until it is linked in a meaningful way to pre-existing concepts (Ausubel, 1963; Gagné, 1985; Novak and Gowin, 1984). All associations, which would include images, expectations, emotions and sensory experience, add to concept meaning and understanding. A concept is the collection of memory elements that are associated with the label (e.g. the photon) and the pattern of their links. Two students' understanding of a particular concept is given by the similarity of their sets of elements, i.e. their concepts will be the same if they have identical sets of images, propositions, episodes and so forth about the label. A possession of a concept (e.g. the electron) is, therefore, not a dichotomy in the sense that the student either has it or has not. It is the elements that are possessed or not possessed, the concept can be held to a greater or lesser degree.
Concepts may be viewed as cognitive devices for classifying objects in an economical way. Meaning is attached to concepts and to the relationships between concepts, and the aim is for students to learn selected networks of meaning. As a consequence Lewis (1973) argues that knowledge in the human and social sciences needs to be seen as a network or 'string bag' rather than as a hierarchy.

White and Gunstone (1992) point out that a viewpoint which defines understanding as the ability to use knowledge and to cope with situations forms the basis of the use of problems in tests, and of transfer tasks in research, as measures of understanding. However, this definition and the tests are to do with overt performance, not with an internal state of mind.

Ausubel and Robinson (1969: 50) refer to two essential factors influencing meaningful learning or understanding:

...the most important factor influencing learning is the quantity, clarity and organization of the learner's present knowledge. This present knowledge, which consists of the facts, concepts, propositions, theories, and raw perceptual data that the learner has available to him at any point in time, is referred to as his cognitive structure...The second important focus is the nature of the material to be learned.

The definition of 'cognitive structure' referred to by Ausubel and Robinson is perhaps a description of the contents of cognitive structure. This definition of cognitive structure needs also to be augmented by White's (1988) suggestion that it should also make reference to the arrangement of knowledge.

This study (along with much of science education research) is making a number of assumptions that need to be made explicit:

(1) that concepts are in some way 'stored' or represented in a learner's brain,
(2) and that there is some form of organisation of these representations (i.e. we accept the existence of cognitive structure);
(3) that therefore the notion of two concepts being more or less closely linked, connected or integrated in cognitive structure is a meaningful and sensible one;
(4) that we do not have access to a learner's cognitive structure;
(5) that a learner's behaviour (statements, responses to questions etc.) may be considered to reflect aspects of her cognitive structure;
(6) that we may construct models to represent cognitive structure in terms such as the various conceptions that a learner holds, and how they appear to be inter-related.

The aim of this study is to try and go behind students' overt performance and describe the organisation of knowledge that underpins overt performance, and define understanding in terms of elements of memory and the pattern of association of these elements (White, 1985 and 1988).

The previous discussion has highlighted the difficulties of describing what could be meant by understanding. The word 'understanding' can have a continuum of meanings depending upon the context. This study has adopted an operational definition or limited 'measure' of understanding at the level of the population group in which understanding is represented by the relationships or groupings of students' ideas (or conceptions). It should be emphasised that the unit of analysis is taken as the population group, and not the individual. The research findings, therefore, reflect the tendencies of the group, and not necessarily the perceptions of individuals.
5 Representing understanding: The conceptual map

To represent the 'understanding' of the population sample required the construction of a 'conceptual map'. The 'metaphor of the map' is a powerful heuristic device to represent the psychological structure of knowledge in the area of quantum physics as perceived by the sample population of A-level physics students.

The general aim of this study is to arrive at a representation of the multidimensional virtual world of students' understanding of quantum physics through the construction of a 'common mental geography'. The generation of a map involves the construction of a bounded graphic representation that corresponds to a perceived reality. Robinson (1982: 1) points out that the act of mapping involves the 'combination of the reduction of reality and the construction of an analogical space', and enables structures to be constructed or discovered that would remain unknown if not mapped.

All maps are approximations and involve distortions of perceived reality, as they inherently involve the use of a projection. The map's intended intellectual function and the desired visual structure are used to determine which projection is most appropriate for a given application. A number of points about maps, however, need to be borne in mind:

(a) mapping and knowing are closely intertwined; (b) maps are excellent heuristic devices; (c) both the map maker and the map reader have important responsibilities to fulfil if communication is to occur; (d) every map reflects both its data and its designer; (e) changes in maps reflect changes in understanding; (f) the prior knowledge of the map maker can have a great influence on the maps he or she produces; (g) all maps distort reality, both because of the very nature of mapping and because map makers have learned how to exploit distortion to achieve their communicative goals; and (h) maps have great cognitive, integrative, summative, and generative power.

Wandersee (1990: 930)

This study is applying the heuristic metaphor of the map to construct graphic representations of a population group of A-level students' understanding of quantum physics, with understanding being represented by the relationships or groupings of students' conceptions. The reference frame being provided by the co-ordinate axes of the map. For instance, as in the diagram below:

The coordinate axes can be interpreted as perceptual dimensions. The dimensions are orthogonal, and their interpretation can be considered independently of each other. The labels given to the dimensions or axes of the map result from interpretations depending on the nature and location of specific conceptions. The post-modern self-consciousness of educational research emphasises that the process of interpretation is the result of an unavoidable interaction between the researcher and the researched.
The aim of this project was, therefore two-fold: to elicit students' conceptions, and investigate the relationships between conceptions.

6 Methodology
With regard to the implementation of this project there were two phases (implemented from May 1993 to May 1995). Phase 1 was concerned with identifying students' conceptions, and Phase 2 with identifying groupings of conceptions and any latent dimensions of thinking.

For Phase 1 the strategy adopted was that of using a series of three studies to elicit students' conceptions. Questionnaires utilising directed or free questions were used, and students encouraged to write freely in their own words. This approach enabled a considerable amount of significant data to be acquired in a relatively short time. The use of a questionnaire maximised the sample size. A large sample size enabled a wide range of students' writing, and consequently a wide spread of students' conceptions to be obtained. Since the study is concerned with understanding at the group population level it was important to obtain as much data as possible from as wide a range of students as practically possible. The usual technique of identifying students' conceptions via interviews with a small number of students was therefore not appropriate with regard to the research questions. The use of a reasonably large sample, and the emphasis on the confidentiality of respondents helps to validate the notion that they are replying honestly. The empirical work, therefore, involved using these studies to test the feasibility of the research, the likelihood of getting useful results, to develop methods for the analysis of data, and to elicit students' conceptions in the required domain area. Each study informed the subsequent study and gave a further insight into the nature of the research question, reflecting the fact that research is not a linear process. It should be borne in mind that the aim of Phase 1 of this study is confirmatory, in the sense of seeing if the conceptions held by the population sample are similar to conceptions identified in previous studies (see Fischler and Lichtfeldt, 1991, 1992; Niedderer, 1987; Niedderer, Bethge and Cassens, 1990; Mashhadi, 1993).

Phase 2 involved representing the conceptions elicited as specific statements in order to develop a structured questionnaire. The students responded to each statement on a 5-point ordinal response scale. The questionnaire, and the data analytical techniques were piloted, and then fully implemented in the final study with a sample population of 319 students (in eight schools and colleges). The final research instrument consisted of 54 statements representing students' conceptions of quantum phenomena, models, and the ontological and epistemological status of theoretical entities. This paper will report on the analysis of students' responses to statements on quantum phenomena (see Appendix A). The process of elicitation of students' conceptions, and the construction of statements is reported elsewhere (Mashhadi, 1995, 1996).

Multidimensional Scaling can be used to determine if there are any underlying structure or 'dimensions' to students' responses to the statements (see Child, 1970; Everitt and Dunn, 1983; O'Muircheartaigh and Payne, 1977). Cluster Analysis can be used to further define and help interpret any groupings. All three methods are used, as confidence in the results is enhanced if different techniques give similar results.
7 Interpretation

7.1 Underlying dimensions of thinking

The responses by the students were entered into an EXCEL spreadsheet, and the data converted into a proximity matrix. Since the grouping of statements is being investigated, not the grouping of students, the statements are treated as variables, and not the respondents. The Multidimensional Scaling program, ALSCAL, represents the structure in a proximity matrix by a geometrical model. A 3-dimensional solution or model is chosen through considerations of 'goodness-of-fit', parsimony and interpretability of the dimensions generated. The dimensions are orthogonal, and their interpretation can be considered independently of each other.

Figure 1 describes the location of statements on quantum phenomena located in the multidimensional space generated by MDS, and provides a plot of Dimension 2 versus Dimension 1.

![Diagram of Dimension 2 versus Dimension 1](image)

The greater distribution of statements along the horizontal Dimension 1 clearly indicates that its influence is greater than the vertical Dimension 2. Successive dimensions account for a smaller proportion of the variance. Overall Dimension 1 is the most influential, then Dimension 2, and Dimension 3 is the weakest.

For the horizontal Dimension 1 the statements at one end of the dimension refer to the definite nature or behaviour of entities (e.g. light is always a wave [B12], electrons are fixed in their shells [B45], and the electron is always a particle [B08])². At the opposite end of Dimension 1 the statements emphasise the indefinite nature of entities (e.g. labelling an electron as a particle or a wave depends on the nature of the experiment [B35], and electron clouds provide a probabilistic picture [B15])³. Dimension 1 is, therefore, interpreted as referring to the Definite to the Indefinite nature of entities.

---

² B12 Light energy always behaves as a wave.
B45 Electrons are fixed in their shells.
B08 The electron is always a particle.
³ B15 Electron clouds provide a probabilistic picture of the likelihood of finding an electron at a particular point.
B35 Whether one labels an electron a 'particle' or a 'wave' depends on the particular experiment being carried out.
For the 'weaker' vertical Dimension 2 the statements at one end of the dimension indicate a certainty in knowledge about the nature of an entity or certainty about a property or behaviour (e.g. electrons have a definite trajectory [B40] or the photon is a spherical entity [B51])\(^4\). The statements at the other end refer to uncertainty in knowledge - for instance, the position of an electron is not known accurately because of its high speed (B27) and the nature of light depends on the experiment (B20)\(^5\). Dimension 2 is, therefore, interpreted as ranging from Certainty to Uncertainty in knowledge about the nature of entities or their property and behaviour.

The Multidimensional Scaling program also generated a plot of Dimension 3 versus Dimension 1 (Figure 2).

Figure 2 Location of statements on quantum phenomena in 3-dimensional space (Dimension 3 v Dimension 1)

From Figure 2 the statements at one end of Dimension 3 are concerned with the visualisability of entities or their behaviour (e.g. an image of the electron [B02] and the planetary model of the atom [B01 and B31])\(^6\). The statements at the other end propose that an atom cannot be visualised (B10) or that electrons are waves (B18) (i.e. refer to non-visualisability)\(^7\). Dimension 3 is interpreted as ranging from Visualisability to Non-visualisability of behaviour and of entities.

The implication of the tentative interpretation of the model generated by ALSCAL is that the location of the statements is 'determined' by three latent dimensions: Definite to Indefinite nature of entity, Certainty to Uncertainty in knowledge of the nature of an

\(\text{\textsuperscript{4}}\) B40 During the emission of light from atoms, the electrons follow a definite path as they move from one energy level to another.

\(\text{\textsuperscript{5}}\) B51 The photon is a small, spherical entity.

\(\text{\textsuperscript{6}}\) B20 How one thinks of the nature of light depends on the experiment being carried out.

\(\text{\textsuperscript{7}}\) B27 Nobody knows the position accurately of an electron in orbit around the nucleus because it is very small, and moves very fast.

\(\text{\textsuperscript{6}}\) B31 Electrons move around the nucleus in (definite) orbits with a high velocity.

\(\text{\textsuperscript{7}}\) B02 It is possible to have a visual 'image' of an electron.

\(\text{\textsuperscript{7}}\) B01 The structure of the atom is similar to the way planets orbit the Sun.

\(\text{\textsuperscript{7}}\) B10 An atom cannot be visualised.

\(\text{\textsuperscript{7}}\) B18 Electrons are waves.
entity or of its behaviour, and Visualisability to Non-visualisability of behaviour or of entities.

The data was also subjected to Factor Analysis. The three dominant factors that account for the largest percentage of variance in students' responses to the statements were consistent with the interpretations of the three principal dimensions identified in the MDS model. Factor Analysis therefore supports the interpretation of the underlying dimensions identified using MDS.

The results indicate that there is an underlying structure to the responses given by the students, determined by three underlying dimensions: Definite-Indefinite, Certainty-Uncertainty, and Visualisable-Non-visualisable:

![Diagram of three-dimensional space showing axes for Definite, Certainty, and Indefinite; Visualisable and Non-visualisable dimensions.]

The dimensions constitute perceptual axes which are implicitly referred to by students in thinking about the behaviour or properties of quantum entities. For instance, in considering the question of how to come to terms with the concept of the electron or photon a number of questions are possibly implicitly posed by students. Does it have a definite or fixed nature? How certain is knowledge about its behaviour? Is it visualisable?
7.2 Clusters of ideas
Cluster Analysis indicated the groupings of statements or conceptions. The Cluster Analysis using the Complete Linkage method produced a dendogram showing how the statements cluster or group together (see Figure 3). Inspection of the dendogram suggested three broad groupings of statements.

Figure 3: Clusters of statements on quantum phenomena

Inspection of the clusters suggests that they are interpretable and internally consistent and coherent. The statements comprising Cluster 1, for instance, describe the 'quantum' behaviour of phenomena - the cluster is therefore labelled as Quantum (see Appendix B for a listing of the statements comprising each of the clusters). Cluster 3 consists of two sub-clusters in both of which the statements describe quantum phenomena in 'mechanistic' terms, and is therefore labelled a Mechanistic cluster. Similarly Cluster 2 consists of two sub-clusters which combine both 'quantum' and 'mechanistic' descriptions of phenomena (i.e. an Intermediate cluster).

In summary, the statements are located within these three perceptual dimensions or group psychological space, and grouped in three broad clusters: Mechanistic, Intermediate, and Quantum (see Figure 4).
8 Dualistic dimensions
An operational definition or limited 'measure' of understanding, at the level of the population group, was adopted in which understanding was represented by the relationships or groupings of ideas (conceptions). This gave rise to the use of the powerful heuristic device of adopting the 'metaphor of the map' to construct 'conceptual maps' to represent the holistic understanding of A-level Physics students, at the level of the population group, of concepts associated with quantum physics. Conceptual maps have been constructed of territory that has had few previous explorers.

The maps produced are approximations and involve distortions of perceived reality, as they inherently involve the use of a projection which constitutes a systematic reference frame (i.e. orthogonal dimensions). The use of a projection is necessary to communicate effectively. Distortion is in fact not only unavoidable, but necessary to allow the map reader to comprehend the meaning of the map. The map's intended intellectual function and the desired visual structure were used to determine which projection was most appropriate. The maps are also a reflection of both the data and the researcher's interpretation of the data. The conceptual maps generated have great cognitive, integrative, summative, and generative power. Figure 5 summarises the conceptual map produced.
The most obvious aspect of the dimensions is their dualistic nature (e.g. Definite versus Indefinite). The model generated by Multidimensional Scaling reflects the pattern of students' responses by the spatial distribution of statements. The fact that the location of the statements leads to a dualistic interpretation of the dimensions emphasises again the primacy of the metaphor of dualism in both science and society. Phenomena are being made sense of in terms of either/or, in terms of polarities. Either-or categories are presumably constructed not simply for convenience, but because by defining one pole as the negation of the other, it is asserted not only what is, but what it is not (i.e. A is defined by being not-B). Conceptions arise from the interpretation of perceptions. In other words the interpretation results from the unconsciously utilised conceptual grid that is being applied. The conceptual maps generated are inherently dualistic. The problem for trying to move from a classical to a quantum framework is that quantum physics is not necessarily inherently dualistic.

9 Additional points

9.1 Dimensions and complexity
In order to gain an insight into phenomena the physicist often copes with the complexity of the situation by using the technique of orthogonality or mutual independence. For instance, in considering the forces experienced by charges moving in magnetic fields or projectile motion vector analysis is used to investigate separately horizontal and vertical components of force or motion. In an analogous manner in order to gain an insight into students' thinking Multidimensional Scaling generated orthogonal dimensions. Each dimension could be considered independently of the other dimensions, and thereby reduce the complexity of the situation. The number of dimensions chosen was guided by statistical measures which indicated the 'goodness-of-fit' in order to obtain the best compromise between fit, parsimony and interpretability.

In responding to the large number of propositions, present in the final research instrument, describing a range of possible behaviours or phenomena a quite startling finding is the small number of underlying dimensions needed to model students' perceptions of quantum phenomena (e.g. Definite versus Indefinite, Certainty versus Uncertainty, Visualisable versus Non-visualisable etc.). This tends to indicate an unexpected level of simplicity in trying to come to terms with what is normally regarded as an incredibly complex phenomena. The implication is not necessarily that student thinking is imprisoned by these dimensions. However the dimensions may constitute reference axes which are implicitly referred to by students. For instance, in considering the question of how to come to terms with the concept of the electron a number of questions are possibly implicitly posed by
students. Does it have a definite or fixed nature? Is it visualisable? How certain is knowledge about its behaviour?

9.2 Range of statements
It could be argued that the range of statements presented to students to respond to determines what dimensions might emerge. Furthermore that additional important dimensions may have been overlooked or distorted. It was precisely for this reason that the statements were informed by the research literature but were primarily developed from an analysis of students' responses to open questions in a series of studies. The studies used open questions with reasonably large samples of students in order to obtain, at the end of a process of interpretative analysis, as wide a range of conceptions as possible.

9.3 Robustness of dimensions
In addition even though, as a result of the pilot study, the number of statements were reduced for the Main Study the same latent dimensions were obtained. For two reasonably large samples of students the same dimensions were identified, and are therefore reasonably robust.

9.4 Description at the level of the group
The description of the common features of the implicit thinking of A-level students has been carried out at the level of the group. It does not follow that each individual student clusters conceptions or has exactly the same dimensions as the group. If the analogy of a fluid is used the macroscopic description of the movement of the fluid will not be reflected by the microscopic motion of a particular molecule. The group conceptual maps constructed, however, can provide an insight into the possible thinking of an individual student.

9.5 Methodology
This project has utilised a quantitative methodology to provide a qualitative insight into students' understanding of complex phenomena. The study has abstracted from the data a hidden structure that results from some basic typology (using Cluster Analysis), and latent dimensions (using Multidimensional Scaling complemented by Principal Components Analysis). It should be pointed out that although quantitative methods were employed the aim was not to arrive at or build quantitative laws.

10 Conclusion
The conceptual map produced illustrates the dualistic dimensions of reasoning underlying students' understanding of quantum physics. A number of insights have been gained into a complex and entangled situation without sacrificing completely the complexity and richness of students' thinking. The construction of conceptual maps enabled what is ultimately a reductionist approach to present an holistic picture. Reality, however, does not necessarily have to be reduced to dichotomies. Virginia Woolf (1957: 114), for instance, expressed her conception of the world poetically:

> What is meant by 'reality'? It would seem to be something very erratic, very undependable - now to be found in a dusty road, now in a scrap of newspaper in the street, now in a daffodil in the sun. It overwhelms one walking home beneath the stars...Sometimes, too, it seems to dwell in shapes too far away for us to discern...But whatever it touches, it fixes and makes permanent. That is what remains over when the skin of the day has been cast into the hedge; that is what is left of past time and of our loves and hates.
11 References


### Appendix A: Statements on quantum phenomena

| Atom | B01 The structure of the atom is similar to the way planets orbit the Sun.  
|      | B04 The atom is stable due to a 'balance' between an attractive electric force and the movement of the electron.  
|      | B07 Coulomb's law, electromagnetism, and Newtonian mechanics cannot explain why atoms are stable.  
|      | B10 An atom cannot be visualised.  |
| Photon | B12 Light energy always behaves as a wave.  
|        | B03 The energy of an atom can have any value.  
|        | B51 The photon is a small, spherical entity.  
|        | B46 Light energy travels from a lamp to a zinc plate as a wave but is absorbed as a packet of energy or photon.  
|        | B17 The photon is a sort of 'energy particle'.  
|        | B39 A photon has no mass or charge.  
|        | B20 How one thinks of the nature of light depends on the experiment being carried out.  
|        | B23 The photon is a 'lump' of energy that is transferred to or from the electromagnetic field.  
|        | B28 It is possible for a single photon to constructively and destructively interfere with itself.  |
| Electron | B08 The electron is always a particle.  
|         | B13 In passing through a gap electrons continue to move along straight line paths.  
|         | B02 It is possible to have a visual 'image' of an electron.  
|         | B18 Electrons are waves.  
|         | B24 Electrons consist of smeared charge clouds which surround the nucleus.  
|         | B32 When a beam of electrons produces a diffraction pattern, it is because the electrons themselves are undergoing constructive and destructive interference.  
|         | B35 Whether one labels an electron a 'particle' or a 'wave' depends on the particular experiment being carried out.  
|         | B42 Individual electrons are fired towards a very narrow slit. On the other side is a photographic plate. What happens is that the electrons strike the plate one by one, and gradually build up a diffraction pattern.  
|         | B29 Since electrons are identical it is not possible to distinguish between them.  |
| Trajectory/ probability | B37 If a container has a few gas molecules in it, and we know their instantaneous positions and velocities then we can use Newtonian mechanics to predict exactly how they will behave as time goes by.  
|        | B31 Electrons move around the nucleus in (definite) orbits with a high velocity.  
|        | B27 Nobody knows the position accurately of an electron in orbit around the nucleus because it is very small, and moves very fast.  
|        | B40 During the emission of light from atoms, the electrons follow a definite path as they move from one energy level to another.  
|        | B45 Electrons are fixed in their shells.  
|        | B47 Orbits of electrons are not exactly determined.  
|        | B33 Electrons move randomly around the nucleus within a certain region or at a certain distance.  
|        | B21 Electrons move along wave orbits around the nucleus.  
|        | B43 It is not possible to continuously observe the motion of an electron.  
|        | B15 Electron clouds provide a probabilistic picture of the likelihood of finding an electron at a particular point.  
|        | B19 When an electron 'jumps' from a high orbital to a lower orbital, emitting a photon, the electron is not anywhere in between the two orbitals.  |
13 Appendix B: Cluster Analysis

Cluster 1 consists of the following statements:

B15 Electron clouds provide a probabilistic picture of the likelihood of finding an electron at a particular point.
B17 The photon is a sort of 'energy particle'.
B43 It is not possible to continuously observe the motion of an electron.
B20 How one thinks of the nature of light depends on the experiment being carried out.
B35 Whether one labels an electron a 'particle' or a 'wave' depends on the particular experiment being carried out.
B27 Nobody knows the position accurately of an electron in orbit around the nucleus because it is very small, and moves very fast.

Cluster 1 statements describe the 'quantum' behaviour of phenomena (Quantum).

Cluster 3 consists of two sub-clusters which comprise the following statements:

B18 Electrons are waves.
B37 If a container has a few gas molecules in it, and we know their instantaneous positions and velocities then we can use Newtonian mechanics to predict exactly how they will behave as time goes by.
B02 It is possible to have a visual 'image' of an electron.
B13 In passing through a gap electrons continue to move along straight line paths.
B51 The photon is a small, spherical entity.
B03 The energy of an atom can have any value.
B27 An atom cannot be visualised.

Cluster 3 consists of two sub-clusters in both of which the statements describe quantum phenomena in 'mechanistic' terms, and is therefore labelled a Mechanistic cluster.

Cluster 2 consists of two sub-clusters which comprise the following statements:

B04 The atom is stable due to a 'balance' between an attractive electric force and the movement of the electron.
B46 Light energy travels from a lamp to a zinc plate as a wave but is absorbed as a packet of energy or photon.
B32 When a beam of electrons produces a diffraction pattern, it is because the electrons themselves are undergoing constructive and destructive interference.
B21 Electrons move along wave orbits around the nucleus.
B23 The photon is a 'lump' of energy that is transferred to or from the electromagnetic field.
B24 Electrons consist of smeared charge clouds which surround the nucleus.
B07 Coulomb's law, electromagnetism, and Newtonian mechanics cannot explain why atoms are stable.
B19 When an electron 'jumps' from a high orbital to a lower orbital, emitting a photon, the electron is not anywhere in between the two orbitals.
B29 Since electrons are identical it is not possible to distinguish between them.
B33 Electrons move randomly around the nucleus within a certain region or at a certain distance.
B47 Orbits of electrons are not exactly determined.
B39 A photon has no mass or charge.

Cluster 2 consists of two sub-clusters which combine both 'quantum' and 'mechanistic' descriptions of phenomena (i.e. an Intermediate cluster).

BEST COPY AVAILABLE
III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

<table>
<thead>
<tr>
<th>Publisher/Distributor:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Address:</td>
</tr>
<tr>
<td>Price:</td>
</tr>
</tbody>
</table>

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

<table>
<thead>
<tr>
<th>Name:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Address:</td>
</tr>
</tbody>
</table>

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
1100 West Street, 2d Floor
Laurel, Maryland 20707-3598

Telephone: 301-497-4080
Toll Free: 800-799-3742
FAX: 301-953-0263
e-mail: ericfac@inet.ed.gov
WWW: http://ericfac.piccard.csc.com

(Rev. 6/96)