This study investigated how the adoption of a constructivist model of teaching and learning and simple item analysis techniques can be used to explore the instructor’s pedagogical content knowledge in teaching elementary statistics. Descriptive data (percent of students responding to multiple-choice test options) are provided that support the case for specific student statistical learning problems on the following topics: calculation and interpretation of measures of central tendency and variability, understanding of reliability and validity, interpretation of correlation coefficients, estimation of correlation coefficients from graphic scatterplots, and the selection of the best test-retest reliability scenarios. It is suggested that item analysis findings from multiple-choice examinations can be used to discover student conceptual misunderstandings, improve classroom instruction, and refine test-item writing. An attached table graphically displays the 10 findings. (Author/ND)
Statistical Content Errors for Students in an Educational Psychology Course

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Running head: Statistical Errors
Abstract

This study explored how the adoption of a constructivist model of teaching and learning and simple item analysis techniques can be used to explore pedagogical content knowledge (PCK) when teaching elementary statistics. Descriptive data (% of students responding to multiple-choice test options) are provided that support the case for specific student statistical learning problems on the following topics: calculation and interpretation of measures of central tendency and variability, understanding of reliability and validity, interpretation of correlation coefficients, estimation of correlation coefficients from graphic scatterplots, and the selection of the best test-retest reliability scenarios. It is suggested that item analysis findings from multiple-choice examinations can be used to discover student conceptual misunderstandings, improve classroom instruction, and refine test-item writing.
The structure of knowledge has long been thought of as an important key to understanding how students learn and how teachers can help students reason at higher levels of cognitive functioning. The cognitive revolution in psychology has rekindled interest in earlier forms of knowledge development such as: discovery learning, transfer of knowledge, and constructivist approaches to teaching and learning.

Bruner (1960) advocated a deeper understanding of the structure of knowledge through discovery learning in specific content areas for improving comprehension, recall, transfer, and reasoning. When teachers can help students understand how knowledge is organized such an organizational framework allows students to advance beyond levels of simply absorbing facts and move toward understanding concepts and principles and applying what they have learned. Since the structure of knowledge is radically different in academic content areas, each discipline must assume the challenge of identifying such structural elements based upon specific content knowledge.

Constructivist approaches to teaching and learning also assert that the structure makes a difference in learning and that students create their own knowledge through personal perceptual processes. Narode (1987) proposed that constructivist "concepts and their symbolic representations contain hidden epistemologies
which must be elucidated by education researchers and then communicated to educators and students" (p. 34).

Although constructivism could be thought of as including two versions (developmental and sociocultural), these ideas are often directly linked to cognitive theorists such as Jean Piaget. DeVries (1997) recently argued that it is inaccurate to assume that Piaget’s work only considered individualistic elements and outlined Piaget’s lesser known social theory. As Airasian and Walsh (1997) reminded us, constructivism is not a theory which offers us an easy or direct instructional application in the classroom and the adoption of such a theoretical view leads to serious issues and problems that must be confronted by educators.

The current interest in constructivism can also be seen as an integration of several different theoretical perspectives. For example, Herman (1995) has suggested that many of the goals of a constructivist approach to teaching and learning are very consistent with the humanistic education movement that rose to prominence in the 1970’s in terms of such concepts as freedom to learn, student-centered learning, facilitation of learning, search for personal meaning, and active involvement in learning.

More recently, Shulman in an interview format (see Shulman & Sparks, 1992) suggested that teachers need to enhance their pedagogical content knowledge (P.C.K.) in order to strive toward excellence in teaching. The P.C.K. element emphasizes the importance of domain specific content in the teaching and learning process and proposes, for example, that teaching poetry
is likely to be very different from teaching mathematics.

Shulman (1988) highlighted how an outstanding teacher's expertise can be distinguished from the knowledge of a subject matter specialist:

The teacher not only understands the content to be learned and understands it deeply, but comprehends which aspects of the content are crucial for future understanding of the subject and which are more peripheral and are less likely to impede learning if not fully grasped. The teacher comprehends which aspects of the content will likely pose the greatest difficulties for the pupil's understanding. The most crucial to learn is not always the most difficult; the most difficult is not always the most crucial. (p. 37)

The teaching of psychology at the college and university level could be advanced if professors thought of teaching as scholarship, critically examined data from their own courses, and shared their findings in research colloquiums which focused upon the sharing of psychological and pedagogical content knowledge. For example, What do we know about how students construct their knowledge of statistical concepts? What types of learning problems, misunderstandings, and points of confusion are they likely to encounter when learning statistics?

Rarely do teaching professors even consider subjecting their own teaching to the scrutiny of research even though such an investigation could offer valuable insights related to how students learn about crucial psychological content such as
statistics. Becker (1996) reviewed some 500 sources in the current literature on statistics teaching and concluded that such resources yielded primarily anecdotal evidence and recommendations based upon the experiences and intuitions of instructors (only 30% of the articles were empirical studies).

The present investigation was undertaken to explore some common errors, misunderstandings, and conceptual problems that were exhibited by students that were asked to apply basic statistical knowledge. Although students frequently experience considerable anxiety when learning statistics, very little research has systematically explored specific conceptual difficulties related to statistical knowledge which could be related to anxiety and poor student performance in this domain.

Method

Subjects

A total of 101 undergraduate students enrolled in three distinct sections of an educational psychology course offered by a Department of Psychology at a small, state university campus in up-state New York served as subjects. The sample was composed of primarily female subjects (75%).

Students over the two previous years had participated in a pilot study and helped the instructor field test and refine the course mastery materials and examinations. The investigator took detailed notes after classes and worked with students in focus groups to explore learning problems on the topic of statistics. Many students taking this course were preparing to become K-12
teachers and only 35% of students had previously taken a statistics course.

**Materials**

The classroom multiple-choice examination questions (four possible options per item) which dealt with statistical topics were subjected to item analysis techniques in order to further develop and test hypotheses related to student learning problems. Students were allowed to use a calculator on the exams. Specific items on two different multiple-choice examinations employed as the regular part of the course requirements were used to measure student progress in comprehending statistical concepts. The two exams differed in both length and comprehensive nature: Exam #3 (75 questions) and the Final Exam (100 questions). Kuder Richardson 21 estimates of reliability over several past exam administrations for subjects averaged .83.

Descriptive statistics for total scores on the two exams are provided in Table 1. Each exam included a different array of statistical test items: Exam #3: 20 items (27% of total exam items on this test) and Final Exam: 13 items (13% of total exam items on this test). Students at the Final Exam had the advantage of already taken similar test items on this content and clarifying mistakes made on Exam #3. All of the multiple-choice items used to evaluate the statistical concepts reported in this report were written by the course instructor/investigator.

It deserves to be noted that students were learning very basic statistical concepts such as: measures of central tendency,
characteristics of the normal distribution, reliability and validity, correlation coefficients, linear and non-linear relationships among variables, scatterplots, and test-retest reliability. The course content did not address inferential statistics or hypothesis testing.

Procedure

Item analysis techniques were used to examine student responses and determine problematic areas within the course content. Descriptive statistics (cumulative % of students marking particular options) were used to identify problematic concepts and relationships that were resistant to direct teaching and student learning. Students were expected to perform at a high level on exams due to the mastery nature of the course design where (1) all course/exam content was covered in the textbook or handouts, (2) all exam content was covered in class, and (3) all students were given parallel practice exams that included answers and detailed written explanations of the answers. In short, students had multiple ways to learn the statistical content covered on the exam and they knew exactly which statistical content they would confront on the exam.

Results

The distribution of student responses for nine exam items (5 items were from Exam #3 and 4 items were from the Final Exam) are depicted in Table 2 which exemplify the following statistical learning problems:
(1) Students often forget to rank order the scores when calculating the median and confuse the mean with the median.

(2) Students confuse measures of central tendency (mean, median, and mode) with measures of variability (range and standard deviation).

(3) Students have problems understanding that extreme outliers in a distribution require the use of the median due to the distortion of the mean under these circumstances.

(4) Students often confuse reliability and validity. They have difficulty conceptualizing that a valid measure is also a reliable measure; however, a reliable measure may or may not be a valid measure.

(5) Students have considerable difficulty realizing that the most optimal test-retest reliability coefficient value must be a positive value. A high negative correlation or a zero-order coefficient does not infer high reliability.

(6) Students become confused if asked to select the option with the most desirable psychometric characteristic from a choice between having high validity or high reliability.

(7) Students experience difficulty when interpreting correlation coefficients in terms of the strongest and weakest predictors when both positive and negative values are provided.

(8) Students demonstrate some degree of difficulty approximating what a scatterplot would look like for a specific correlation coefficient. They often confuse the + and -
directionality and degree of relationship elements.

(9) Students have great difficulty picking out a scatterplot which demonstrates high test-retest reliability.

(10) Students have considerable difficulty understanding that correlation coefficients only depict linear relationships while scatterplots can represent linear and non-linear relationships between variables.

The proportional loadings (% of students attracted to the correct options) for the nine sample items indicated various levels of item difficulty (n=9 correct answers; mean:66.33%; sd=11.18; range: 52%-83%). The proportional loadings for the 27 incorrect options (each item had three incorrect options) demonstrated that a wide range of subjects were attracted to these items (n=27 incorrect answers; mean=10.89%; sd=12.00; range: 0%-44%). Many of these loadings on incorrect options were small and inconsequential; however, eleven of the 27 loadings included 10% or more of the class (as many as 44% in one case) selecting an incorrect option. These popular loadings on incorrect responses were interpreted as key elements to understanding how large numbers of students misunderstood statistical concepts or became confused about the issue under scrutiny.

Discussion and Conclusion

The findings of this study make it clear that proportional responses to incorrect answers can offer a useful guide to uncovering misconceptions and misunderstandings. It is crucial
to make the empirical findings of a relatively large sample of students in a course drive the exploration of understanding how students conceptualize knowledge and how to make improvements in instructional design and evaluation techniques.

If college instructors need to teach these statistical concepts in courses, they may wish to offer students special instruction in class highlighting such problematic elements of learning elementary statistics. When instructors better understand how students think about statistical content, they can promote learning, improve instruction, and advance evaluation all at the same time based upon detailed descriptive analyses of exam results. Professors who teach more advanced statistics courses might also wish to make certain that students do not harbor confusion in these and other essential concepts before teaching more complex subject matter based upon such fundamental ideas.

Test items written around the ten problematic issues identified in this paper are likely to challenge students to think critically about these statistical concepts. The process of using item analysis results from classroom examinations to improve test item writing and class instruction is generalizable to all content areas of statistical knowledge.

This is but a humble beginning in the quest to better understand how constructivism and pedagogical content knowledge can better inform teaching, learning, and evaluation. Undoubtedly, many other conceptual problems baffle students as they rapidly attempt absorb course content and think critically
about subject matter. Professors should consider using their research skills to uncover such problems in student perceptions and sharing their findings with a scholarly community devoted to understanding student learning and outstanding teaching. Let this paper become one small step in promoting teaching as a unique form of scholarship, fostering critical thinking among students, and striving for excellence in teaching.
References


Table 1

**Descriptive Statistics for Total Exam Results**

<table>
<thead>
<tr>
<th>Exam #3</th>
<th>Final Examination</th>
</tr>
</thead>
<tbody>
<tr>
<td>n= 101</td>
<td>n= 100</td>
</tr>
<tr>
<td>M= 56.02 (75%)</td>
<td>M= 77.35 (77.35%)</td>
</tr>
<tr>
<td>sd= 8.67</td>
<td>sd= 13.42</td>
</tr>
<tr>
<td>range= 39</td>
<td>range= 58</td>
</tr>
</tbody>
</table>

Range of Scores:
32 (43%) - 71 (95%)

Range of Scores:
39 (39%) - 97 (97%)

Question Format:
75 multiple-choice items

Question Format:
100 multiple-choice items
Empirical Support for Findings Based Upon Question Item Analysis

Finding #1  Students often forget to rank order the scores when calculating the median and confuse the mean with the median.

Test Item from Exam #3
What is the median for the following set of scores?
91, 83, 78, 95, 88, 87, 80
a. 86  
b. 95  
c. 89  
d. 87

Finding #2  Students confuse measures of central tendency (mean, median, and mode) with measures of variability (range and standard deviation).

Finding #3  Students have problems understanding that extreme outliers in a distribution require the use of the median due to the distortion of the mean under these circumstances.

Test Item from Exam #3
Consider the task of selecting the best measure of central tendency for the following scores:
52, 65, 72, 44, 82, 234, 67, 77, 58, 62
Which measure of central tendency would most accurately describe the above data set?

a. mean  
b. median  
c. mode  
d. range
Finding #4 Students often confuse reliability and validity. They have difficulty conceptualizing that a valid measure is also a reliable measure; however, a reliable measure may or may not be a valid measure.

Test Item from Exam #3
Which of the following is true?

a. Validity is easier to determine and calculate than reliability.
b. A reliable instrument must also be valid.
c. Validity refers to consistency across testing situations.
d. Reliability is a necessary, but not sufficient condition for validity.

Finding #5 Students have considerable difficulty realizing that the most optimal test-retest reliability coefficient value must be a positive value. A high negative correlation or a zero-order coefficient does not infer high reliability.

Test Item from Final Exam
Which of the following correlation coefficients represents the best test-retest reliability?

a. $r=+.79$
b. $r=+.58$
c. $r=-.06$
d. $r=-.89$

Finding #6 Students become confused if asked to select the option with the most desirable psychometric characteristic from a choice between having high validity or high reliability.

Test Item from Final Exam
Which of the following would be considered the most valuable characteristic of a test from a psychometric perspective?

a. High validity
b. Low validity
c. High reliability
d. Low reliability
Finding #7  Students experience difficulty when interpreting correlation coefficients in terms of the strongest and weakest predictors when both positive and negative values are provided.

**Test Item from Final Exam**

Which of the following represents the strongest relationship between two variables?

- a. $r=+.84$
- b. $r=+.12$
- c. $r=-.35$
- d. $r=-.89$

Finding #8  Students demonstrate some degree of difficulty approximating what a scatterplot would look like for a specific correlation coefficient. They often confuse the + and - directionality and degree of relationship elements.

**Test Item from Exam #3**

Study the graph below and respond thoughtfully to the question.

Determine the nature of the relationship and estimate the value of the correlation coefficient statistic that is represented in the graph. Which of the following provides the best description of this relationship?

- a. positive relationship  $r=+.98$
- b. negative relationship  $r=-.53$
- c. zero-order relationship  $r=+.03$
- d. positive relationship  $r=+.57$
Finding #9  Students have great difficulty picking out a scatterplot which demonstrates high test-retest reliability.

Test Item from Final Exam
Which of the following represents the most desirable test-retest reliability?

![Graphs of scatterplots]

Finding #10  Students have considerable difficulty understanding that correlation coefficients only depict linear relationships while scatterplots can represent linear and non-linear relationships between variables.

Test Item from Exam #3
A scatterplot
a. can only display the relationship between two non-linear variables.
b. refers to a random depiction of the relationship between two variables.
c. can only display the relationship between two linear variables.
d. can depict linear and non-linear relationships between variables.

SPECIAL NOTES: The possible test item options labeled a, b, c, and d are coded as follows by the computer: a=1, b=2, c=3, and d=4. The correct answers are denoted by the dark bar graphs.
Title: STATISTICAL CONTENT ERRORS FOR STUDENTS IN AN EDUCATIONAL PSYCHOLOGY

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Corporate Source: NA

Publication Date: AUGUST 1997

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