Mathematics software can be a great aid in understanding difficult mathematics concepts at all levels. This paper presents nine exercises on calculus concepts by using different software used in mathematics education. Each exercise includes instruction on how to use software in order to highlight a specific concept in mathematics. This paper also presents a technology review by the comparison of ISETL, Derive, Geometer's Sketchpad, TI-82, TI-83, TI-85, TI-86, TI-92 and CBL along with discussing the advantages and disadvantages of each system. The mathematical concepts and software used in this paper include: (1) vertices of a triangle, midpoint formula, median of a side, equation of a line, and slope using the software "The Geometer's Sketchpad"; (2) writing the equation of a line using slope-intercept form and point-slope form and writing equations of parallel and perpendicular lines to a given line and passing through a given point using the software "The Geometer's Sketchpad"; (3) visualizing trigonometric identities for sine, cosine, and tangent functions using the software "The Geometer's Sketchpad"; (4) using equations and graphs to identify families of functions including linear, quadratic, and exponential using the software "Derive"; (5) functions, vertical line test and domain, using the software "Power Point"; (6) Pascal's triangle, binomial expansion, and pattern recognition using the software "Power Point"; (7) graphing factored polynomials of degree n>1 and solving higher degree inequalities by factoring and graphing using the software "Microsoft PowerPoint"; (8) review of quadratic functions and application problems using quadratic functions using the software "Microsoft PowerPoint and Microsoft Excel"; and (9) shifting graphs including horizontal, vertical, and combination graph shifts using the software "Derive". (ASK)
EXERCISE #1

Vertices of a Triangle
Midpoint Formula
Median of a Side
Equation of a Line
Slope
Software: The Geometer’s Sketchpad
EXERCISE #1

Vertices of a Triangle, Midpoint Formula, Median of a Side,
Equation of a Line, and Slope

Date: ____________________

Group Members: ____________________

Software Required:
The Geometer's Sketchpad

The student will learn how to use Sketchpad's coordinate system to measure coordinates and plot points. The student will also write the equation of a line and check that the coordinates of a given point satisfy the equation. The student will also need to review the midpoint formula before beginning this exercise.

PART I

Vertices of a Triangle, Midpoint Formula, Median of a Side:

1. Start with a new sketch. Draw a triangle in the first quadrant and find the coordinates of its vertices.
   - Use the segment tool to draw the triangle in the first quadrant.
   - Select the three vertices and choose Coordinates from the Measure menu.

Your sketch should look similar to the one below. The coordinates and a pair of axes also appear. Your coordinates should not match the example below.

A: (0.58, 0.71)
B: (2.00, 1.23)
C: (0.97, 1.36)

- Try dragging the vertices of the triangle and notice how the coordinates change.
2. Write down the coordinates of the points of the triangle that you have currently on the screen.

Point A (___, ___) Point B (___, ___) Point C (___, ___)

Use your calculator to compute the mean of the x-coordinates of the three points Show all of your work in the next few steps:

\[
\frac{x_A + x_B + x_C}{3} = \frac{___}{3} = \text{_______ (final answer)}
\]

Use your calculator to compute the mean of the y-coordinates of the three points Show all of your work in the next few steps:

\[
\frac{y_A + y_B + y_C}{3} = \frac{___}{3} = \text{_______ (final answer)}
\]

3. Verify your answer from step #2 by using Sketchpad:
   - Select the three coordinates by clicking on the summary of what the coordinates are for the vertices of the triangle. (Use shift-select to select all three points)
   - Choose Calculate from the Measure menu.
   - Start by pressing the **left open parenthesis** key. You want to make sure that the sum of the x-values is what you get in the numerator of the fraction.
   - Hold down the mouse pointer on the **Values** pop-up menu. Move the pointer to Point A, then move to the right to choose x in the cascading menu. The calculator display shows \( x_A \).
   - Press the + key (plus) on the calculator keypad.
   - Next choose \( x_B \) on the **Values** pop-up menu.
   - Press + key (plus) on the calculator keypad once again.
   - Choose \( x_C \) on the **Values** pop-up menu.
   - Press the **right open parenthesis**.
   - Press / and then 3 on the calculator keypad.
   - Press the OK button. The calculation should appear in the sketch window.

\[
\frac{x_A + x_B + x_C}{3} = \text{_______}
\]

4. Now repeat the above calculation for the mean of the y-coordinates. After following the steps, the calculation you needed should appear in the sketch window.

\[
\frac{y_A + y_B + y_C}{3} = \text{_______}
\]

5. Now you are going to plot another point by using the mean of the x-coordinates and mean of the y-coordinates.
Select on your screen the calculation that represents the mean of the x-coordinates, then use the Shift-select to select the calculation that represents the mean of the y-coordinates.

Choose Plot as \((x,y)\) from the Graph menu.

A new point appears in the middle of the triangle. Drag the vertices of the triangle and write below how the new point seems to respond:

6. The plotted point represents the centroid of the triangle. You should check this guess by drawing some of the median lines of the triangle.

- Consider segment AB.
- Use your calculator to find the midpoint of segment AB.

\[
\text{midpoint of segment AB} = \left( \frac{x_A + x_B}{2}, \frac{y_A + y_B}{2} \right)
\]

\[
= \left( \ , \ , \right)
\]

- Next, select the segment AB on your screen.
- Choose Midpoint from the Construct menu.
- Select point C and the midpoint of segment AB.
- Choose Line on your Toolbar (not segment - hold down the segment button until you see other options on the Toolbar) then select Line from the Construct menu.
- Notice that the median line appears to go through the plotted point no matter how you drag the triangle.
- Next, select the segment BC on your screen.
- Choose Midpoint from the Construct menu.
- Select point A and the midpoint of segment BC.
- Choose Segment from the Construct menu.
- Notice that the median line still appears to go through the plotted point.
- Label the plotted point Centroid
- Label the midpoint of the line segment AC by using the label M (Select the Rename Label in the Display menu and type in M)
PART II

Equation of a Line and Slope

1. Next you will find the equation for a median line for the triangle you left off with in Part I. You will determine whether the coordinates of the centroid satisfy the equation.

- Select a median line that you constructed from Part I.
- Choose Equation from the Measure menu. (Notice that if you drag the vertices of the triangle, the equation of the median will change dynamically.
- Select the mean of the x-coordinates and the equation of the line.
- Choose Calculate in the Measure menu.
- In the Value pop-up menu, choose the slope of the line.
- Press * key (asterisk) on the keypad.
- In the Value menu, choose the mean of the x-coordinates.
- Press + key (plus) on the keypad.
- In the Value pop-up menu, choose the intercept of the line.
- Press OK.

Here is an example of what your screen might look like now:

\[
\frac{y_A + y_B + y_C}{3} = 1.26
\]

The result of substituting a particular x into the equation of the line appears. Compare it with the mean of the y-coordinates. Drag the vertices of the triangle to check that equality of the two computations continues to hold true. You could use your calculator and prove that the medians of a triangle all go through a single point.
REVIEW QUESTIONS:

1. How do you get the coordinates of a point and the equation of a line?

2. How do you get the x-coordinate of a point in a calculation?

3. Given the computed coordinates of a point, how would you plot the point in a coordinate system?

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EXERCISE #2

Writing the Equation of a Line using Slope-Intercept Form and Point-Slope Form

Writing equations of Parallel and Perpendicular lines to a given line and passing through a given point.

Software: The Geometer’s Sketchpad
EXERCISE #2

Writing the Equation of a Line using Slope-Intercept Form and Point-Slope Form

and

Parallel Lines and Perpendicular Lines to a Fixed Line and a Given Point

Date: ________________

Group Members: ___________________________

Software Required:
The Geometer's Sketchpad

The student will use Sketchpad’s coordinate system to plot two points and then write the equation of the line passing through the two points. The student will also write the equation of a line using the slope-intercept form and the point-slope form of a line. The student will also need to review the slope-intercept form and the point-slope form of a line as well as concepts related to parallel and perpendicular lines to a given line and a point.

Plotting Two Given Points

1. Start with a new sketch. Plot the points (3, -1) and (2, 1) on a coordinate system.
   - Select Create Axes from the Graph menu.
   - Select the point at (1,0) and drag it left to change the scale on your axes.
   - Select Plot Points from the Graph menu.
   - Type in the window that appears the two points listed above: (3, -1) and (2, 1).
   - Click on the Line tool in the Toolbar (hold down the Segment tool until you see the option of drawing a line)
   - Select Line in the Construct menu.
   - Label each point by selecting Relabel objects in the Display menu.
     Type in Q (3, -1) for the appropriate point and R (2, 1) for the coordinates of the other point.
   - Select the point where the line intersects the y-axis. Select Coordinates from the Measure menu. How will you be sure that it is the y-intercept?

Your sketch should look similar to the one on the next page.
2. Write down the coordinates of the points and use the Point-Slope Form to write the equation of the line shown above. **SHOW ALL OF YOUR WORK.**

   a) Use the slope formula you have learned to find the slope of the line.

   \[
   \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}
   \]

   b) Use one of the points Q or R that you plotted and the slope of the line to write the equation of the line using the Point-Slope Form of a line.

\[
\text{Write your equation in standard form: } Ax + By = C
\]

Write it on the line below:

*Equation of the line* = _____________________________
3. Write down the y-intercept of the line and also the slope you found in step #2 to write the equation of the line using the slope-intercept form of a line.

SHOW ALL OF YOUR WORK.

a) Use the slope you found in step #2: slope: ________

b) Write down the y-intercept of the line that you have found in Sketchpad.

y-intercept: ________

c) Use the slope-intercept form of a line to write the equation of the line passing through Q and R.

Slope-Intercept Form of the line = __________________________

4. Verify your answer from step #2 by using Sketchpad:

- Select the line passing through the two points (3, -1) and (2, 1).
- Select Slope from the Measure menu. Write the answer below:

  slope: ________

Write your slope answer from step #2. ________

How do they compare?

- Select Equation Form from the Graph menu.
- Select Standard form from the options.
- Select Equation from the Measure menu. Write the answer below:

  standard form of the line:
  __________________________

  Write the standard form of the line you wrote in Step #2:
  __________________________

  How do the two answers compare?
5. Verify your answer from step #3 by using Sketchpad:
   - Once again, select the line passing through the two points (3, -1) and (2, 1).
   - Select **Slope** from the **Measure** menu. Write the answer below:
     
     \[
     \text{slope: } \quad \text{__________}
     \]
   - Select **Equation Form** from the **Graph** menu.
   - Select **Slope-Intercept** form from the options.
   - Select **Equation** from the **Measure** menu. Write the answer below:
     
     \[
     \text{slope-intercept form of the line:}
     \]

     \[
     \text{______________________}
     \]
   - Write your answer from step #3.
     
     \[
     \text{slope-intercept form of the line:}
     \]

     \[
     \text{______________________}
     \]
   - How do the two answers compare?
PART II

Equation of a Line Parallel to a given line and a given point.

1. Next you will find the equation for a line parallel to a given line and a given point.
   - Select the line QR that you have on the screen.
   - Plot the point (-1,4).
   - Select the point and the line.
   - Select Parallel Line from the Construct menu.
   - Find the slope of this new line by selecting Slope in the Measure menu.
     \[ \text{Slope} = \underline{\text{______}} \]
   - How does it compare to the slope of the line passing through Q and R?

2. Use the point you plotted and the slope of the new line to find the equation of the new line. SHOW ALL YOUR WORK.
   Write the equation of the line in slope-intercept form.

3. Verify your answer from step #2.
   - Select the new line.
   - Select Equation Form in the Graph menu. Choose slope-intercept form.
   - Select Equation in the Measure menu.
   Write the equation of the line in slope-intercept form that you obtained from Sketchpad:

   How do the two answers compare?

4. Next you will find the equation for a line perpendicular to a given line and a given point.
   - Select the line QR that you have on the screen.
   - Plot the point (2,-3)
   - Select the point and the line.
   - Select Perpendicular Line from the Construct menu.
   - Find the slope of this new line by selecting Slope in the Measure menu.
     \[ \text{Slope} = \underline{\text{______}} \]
   - How does it compare to the slope of the line passing through Q and R?
5. Use the point you plotted and the slope of the new perpendicular line to find the equation of the new line. **SHOW ALL YOUR WORK.**

Write the equation of the line in slope-intercept form.

6. Verify your answer from step #4.
   - Select the new line.
   - Select **Equation Form** in the **Graph** menu. Choose slope-intercept form.
   - Select **Equation** in the **Measure** menu.

Write the equation of the line in slope-intercept form that you obtained from Sketchpad:

How do the two answers compare?

**Review Questions concerning Sketchpad:**

1. How do you plot a given point?

2. How do you get the slope of a line?

3. How do you get the equation of a line to appear on the screen?

4. Given a fixed point and a line, how do you construct a line perpendicular to the given line.

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EXERCISE #3

Visualizing Trigonometric Identities for Sine, Cosine, and Tangent Functions

Software: The Geometer's Sketchpad
EXERCISE #3
Constructing Trigonometric Identities for Sine, Cosine, and Tangent

Date: ____________________  Group Members: ____________________

Software Required:
The Geometer's Sketchpad

The student will learn how to construct a perpendicular line, measure angles, and compute ratios on the Sketchpad. The student will use these features in order to construct a right triangle and measure the ratio of the lengths of the sides in order to visualize trigonometric identities for the sine, cosine, and tangent functions.

PART 1
Measurement of Angles and Sides of a Right Triangle to Explore the Sine Identity as a Ratio of Opposite over Hypotenuse:

1. Open the file “Sine1” in The Geometer’s Sketchpad. Your screen should contain the following sketch.

2. Use Sketchpad to measure the following:
   - angle BAC
   - the length of side AB
   - the length of side AC
   - the length of side BC
Your sketch should now contain all of the following data:

- $m\angle BAC = 35^\circ$
- $m\overline{BA} = 0.86$ inches
- $m\overline{CA} = 1.05$ inches
- $m\overline{BC} = 0.60$ inches

3. Now use Sketchpad to Calculate the following:
   - The ratio of the length of the opposite side to angle $BAC$ to the length of the hypotenuse
   - The sine of angle $BAC$

Your sketch should now contain all of the following data:

- $m\overline{BC} / m\overline{CA} = 0.57$
- $\sin(m \angle BAC) = 0.57$
4. Now grab point A and begin to move it to answer the following questions.

a. What do you notice about the length of the sides of the triangle as you move point A?

b. What do you notice about the measurement of Angle BAC as you move point A?

c. What do you notice about the ratio of length of the opposite side to the length of the hypotenuse as you move angle A?

d. Does the ratio of the lengths of the given sides change as the length of the sides change?

e. What do you notice about the sine of angle BAC as you move point?

f. What do you notice about the sine of angle BAC and the ratio of the length of the opposite side to the length of the hypotenuse for angle BAC?

5. What can you tell me about the sine of an angle?
PART II

Exploration of the Cosine and Tangent Functions

1. Delete the ratio and sine off of your sketch for Part I. (or open "sine2" in the Sketchpad if you do not have a sketch from Part I). Your sketch should look like the following:

\[ m\angle BAC = 35^\circ \]
\[ m\overrightarrow{BA} = 0.86 \text{ inches} \]
\[ m\overrightarrow{CA} = 1.05 \text{ inches} \]
\[ m\overrightarrow{BC} = 0.60 \text{ inches} \]

2. Now use Sketchpad to Calculate the following:
   - The ratio of the length of the adjacent side to angle BAC to the length of the hypotenuse
   - The cosine of angle BAC

Your sketch should now contain all of the following data:

\[ m\angle BAC = 35^\circ \]
\[ \frac{m\overrightarrow{BA}}{m\overrightarrow{CA}} = 0.82 \]
\[ \cos (m\angle BAC) = 0.82 \]
3. Now grab point A and begin to move it to answer the following questions.

a. What do you notice about the length of the sides of the triangle as you move point A?

b. What do you notice about the measurement of Angle BAC as you move point A?

c. What do you notice about the ratio of length of the adjacent side to the length of the hypotenuse as you move angle A?

d. Does the ratio of the lengths of the given sides change as the length of the sides change?

e. What do you notice about the sine of angle BAC as you move point A?

f. What do you notice about the cosine of angle BAC and the ratio of the length of the adjacent side to the length of the hypotenuse for angle BAC?

4. What can you tell me about the cosine of an angle?
5. Repeat the above set of exercises for the ratio of the length of the side opposite angle BAC to the length of the side adjacent to angle BAC. This time calculate the tangent of angle BAC rather than the cosine.

6. What can you tell me about the tangent of an angle?

JUST FOR FUN What do you think SOH - CAH - TOA stands for?
EXERCISE #4

Using Equations and Graphs to Identify Families of Functions Including: Linear, Quadratic, and Exponential

Software: Derive
EXERCISE #4

Using Equations and Graphs to Identify Families of Functions Including: Linear, Quadratic, and Exponential

Date: ____________________  
Group Members: ____________________

Software Required:  
Derive

The student will develop the concept of qualities that are similar within each family of functions. First, the student will classify a set of equations by similarities they choose. Next, Derive will be used to graph each equation so that the graphs can be classified into groups. Finally, the student will match each equation to its graph and classify the pairs into similar groups.

PART 1

Linear, Quadratic, and Exponential Equations and Graphs

1. Linear, Quadratic, and Exponential Equations and Graphs

\[ f(x) = 2x^2 - 3x + 1 \quad g(x) = 5 - 9x \]

\[ s(t) = -5t^2 + 7t \quad h(x) = -2x^2 - 5 \quad f(x) = 3^x \]

\[ f(x) = 6^{|x|} - 7 \quad f(x) = 1 - 2^x \quad h(t) = (t + 1)(t - 1) \]

\[ g(t) = -9t + 5 \quad f(x) = 6x + 8 \quad g(x) = (-6)^{x-3} \]

\[ h(t) = 5t^2 - 7t - 1 \quad s(t) = 4^t - 2 \]

How did you choose the three groups you use?
2. Open Derive and sketch each of the prior functions in the space provided.
3. Classify each of your graphs into 3 groups in the space provided.

How did you choose the three groups?

4. Do the three groups of equations match the three groups of graphs? If not, put each equation with its graph and break the pairs into three groups where the equations and the graphs all have qualities in common within each group.

Share your results with the class and determine three names for families of functions that fit the given functions.
PART II

Practicing Classification of Functions by Equation and Graph.

1. Classify each of the following equations and/or graphs as either:
   - Linear
   - Quadratic
   - Exponential

Give a reason for each of your answers.

\[ f(x) = x^2 + 3x \]

\[ h(t) = (t + 1)(t + 5) \]

\[ s(t) = 5 + 9^x \]

\[ f(x) = 1 \]

\[ f(x) = 2^{3x-5} \]

\[ h(x) = 9 - x^2 \]

\[ g(x) = -3x + 1 \]
EXERCISE #5

Functions:
Vertical Line Test
And Domain
Software: Power Point
EXERCISE #5

Functions: Vertical Line Test and Domain

Date: ___________________  Group Members: ___________________

Software Required:  
Power Point

The student will learn how to identify a function by looking at the graph of equations. The student will also practice identifying the domain of various functions.

PART 1

Functions: Vertical Line Test

1. Open the PowerPoint file titled “functions” and read slides #1-3 to answer the following questions.
   - What is a function?
   - How can you identify a function from the graph of an equation?

2. Follow the directions for slides #4-15 and answer the following questions.
   - Is this the graph of a function?

Why or Why not?
3. Is this the graph of a function?

Why or Why not?

4. Is this the graph of a function?

Why or why not?
5. Is this the graph of a function?

Why or why not?

6. Is this the graph of a function?

Why or why not?
PART II

Domain of a Function

1. Continue exploring PowerPoint file “functions”, looking at slides #16-32 to answer the following questions.

   - What is one restriction on the domain of a function?

   - What is the domain of this function?
     
     $f(x) = \frac{x^2 + 1}{x - 2}$

   - What is the domain of this function?
     
     $f(x) = \frac{x^2 + 2x - 1}{x(x + 3)(x - 4)}$

2. What is another restriction on the domain of a function?

   - What is the domain of this function?
     
     $g(x) = \sqrt{2x + 3}$
- What is the domain of the function $f(x) = x - 2$? $f(x) = 4x - 7$? $f(x) = (x + 5)(x - 2)$?

Why?

- What is the domain of this function?

$$s(t) = \frac{t + 4}{t^2 - 9}$$

- What is the domain of this function?

$$f(x) = x^3 - 2x + 9$$

- What is the domain of this function?

$$g(x) = 2 - \sqrt{3x}$$
A function is a rule that assigns a single output to each input.

Definition: A relation that assigns to each member of its domain exactly one member, its range.

Vertical line test: If it is possible for a vertical line to intersect a graph more than once, the graph is not the graph of a function.

This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.

This example is not a function because if you draw a vertical line anywhere near the middle of the graph, it will cross more than once.

Practice: Is this the graph of a function?
This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.

Practice: Is this the graph of a function?

This example is not a function because if you draw a vertical line anywhere, it will cross the graph more than one time, near the middle.

Practice: Is this the graph of a function?

This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.

Practice: Is this the graph of a function?
This example is a function because no matter where you draw a vertical line it crosses the graph no more than 1 time.

This example is not a function because if you draw a vertical line anywhere on the right side of the graph, it will cross more than one time.

Examples

- The domain of this function is the set of all real numbers not equal to 3.

\[ f(x) = \frac{x + 7}{x - 3} \]

What is the domain of \( f(x) \)?

- The Denominator cannot equal zero. So the domain is made up of all real numbers that will not make the denominator equal to zero.

\[ f(x) = \frac{x^2 + 1}{x - 2} \]
Answer

- The domain of the prior function is the set of all real numbers not equal to 2.

Another way to write the answer is:

$$\{ x \in \mathbb{R} : x \neq 2 \}$$

Another example of a function and the domain.

$$f(x) = \frac{x^3 - 2x}{x^2 - 1} = \frac{x(x^2 - 2)}{(x+1)(x-1)}$$

domain

$$\{ x \in \mathbb{R} : x \neq 1, -1 \}$$

Practice: What is the domain of the following function?

$$f(x) = \frac{x^2 + 2x - 1}{x(x+3)(x-4)}$$

Your answers should be:

- The set of all real numbers not equal to 0, 4, and -3.

$$\{ x \in \mathbb{R} : x \neq 0, 4, -3 \}$$

Domain

- An even radicand must be greater than or equal to zero. In other words, an even radicand can never be negative.
The following function is a square root function. Because square root is even, the part of the function under the square root sign must be greater than or equal to zero.

\[ f(x) = \sqrt{x - 4} \]

To find the domain of a function that has an even radicand, set the part under the radical greater than or equal to 0 and solve for \( x \).

\[ x - 4 \geq 0 \]

\[ x \geq 4 \]

Therefore, the answer is all real numbers \( x \geq 4 \).

What is the domain of \( f(x) = x - 2 \)?

What is the domain of \( g(x) = 4x + 7 \)?

What is the domain of \( s(t) = (x + 5)(x - 2) \)?

The solution is all real numbers for each of the prior three examples because there are no denominators and no radicals.
In the first problem you have to factor the denominator to see when it will equal zero. The second function has no fractions (denominator) and no radicals so the answer is all real numbers. In the last problem you must set the 3x that is under the radical sign greater than or equal to zero and solve for x by dividing by 3 on both sides.

Practice: What are the domains of the following functions?

1. $x(t) = \frac{t + 4}{t^2 - 9}$
2. $f(x) = x^2 - 2x + 9$
3. $g(x) = 2 - \sqrt{3x}$

Your answers should be:

1. $\{t \in \mathbb{R} : t \neq \pm 3\}$
2. All real numbers
3. $\{x \in \mathbb{R} : x \geq 0\}$

Reason for the last three answers.

- In the first problem you have to factor the denominator to see when it will equal zero.
- The second function has no fractions (denominator) and no radicals so the answer is all real numbers.
- In the last problem you must set the 3x that is under the radical sign greater than or equal to zero and solve for x by dividing by 3 on both sides.
EXERCISE #6

Pascal’s Triangle, Binomial Expansion, and Pattern Recognition
Software: Power Point
EXERCISE #6

Pascal's Triangle, Binomial Expansion, and Pattern Recognition

Date: ___________________________ Group Members: ___________________________

Software Required:
Power Point

The student will explore the relationship between Pascal's triangle and binomial expansion. These ideas will be used to recognize patterns that lead to infinite sequences and series.

PART 1

Pascal's Triangle

1. Open the PowerPoint file titled “Pascal” and read slides #1-9 to answer the following questions.
   - What is Pascal's triangle?
   - How can Pascal's triangle be used for expanding binomials?
   - What is another method for expanding binomials?

2. Expand the following binomials.

   \[ f(x) = (x + 1)^0 \quad f(x) = (x + 2)^2 \]
\[ f(x) = (x + y)^3 \quad f(x) = (y - 1)^3 \]
\[ f(x) = (2y + 2)^4 \]
PART II

Pattern Recognition and Area of a Triangle

This activity explores some of the number patterns found in the Sierpinski triangle. Conclude the Power Point file “Pascal”.

Directions: The first four stages of the construction of the Sierpinski triangle are shown below. In subsequent stages, the subdivision continues into smaller and smaller triangles. Use these figures to explore number patterns that emerge as the Sierpinski triangle is developed through successive iterations.

1. Count the number of shaded triangles at each stage 0 through 4.

2. Extend the pattern to predict the number of triangles at stage 5. What constant multiplier can be used to go from one stage to the next?

3. Generalize to find the number of triangles for level $n$. As $n$ becomes large without bound, what happens to the number of triangles?
AREA OF TRIANGLES

4. Let the area at stage 0 be 1. Find the total shaded areas at stage 1 through 4.

5. Extend the pattern to predict the total area at stage 5. What constant multiplier can be used to go from one stage to the next?

6. Generalize to find the total area at stage \( n \). As \( n \) becomes large without bound, what happens to the shaded area?
Patterns and Fractals

In a certain sense, an acorn is the whole of the tree that grows from it. The essential features of the tree derive from the code carried in the cells of the acorn. However, some plants exhibit dependency upon their parts in an even more graphic manner. For example, a single stalk of broccoli will be a replication of the entire bunch while a single florette contains the same repeating pattern.

This repeating pattern can be found in many places such as a seashell.

Binomial Expansion

Pascal’s triangle is commonly seen as a triangular array of numerical coefficients in the binomial expansion where the exponent increases through the whole numbers from 0 to n. This triangular array of numbers offers a rich setting for studying both numerical and geometric patterns.
A simple iterative algorithm for generating an entry in the numerical array of numbers in Pascal's triangle is to add the two numbers in the level above it. Unfortunately, the numbers imbedded deeply within the triangle are very large, and this ultimately makes the numerical iteration process increasingly laborious.

Here are the expansions from rows 1 and 2 on Pascal's triangle.

```
1
1 1
1 2 1
1 3 3 1
```

Here are the expansions from rows 3-5 on Pascal's triangle.

```
expand((x + y)^2)  x^2 + 2*x*y + y^2
expand((x + y)^3)  x^3 + 3*x^2*y + 3*x*y^2 + y^3
expand((x + y)^4)  x^4 + 4*x^3*y + 6*x^2*y^2 + 4*x*y^3 + y^4
```

Number Patterns and Variations

- Many number patterns can be explored using the Sierpinski triangle.
- The Sierpinski triangle is created through a process that connects the midpoints of each side of a triangle to divide each triangle into 4 equivalent triangles. This process is repeated within each new triangle.

SIERPINSKI TRIANGLE

BEST COPY AVAILABLE
EXERCISE #7

Graphing Factored Polynomials of Degree $n > 1$

and

Solving Higher Degree Inequalities By Factoring and Graphing

Software:
Microsoft PowerPoint
EXERCISE #7

Graphing Factored Polynomials of Degree $n > 1$ and Solving Factored Inequalities in One Variable using Graphing Techniques learned in Part I.

Date: ___________________  Group Members: ___________________

Software Required:
PowerPoint

The student will learn how to find real zeros of a factored polynomial function and their multiplicities. The student will also learn to sketch factored polynomials and then use the graphing techniques to solve factored inequalities in Part II.

PART I

Graphing Factored Polynomials:

1. Open the PowerPoint file titled “POLYN” and read slides #1-3 to answer the following question.

Which graphs below are “continuous” functions?

A

B

C

D
2. Read slides #4-11 and answer the following questions from slide #11:

Use the Leading Coefficient Test to determine the behavior of the graphs below when \( x \to \infty \) and \( x \to -\infty \).

a) \( f(x) = x^3+2 \)

as \( x \to \infty \) \( f(x) \to \) __________

as \( x \to -\infty \) \( f(x) \to \) __________

b) \( f(x) = 3x^4+2x-5 \)

as \( x \to \infty \) \( f(x) \to \) __________

as \( x \to -\infty \) \( f(x) \to \) __________

c) \( f(x) = -2x^3-5x+1 \)

as \( x \to \infty \) \( f(x) \to \) __________

as \( x \to -\infty \) \( f(x) \to \) __________

d) \( f(x) = 2x^5-x \)

as \( x \to \infty \) \( f(x) \to \) __________

as \( x \to -\infty \) \( f(x) \to \) __________

e) \( f(x) = -(x+2)(x-3)^2(x+4)^2 \)

as \( x \to \infty \) \( f(x) \to \) __________

as \( x \to -\infty \) \( f(x) \to \) __________

3. Read slides #12-17 and answer the following questions:

Find each real zero and its multiplicity for the polynomial functions. (Be sure to factor each polynomial before you find its zeros)

a) \( f(x) = x(x-3)(x+2) \)

zeros multiplicity

b) \( g(x) = x^5-5x^3+4x \)

zeros multiplicity

c) \( h(x) = 3x^2(x-1)(x-1) \)

zeros multiplicity

d) \( p(x) = 7x^3-4x^2 \)

zeros multiplicity

e) \( r(x) = -2x(x-2)^3(x+4)(x-5) \)

zeros multiplicity
4. Read slides #18-23 and answer the following questions:

Use the sketching techniques described in the slides to sketch the following polynomial functions:

a) \( f(x) = (x-3)(x+2)^3(x-4) \)

b) \( f(x) = -4x(x+2)(x-5)^2(x-3) \)

c) \( f(x) = -0.5x^2(x+3)(x-1)^2(x-2) \)

d) \( h(x) = -2x^3(x-3)^2 \)
e) $f(x) = 0.5x^2(x-2)(x+3)$
Shown above is the graph of a polynomial function.

a) Is the degree of the polynomial even or odd?

b) Is the leading coefficient positive or negative?

c) Why is $x^2$ a factor of the polynomial?

d) What is the minimum degree of the polynomial?

e) Formulate three different polynomials whose graphs could look like the one shown above.

Compare yours to other group members. What do you see?
PART II

Solving Factored Inequalities in One Variable

1. Read slides #24-30 and answer the following questions:

   a) Solve: 
      \[-x(x-3)(x+2) > 0\]

   b) Solve: 
      \[x^5 - 10x^4 + 9x < 0\]

   c) Solve: 
      \[-3x^2(x-1)(x+1) > 0\]

   d) Solve: 
      \[x^2(x-3)^2(x+1)(x+2) < 0\]
2. Answer question #1 again if the inequalities contain either ≤ or ≥ in the problem:

a) Solve: \(-x(x-3)(x+2) \geq 0\)  
   answer: __________________

b) Solve: \(x^5 - 10x^4 + 9x \leq 0\)  
   answer: __________________

c) Solve: \(-3x^2(x-1)(x-1) \geq 0\)  
   answer: __________________

d) Solve: \(x^2(x-3)^2(x+1)(x+2) \leq 0\)  
   answer: __________________

e) Solve: \(5x^4 - 2x^3 \geq 0\)  
   answer: __________________
The graph of a polynomial function is a smooth curve. (We say continuous and you will study this concept in great detail in calculus). This means that the graph of a polynomial function has no breaks or sharp turns.

Consider the graphs below:

Here are some more polynomial functions so that n is even:

A: \( f(x) = x^4 + 1 \)  
B: \( g(x) = (x-3)^4 \)  
C: \( h(x) = (x+2)^2 \)

Let's consider the graph of \( f(x) = x^n \) and focus on polynomial functions with degree > 1. Suppose n is even and observe some simple graphs:

\#1: \( f(x) = x^2 \)  
\#2: \( f(x) = x^4 \)  
\#3: \( f(x) = x^6 \)

Notice that if n is even, the graph of \( x^n \) touches the x-axis at the x-intercept. Also notice that when n is even, the graph is similar to the graph of \( f(x) = x^2 \).

Here are some more polynomial functions so that n is even:

A: \( f(x) = x^4 + 1 \)  
B: \( g(x) = (x-3)^4 \)  
C: \( h(x) = (x+2)^2 \)

Let's consider n is odd and observe some simple graphs:

Since we are considering the graph of \( f(x) = x^n \), let's consider n is odd and observe some simple graphs:

Notice that if n is odd, the graph of \( x^n \) crosses the x-axis at the x-intercept. Also notice that when n is odd, the graph is similar to the graph of \( f(x) = x^3 \)
Here are some more polynomial functions so that \( n \) is odd.

A: \( f(x) = -x^3 \)
B: \( g(x) = x^3 + 2 \)
C: \( h(x) = (x-3)^7 \)

1. When \( n \) is even and the leading coefficient \( a_n > 0 \)
   \( f(x) \to \infty \) as \( x \to \infty \)
   \( f(x) \to \infty \) as \( x \to -\infty \)

2. When \( n \) is even and the leading coefficient \( a_n < 0 \)
   \( f(x) \to -\infty \) as \( x \to \infty \)
   \( f(x) \to -\infty \) as \( x \to -\infty \)

Consider the general form of a polynomial function:
\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]

The algebra may seem complicated, but suppose we factored out the leading term \( a_n x^n \) so that we get:
\[ f(x) = a_n (x-1)^7 \]

Each term that contains \( x \) in the parenthesis will be very small numbers when \( |x| \) gets large. The term that dominates this polynomial will be the term same, or the leading coefficient.

NOW YOU TRY IT!!!

Use the Leading Coefficient Test to determine the behavior of the graphs below when

- \( f(x) = x^4 + 2 \)
- \( f(x) = -3x^4 + 2x - 5 \)
- \( f(x) = -2x^3 - 5x + 1 \)
- \( f(x) = 2x^2 - x \)
- \( f(x) = -x^2 + 2(\pm 3)^2 \pm 4) \)

REAL ZEROS OF POLYNOMIAL FUNCTIONS

Remember that a real zero of a function \( f \) is a number \( c \) for which \( f(c) = 0 \)

Example:
\[ f(x) = x^3 + x - 12 \]
\( (x-2)(x+4) \)

3 is a zero of \( f \) because
\( f(3) = (3-2)(3+4) \)
\( f(3) = 1 \cdot 7 \)
\( f(3) = 0 \)

-4 is a zero of \( f \) because
\( f(-4) = (-4-2)(-4+4) \)
\( f(-4) = (-7)(0) \)
\( f(-4) = 0 \)

Find the zeros for the following functions:

a) \( f(x) = x^3 - 4x \) zeros:

b) \( f(x) = (2x-3)(x+1) \) zeros:
REAL ZEROS OF POLYNOMIAL FUNCTIONS

If \( f \) is a polynomial function and \( c \) is a real zero of \( f \), then the statements below are equivalent:

1. \( x = c \) is a zero of the function \( f \).
2. \( x = c \) is a solution of the equation \( f(x) = 0 \).
3. \( (x-c) \) is a factor of the polynomial function \( f \).
4. \( (c,0) \) is an \( x \)-intercept of the graph of \( f \).

We can often use information about the zeros of a function to help sketch its graph. We can also use information about the graph of a function to help find its zeros.

In some examples you should have seen real zeros that we call repeated zeros.

- \( h(x) = 3x^2(x-1)(x-1) \)
  - zero is a repeated zero
    - and
  - \( p(x) = (x-3)^2(x+1)(x+1)(x+4) \)
  - three is a repeated zero
  - \( r(x) = x^3(5x-2) \)
  - zero is a repeated zero

Suppose the polynomial function \( f \) has a factor \((x-c)^k\).
That means that \( c \) is a zero of multiplicity \( k \).

IMPORTANT !!!!

If \( k \) is odd, the graph will cross the \( x \)-axis at \( x = c \) and the "behavior" of the graph at \( c \) will be very much like the graph of \( f(x) = (x-c)^k \) as it crosses the point \((c,0)\).

If \( k \) is even, the graph will touch (but will not cross) the \( x \)-axis at \( x = c \) and the "behavior" of the graph at \( c \) will be very much like the graph of \( f(x) = (x-c)^k \) as it touches \((c,0)\).

SEARCHING FOR ZEROS...

Do the following problems. Remember to factor each polynomial completely.

1) \( f(x) = x(x-3)(x+2) \)
2) \( g(x) = x^2 - 5x + 4 \)
3) \( h(x) = 3x^3(x-1)(x+1) \)
4) \( p(x) = (x-3)^2(x+1)(x+4) \)
5) \( r(x) = 6x^4 - 2x^3 \)

In general, we say that a factor \((x-c)^k\) will give us a repeated zero \( x = c \) of multiplicity \( k \).

- \( h(x) = 3x^2(x-1)(x-1) \)
  - zero is a repeated zero
    - and
  - \( p(x) = (x-3)^2(x+1)(x+1)(x+4) \)
  - three is a repeated zero
  - \( r(x) = x^3(5x-2) \)
  - zero is a repeated zero

LET'S PUT ALL THIS TOGETHER NOW AND SEE HOW EASY IT IS TO SKETCH FACTORED POLYNOMIALS

Sketch \( f(x) = x(x-1)(x+1) \)
- Leading term: \( x^3 \)
- Degree: 3
- Multiplicity 2
- Graph crosses \((1,0)\) and \((-1,0)\)
- Graph contains point \((0,0)\)

Graph of some polynomial curves touch and cross at \((0,0)\) like a linear function.
Sketch: \( f(x) = -\frac{1}{2}x^2(x+2)^2(x-3) \)

- Leading term: \(-\frac{1}{2}x^6\)
- Zeroes: 0 - multiplicity 3
  -2 - multiplicity 2
  3 - multiplicity 1
- y-intercept: 0

Let's start in Quadrant II for you can start in Q II

SKETCH THE FOLLOWING GRAPHS ON YOUR WORKSHEET.

1) \( f(x) = (x - 3)(x + 2)^2 (x - 4) \)
2) \( f(x) = -4(x + 2)(x - 6)^2(x - 3) \)
3) \( f(x) = -\frac{1}{2}x^2(x + 2)(x - 1)^2(x - 2) \)

DO NOT BEGIN PART II OF THIS EXERCISE UNTIL YOU HAVE FINISHED YOUR WORKSHEET FOR PART I.
SOLVING FACTORED INEQUALITIES IN ONE VARIABLE

Let's look at a method for solving inequalities such as

\[(x - 3)(x + 2)(x - 1) > 0\]

or

\[x(x + 2)'(x + 1)'(x - 2) < 0\]

Let's use our new graphing technique to solve these inequalities:

So where is \(x(x + 3)'(x + 1)'(x - 2) < 0\)

The expression is less than zero where the graph of the function is below the x-axis, right?

The answer is

\[-3 < x < -2\] or \[-2 < x < -1\] or \[0 < x < 2\]

In interval notation, the answer would be:

\[(-\infty, -2) \cup (-2, -1) \cup (0, 2)\]

Solve the inequality \(x(x + 2)'(x + 1)'(x - 2) < 0\)

Suppose we think about it like this...

Let \(f(x) = x(x + 2)'(x + 1)'(x - 2)\)

Sketch:

Since the leading coefficient is positive and the degree is odd, the function should end up in Quadrant III and I.

leading term: \(x^3\)

zeros: 0 - multiplicity 1
-2 - multiplicity 2
-1 - multiplicity 3
2 - multiplicity 1

y-intercept: 0

Notice how the graph touches the point \((-2, 0)\) like a parabola, crosses the point \((-1, 0)\) like a cubic function, crosses the point \((0, 0)\) and the point \((2, 0)\) like a linear function.

So where is \(-0.5(x - 3)(x - 1)'(x + 2) > 0\)

The expression is greater than zero where the graph of the function is above the x-axis, right?

The answer is

\[1 < x < 3\]

In interval notation, the answer would be the interval: \((1, 3)\)

Solve the inequality \(-0.5(x - 3)(x - 1)'(x + 2) > 0\)

Suppose we think about it like this...

Let \(g(x) = -0.5(x - 3)(x - 1)'(x + 2)\)

Sketch:

Since the leading coefficient is negative and the degree is even, the function should end up in Quadrant III and IV.

leading term: \(-0.5x^3\)

zeros: 3 - multiplicity 1
1 - multiplicity 3
2 - multiplicity 2

y-intercept: \(f(0) = -6\)

Notice how the graph touches the point \((-2, 0)\) like a parabola, crosses the point \((1, 0)\) like a cubic function, and crosses the point \((0, 0)\) \((3, 0)\) like a linear function.

SOLVING MORE INEQUALITIES...

Use the sketching techniques from Part 1 to solve the following inequalities. Remember to factor each polynomial expression completely.

1) \(x(x - 3)(x + 2) > 0\)
2) \(x^2 - 10x + 9x < 0\)
3) \(3x^2(x - 1) > 0\)
4) \(x^2(x - 2)^2 (x - 1)^2 (x + 2) < 0\)
5) \(8x^2 - 2x > 0\)

Do these problems on your worksheet!
EXERCISE #8

Review of Quadratic Functions and Application Problems Using Quadratic Functions

Software:
Microsoft PowerPoint
Microsoft Excel
In Part I, the student will review the standard equation \( f(x) = ax^2 + bx + c \) of a quadratic function and the graph of a quadratic function. The student will identify the vertex and whether it is the lowest or highest point (minimum or maximum point) on the graph when the function is written in the form \( f(x) = a(x-h)^2 + k \). The student will review how to complete the square to rewrite the parabola in the form \( f(x) = a(x-h)^2 + k \). The student will utilize this information to solve application problems in Part II.

PART I
Quadratic Functions

1. Open the PowerPoint file titled "QuadFun" and read slides #1-3 to answer the following questions from slide #4.

Identify the vertex of each parabola that appears below in standard form. State whether the vertex is the highest or lowest point on the parabola?

a) \( f(x) = (x-2)^2 + 3 \)  
vertex _____  highest or lowest point? _____

b) \( f(x) = -3x^2 + 1 \)  
vertex _____  highest or lowest point? _____

c) \( f(x) = -2(x+3)^2 - 1 \)  
vertex _____  highest or lowest point? _____

d) \( f(x) = -4(x-5)^2 \)  
vertex _____  highest or lowest point? _____

e) \( f(x) = (x+2)^2 + 5 \)  
vertex _____  highest or lowest point? _____

f) \( f(x) = 3(x-6)^2 - 14 \)  
vertex _____  highest or lowest point? _____
2. Read slides #5-7 and answer the following questions from slide #8:

- Complete the square to rewrite the function in the form \( f(x) = a(x-h)^2 + k \)
- Identify the vertex of each parabola
- State whether it is the lowest or highest point of the graph
- Identify the maximum or minimum value of the function

a) \( f(x) = x^2 - 12x + 1 \)
   - New equation: 
   - Vertex: 
   - Highest or lowest point?: 
   - Maximum value: 

b) \( g(x) = 4x^2 - 24x - 3 \)
   - New equation: 
   - Vertex: 
   - Highest or lowest point?: 
   - Maximum value: 

c) \( f(x) = -2x^2 - 8x + 5 \)
   - New equation: 
   - Vertex: 
   - Highest or lowest point?: 
   - Maximum value: 

d) \( g(x) = -3x^2 + 30x - 4 \)
   - New equation: 
   - Vertex: 
   - Highest or lowest point?: 
   - Maximum value: 

Page 2
3. Read slides #9-11 and answer the following questions from slide #10:

Use the information that the vertex of the parabola \( f(x) = ax^2 + bx + c \quad (a \neq 0) \)
can also be found by using the formula \( \left( -\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right) \) to verify that your answers in #2 are correct. **Show your work.**

a) \( f(x) = x^2 - 12x + 1 \)  
   \( a = \_\_\_ \quad b = \_\_\_ \)  
   vertex \_\_\_

b) \( g(x) = 4x^2 - 24x - 3 \)  
   \( a = \_\_\_ \quad b = \_\_\_ \)  
   vertex \_\_\_

c) \( f(x) = -2x^2 - 8x + 5 \)  
   \( a = \_\_\_ \quad b = \_\_\_ \)  
   vertex \_\_\_

d) \( g(x) = -3x^2 + 30x - 4 \)  
   \( a = \_\_\_ \quad b = \_\_\_ \)  
   vertex \_\_\_
PART II
Application Problems Using Quadratic Functions

1. Read slides #12-25 and work Problem #1 as shown in the lesson.

PROBLEM #1

A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?

Step #1

The width of the base is x inches
The length of the cross-section rectangle is (12 - 2x) inches
Recall: The area of a rectangle is given by Area = (length)(base)

Remember, we want to determine the number of inches we should turn up to give the gutter its greatest capacity. That means the capacity will be greatest when the cross-sectional area of the rectangle with sides of lengths x and 12-2x has its greatest value.

Let f(x) denote this area: f(x) = x(12-2x)

Finish problem by finding the vertex of the quadratic function now. SHOW WORK.

Answer: ________________________________
2. Read slide #26 to begin your work on Problem #2 as shown in the lesson.

**PROBLEM #2**

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

Let \( x \) represent the number of $15 decreases in monthly rent. (For example, if \( x = 2 \), the rent is $420).

A) Write expressions that represent
   (a) the resulting rent per unit
   (b) the resulting number of tenants.

B) Let \( R \) represent the total rental income and \( x \) represent the number of $15 decreases in monthly rent. Using what you found earlier, find an algebraic expression of \( R \) as a function of \( x \).

C) Find the rent that will yield the rental company the maximum monthly income. (What is the domain of this function for this problem?)

**SOLUTION:**

A) Read slides #27-30 carefully to write expressions that represent the
   (a) the resulting rent per unit
   (b) the resulting number of tenants (units)

Let's construct a spreadsheet that will help us think about this problem.
You might use the spreadsheet below from the lesson to help you determine the formulas that would go in each cell in the missing rows...

<table>
<thead>
<tr>
<th># of Decreases</th>
<th>Avg Rent/ Month</th>
<th>Total Units</th>
<th>Total Rented</th>
<th>Total Rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>450</td>
<td>800</td>
<td></td>
<td>360000</td>
</tr>
<tr>
<td>1</td>
<td>435</td>
<td>850</td>
<td></td>
<td>369750</td>
</tr>
<tr>
<td>2</td>
<td>420</td>
<td>900</td>
<td></td>
<td>378000</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Rent?
- 0 decrease? \( \text{Rent} = 450 - (0)(15) = 450 \)
- 1 decrease? \( \text{Rent} = 450 - (1)(15) = 435 \)
- 2 decreases? \( \text{Rent} = 450 - 2(15) = 420 \)
- 3 decreases? \( \text{Rent} = 450 - 3(15) = 405 \)
- \( x \) decreases? \( \text{Rent} = 450 - x(15) \)

Units Rented?
- 0 decreases \( 800 + (0)(50) = 800 \) units
- 1 decrease \( 800 + (1)(50) = 850 \) units
- 2 decreases \( 800 + (2)(50) = 900 \) units
- 3 decreases \( 800 + (3)(50) = 950 \) units
- \( x \) decreases \( 800 + (x)(50) \)
B) Read slides #31-33 carefully to write an algebraic expression of R as a function of x.

**Total Rental Revenue?**

<table>
<thead>
<tr>
<th>Decreases</th>
<th>Rent</th>
<th>Price</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 decreases</td>
<td>800</td>
<td>$450</td>
<td>$360,000</td>
</tr>
<tr>
<td>1 decrease</td>
<td>850</td>
<td>$435</td>
<td>$367,500</td>
</tr>
<tr>
<td>2 decreases</td>
<td>900</td>
<td>$420</td>
<td>$378,000</td>
</tr>
<tr>
<td>3 decreases</td>
<td>950</td>
<td>$405</td>
<td>$388,500</td>
</tr>
<tr>
<td>x decreases</td>
<td>800 + 100x</td>
<td>$450 - 15x</td>
<td></td>
</tr>
</tbody>
</table>

\[
R(x) = (800 + 50x) (450 - 15x)
\]

**Notice that our Revenue Income R is a quadratic function...**

If we simplify the equation, we get

\[
R(x) = 360000 - 12000x + 22500x - 750x^2
\]

\[
= -750x^2 + 10500x + 360000
\]

When will we get the maximum value for \( R(x) \)?

Compare it to your spreadsheet to see if the spreadsheet agrees?

---

**SOLVING MORE PROBLEMS**

You have been working problems that are sometimes referred to as optimization problems in mathematics. Do the following problems with your group members for additional practice.

**PROBLEM #3**

A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). What dimensions will produce a maximum enclosed area? Show all work.
PROBLEM #4  (Create a spreadsheet to help you solve this problem. See problem #2 and the sample spreadsheet below as an example)  Show all work.
The owner of a 60-unit motel, by checking records of occupancy, knows that when the room rate is $50/day, all units are occupied. For every increase of $x$ dollars in the daily rate, $x$ units are left vacant. Each unit occupied costs $10 per day to service and maintain. Let $P$ denote the total daily profit.
a) Determine a formula in one variable for $P(x)$.
b) What is the maximum profit according to your spreadsheet?
c) Find the maximum value using algebraic methods and compare your answer with your spreadsheet.

$$P(x) = \text{ }$$

Maximum profit =

<table>
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<tr>
<th># of x dollars increases</th>
<th># of rooms rented</th>
<th>Income per Room</th>
<th>Total Rental Income</th>
<th>Expense Rented Profit</th>
</tr>
</thead>
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<td>59</td>
<td>51</td>
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<td>15</td>
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</tbody>
</table>

PROBLEM #5
An object is thrown straight upward from a height of 6 feet with an initial velocity of 32 feet per second. The height at any time is given by

$$s(t) = -16t^2 + 32t + 6$$

where $s(t)$ is measured in feet and $t$ in seconds. Find the maximum height attained by the object before it begins falling to the ground.  Show work.

Maximum height =
When given a parabola in the form \( y = ax^2 + bx + c \) remember this method can be used to rewrite the parabola in the form \( y = a(x-h)^2 + k \):

\[
a = 1
7 = x^2 + 6x + \frac{9}{2}
\]

**EXAMPLE #1:**

Add and subtract the square of one-half the coefficient of \( x \) inside the parenthesis.

Parabola now in required form

Vertex: \((-3, -5)\) and opens up

**EXAMPLE #2:**

Parabola now in required form

Vertex: \((-3, -51)\) and opens up

**Identify the vertex of each parabola that appears below in standard form. State whether the vertex is the highest or lowest point on the parabola.**

a) \( f(x) = (x-2)^2 + 3 \)
b) \( f(x) = -3x^2 + 1 \)
c) \( f(x) = 2(x+3)^2 - 1 \)
d) \( f(x) = -4(x-5)^2 \)
e) \( f(x) = (x+2)^2 + 5 \)
f) \( f(x) = 3(x-6)^2 - 14 \)
EXAMPLE #3:

1) \[ y = -3x^2 + 24x + 7 \]
2) \[ y = -3(x^2 - 8x + 16) + 7 \]
3) \[ y = -3(x^2 - 8x + 16) + 48 + 7 \]
4) \[ y = -3(x-4)^2 + 55 \]

The statement can be proven by completing the square of a parabola in standard form. When you use the formula above, once the x-coordinate is found, you can calculate the y-coordinate by substituting \(-b/(2a)\) for \(x\) in the equation of the parabola.

END OF PART I

DO NOT BEGIN PART II OF THIS EXERCISE UNTIL YOU HAVE FINISHED YOUR WORKSHEET FOR PART I.
A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up to give the gutter its greatest capacity?
Writing the Equation for the Problem

Remember, we want to determine the number of inches we should turn up to give the gutter its greatest capacity. That means the capacity will be greatest when the cross-sectional area of the rectangle with sides of lengths $x$ and $12-2x$ has its greatest value.

- The length of the cross-section rectangle is $12 - 2x$ inches
- The width of the base is $x$ inches
- The area of a rectangle is given by $\text{Area} = \text{(length)} \times \text{(base)}$
Solving the problem

\[ f(x) = x(12-2x) \]

= 12x - 2x^2

Distributive Property

Write right side in standard form

Area of the cross-section of the rectangle

Since this is a quadratic function, \( a = -2 \), \( b = 12 \), and \( c = 0 \).

The graph of the function is a parabola that opens downward. The vertex of the parabola is at \( (\frac{12}{-2}, -\frac{12(-2)}{-4}) \) or \( (3, 3) \). So the parabola has its maximum value at \( x = 3 \).

Statement of Problem #2 again...

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

(a) Write expressions that represent the resulting rent per unit and the resulting number of tenants.

1) Write expressions that represent the (a) resulting rent per unit and (b) resulting number of tenants.

2) Let \( R \) represent the total rental income and \( x \) represent the number of $15 decreases in monthly rent. Using what you found in (a), find an algebraic expression of \( R \) as a function of \( x \).

3) Find the rent that will yield the rental company the maximum monthly income. (What is the domain of this function for this problem situation?)

Where did these numbers come from in this spreadsheet?

Determine the formulas that would go in each cell from row 5 down...

(b) the resulting number of tenants.

0 decreases \( 800 + (0)(50) = 800 \) units

1 decrease \( 800 + (1)(50) = 850 \) units

2 decreases \( 800 + (2)(50) = 900 \) units

3 decreases \( 800 + (3)(50) = 950 \) units

\( x \) decreases \( 800 + (x)(50) = \)

Statement of Problem #2 again...

An apartment rental company has 2300 units available, and 800 are currently rented at an average of $450/month. A market survey indicates that each $15 decrease in average monthly rent will result in 50 new tenants.

1) Write expressions that represent the (a) resulting rent per unit and (b) resulting number of tenants.

2) Let \( R \) represent the total rental income and \( x \) represent the number of $15 decreases in monthly rent. Using what you found in (a), find an algebraic expression of \( R \) as a function of \( x \).

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Determine the formulas that would go in each cell from row 5 down...

(b) the resulting number of tenants.

0 decreases \( 800 + (0)(50) = 800 \) units

1 decrease \( 800 + (1)(50) = 850 \) units

2 decreases \( 800 + (2)(50) = 900 \) units

3 decreases \( 800 + (3)(50) = 950 \) units

\( x \) decreases \( 800 + (x)(50) = \)
2) Let \( R \) represent the total rental income and \( x \) represent the number of $15 decreases in monthly rent. Using what you found in #1, find an algebraic expression of \( R \) as a function of \( x \).

\[
\begin{align*}
R(0) &= 800 \times 450 = [800 + 0(50)] [450 - 0(15)] \\
R(1) &= 850 \times 435 = [800 + 1(50)] [450 - 1(15)] \\
R(2) &= 900 \times 420 = [800 + 2(50)] [450 - 2(15)] \\
R(3) &= 950 \times 405 = [800 + 3(50)] [450 - 3(15)] \\
R(x) &= \text{?} = [800 + x(50)] [450 - x(15)] \\
\end{align*}
\]

\[ R(x) = (800 + 50x)(450 - 15x) \]

Notice that our Revenue Income \( R \) is a quadratic function...

If we simplify the equation, we get

\[ R(x) = 360000 - 12000x + 22500x - 750x^2 \]

\[ R(x) = -750x^2 + 10500x + 360000 \]

\[ R(x) \text{ will be at its maximum value at the vertex, or where } x = -b/(2a) = -10500/(-750) = 7 \]

Why are we NOT surprised?!! With 7 decreases, the revenue was at its highest in the spreadsheet. So the spreadsheet is correct.

3) Find the rent that will yield the rental company the maximum monthly income. (What is the domain of this function for this problem?)

Now we want to maximize the total rental revenue for the company. From the spreadsheet, it looks like the revenue would be maximized with 7 decreases. Is the spreadsheet correct?

Also, the domain \([0, 30]\) for this function would be:

\[ 0 \leq x \leq 30 \]

SOLVING MORE PROBLEMS

In mathematics, you have been working problems that are sometimes referred to as optimization problems. Do these problems in your worksheet.

**PROBLEM 12**

A farmer has 300 feet of fencing to enclose two adjacent rectangular enclosures as shown. What dimensions will provide a maximum enclosed area?

**PROBLEM 14**

The owner of a 65-room motel, by checking records of occupancy, knows that when the rates are $65/day, all rooms are occupied. For every increase of a dollar in the daily rate, a unit (one room) is unbooked. Each unoccupied room costs $10 per day to maintain. Let \( P \) denote the total daily profit. Create a spreadsheet for this problem.

- a) Determine a formula to use within the spreadsheet.
- b) What is the maximum profit according to your spreadsheet?
- c) Approximate the maximum value of the function \( P \).
- d) Compare your answers.

**PROBLEM 15**

An object is thrown downward upward from a height of 6 feet with an initial velocity of 22 feet per second. The height at any time \( t \) is given by:

\[ h = -16t^2 + 22t + 6 \]

where \( h \) is measured in feet and \( t \) is seconds. Find the maximum height attained by the object before it begins falling to the ground.
EXERCISE #9

Shifting Graphs Including Horizontal, Vertical, and Combination Graph Shifts
Software: Derive
EXERCISE #9

Shifting Graphs

Date: ___________________________                    Group Members: ___________________________

Software Required:  
Derive ____________________________

The student will explore connections between constants in equations and associated shifts in a basic graph. These shifts will include horizontal, vertical, and combinations of these shifts.

PART 1

Single Shifts

1. Use Derive to help you sketch the following functions.

   \[ f(x) = x^2 \]
   \[ f(x) = x^2 + 2 \]

   \[ f(x) = x^2 + 5 \]
   \[ f(x) = x^2 - 3 \]

   \[ f(x) = x^2 + 9 \]
   \[ f(x) = x^2 - 4 \]
2. What do you notice about these graphs? Are the shapes the same? How could you use the first graph to help you sketch the other equations?

3. Make a rule to generalize what is happening to the graph when you have

\[ f(x) = x^2 + c \]

4. Does it matter if the constant is positive or negative? If so, how?

5. Do you think your rule will generalize to any function where \( y=f(x)+c \)?

6. Use Derive to help you sketch the following functions.

\[ f(x) = \sqrt{x} \]

\[ f(x) = \sqrt{x+1} \]

\[ f(x) = \sqrt{x+8} \]

\[ f(x) = \sqrt{x-5} \]
7. What do you notice about these graphs? How are the graphs similar? How are they different?

8. Make a rule to generalize what is happening to the graph when you have
   \[ f(x) = \sqrt{x + c} \]

9. Does it matter if the constant is negative?

10. Do you think your rule will generalize to \( y = f(x + c) \)?
PART II

Graph Shift Combinations

1. Use your rules from Part I to predict what will happen when you sketch the graph for each of the following equations.

\[ f(x) = (x - 2)^2 + 1 \quad f(x) = (x + 4)^2 - 8 \]

\[ f(x) = \sqrt{x - 1} - 2 \quad f(x) = \sqrt{x + 5} - 6 \]

\[ f(x) = (x + 4)^2 + 4 \quad f(x) = \sqrt{x - 7} + 1 \]

2. Do your rules hold true? If no, revise your rules.

3. Discuss your rules with another group and revise again if necessary.
Technology Review

by

Vicki Norwich
and

Jacci White

ISETL
A Mathematical Programming Language

TECHNOLOGY COMPARISON

♦ ISETL
♦ DERIVE
♦ Geometer’s Sketchpad
♦ TI-82, TI-83
♦ TI-85, TI-86
♦ TI-92
♦ CBL

FREEWARE
Available from the Internet, or West Publishing
IBM DOS, Windows, or Mac

VERSION AVAILABLE

SAMPLE ISETL PROGRAM

♦ f:=func(x);
  − If x>0 then return x^2-1;
  − elseif x=0 then return 1;
  − else return x+1;
  − end;
  − end;

♦ Write programs to create functions.
♦ Write programs to graph functions.
♦ Write programs to evaluate functions.
♦ Write programs for all operations.
**ISETL ADVANTAGES**
- FREE
- EASY TO INSTALL
- NO ADDITIONAL PURCHASE NECESSARY

**ISETL DISADVANTAGES**
- Must learn the programming language.
- Operations must be programmed.
- Assistance is hard to find.
- Time consuming to use until it becomes familiar and a store of programs have been developed.

**DERIVE**
SYMBOLIC COMPUTER SYSTEM

**DERIVE**
- Must be purchased
- Can be ordered from local dealers
- Student version available

**DERIVE**
- Enter problems the way they appear in the text
- Evaluate functions
- Graph functions
- Perform all basic Calculus computations and operations
- Perform algebraic manipulations

**SAMPLE DERIVE APPLICATIONS**
- Evaluate a function at different values
- Graph any function in 2 or 3 dimension
- Solve equations
**DERIVE ADVANTAGES**
- Easy to use, no assistance necessary
- Can display the answer in different forms
- Fast
- Problems appear in actual form

**DERIVE DISADVANTAGES**
- COST
  - Need a computer

**GEOMETER'S SKETCHPAD**
A Dynamic Geometry System

**GEOMETER'S SKETCHPAD**
- Must be purchased
- Can be ordered from local dealers
- Student version available

**GEOMETER'S SKETCHPAD**
- Sketch geometric figures including angles, bisectors, perpendicular lines, triangles, circles, and more
- Evaluate functions
- Graph functions
- Perform numeric computations

**SAMPLE GEOMETER'S SKETCHPAD APPLICATIONS**
- Sketch geometric figures
- Calculate angles, midpoints, lengths, and more
- Animate sketches to illustrate identities and theorems
GEOMETER'S SKETCHPAD

**ADVANTAGES**
- Easy to use, directions are fairly straightforward
- Little background in Geometry is necessary to get started
- Fast
- Very useful as a discovery tool for visualizing new concepts

**DISADVANTAGES**
- Cost
  - Need a computer
  - Designed for Geometry so not as useful in other mathematics courses

**COST**
- Approximately $89.00
- Available from local dealers as well as office supply stores, and Target, Service Merchandise, etc...
- Designed for use up to College Algebra

**APPLICATIONS**
- Programmable
- Can printout on the computer using Graphlink
- Links to CBL
- Shares viewscreen with TI-83

**TI-82**
- Evaluate functions
- Graph functions
- Perform basic Calculus operations and functions
**TI-82 ADVANTAGES**
- Cost is low for a graphing calculator
- Displays data in tabular form
- Menus are easy to use

**TI-82 DISADVANTAGES**
- Problems must be entered using the correct order of operations
- Less memory space than more expensive models
- Limited number of advanced functions
- No algebraic manipulations

**TI-85**
- Approximately $99.00
- Available from local dealers as well as office supply stores, Target, Walmart, etc...
- Designed for use up to Calculus II

**Sample TI-85 Applications**
- Graph functions
- Perform many Calculus I and II operations
- Answers in decimal, fraction, or complex number form

---

- Programmable
- Can printout on the computer using Graphlink
- Links to CBL
- Shares Viewscreen with TI-86
**TI-85 ADVANTAGES**

- More memory than the TI-82
- All commands can be found under the catalog key
- Many advanced engineering capabilities

**TI-85 DISADVANTAGES**

- No table feature
- Slightly more expensive than the TI-82
- No algebraic manipulations

**TI-86**

**GRAPHING CALCULATOR**

- Programmable
- Can printout on the computer using Graphlink
- Links to CBL
- Shares Viewscreen with TI-85

**Sample TI-86 Applications**

- Graph functions
- Perform many Calculus I and II operations
- Answers in decimal, fraction, or complex number form
- Performs many procedures from Differential Equations
**TI-86 ADVANTAGES**
- More memory than the TI-85
- All commands can be found under the catalog key
- Many advanced engineering and Differential Equations capabilities
- Contains the table feature to display data

**TI-86 DISADVANTAGES**
- Slightly more expensive than the TI-82
- No algebraic manipulations
- No 3-D graphing

**TI-92**
- Approximately $190.00
- Available from most dealers and a few office supply stores
- Designed for use up to Calculus III and advanced mathematics courses

**TI-92**
- Programmable
- Has Derive built in
- Has Cabri Geometry capabilities
- Can printout on the computer

**Sample TI-92 Applications**
- Algebraic manipulations
- Graphing including 3-space
- Does all Calculus I, II, and III applications
- Geometry
**TI-92 ADVANTAGES**

- Algebraic manipulations
- Derive command structure
- 3-D graphing
- Geometry applications

**TI-92 DISADVANTAGES**

- Cost is higher than other calculators

**TI-83**

**GRAPHING CALCULATOR**

- Approximately $100.00
- Available in the same locations as other Texas Instruments calculators
- Designed for use up to College Algebra and Statistics

**Sample TI-83 Applications**

- Graph split with table feature
- Spreadsheet capabilities
- Statistics
- Business

- Split screen
- Programmable
- Can be used with Graphlink
TI-83 ADVANTAGES
- Designed with more advanced statistical and business menu options.

TI-83 DISADVANTAGES
- No new Calculus features
- No algebraic manipulations
- Slightly more expensive than the TI-82

CBL
- Approximately $200.00
- Available from local Dealers and Vernier Software

Sample CBL Applications
- Motion
- Temperature
- Light
- Many more

CBL ADVANTAGES
- Visualize real applications
- Experiment with new ideas
- Portable
CBL DISADVANTAGES

- Limited use for calculations
- Applications are not obvious to students
- Designed for classroom demonstration or group exploration
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<td>Jacci White and Vicki Norwich</td>
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