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ABSTRACT

The National Council of Teachers of Mathematics (NCTM) has published three documents that propose standards related to mathematics education called "The Curriculum and Evaluation Standards for School Mathematics" (1989), "The Professional Standards for Teaching Mathematics" (1991), and "The Assessment Standards for School Mathematics" (1995). The Pacesetter mathematics program and the course through which it is implemented, Precalculus through Modeling, embody the standards related to secondary school mathematics set forth in these documents. This booklet gives a detailed explanation of the relationship between these standards and Pacesetter Mathematics. Unit 1 is based on students' previous experience with linear equations and focuses on problems associated with real-life situations to introduce mathematical modeling. The secondary focus is on the concept of function in general mathematics. Unit 2 examines multiplicative changes in art, finance, and social sciences. Sample activities include developing a savings plan for a person who has won a lottery and developing a savings plan for buying a car. In Unit 3 students build from quadratic and cubic equations to investigate the properties of power, polynomial, and rational functions. The sine, cosine, and tangent functions are introduced in Unit 4 which also utilizes graphing calculators. Unit 5 covers modeling with matrices, and Unit 6 addresses modeling with various other functions. (AIM)

CROSSWALKS:
Pacesetter Mathematics and the National Standards

CROSS WALKS

PACESETTER® MATHEMATICS

and the National Standards

THE COLLEGE BOARD

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Crosswalks: PACESETTER Mathematics and the National Standards

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Crosswalks PACESETTER® Mathematics and the National Standards

"Pacesetter Mathematics is a working example of a standards-based mathematics curriculum. Its focus on the featured processes of problem solving, reasoning, communicating, and connecting are immediately evident on investigation of the materials designed for student use" (Introduction, *Precalculus through Modeling*, Teacher's Ed.).

Introduction

The National Council of Teachers of Mathematics has published three documents in the past few years that propose standards related to mathematics education—the *Curriculum and Evaluation Standards for School Mathematics* (1989), the *Professional Standards for Teaching Mathematics* (1991), and the *Assessment Standards for School Mathematics* (1995). The Pacesetter mathematics program and the course through which it is implemented, Precalculus through Modeling, embody the standards related to secondary school mathematics set forth in these documents. This document will give a detailed explanation of the relationship between these standards and Pacesetter Mathematics.

PACESETTER Mathematics Units and Task Sets

UNIT 1—Modeling with Linear Functions

This unit is based on students' previous experience with linear equations and focuses on problems associated with real-life situations as a way of introducing students to mathematical modeling. A second area of focus is the concept of function as a general mathematical idea apart from any particular application.

- Stacks of Cups—Students are asked to generate a mathematical model for the height of a box that is to contain a stack of a certain number of cups. They must generate their own data through experimentation. The topics addressed in this task set are: building a linear model from student-generated data, graphing linear functions, slope, *y*-intercept, equations of lines, points of intersection, discrete value functions, direct variation, and arithmetic sequences.

- The Mile Run—Students are asked to generate a model to help predict when the record time for the mile run will reach a certain value given a set of data for previous records. The topics addressed in this task set are: using "dirty data," lines of best fit, making predictions from data, slope, intercepts, equations of lines, concept of linear regression, and median-median line (optional).

- Ships in the Fog—Students are asked to determine whether two ships that are being tracked on a screen will collide. In the process, they will generate a mathematical model for the paths of the ships and use their model to answer questions about the paths of the ships. The topics addressed in this task set are: linear relationships defined parametrically and graphs of these relationships, motion in the plane, intersection of lines, distance formula, and inverse functions.

- Temperature Scales—Students are asked to examine the relationships among the Celsius, Fahrenheit, and Kelvin temperature scales. The topics addressed in this task set are: slope, intercepts, inverse functions, composition of functions, and translations of graphs.

UNIT 2—Modeling with Exponential and Logarithmic Functions

This unit requires students to examine multiplicative changes in the areas of art, finance, and social sciences. As contrasted with the arithmetic growth seen in Unit 1, the regularity of geometric growth is a constant ratio between successive terms as change takes place from one setting or time frame to the next.

- The Square Craft Design—Students are asked to develop a model that can be used to determine the sizes of the triangular pieces in a craft design. The topics included are: development of an exponential function through student-generated data, isosceles right triangles, ratio, similarity, scale, and area.

- The Pennsylvania Lottery—Students are asked to develop a savings plan for a person who has won a lottery. The topics included are: compound interest (with varying compounding periods), exponential growth, the ratio property of exponential functions, graphs of exponential functions, and nominal and effective interest rates.

- A Powerful Function—This task set is used to convey to the student the importance of logarithmic functions as inverses of exponential functions in solving exponential equations. Students use the calcula-

tor as a trial-and-error tool to first make a connection between exponents and logarithms. Discovery exercises lead to the properties of logarithms. The topics included are: common logarithms, other base logarithms and the change-of-base formula, logarithmic functions, inverse functions, composition of functions, and solution of exponential equations.

- Population Growth — Students are asked to estimate when the world's population will reach ten billion. Topics included are: making predictions using "dirty data," exponential functions, and the concept of exponential regression.

- Saving to Buy a Car — Students are asked to develop a savings plan that would provide enough money to buy a car. The topics included are: parameters of an exponential function, sum of a geometric series, graphs of exponential functions, and analysis of these graphs.

UNIT 3 — Modeling with Polynomial and Rational Functions

This unit extends the ideas of modeling beyond linear and exponential functions. Building from quadratic and cubic equations, students investigate the properties of power, polynomial, and rational functions.

- Fences — Students are asked to determine for a customer the largest possible rectangular area for a dog enclosure, given a certain length of fencing. The topics included are: quadratic functions and their graphs; the relationship among the graphs of these functions, their roots, and the discriminant; the relationship between area and perimeter; introduction to rational functions; and maximizing volume.

- Highway Safety — Students are asked to determine how many feet it would take to bring a car that is traveling at a speed of fifty miles per hour to an emergency stop, given an average reaction time. The topics included are: modeling with linear and quadratic functions, data analysis, optimization, dimensional analysis, and analysis of the parameters of a function.

- Planning a Summer Camp — Students are asked to determine the per camper cost for children to attend a summer camp, given certain conditions. The topics included are: rational functions, translations of rational functions, vertical and horizontal asymptotes, and inverse functions.

- Graphs of Rational Functions—Students are asked to examine and interpret the graphs of various forms of rational functions. The topics included are: increasing and decreasing functions, domain, boundedness, end behavior, asymptotes, graphs of sums and quotients of functions, and infinite geometric series.

- The Ideal Traffic Pattern—Students are asked by the City Council to design a highway traffic flow pattern that meets the requirements of several special interest groups. This task set is based on the information found in the Highway Safety task set. The topics included are: optimization, dimensional analysis, graphical analysis of functions, and exploration of a rational function and a piecewise-defined function.

UNIT 4—Modeling with Trigonometric Functions

The sine, cosine, and tangent functions are introduced through periodic phenomena such as the horizontal and vertical projection of a rotating bicycle wheel. Students use graphing calculators to explore the graphs of these functions.

- Bicycle Wheels—A two-wheeler with training wheels runs over a freshly painted parking strip. Students are asked to determine when the paint strip on the front wheel is the same height above the pavement as the paint strip on the training wheel. The graph of a sine function is developed. The topics included are: circular motion, right triangle trigonometry, domain, periodicity, amplitude, frequency, vertical translation, phase shift, degree measure versus radian measure, and the graph of a cycloid, using parametric equations.

- Daylight Hours—Students are asked to determine how many daylight hours a specific city would experience on a specific date of the year. The topics included are: periodic phenomena, the sine function, parameter analysis, data analysis, geometry of the sphere, right triangle trigonometry, and connections to astronomy and geography.

- The Chocolate Factory—Given certain specifications, students are asked to design a vat that is used as part of the mechanical stirring equipment in a chocolate factory. The topics included are: the unit circle, definition of sine and cosine, the Pythagorean theorem, similar triangles, right triangle trigonometry, law of cosines, angular velocity, and the intuitive meaning of boundedness (limits).

- The Discus Throw—Students are asked to derive a formula that will

determine the distance a discus has been thrown based on angle measurements made by electronic devices. The topics included are: right triangle trigonometry, the law of sines, and the law of cosines.

UNIT 5—Modeling with Matrices

Students study how matrices can be used to solve systems of equations and to model inventory-type settings. Problems involve handling and summarizing large arrays of data.

- The Sound Store—Students are asked to develop an efficient mathematical model for organizing and managing inventory data related to the growth of a small chain of retail music outlets selling CDs, tapes, records, and related products. The topics included are: organization and analysis of data and the basic matrix operations of addition, subtraction, scalar multiplication, matrix transposition, and multiplication.

- Fishing Survey—Students are asked to design a management program for fish that would allow the species to prosper but also would allow for a successful fishing industry. The topics included are: inverse of a square matrix; solution of $n \times n$ systems of linear equations using matrices; and the relationship among the existence of an inverse, the unique solution of an $n \times n$ linear system, and the value of the determinant of a square matrix.

- Sizing Up Real Estate—Students are asked to determine the areas of irregularly shaped lots in order to help a real estate development company determine selling prices and estimate property taxes. The topics included are: areas of geometric figures using coordinates and determinants and volumes of parallelepipeds whose vertices are known.

- Markov's Mini-Mall—Students are asked to analyze movement among several stores in a mall. The topics included are: powers of matrices, elementary probability, limit matrices, and the mathematics of Markov chains.

UNIT 6—Modeling with Other Functions

This unit asks students to pull together what they've learned about various types of functions and to see the effects of combining functions by addition, subtraction, multiplication, division, and composition.

- Rates of Change—Students are asked to calculate the values for

the rates of change for various functions, then to look at these rates of change geometrically, and finally to develop the rate-of-change functions using the difference quotient. This task set gives the conceptual foundation for the development of derivatives. The topics included are: difference quotients, constant rate of change vs. varying rates of change, increasing and decreasing functions, and rate-of-change functions.

- Going Forward by Looking Back—Students are asked to investigate three scenarios, one involving drug dosage and two others involving animal populations. The problems presented in these scenarios can be solved using recursive function models. The students also investigate the effects of the initial values and various parameters on the functions they have developed. The topics included are: arithmetic and geometric sequences, recursion, and iteration.

- Bungee Jumping—Students are asked to develop a model for an amusement park ride. They must also evaluate the safety of the ride. The topics included are: linear, exponential, and trigonometric functions; damped harmonic motion; composition of functions; addition and multiplication of functions; and parametric equations.

The Curriculum and Evaluation Standards for School Mathematics

The Curriculum Standards for grades 9 through 12 are divided into 14 groups. Following is a list of those groups and the specific NCTM standards for the mathematics curriculum in grades 9–12 that apply to Pacesetter Mathematics.

1. Mathematics as Problem Solving—"The curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content; apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics; ... apply the process of mathematical modeling to real-world problem situations."
   See Pacesetter Units 1, 2, 3, 4, 5, and 6.

2. Mathematics as Communication—"The curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can reflect upon and clarify their thinking about mathematical ideas and relationships; formu-

late mathematical definitions and express generalizations discovered through investigations; express mathematical ideas orally and in writing; read written presentations of mathematics with understanding; ...appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas."

See Pacesetter Units 1, 2, 3, 4, 5, and 6.

3. Mathematics as Reasoning—"The curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can make and test conjectures; formulate counterexamples; follow logical arguments; judge the validity of arguments; construct simple valid arguments."

   See Pacesetter Units 1, 2, 3, 4, 5, and 6.

4. Mathematical Connections—"The curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can recognize equivalent representations of the same concept; relate procedures in one representation to procedures in an equivalent representation; use and value the connections among mathematical topics; use and value the connections between mathematics and other disciplines."

   See Pacesetter Units 1, 2, 3, 4, 5, and 6.

5. Algebra—"The curriculum should include the continued study of algebraic concepts and methods so that all students can represent situations that involve variable quantities with expressions, equations, inequalities, and matrices; use tables and graphs as tools to interpret expressions, equations, and inequalities; operate on expressions and matrices, and solve equations and inequalities; appreciate the power of mathematical abstraction and symbolism; use matrices to solve linear systems."

   See Pacesetter Units 1, 2, 3, 4, 5, and 6.

6. Functions—"The curriculum should include the continued study of functions so that all students can model real-world phenomena with a variety of functions; represent and analyze relationships using tables, verbal rules, equations, and graphs; translate among tabular, symbolic, and graphical representations of functions; recognize that a variety of problem situations can be modeled by the same type of functions; analyze the effects of parameter changes on the graphs of functions; understand operations on, and the general properties and behavior of, classes of functions."

   See Pacesetter Units 1, 2, 3, 4, and 6.

7. Geometry from a Synthetic Perspective—"The curriculum should include the continued study of the geometry of two and three dimensions so that all students can represent problem situations with geometric models and apply properties of figures."
   See Pacesetter Units 1, 2, 4, and 5.

8. Geometry from an Algebraic Perspective—"The curriculum should include the study of the geometry of two and three dimensions from an algebraic point of view so that all students can translate between synthetic and coordinate representations; deduce properties of figures using transformations and using coordinates; apply transformations, coordinates, ... in problem solving."
   See Pacesetter Unit 5.

9. Trigonometry—"The curriculum should include the study of trigonometry so that all students can apply trigonometry to problem situations involving triangles; explore periodic real-world phenomena using the sine and cosine functions; understand the connection between trigonometric and circular functions; use circular functions to model periodic real-world phenomena; apply general graphing techniques to trigonometric functions; solve trigonometric equations."
   See Pacesetter Units 4 and 6.

10. Statistics—"In grades 9–12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations; use curve fitting to predict from data."
   See Pacesetter Units 1, 2, 3, 4, 5, and 6.

11. Probability—In Pacesetter Mathematics, students use the basic concepts of elementary probability in the task set involving Markov chains.
   See Pacesetter Unit 5.

12. Discrete Mathematics—"In grades 9–12, the mathematics curriculum should include topics from discrete mathematics so that all students can represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations; represent and analyze finite graphs using matrices."
   See Pacesetter Units 2, 5, and 6.

13. Conceptual Underpinnings of Calculus—"In grades 9–12, the mathematics curriculum should include the informal exploration of calculus concepts from both a graphical and a numerical perspective so that all students can determine maximum and minimum points of a graph and interpret the results in problem situations; investigate limiting processes by examining infinite sequences and series and areas under curves; understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line ... analyze the graphs of polynomial, rational ... and transcendental functions."
    See Pacesetter Units 1, 2, 3, 4, 5, and 6.

14. Mathematical Structure—In Pacesetter Mathematics, mathematical structure is addressed in the Fishing Survey task set.
    See Pacesetter Unit 5.

The charts that follow show the correlation between the 25 Pacesetter task sets and the Curriculum Standards. The fourteenth standard, Mathematical Structure, is not included in the charts.

The Professional Standards for Teaching Mathematics

The Professional Standards are divided into four groups: Standards for Teaching Mathematics, Standards for the Evaluation of the Teaching of Mathematics, Standards for the Professional Development of Teachers of Mathematics, and Standards for the Support and Development of Mathematics Teachers and Teaching. Although the Pacesetter Mathematics Program embodies the principles of all of the groups through its teacher training and support, Pacesetter's written materials most closely reflect the first group of standards. Therefore these are the standards that will be addressed in this document.

Standards for Teaching Mathematics

The following assumptions, upon which the *Professional Standards for Teaching Mathematics* are based, are an integral part of the philosophy of Pacesetter Mathematics.

1. The goal of teaching mathematics is to help all students develop mathematical power.

2. What students learn is fundamentally connected with how they learn it.

3. All students can learn to think mathematically.

NCTM CURRICULUM STANDARDS

PACESETTER TASKS	1	2	3	4	5	6	7	8	9	10	11	12	13
Stacks of Cups	◆	◆	◆	◆	◆	◆	◆			◆		◆	
The Mile Run	◆	◆	◆	◆	◆	◆				◆			
Ships in the Fog	◆	◆	◆	◆	◆	◆							◆
Temperature Scales	◆	◆	◆	◆	◆	◆							
Square Craft Design	◆	◆	◆	◆	◆	◆	◆					◆	◆
Pennsylvania Lottery	◆	◆	◆	◆	◆	◆				◆		◆	◆
A Powerful Function	◆	◆	◆	◆	◆	◆							◆
Population Growth	◆	◆	◆	◆	◆	◆				◆			
Saving to Buy a Car	◆	◆	◆	◆	◆	◆						◆	
Fences	◆	◆	◆	◆	◆	◆		◆					◆
Highway Safety	◆	◆	◆	◆	◆	◆				◆			◆
Planning a Summer Camp	◆	◆	◆	◆	◆	◆							◆
Graphs of Rational Functions	◆	◆	◆	◆	◆	◆							◆
Ideal Traffic Pattern	◆	◆	◆	◆	◆	◆							◆
Bicycle Wheels	◆	◆	◆	◆	◆	◆			◆				
Daylight Hours	◆	◆	◆	◆	◆	◆	◆		◆	◆			◆
Chocolate Factory	◆	◆	◆	◆	◆	◆	◆		◆				◆
Discus Throw	◆	◆	◆	◆	◆	◆	◆		◆				
Sound Store	◆	◆	◆	◆						◆		◆	
Fishing Survey	◆	◆	◆	◆	◆					◆		◆	
Sizing Up Real Estate	◆	◆	◆	◆	◆		◆	◆				◆	
Markov's Mini-Mall	◆	◆	◆	◆	◆					◆	◆	◆	◆
Rates of Change	◆	◆	◆	◆	◆	◆			◆	◆			◆
Going Forward by Looking Back	◆	◆	◆	◆	◆	◆				◆		◆	◆
Bungee Jumping	◆	◆	◆	◆	◆	◆			◆				◆

PACESETTER MATHEMATICS UNITS

CURRICULUM STANDARD	1 – Linear	2 – Exp/Log	3 – Poly/Rat	4 – Trig	5 – Matrices	6 – Other
Mathematics as Problem Solving	◆	◆	◆	◆	◆	◆
Mathematics as Communication	◆	◆	◆	◆	◆	◆
Mathematics as Reasoning	◆	◆	◆	◆	◆	◆
Mathematical Connections	◆	◆	◆	◆	◆	◆
Algebra	◆	◆	◆	◆	◆	◆
Functions	◆	◆	◆	◆		◆
Geometry – Synthetic Perspective	◆	◆		◆	◆	
Geometry – Algebraic Perspective					◆	
Trigonometry				◆		◆
Statistics	◆	◆	◆	◆	◆	◆
Probability					◆	
Discrete Mathematics		◆			◆	◆
Conceptual Underpinnings of Calculus	◆	◆	◆	◆	◆	◆

4. Teaching is a complex practice and hence not reducible to recipes or prescriptions.

These assumptions are reflected in the six teaching standards discussed below.

Standard 1. Worthwhile Mathematical Tasks

The Pacesetter mathematics course framework is built around the 25 task sets or problem situations that were described earlier. The mathematical concepts are developed as the students work through the tasks. Students are initially asked to explore a problem in pairs or groups, and thus are given an opportunity to use their previous knowledge of relevant mathematical skills and concepts. This approach allows for diverse approaches and individual problem-solving techniques.

For example, when students are asked in a task in Unit One to determine whether two ships collide, they can use a range of methods, from basic arithmetic to complex algebra, to answer the question, depending on their past experiences and problem-solving styles. As they progress through the tasks, previous knowledge developed in the course can be used in the problem-solving process. In Unit Two, students can use the knowledge of logarithms developed in the Powerful Function task set to answer questions posed in the Population Growth and Saving to Buy a Car task sets.

The diverse scenarios among the 25 task sets appeal to the various interests of students. Those interested in sports are intrigued by the Mile Run and Discus Throw task sets, while those with financial interests find the Pennsylvania Lottery and Saving to Buy a Car tasks interesting. Scientific interests are addressed in the Daylight Hours task set and the medicine problem in the Going Forward by Looking Back task set. Many students find something of interest in each task set.

Communication about mathematics plays a major role in Pacesetter Mathematics. Students are asked to write in each of the task sets. The writing assignments range from explanations of the methods used to arrive at their solutions to preparation of an investment proposal outlining the pros and cons of various investment scenarios.

Standard 2. The Teacher's Role in Discourse

In the detailed teacher notes that accompany each Pacesetter task set, teachers are encouraged to orchestrate discourse by posing questions that challenge each student's thinking. They are also encouraged to listen to and respect students' ideas. Teachers should ask students to clarify and justify their ideas orally and in writing. In the Square Craft Design task set, students are asked to reproduce a given design and then to summarize their approach, using enough detail so that someone reading the summary could

reproduce the design. The teacher notes point out that individuals and/or groups will approach re-creating the design in different ways. Sharing of these ways among students helps assure them that different ways of approaching a problem are encouraged, valued, and respected.

The writing assignment mentioned above also provides an opportunity for the teacher to ascertain the mathematical language level of the students, particularly with respect to geometry. Based on this information, appropriate language assignments can be made.

As the students are working in their groups, the teacher's role is to circulate around the room, monitoring the interactions within the groups and asking probing questions when necessary to direct or redirect the discussion. The teacher must decide when to provide information, when to clarify an issue, when to model, when to lead, and when to let the students struggle with a difficulty.

Standard 3. Students' Role in Discourse

Much of the exploration of the initial problem posed in each task set takes place in small-group discussion. Students are encouraged to listen to, respond to, and question each other. They are also encouraged to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers.

Students use a variety of tools to reason, make connections, solve problems, and communicate. For example, in Unit Four—Modeling with Trigonometric Functions—the first task set, Bicycle Wheels, involves developing a model using geometric concepts; the second task set, Daylight Hours, involves developing a model from real-life data; the third task set, The Chocolate Factory, lends itself to the development of a physical model to aid in understanding the mathematics needed to solve the problem; and the fourth task set, The Discus Throw, uses the mathematics developed in the preceding task set to develop a model.

Standard 4. Tools for Enhancing Discourse

A graphing calculator is a required tool for the course. It is used for the exploration of data and data analysis, for the study of graphs, and for the development of models, among other things. Teachers are also encouraged to have available for student use other tools such as rulers, tape measures, straight edges, graph paper, etc. Some task sets provide the opportunity for exploration by computer. Students are encouraged to use the information highway for data collection and sharing of ideas about the tasks with other Pacesetter students. As mentioned in the discussion of Standard 3, concrete models are used in some task sets. Data are explored using tables and graphs, as well as the graphing calculator. In the Mile Run and Population Growth task sets the graphing calculators are used to develop the models.

Standard 5. Learning Environment

The Pacesetter Mathematics teacher is expected to create a learning environment that fosters the development of each student's mathematical power by adhering to the specific behaviors listed in this standard, namely by providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems; using the physical space and materials in ways that facilitate students' learning of mathematics; providing a context that encourages the development of mathematical skill and proficiency; and respecting and valuing students' ideas, ways of thinking, and mathematical dispositions.

The issue of development of students' mathematical skills is addressed in the teacher notes by suggesting when it is appropriate to use the text for this purpose. For example, in the Bicycle Wheels task set, the text is used to study the effect of the various parameters on the graphs of the trigonometric functions after the graphical model is developed in the task set, but before the function model is determined.

As mentioned earlier, much of the initial exploration in the task sets is done in small groups. However, since each student is responsible for his or her own learning, the teacher is encouraged to balance the group activities with opportunities for individual study.

Standard 6. Analysis of Teaching and Learning

The Pacesetter Mathematics teacher is expected to engage in ongoing analysis of teaching and learning by observing, listening to, and gathering other information about students to assess what they are learning, and by examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions. Assessment in Pacesetter Mathematics will be discussed in detail in the section on the relationship between Pacesetter Mathematics and the NCTM's Assessment Standards document.

Assessment Standards for School Mathematics

The Assessment Standards were designed to expand on and complement the Evaluation Standards in the NCTM's *Curriculum and Evaluation Standards* document. Assessment is defined as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes" (NCTM, 1991, p. 3). Evaluation refers to "the process of determining the worth of, or assigning a value to, something on the basis of careful examination and judgment" (NCTM, 1991, p. 3).

Assessment, as defined above, is an integral part of the Pacesetter Mathematics course. The primary purpose of assessment is to foster learning and

enhance instruction. It is expected that assessment will be a part of instruction and will be continuous throughout the course, not just a periodic event.

Three dimensions are assessed throughout the course: Mathematical Knowledge, Communication, and Application/Modeling. Individual learning is assessed and is the primary focus. However, the individual's participation in and contribution to the group learning process are also assessed.

In addition to the various assessments that occur throughout the course, an annual standardized assessment is provided to teachers toward the end of the course. This assessment is expected to be given to all Pacesetter Mathematics students. The standardized assessment consists of two parts: a multiple-choice section that focuses on mathematical knowledge and a free-response section that focuses on modeling and communication.

Mathematics Assessment Standards

Standard 1. The Mathematics Standard

Assessment should reflect the mathematics that all students need to know and be able to do.

The target audience for Pacesetter Mathematics consists of those students who have successfully completed the equivalent of two years of algebra and one year of geometry. Therefore, the content is at the level of the fourth year of secondary school mathematics. Since the central focus of the course is to introduce students to the nature and use of mathematical models to reason about quantitative problems in their lives and the world, the subject matter includes what all of these students need to know and be able to do. The mathematical life skills developed in the course include communication about mathematical ideas and concepts and their application. The assessments that accompany the task sets, as well as the standardized assessment, address these life skills.

The assessments engage students in realistic and worthwhile mathematical activities. For example, in the Highway Safety task set in Unit Three, students are asked to present an analysis of the "two-second" rule and the "one-car-length" rule for safe stopping distance in a format that could be handed out to students in a driver education class to help them become safer drivers. This activity tests the student's ability to communicate and to apply the mathematical knowledge gained by working through the task set to a realistic situation.

Standard 2. The Learning Standard

Assessment should enhance mathematics learning.

The example of the assessment explained in the discussion of Standard 1 illustrates how assessment is linked to instruction and how assessment allows students to demonstrate what they know and what they can do in a novel situation. Assessments such as this occur throughout the course.

Standard 3. The Equity Standard
Assessment should promote equity.

Since many of the assessments throughout the course and some part of the standardized assessments are open-ended and require explanations of methods, students have the opportunity to demonstrate what they know and can do. Credit can be given for different approaches and problem-solving methods.

Standard 4. The Openness Standard
Assessment should be an open process.

Pacesetter Mathematics teachers are encouraged to share the definitions of the three assessment dimensions and the scoring standards for assessment tasks with the students. They are also encouraged to have each student develop a portfolio of work that includes his or her reflection on the mathematics learned in each unit and the progress being made along each of the dimensions.

Standard 5. The Inference Standard
Assessment should promote valid inferences about mathematics learning.

In Pacesetter Mathematics, multiple sources of evidence are used for making inferences about student performance: program-provided tasks in the units; teacher-developed written assessments; teacher observation, questioning, and listening; portfolios; and a standardized assessment given under timed conditions. Efforts are made in the initial training and in the midyear and refresher meetings to have teachers share student papers and discuss the evaluation of these papers. Teachers in individual schools and districts are also encouraged to meet to discuss assessment issues. In these ways it is hoped that some modicum of consistency of scoring can be achieved.

Standard 6. The Coherence Standard
Assessment should be a coherent process.

In Pacesetter Mathematics, the assessments are an integral part of the instruction. Many are embedded within the task sets. Collectively, they address each of the three assessment dimensions—mathematical knowledge, communication, and application/modeling. The standardized assessment consists of two parts in the free-response portion: a group activity and individual activities. In this way, the standardized assessment reflects the instructional practice to which the students have been exposed throughout the course.

References

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Reston, Va.: The Council.

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