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ABSTRACT

Contemporary constructivist views of mathematical learning have encouraged curriculum developers to devise instructional materials that help students build their own understanding and procedures for doing rational number computations, solving proportions, and applying those skills to real and whimsical problems. The Connected Mathematics Project (CMP) curriculum supports construction of rational number knowledge by presenting students with a series of units based on contextual problems that require proportional reasoning and computation. The goal of this study was to describe the character and effectiveness of proportional reasoning by students with different curricular experiences as they face problems in which ratio and proportion ideas are appropriate and useful. Performance task papers and follow-up interviews with selected students from the study indicated that, in addition to a greater frequency of correct answers and reasoning compared with control group students, CMP students appeared to have developed greater ability to articulate their thinking. Students from CMP classes had a generally broader and more flexible repertoire of strategies available for problem solving. The results suggest that problem-based curriculum and instruction can be effective in helping students construct effective personal understanding and skill in one of the core strands of middle grade mathematics. (PVD)

Development of Proportional Reasoning in a Problem-Based Middle School Curriculum

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Many important kinds of arithmetic problems require operations on and interpretation of common fractions, decimals, percents, ratios, and proportions. In traditional middle school curricula, each arithmetic operation with each type of rational number is taught with a focus on developing student proficiency in well-defined computational algorithms that are then practiced to ensure speed and accuracy of execution. Only when that computational proficiency is attained are students challenged to apply their computational skill to practical or fanciful "word problems."

Contemporary constructivist views of mathematical learning have encouraged curriculum developers to devise instructional materials that help students to build their own understanding and procedures for doing rational number computations, solving proportions, and applying those skills to real and whimsical problems. The Connected Mathematics Project (CMP) curriculum supports that construction of rational number knowledge by presenting students with a series of units based on contextual problems that require proportional reasoning and computation. Students collaborate in work on the problems, sharing their insights and approaches with partners and then with the whole class through *Mathematical Reflection* discussions and journal writing. At no point in the CMP curriculum materials are students shown any standard algorithms for fraction or decimal computations or for solving problems involving percents, ratios, or proportions. However, students focus on situations requiring proportional reasoning in at least eight of the 24 units comprising the CMP middle grades program.

The striking difference between traditional and CMP approaches to rational number and proportional computation and problem solving raises a very natural and fundamental question:

How do the conceptual understanding, computational skills, and problem solving strategies and success of CMP and traditional curriculum students compare?

In particular, it is natural to wonder whether the new CMP approach does successfully lead students to construct effective (accurate and/or efficient) strategies for fraction, decimal, percent, and proportional computation and whether CMP students develop flexible and/or effective strategies for solving contextual problems involving rational numbers and proportions. During the 1994-95 school year the researchers conducted an extensive study of proportional reasoning in seventh grade CMP and control classes to get data that might help to answer the important questions about CMP approaches to rational numbers and proportional reasoning. We made a follow-up study in eighth grade classes during the 1995-96 school year.

We began with the notion that ratio and proportion questions are fundamentally about comparison. Following ideas of Freudenthal, we focused on three broad families of situations in which ratio and proportion ideas are appropriate and useful:

- Comparing two parts of a single whole, as in the "the ratio of girls to boys in a class is 15 to 10" or "a segment is divided in the golden ratio."

- Comparing magnitudes of different quantities with an interesting connection, as in "miles per gallon" or "people per square kilometer" or "kilograms per cubic meter". These comparisons are not generally called ratios, but rates or densities.

- Comparing magnitudes of two quantities that are conceptually related, but not naturally thought of as parts of a common whole, as in "the ratio of sides of two triangles is 2 to 1." These comparisons are sometimes referred to as scalings and they include questions of stretching and shrinking in similarity transformations.

The basic goal of our study was to describe the character and effectiveness of proportional reasoning by students with different curricular experiences as they face problems in these three broad categories.

Data Collection

Data for the study came from three main sources:

(1) Samples of CMP and control students were given performance tasks requiring proportional reasoning in three primary contexts—one involving reasoning about scaling such as is common in geometric questions of similarity (5 problems); one involving reasoning about a conventional situation involving ratios of discrete quantities (4 problems); and one involving situations that are often analyzed using concepts of unit rates (5 problems). These paper and pencil instruments were administered in seventh

grade (1995) and eighth grade (1996) CMP field test and control group classes as part of the regular CMP evaluation studies.

(2) To get more personal and in-depth understanding of the student thinking represented in written responses to the performance tasks, the investigators conducted individual follow-up interviews with a 20% sub-sample of the seventh grade subjects. Following questions about a sample of contextualized problems, we asked students to demonstrate their thinking on context-free computations paralleling those in the problem-based tasks to see the strategies they use (or fail to use) in those purely computational tasks.

(3) To understand the curricular and instructional experiences of CMP and control students, as a backdrop for interpreting the performance task and interview results, the investigators conducted interviews with the CMP and control class teachers to assess proportional reasoning topics they covered and the point of view with which those topics were presented.

Our data analysis looked for the following kinds of understanding and skill in each proportional reasoning problem family: (1) Can students effectively compare ratios to determine equivalence or to identify the larger or smaller of two ratios? (2) Can students solve "missing value" problems in the various forms they occur? (3) Can students recognize situations in which ratio or proportion comparisons are appropriate and others in which different sorts of comparison are more meaningful, and can they represent ratio and proportion information flexibly and accurately? We have sought to characterize the accuracy of student reasoning in various contexts, varieties of strategies employed by students with different curricular experience, frequent and especially interesting misconceptions, and connections between student performance and the intended and implemented curricula of their classes.

Results

Student responses to each performance task item were categorized using the following six part scheme:

Correct

- Correct answer without supporting work or reasoning
- Correct answer with appropriate supporting work or reasoning
- Correct answer with incorrect or inappropriate support work or reasoning

Incorrect

- Incorrect answer without supporting work or reasoning
- Correct thinking but incorrect conclusion
- Incorrect answer and incorrect reasoning

Data from the papers of seventh grade CMP and control class students are summarized in Table 1. They suggest markedly better performance by CMP students, especially if one looks at the category "correct answer and appropriate reasoning" where the advantage is roughly 2 to 1 in percent correct.

Table 1

Overall Results on Seventh Grade Proportional Reasoning Assessment

(All numbers in this table are in percents.)

	RATE		RATIO		SCALING		OVERALL	
	CMP N=124 P=496	CNT N=91 P=364	CMP N=124 P=372	CNT N=85 P=255	CMP N=126 P=630	CNT N=80 P=400	CMP P=1498	CNT P=1019
Correct Responses:								
Answer Only	2	5	5	4	4	4	3	4
Correct Reasoning	51	28	43	21	36	16	43	21
Incorrect Reasoning	7	14	9	20	1	4	5	12
Incorrect Responses:								
Answer Only	2	6	4	5	17	17	9	10
Correct Reasoning	13	10	6	5	2	2	7	6
Incorrect Reasoning	18	28	19	20	23	30	20	27
No Response	7	9	14	25	17	27	13	20

To see whether differences observed in the seventh grade data were an artifact of greater CMP emphasis on proportional reasoning in grade seven, we repeated the paper-and-pencil performance assessment with a somewhat smaller eighth grade sample in 1996. Results from that study are summarized in Table 2. Again, the CMP students outperformed students in control classes.

Table 2

Overall Results on Eighth Grade Proportional Reasoning Assessment

(All numbers in this table are in percents.)

	RATE		RATIO		SCALING		OVERALL	
	CMP N=105 P=525	CNT N=37 P=185	CMP N=107 P=535	CNT N=39 P=195	CMP N=108 P=648	CNT N=38 P=228	CMP P=1708	CNT P=608
Correct Responses:								
Answer Only	1	2	4	4	2	5	3	4
Correct Reasoning	68	29	39	18	33	14	46	20
Incorrect Reasoning	5	16	10	14	3	4	6	11
Incorrect Responses:								
Answer Only	1	3	2	14	7	25	3	15
Correct Reasoning	4	4	11	3	5	1	6	2
Incorrect Reasoning	17	34	25	29	38	32	27	32
No Response	4	12	9	18	12	19	9	16

Inspection of the performance task papers and follow-up interviews with selected students in the seventh grade study suggested that, in addition to a greater frequency of correct answers and reasoning, CMP students seem to have developed greater ability to articulate their thinking.

In an effort to see how performance of CMP and control class students compared to findings of prior research on proportional reasoning, the first author made a detailed strategy analysis of work on the seventh grade performance tasks. He discovered that common correct and incorrect procedures were those predicted by the literature (Tourniaire and Pulos, 1985; Behr, Post, and Lesh, 1992). However, he also discovered that students from CMP classes had a generally broader and more flexible repertoire of strategies available.

In general, the results of our studies suggest that the sort of problem-based curriculum and instruction recommended by recent reform proposals like the NCTM *Curriculum and Professional Teaching Standards* can be effective in helping students construct effective personal understanding and skill in one of the core strands of middle grades mathematics. It appears that students can learn in ways other than the traditional American "show and tell and practice" style, developing their own understanding which is then articulated and formalized in reflective discourse with other students and their teacher. Of course, it is also quite clear that, even for CMP students, there is considerable room for growth in both understanding and skill. The various facets and applications of proportional reasoning are absolutely fundamental mathematical tools that students should acquire from middle grades instruction. Unfortunately, the topic has always been a great challenge for many students, and further research and development work would be well-advised.

References

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[More detailed discussion of the procedures and results of the studies are being prepared for submission to research journals. Further information can be obtained from the second author at the Department of Mathematics, University of Maryland, College Park, MD 20742]

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