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ABSTRACT

This paper reports on a study of seventh grade students (N=4) who attend a rural K-12 school. Students participated in a 5-week teaching experiment designed to build on their existing knowledge of the unit concept and extend it to the rational number operations of multiplication and division. Data collected comes in the form of videotapes, audiotapes, researcher journal, and students' written work. Other data consist of individual student interviews conducted prior to and at the conclusion of the teaching experiment. Analysis of the results reveals that students' concepts of unit are enriched by participating in the teaching experiment. Among the conclusions drawn were that students developed a flexible concept of unit, modeling provided continuity between conceptual domains, equipartitioning remained a persistent difficulty, sustained focus on the measuring unit is hard to achieve, development of models is impeded by students' selection and use of measurement units, unitizing skills endure and are extendible, and models can inform procedural methods and/or provide alternative solution methods. (DDR)

Extending Meaning of Multiplication and Division of Rational Numbers

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Rational number concepts are generally considered to be among the important mathematical ideas children encounter during the middle grades. Yet the rational number domain is known for great difficulties it presents for both students and teachers. A major factor contributing to the complexity of this domain concerns the limited models students and teachers hold for both multiplicative reasoning (Greer, 1992) and rational number understanding (Behr, Harel, Post, & Lesh, 1992). As students begin to construct knowledge of fractions, they search for connections to existing mathematical knowledge. They may try to use procedures acquired from the whole number domain, only to become frustrated because these old rules won't work on the new numbers! Recent research has focused on the basic concept of unit as a way to link the whole number and rational number domains (Alexander, 1996; Behr, Harel, Post, & Lesh, 1994; Golding, 1994).

Studies have suggested that students can intuitively form units, and further, that this intuitive knowledge of units can be used as a foundation for rational number understanding (Lamon, 1994; Mack, 1993). One recent study revealed that a focus on the unit can serve as a link between whole number and rational number domains for addition and subtraction (Golding, 1994). This study examined the role of the unit as a link between whole number and rational number domains for multiplication and division. Additionally, it explored whether students' learning could carry over from the group setting to individual performance and whether their new understandings could be applied to standard school tasks.

Four students of varying mathematical ability were selected from one seventh-grade mathematics class from a rural K-12 school. The group participated in a five week teaching experiment designed to build on their existing knowledge of the unit concept and then to extend this to the rational number operations of multiplication and division. Data were collected daily through videotapes, audiotapes, researcher journal, and students' written work. Other data consist of individual student interviews conducted both prior to and at the conclusion of the teaching experiment.

Individual initial interviews were conducted during the first week of the teaching experiment in order to determine students' existing fraction concepts and abilities. Students participated in fifteen lessons during the experiment, each designed to illustrate various aspects of the unit concept such as unitizing, reunitizing, and norming. The lessons focused on four areas on instruction: (a) unitizing in whole number situations, (b) model-building for multiplication and division of whole numbers, (c) unitizing in rational number situations, and (d) model-building for multiplication and division of rational numbers. Analysis of the unit as a connector between the whole number and rational number domains involved examination of students' procedures and responses during the final phase of lessons and on the exit interviews. Additionally, the exit interviews during the last week of the study sought to determine the degree of independence of thinking students achieved through the instructional experience and the degree to which the concept of unit could inform students' procedural methods of the usual school curriculum.

Analysis of the results from this teaching experiment reveal that students' concepts of unit were enriched by participating in the teaching sequence. Several conclusions were drawn from the research findings.

1. *The students developed a flexible concept of unit*. The lessons were successful in enabling students to extend their existing knowledge of fractions to include awareness of the unit in fractional situations, verbalization of the meaning of the unit fraction, the necessity of equality in partitioning, and ability to rename a unit as the unit whole changed. One of the most critical understandings in rational numbers is that the numerical symbol, such as 1/3, can represent different amounts, depending upon what the unit whole happens to be. Students used rods and fraction circles to explore such relationships. Students were successful in building on their existing skills in unitizing and norming to extend their part-whole interpretation of fractions so as to envision composite units as iterations of unit fractions. In subsequent activities, students used decomposition skills to develop a model for unit reconstruction tasks. Such an ability, indicating that the student perceives the unit as an amount and not simply as a command for the action of dividing up the whole into parts, is considered critical by many researchers for development of a mature concept of rational numbers (Behr et al., 1992).

2. *Modeling provided continuity between conceptual domains*. By focusing on the unit in each fractional situation, the students were successful in expanding models for multiplication and division of whole numbers and in extending these models to the rational number domain. The use of a variety of manipulatives during the study served to broaden students' concepts of unit and to facilitate linkage of concepts from whole number to rational number domains. For example, students developed the area model for multiplication of whole numbers. When the lessons progressed to models for rational numbers, embedded units were conceptualized through the use paperfolding tasks which initially utilized folding techniques from only one direction. As the folding strategies evolved into folding from two directions, the students quickly recognized the similarity of the resulting model for rational numbers to their area model for whole numbers. Thus they associated the task of finding a unit fraction of a unit fraction with the operation of multiplication of rational numbers.

3. *Equipartitioning remained a persistent difficulty*. Students' concepts about the part-whole relationship were limited initially because of their unconcern with equality in partitioning and their weak skills in partitioning. The lack of awareness of equality in partitioning continued to impact unit formation abilities of two students during much of the study. While this problem was overcome for all but one student during the study, it persisted far longer than anticipated.

4. *A sustained focus on the measuring unit is hard to achieve*. A persistently recurring problem emerging in the process of expanding models for multiplication and division concerned students' loss of sight of the measuring unit. As students engaged in tasks requiring the comparison of two units, they frequently confused the measuring unit or ruler with the object being measured. Such confusion resulted in an answer which was the inverse of the one being requested. Only with a concerted effort of focusing on the measuring unit were students able to minimize the interference of this obstacle. Division situations in which students were unable to overcome loss of sight of the unit involved problems in which the rulers were longer than the rods being measured. In such situations, the tendency of students was to exchange the units in the problem situation, thus illustrating a constraint of primitive models of division: the divisor must be less than the dividend. Confusion also arose in the development of the area model for multiplication because of the change in unit structure from linear to area units. Students' instinctive response

was that, instead of square units, the "answer" should be of the same unit as the sides of the rectangle.

5. *The development of models was handicapped by students' selection and use of measurement units*. Students experienced difficulties in selecting the appropriate measurement unit for linear or area measurement tasks. For example, in determining the length of a side, students debated between counting the points or the segments or the squares along the side. If students chose to count the points, they began their count with "one." If they counted the squares or boxes, they displayed the propensity not to count the corner square twice. Students' modeling efforts were further hampered by weak measurement skills which caused impreciseness in measurements.

6. *Unitizing skills endure and are extendable*. All of the students were successful in solving most of the problems in the exit interview. In fact, the students' performance in the exit interviews often demonstrated a deeper understanding of unit concepts than was displayed during the group sessions. For example, Task 5 of the exit interview presented three dots which represented 1/4-unit and required students to determine 2/3-unit. Melanie, who had experienced no success with similar tasks during the teaching sequence, demonstrated sound understanding of the underlying unit structure of the task. She immediately selected twelve chips and formed a 3 x 4 array. She explained "that's my whole [and] I'm doing this so I can separate it into thirds." She refocused attention from the four columns, each representing 1/4-unit, to the three rows by widening the space between the rows. She explained the arrangements of rows and columns as two different unit interpretations. Pointing out the columns, Melanie remarked, "this is how they were in fourths." Shifting her attention to the rows, Melanie concluded, "and this is how they were in thirds."

Results from the exit interview provide evidence that students do internalize unit concepts and rational number concepts acquired through the group learning experience. Further, the knowledge students acquired in unitizing and norming is not only strong it is extendable in that some of the students could use their knowledge to solve more complex problems than were presented during the instructional sequence. The embedded units tasks during the teaching sequence were limited to finding a unit fraction of a unit fraction. The exit interview addressed such tasks as well by asking students to find 1/3 of 1/5-unit. As all the students were successful in this endeavor, they were then asked to find 1/3 of the composite fraction 2/5. While only one student was unable to quantify the result, all students successfully modeled the extension task. In another example, students were required to equipartition an already partitioned unit whole -- a problem not addressed during the teaching sequence. Efforts on a similar task in the initial interview were unsuccessful for all students except one, and this student required prodding to complete the task. Whereas this same student again required assistance on the exit interview, the other students were able to solve the task alone. Likewise, reconstitution tasks during the teaching sequence were confined to finding the unit whole from a unit fraction; however, the exit interview required students to find a specified composite unit, 2/3, when given only a unit fraction, 1/4. All students recognized the immediate necessity of establishing the unit whole and then sought to reunitize this unit whole into the necessary norming unit to obtain the composite fraction 2/3.

7. *Models can inform procedural methods and/or provide alternative solution methods*. None of the students could model rational number problems at the outset of the teaching experiment, yet all of them were successful on the exit interview in using models to solve the problems. Additionally, students' confidence in their models was stronger than their confidence in procedural solution methods, as evidenced by their consistent efforts to alter their procedural methods in order to obtain the same answer dictated by their model. For one student, this was in stark contrast with her performance on the initial interview. At that time, Judy was successful in solving rational number problems, but had difficulty determining appropriate models. In choosing which answer to accept -- the one determined by her model or the one determined by her calculation -- she preferred her calculation. Judy had strong confidence in her procedural solution while little confidence in the answer displayed by her model. Having the correct model in the exit interview did not always enable students to solve the task procedurally; however, the ability to model problem situations served to provide them with an alternative solution method.

This study points to the need for more school practice in partitioning and measurement activities and more extensive use of modeling to facilitate development of unit concepts. Future research should investigate strategies for students to overcome constraints of primitive models of division.

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