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ABSTRACT

This study examines teachers' mathematical understanding of various representations of slope and their knowledge for teaching the concept of slope. Specifically examined in this research are the concept images and concept definitions of slope held by both preservice (N=18) and inservice (N=21) secondary mathematics teachers. Data collected include transcriptions of audiotaped interviews and surveys which are designed to probe teachers' knowledge of slope, their concept images and concept definitions, and their mathematical understanding. The process of analytic induction is the method used for analysis of the transcripts. Transcript analysis revealed patterns in individual teachers' responses as well as patterns in comparisons among various teachers' responses. The results of the mathematics survey provide a broad picture of secondary mathematics teachers' knowledge of the concept of slope and indicate general trends concerning the representations of slope. Contains 25 references. (DDR)

Secondary Mathematics Teachers' Knowledge of the Concept of Slope

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Among the recommendations of the National Council of Teachers of Mathematics' (NCTM) Professional Standards for Teaching Mathematics (NCTM, 1991), the education of teachers of mathematics should help them develop their knowledge of the following:

- mathematical concepts and procedures and the connections among them ("Standard 2: Knowing Mathematics and School Mathematics," p. 132);
- multiple representations of mathematical concepts and procedures ("Standard 2: Knowing Mathematics and School Mathematics," p. 132);
- ways to represent mathematics concepts and procedures ("Standard 4: Knowing Mathematical Pedagogy," p. 151).

To explore these various aspects of knowledge, several investigations have focused on teachers' knowledge of functions. Research has focused on preservice and experienced teachers. Stein, Baxter, and Leinhardt (1990) investigated a fifth-grade teacher's knowledge of functions and graphing and found that his limited understanding of functions affected his teaching. Norman (1992) found that experienced secondary mathematics teachers had gaps in their understanding of functions. Even (1993) and Wilson (1994) found that preservice secondary teachers also had a limited understanding of functions.

This investigation focused on another fundamental mathematical idea: the concept of slope. Slope is typically introduced in first-year algebra and then reappears in various forms throughout the secondary mathematics curriculum. A conceptual understanding of slope is especially crucial for the study of calculus or physics. Calculus typically begins with the study of derivatives and rates of change, using slopes of lines to develop these concepts. Physics assumes the ability of students to interpret slope as a functional relationship between two quantities. It is important that secondary mathematics teachers have a robust mathematical understanding of slope and be able to communicate ideas about slope to their students. This study examined teachers' mathematical understanding of various representations of slope and their knowledge for teaching the concept of slope.

Representations of Slope

According to Dreyfus (1991), to represent a mathematical concept is to generate an instance, specimen, example, or image of it. A symbolic representation is written or spoken; a mental representation refers to internal frames of reference (p. 31). Although several competing mental representations of a given concept may coexist for an individual, different ones may be called up for different mathematical situations. It is important to have many representations of a concept. Moreover, to support successful problem solving, the various representations must be correctly and strongly linked. The process of switching among representations is essential (p. 32).

We may consider the following representations of slope: geometric, algebraic, physical, trigonometric, functional, and ratio. Slope is defined geometrically as a property of a line: given two points (x_1, y_1) and (x_2, y_2), the slope of the line connecting the two points is $\frac{y_2 - y_1}{x_2 - x_1}$. Slope is represented algebraically as one of the parameters in a linear equation; slope is the m in the equation $y = mx + b$. Slope is also used to describe the steepness of physical objects such as mountain roads, ski slopes, and wheelchair ramps. Although slope may be referred to in everyday language as the angle of inclination, slope is mathematically the trigonometric tangent of the angle of inclination: $m = \tan \theta$. When functions are understood to be composed of two variables that have a dynamic relationship, slope represents the rate of change in one variable that results from a change in the other variable. Slope, of course, is a ratio, which the result of comparing two quantities multiplicatively (Thompson, 1994).

Theoretical Perspective

This investigation focused on three aspects of teachers' knowledge of slope: their concept images and concept definitions, their mathematical understanding, and their pedagogical content knowledge.

Concept Images and Concept Definitions

The theory of concept images and concept definitions (Tall & Vinner, 1981) is useful in analyzing teachers' knowledge of slope. A concept image is the total cognitive structure that is

associated with a given concept. A concept definition is a set of words used to specify a given concept. This investigation did not attempt to distinguish between teachers' concept images and concept definitions of slope. Rather, it considered concept images and concept definitions together as one component of teachers' knowledge, and it examined the various representations included in teachers' concept images and concept definitions of slope.

Mathematical Understanding

Flexible understanding of mathematics entails the ability to form relationships within mathematics as well as across disciplinary fields and to make connections to the world outside school (McDiarmid, Ball, & Anderson, 1989, p. 193). Meaningful learning of mathematics includes forming relationships between conceptual and procedural knowledge (Hiebert & Lefevre, 1986). Conceptual knowledge is knowledge that is rich in relationships. It is a connected web of knowledge, a network that contains prominent linking relationships. Procedural knowledge consists of formal language and symbol systems as well as algorithms and rules.

Pedagogical Content Knowledge

Shulman (1986) used the term pedagogical content knowledge to describe "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Included in this category is knowledge of representations, analogies, illustrations, examples, explanations, and demonstrations for topics in mathematics. An understanding of the potential misconceptions of students associated with specific mathematical topics is also part of pedagogical content knowledge.

McDiarmid, Ball, and Anderson (1989) defined instructional representations as "a wide range of models that may convey something about the subject matter to the learner: activities, questions, examples, and analogies, for instance" (p. 194). Mathematics pedagogy may be viewed as a repertoire of instructional representations. By shifting the emphasis from methods or strategies of teaching to instructional representations, the focus of teaching mathematics moves from the teacher to the mathematics, and the connection between what the teacher knows and

what the teacher does is tightened. To develop appropriate instructional representations, teachers must understand the content they are representing, the ways of thinking associated with the content, and the students they are teaching (pp. 197-198).

The following research questions guided this study of secondary mathematics teachers' knowledge of the concept of slope:

1. What representations of slope are included in preservice and inservice secondary mathematics teachers' concept images and concept definitions?
2. Do preservice and inservice secondary mathematics teachers have a robust mathematical understanding of various representations of slope?
3. What representations of slope are included in preservice and inservice secondary mathematics teachers' descriptions of classroom instruction?
4. How does preservice secondary mathematics teachers' knowledge of slope compare with the knowledge of inservice secondary mathematics teachers?

Methodology

This study examined the concept images and concept definitions of slope held by both preservice and inservice secondary mathematics teachers, their mathematical understanding of slope, and their pedagogical content knowledge of slope. This study also compared the knowledge of preservice teachers with the knowledge of inservice teachers in relation to the concept of slope. Thus, the term "teachers" represents both preservice and inservice teachers.

Research Procedures

Initial data from both preservice and inservice teachers were collected from paper-and-pencil surveys. Eighteen preservice teachers and twenty-one inservice teachers completed surveys. From the sets of those preservice teachers and inservice teachers willing to be interviewed, a random subset of four females and four males from each group was selected. The length of the interviews varied from about 45 minutes to one hour. The interviews were audio-recorded and the tapes were later transcribed. This paper discusses the results of the

mathematics survey and half the interviews. For a complete description of the results, see Stump (1996).

Participants

The preservice participants in this study included 10 males and 8 females enrolled in a secondary mathematics methods course. All but one of the preservice teachers had done some teaching in either junior high or high school mathematics classrooms as part of the clinical experiences included in a previous course. All the preservice teachers planned to teach high school mathematics. Three of them planned to also teach physics, one planned to teach chemistry and computers, and one planned to teach physical education and driver education.

The 12 male and 9 female inservice teachers who completed the mathematics survey were from four high schools in central Illinois. All but 2 of the teachers had Bachelor degrees in mathematics or mathematics education. Of the 15 teachers who had Master degrees, 8 had degrees in either Mathematics or Mathematics Education. Their teaching experience ranged from first-year to 32 years.

Instruments

The mathematics survey and the interview protocol were designed to probe teachers' knowledge of slope: their concept images and concept definitions, their mathematical understanding, and their thoughts about teaching the concept of slope. Both the mathematics survey and the interview protocol contained questions about concept definitions of slope, problems involving various representations of slope, and questions about mathematics pedagogy.

Data Analysis

Surveys were analyzed to provide a broad picture of teachers' knowledge of slope. Teachers' responses to questions involving concept definitions and mathematics pedagogy were counted and coded according to the types of mathematical representations included: geometric, algebraic, physical, trigonometric, functional, ratio, and arithmetic. Responses to questions assessing mathematical understanding were checked for correctness and, in one case, for the types of mathematical representations employed to solve the problem.

The transcripts of interviews were first analyzed using predetermined codes. Teachers' responses to questions involving concept images and concept definitions and mathematics pedagogy were coded according to the types of mathematical representations included, using the same codes that were used to analyze the mathematics surveys.

Data sets (surveys and interview transcripts) for two preservice teachers and two inservice teachers were independently coded by a second researcher using the same codes. Agreement between the two researchers was 91%, determined as follows: each researcher assigned a set of codes to each item of data; they counted the number of codes contained in the union of their two sets of codes and the number of codes contained in the intersection of their two sets; and they calculated the sums of the unions and the intersections over all of the coded data items and then divided the sum of the intersections by the sum of the unions.

The transcripts were then analyzed using the process of analytic induction as described by Erickson (1986). After repeated readings of the data, various assertions were generated. The validity of those assertions was then tested by reviewing the transcripts repeatedly, seeking confirming as well as disconfirming evidence. Patterns were revealed through analysis within individual teachers' responses as well as through comparisons among various teachers' responses.

Results of the Mathematics Survey

The results of the mathematics survey provide a broad picture of secondary mathematics teachers' knowledge of the concept of slope. They indicate general trends concerning the representations of slope included in teachers' concept definitions. They also reveal various aspects of teachers' mathematical understanding of slope. Furthermore, they provide a general portrait of the representations of slope included in their descriptions of classroom instruction.

Concept Images and Concept Definitions

Teachers' responses to the questions "What is slope?," and "What does slope represent?," as well as the examples that the teachers used to illustrate their responses, provided information about their concept images and concept definitions of slope. Most teachers' responses included

more than one representation of slope. Figure 1 shows examples of teachers' responses and the corresponding representations of slope.

In their responses to the two questions about concept images and concept definitions, the mean number of representations per teacher was 2.1 for preservice teachers and 2.0 for inservice teachers. As shown in Figure 2, geometric representations of slope dominated both preservice and inservice teachers' responses. Functional representations were the second most frequently mentioned in teachers' concept definitions, more often by preservice teachers than by inservice teachers. Physical representations were mentioned more often by inservice teachers than they were by preservice teachers, and algebraic representations were mentioned more often by preservice teachers. Ratio was mentioned by both preservice and inservice teachers, but the trigonometric representation was included in only one inservice teacher's concept definition.

Mathematical Understanding of Slope

Various aspects of teachers' mathematical understanding of slope were assessed with the mathematics survey. The problems included geometric, algebraic, functional, and trigonometric representations of slope. Figure 3 shows the problems on the mathematics survey. The representation of slope assessed by each problem appears in brackets.

Problems 1, 2, and 3 in Figure 3 were taken from the first edition of UCSMP Algebra (McConnell et al., 1990), from the chapter entitled "Slopes and Lines." All the teachers, both preservice and inservice, correctly calculated the rate of change in Problem 1 and correctly represented the rate of change in Problem 2 with an algebraic equation. For Problem 3, all the teachers, both preservice and inservice, correctly calculated the slope in Part A to be 1.1 dollars per minute. In Part B, all the teachers interpreted the slope as a rate of change, but the interpretations of two inservice teachers were incorrect. One of these teachers wrote, "As time increases, the cost per minute is increasing, i.e., the cost per minute is not constant." The other teacher wrote, "The more minutes you talk, the more expensive each minute gets." A correct interpretation would be, "For each additional minute, the cost increases $1.10."

McDermott, Rosenquist, and van Zee (1981) presented Problem 4 in Figure 3 to undergraduates in an introductory physics course to determine if students could discriminate between the slope and the height of a graph. In this investigation, 67% of the preservice teachers and 71% of the inservice teachers answered the problem correctly. We determine that the speed of Object A is greater than the speed of Object B by noticing that the slope of the line for Object A is greater than the slope of the line for Object B. The two objects will never have the same speed because the slopes of the lines represent constant speeds. Four preservice teachers and three inservice teachers said the speed of Object A was less than the speed of Object B. Two preservice teachers and one inservice teacher answered Part A correctly, but indicated in Part B that the two objects would have the same speed at some point. Two inservice teachers left the question blank.

One first-year inservice teacher admitted that the notion of slope as a rate of change was new to her. On the back of her survey, she wrote, "When I answered the first page I hadn't thought of slope as a rate of change, even though I've spent loads of time with all sorts of velocity graphs, etc."

Problem 5 in Figure 3 was the first part of an exercise in the first chapter of the Harvard Consortium's *Calculus* (Hughes-Hallet et al., 1992) in a section on linear functions. Only 33% of the preservice teachers and 57% of the inservice teachers answered both parts of the problem correctly. A correct equation for Part A is $l = a l_0 t + l_0 (1 - a t_0)$. For Part B, the slope is $a l_0$, and the y-intercept is $l_0 (1 - a t_0)$. Although 83% of the preservice teachers provided a correct equation, only 61% wrote a correct expression for the slope, and only 33% wrote the correct y-intercept. Of the inservice teachers, 81% wrote the correct equation, 62% wrote the correct slope, and 57% wrote the correct y-intercept. Two preservice teachers and three inservice teachers gave no response for either part of the question.

Problem 6 in Figure 3 was answered successfully by 33% of the preservice teachers and 67% of the inservice teachers, a noticeable difference. It is possible to find the slope of the line; it is tan 60°. Teachers tried geometric and trigonometric approaches to solve the problem. A typical

geometric response was the following: "We need another point in order to determine the change in y over the change in x."

Pedagogical Content Knowledge

Responses to two questions on the mathematics survey provided information about teachers' pedagogical content knowledge in relation to the concept of slope: "What mathematical concepts must students have experience with before they can truly understand slope?," and "What analogies, illustrations, examples, or explanations do you think are most useful or helpful for teaching the concept of slope?"

Prerequisites for students' understanding. As shown in Figure 4, both preservice and inservice teachers mentioned geometric concepts and skills most often as prerequisites for students' understanding of slope. The Cartesian-coordinate system, plotting points, and graphing were among the geometric concepts and skills identified by both preservice and inservice teachers. Variables, formulas, and solving equations were among the algebraic concepts and skills mentioned. Graphing linear equations was categorized as both geometric and algebraic. Both preservice and inservice teachers listed arithmetic concepts such as fractions, addition, and subtraction. Inservice teachers additionally included the following arithmetic concepts: percentages, positive and negative numbers, and division involving zero. A few preservice and inservice teachers mentioned the importance of knowledge of functional relationships. A few from each group named the concept of ratio as a prerequisite for students' understanding of slope.

Instructional representations. As indicated in Figure 5, physical representations were mentioned most often as useful instructional representations for slope. Teachers mentioned ski slopes, mountain roads, stairs, wheelchair ramps, and roofs as useful examples for teaching the concept of slope. Graphs of lines were mentioned as geometric representations. Functional analogies that teachers considered to be useful were situations involving unit prices, velocity, or acceleration.

Summary of Survey Results

Data from the mathematics survey revealed that both preservice and inservice teachers' concept definitions were dominated by geometric representations of slope. Physical, functional, algebraic, trigonometric, and ratio representations were mentioned less frequently.

Various aspects of teachers' mathematical understanding of slope were assessed with the mathematics survey. Figure 6 summarizes the success of preservice and inservice teachers with these various representations of slope. All teachers, both preservice and inservice were successful with a problem involving a geometric representation of slope, but they were less successful with problems involving algebraic, functional, and trigonometric representations of slope. Inservice teachers were more successful than preservice teachers with a problem involving a trigonometric representation slope.

Preservice and inservice teachers mentioned geometric concepts and skills most frequently as mathematical prerequisites for students' understanding of slope. They identified physical representations, however, as being most useful for instruction.

Results of the Interviews

The data obtained from the interviews provide profiles of individual teachers. They reveal the various representations of slope included in individual teachers' concept images and concept definitions. They also provide information about individual teachers' mathematical understanding of slope and their pedagogical content knowledge.

The teachers' actual names were replaced with pseudonyms. First names were assigned to preservice teachers, and last names were assigned to inservice teachers. The preservice teachers were Calvin, Diana, George, and Holly. George had a previous degree in mathematics and had returned to school to complete requirements for a teaching certificate. The inservice teachers, Ms. Ball, Mr. Car, Ms. Day, and Mr. Eno, had been teaching mathematics 25 years (off and on), 5 years, 12 years, and 28 years respectively. Ms. Ball had a Master's degree in mathematics education, and Mr. Car and Mr. Eno had Master's degrees in mathematics.

Concept Images and Concept Definitions

Table 1 shows the representations of slope included in the eight teachers' concept images and concept definitions as indicated by their responses to the following questions from the mathematics survey and the interview protocol:

1. What is slope? Give one or more examples to illustrate.
2. What does slope represent? Give one or more examples to illustrate.
3. When someone says "slope", what do you think of?
4. How is this thought of yours related to the mathematical definition of slope?

All eight teachers, both preservice and inservice, included geometric representations of slope. George was the only preservice teacher to include more than two representations of slope in his concept definition. Holly was the only preservice teacher to include a functional representation. Diana and Ms. Day were the only two teachers to include algebraic representations. Ms. Ball was the only inservice teacher with fewer than four representations of slope in her concept definition. She was also the only inservice teacher to not include a functional representation. Mr. Eno was the only teacher in the entire study to include a trigonometric representation in his concept definition.

Mathematical Understanding of Slope

Teachers responded to various questions designed to probe their mathematical understanding of slope. The questions examined the teachers' understanding of geometric, algebraic, functional, and trigonometric representations of slope, and their abilities to make connections between various representations.

Connecting geometric, algebraic, and functional representations. The problem shown in Figure 7 required teachers to remember the formula for the circumference of a circle, to interpret the formula as a linear equation, and to identify the slope of the line representing the situation. The equation of the line representing this situation is $c = \pi d$, where c is the circumference and d is the diameter. The slope of the line is π.

Mr. Eno described the connections among the geometric, algebraic, and functional representations.

Mr. Eno: We're plotting the circumference versus the diameter. Yes, it will [be a line]. And the reason it will be a line is because the equation is $c = \pi d + 0$, which indicates it will go through the origin, and your slope should be π, because that would be your $mx + b$. If this was in our Algebra II book, we would even say that was a direct variation. Because if you have a line that passes through the origin, then we call that direct variation, where the slope is m and the y-intercept is zero. And we just say the circumference would vary directly with the diameter.

Holly and Ms. Ball had difficulty with this problem.

Holly: I would guess around π. . . . Well, this is increasing. This is π times larger than this. So for every diameter, your circumference is π times that. I'm not really sure if it would be the slope, but I'm thinking it would be related to it.

Ms. Ball drew pictures of two circles, one with a diameter of 10 and one with a diameter of 20. She calculated the circumferences of the two circles, and then she drew a graph of a line using the points ($10, 10\pi$) and ($20, 20\pi$). She then determined that the slope of the line was 2 by making the following calculation: $\frac{10 - 20}{5 - 10}$.

The problem shown in Figure 8 requires a connection between slope as an algebraic parameter and slope as a rate of increase. Orton (1983) presented this problem to high school students and preservice teachers. Part D asks for the rate of increase between two points on a line, and Part E asks for the rate of increase at a single point. The answer to Part D and to Part E is 3.

George, Holly, Mr. Car, and Mr. Eno were the only teachers to give correct responses for Parts D and E. Mr. Car explained how he applied his understanding of the relationship between slope and rate of increase.

Mr. Car: Part D, I took the ratio of change in y to change in x. And then realized that I didn't need to do that, because I understand the concept of slope. I should have know that the rate of increase was 3. And then similarly for part E, the rate of increase is constant. So I knew that was also 3.

Diana explained her thinking and revealed an interesting misconception about rate of increase.

Diana: In C, they are asking for a number or a letter increase. In D, they are asking for rate, so that's like a fractional or a decimal-type thing. Rate is a proportion. And actual increase, they are just asking for a single figure. . . . I think that $\underline{y}$ increases by $3\underline{h}$. And the rate of increase would be like rise over run. . . . (Writes $\frac{\underline{a} + \underline{h}}{\underline{h}}$.) . . . I'm not sure that E has an answer. . . . There is only one point. For the rate of increase, you need to have a comparison from one point to the other. There is no rate of increase at one point.

Connecting geometric and functional representations. The teachers were given the following set of three collinear points representing the monthly rents for high-rise apartments: (13th floor, $325), (17th floor, $365), (19th floor, $385), a problem appearing in the first edition of UCSMP Algebra (McConnell et al., 1990). All the preservice and inservice teachers were able to calculate the slope, which had a value of 10. Only one inservice teacher, Ms. Ball, struggled to describe what the slope represented within the context of the situation, that moving up in the building results in a $10-per-floor increase in rent. She expressed her lack of experience with interpreting slope as a rate of change.

Ms. Ball: Well, in general, as the apartments are on higher floors, the rent is higher. And now I have to decide how much higher. It isn't 10 times as much. I probably want to see the pattern. As you go up every 4 floors, the rent increases 10, something like that. Is that what you mean? If the kids were talking like that, I would say let's fill in some more, and see what happens. There's 40 there, so 14 would be 335, 15 would be 345, 16 would be 355. So we can see, really, what's happening. So as the floors go up 1, the rent goes up 10. That's sort of like a rate of change, I guess, which is a new concept for me just today (emphasis added).

Teachers were less successful with a problem requiring the interpretation of a graph involving velocity versus time. Figure 9 shows the problem, which asked teachers to find the acceleration of three rocket engines. McDermott, Rosenquist, and van Zee (1981) presented a variation of the problem to undergraduates in an introductory physics course to examine their ability to match narrative information with relevant features of a graph. This investigation used the problem to examine teachers' connections between geometric and functional representations of slope.

We find the acceleration of the first two engines by calculating the slope of two line segments on the graph. It is impossible to find the acceleration of the third engine due to lack of

essential information. Diana, George, Mr. Car, and Mr. Eno provided correct explanations for all three engines. George and Mr. Car articulated their understanding of slope as a rate of change.

> George: Acceleration is the change in velocity per unit time. So, the acceleration when Engine 1 is on, which is from P to Q, will be the slope of the line PQ. And the acceleration when Engine 2 is on, which is from Q to R, is going to be the slope of that line QR. As far as Engine 3 goes, if we don't have any information after that engine is turned on, there's no way we can have any idea what the acceleration is going to be.

> Mr. Car: I found the slope of the line. Because acceleration is the rate of change of velocity with respect to time. And I recognized that is representable by the idea of slope. I essentially then found the coordinates of points P, Q, and R, and calculated the slopes based upon that information.

Ms. Day seemed less sure about the connection between the slopes of the line segments and the accelerations of the rocket engines. She successfully calculated the accelerations of Engines 1 and 2, but she mistakenly calculated the slope of line segment PR to determine the acceleration of Engine 3.

> Ms. Day: Well, I would go this way and find the slope of that line. So that would be the acceleration of Engine 1. Then I would do the same thing for Engine 2. And now Engine 3. [I calculated the slope] just because I didn't know what else to do. . . . I'm thinking of an acceleration rate, which would mean the comparison of two things, which would be the time and the velocity.

Two teachers exhibited serious misconceptions about velocity and acceleration in relation to the graph. Ms. Ball calculated accelerations for all three engines by dividing the y-coordinate by the x-coordinate for each of the three points, P, Q, and R.

> Ms. Ball: Is that a single number, acceleration? So in 6 minutes, it goes 300 kilometers per hour, so it must be going 50 kilometers per hour. And the Number 2 engine is . . . so it's $33\frac{1}{3}$ kilometers per hour. (Divides 700 by 14 to find an acceleration of 50 kilometers per hour for Engine 3.) That's interesting that's it's the same as the first one and I didn't notice that.

Holly made a connection between slope and acceleration, but she thought she needed to use derivatives to find acceleration, and so she was not able to solve the problem.

> Holly: I would say acceleration is your rate of increase, your increase in velocity. We have a point here. Have them find the point, (6, 300). And have them find an equation of this line using the points. Have them solve for the line. Find slope. Have them relate acceleration to slope. The steeper the slope, the faster the acceleration produced by the engine. Show the difference produced by the three. . . . To find acceleration, I have always learned that it

> was the double derivative, and velocity was the derivative. I guess it would depend on the class. If it was a precalculus or calculus class, they could probably find it. Find it from the equations of the lines, taking the derivatives. But if this was just an algebra class, I don't think they would be able to find the acceleration.

Connecting geometric and trigonometric representations. Two questions, shown in Figure 10, were designed to determine if teachers thought of the slope of a line as the tangent of the angle of inclination. In response to first question, if the scales on the x- and y-axes are the same, then the slope of the line is the tangent of the angle. For the second question, if the scales on the x- and y-axes are the same, then the slope of line l is tan θ, and the slope of line k is tan 2θ. Because tan 2θ is not the same as 2 tan θ, the slope of line k is not twice the slope of line l.

Mr. Car, Ms. Day, and Mr. Eno all used the word "tangent" in their responses to at least one of these questions. When responding to the first question in Figure 10, Ms. Day immediately made a connection between the geometric and the trigonometric representations of slope, but suggested that a student who makes this connection has greater understanding than a typical beginning algebra student.

> Ms. Day: I'd move him into an advanced honors class first thing, because it would take more of a discussion than that. First, we would have to see where he thinks this angle is being formed with the x-axis. Then I would want to know what connections he's making with that, how he thinks that's related. Because if he's moving on into tangent, then you have someone who doesn't belong in their math class. But, yeah, there is a relationship.

George was the only preservice teacher to allude to the synonymous relationship that exists between the angle and the slope of the line although he made no specific reference to the word "tangent." He pointed out the benefit of having two ways of looking at the concept of slope.

> George: I think that would be a really good opportunity to show that there is a different way of looking at how steep a line or something is. You can look at the angle. That tells you how steep it is also. And to see that sometimes one representation may be more useful than the other, but that you can convert between them.

Some teachers identified a direct relationship between the angle and the slope. Calvin and Ms. Ball expressed uncertainty about being able to determine the actual slope of the line if they knew the measure of the angle.

Calvin: I don't know. I haven't really been taught that. I honestly haven't. Maybe I don't remember, or I don't know. I'm sure there is a relationship.

Ms. Ball: So if it's a positive slope, it would be an acute angle with the x-axis. If it's a negative slope, it would have an obtuse angle with the x-axis. So there probably is a relationship, but I've never thought of it before.

Holly expressed a similar uncertainty about responding to the first question in Figure 10.

Holly: If I had the equations of the line it would be helpful. I'm not really sure. To tell you the truth, I wouldn't really know if he was correct without the equation of the line to analyze. . . . I've never seen anything like this. When I learned slope, we never talked about the angle.

Pedagogical Content Knowledge

The interviews investigated two aspects of pedagogical content knowledge: knowledge of instructional representations for teaching slope and knowledge of students' difficulties with learning slope.

Instructional representations. Table 1 shows the representations of slope included in teachers' descriptions of classroom instruction. All eight teachers included geometric and physical representations, but only four teachers--Diana, George, Ms. Day, and Mr. Eno--included functional representations.

Inservice teachers described what they wanted their students to know about slope and the instructional representations they actually used in their classrooms. Ms. Ball described the geometric and algebraic representations that she would use, making connections between lines and their equations.

Ms. Ball: I would probably put some lines on the board that all have the same slope and have them see the similarities. And then I might look at some that have different, some going up and some going down, and what's the difference about them. And maybe even show them a horizontal line. Then I would have them, as we had done earlier, make a chart of the x and y values. . . . Then look at the equation that those ordered pairs came from and see the similarity of the changes in the x's, changes in the y's, to the number before the x term.

Ms. Day described an unusual geometric representation that she devised in which she

related slope to the angle of inclination.

Ms. Day: Well, usually what I do is to bring in one of those collapsible yardsticks, the ones on the hinges. And hold it up so I'm only using one hinge. And usually all I do is take that and change the angle, and say something to them about, in that respect, the horizontal change, the horizontal difference is always the same. But that vertical change becomes bigger as the angle becomes bigger.

Mr. Eno referred to geometric, physical, and functional representations as he explained what he wanted his students to know about slope.

Mr. Eno: I want them to have a common working knowledge of slope. So for the practical side of it, the student is traveling and they talk about a grade, or they talk about a ramp, or they talk about a house having a certain pitch in the roof, they have an idea about what slope means in a very practical sense. And also, I guess, as far as math is concerned, too, I want them to understand that it has a lot of different uses as far as a rate of change. I want to think of velocity of a rate of change, that it is really a slope. And just to know how to calculate slope and how to use it in different applications.

Preservice teachers contemplated how they might teach the concept of slope. Calvin used geometric representations in his description of an introductory lesson on slope.

Calvin: What I would probably do is start plotting points and plotting lines. And say, well, I have a couple lines on the board. And have a couple points on each line. And start comparing. I would just say here is the slope of the line. It's either rising or falling. I would probably just give the definition rise over run. The difference of the y's over the difference of the x's.

George said he was interested in applications, and he mentioned several physical representations of slope.

George: I would probably introduce the idea by bringing in lots of examples. Like I said, the roof of a house, a mountain road, a line on a graph. Although the line on the graph is more of an abstract thing. You can use the slope there to represent all sorts of different kinds of ratios. But, I think I would start with the real concrete things, out in the real world, like a stairway.

Holly described how she would connect geometric and algebraic representations.

Holly: I would probably have like a worksheet with different graphs of lines, and have them look at them and write down what they think about the steepness of each line. And have the equation of the line, and somehow relate the two. What do you notice about the graph of the line and the equation, and try to see if they can pick out what the slope is.

Student's difficulties with slope. Inservice teachers spoke about the actual difficulties they had witnessed during their years of experience with students. They echoed the difficulties with symbolic interpretation and manipulation documented in the literature on students' understanding of slope (Barr, 1980, 1981; Schoenfeld, Smith, & Arcavi, 1993). Almost all the inservice teachers mentioned students' confusions between rise and run or between y's and x's in the formula for slope.

Ms. Ball: Well, if they don't know the formula, one main thing, they put the x's over the y's. They do that all the time.

Mr. Car: Reversing the numerator and the denominator. In terms of the definition, change in y over change in x. . . . And I would say the order in which they subtract them. It takes a little bit of time to get used to the idea that you can do it either way as long as we do it in the same order.

Preservice teachers hypothesized about potential difficulties. Calvin mentioned difficulties with symbolic manipulation.

Calvin: Possibly positive and negative slope. The term, "m equals." At first I was like, m? Just the variable m. When you're graphing, if the slope is 3, you might do 3 x's and 1 y. They might have difficulties getting that figured out.

Preservice teachers also suggested possible difficulties with conceptual understanding. Such comments were not found among inservice teachers.

Holly: If they didn't know what an unknown represented, that there could be several different values of x in this equation that would work. . . . If they didn't really understand that there could be different values for x, then I don't think they would understand that the line is changing, that's what the slope is, the rate of change of the y over the x.

George: My guess is that some might be frightened off as soon as you introduce a mathematical definition or a formula for a line, like the slope-intercept of the equation. As soon as some people see equations, they just go nuts, especially with symbols instead of numbers. . . . Not because they don't understand what slope is, but because they are not making the connection between the intuitive and even the not-so-intuitive idea of taking the ratio of this to this. Not making the connection between that and the symbolic abstract equation on paper. That's just a guess. I haven't had experience with that.

Diana: I think seeing the big picture. . . . I don't think it's difficult to find m. I think it's a really basic -- it's basic arithmetic. But I think actually being able to go further with slope, seeing that it does mean something.

Summary of Interview Results

All eight teachers included geometric representations of slope in their concept images and concept definitions. They cited functional and physical representations less frequently but more often than algebraic, trigonometric, or ratio representations.

The mathematical understanding of both groups of teachers, preservice and inservice, varied from weak to strong. Although some teachers demonstrated a robust understanding of slope, other teachers exhibited difficulties with several mathematical representations of slope. Teachers had the most difficulty with problems involving functional and trigonometric representations of slope.

The functional representation was also missing from many teachers' descriptions of classroom instruction. None of the teachers mentioned using the trigonometric representation of slope in their classrooms. Teachers focused on geometric representations of slope as sources of potential student difficulties. Although some teachers had difficulties themselves with functional representations of slope, none of the teachers mentioned functional representations as potential sources of confusion for their own students. Both preservice and inservice teachers mentioned potential student difficulties with procedures, and preservice teachers also mentioned potential misunderstandings of concepts.

Discussion

Concept Images and Concept Definitions

As evidenced by their concept definitions, almost all teachers, both preservice and inservice, think of slope as a geometric concept, as a property of the graphs of lines. Only half of the preservice teachers and slightly more than a third of the inservice teachers think of slope as a functional concept. In other words, the majority of teachers do not think of slope as a rate of change. These findings are comparable to those of Azcarate (1992), who investigated high school students' definitions of slope. The majority of students in her study used geometric language to define slope, but the use of functional language was much less prevalent. Very few teachers think

of slope as a ratio, and the trigonometric representation of slope is virtually non-existent in teachers' concept images. Although teachers may be capable of making connections between the various representations of slope, few have incorporated these representations into their concept images.

Mathematical Understanding of Slope

Livingston and Borko (1990) found differences in the teaching practices of preservice and inservice teachers and hypothesized that preservice teachers were hindered by a lack of connections in their content knowledge. The inservice teachers in this study have a better understanding of the trigonometric representation of slope. In this respect, their content knowledge does contain more connections. Whereas preservice teachers think that two points are necessary to determine the slope of a line, more inservice teachers know that the slope also may be determined by the angle of inclination. Perhaps that is because so many of the experienced teachers have taught trigonometry. Furthermore, the study of trigonometry is not typically part of undergraduate mathematics teacher preparation.

In relation to their mathematical understanding of other representations of slope, inservice teachers demonstrate no particular advantage over preservice teachers. Algebraic and functional representations of slope prove to be difficult for both preservice and inservice of teachers.

Overall, gaps exist within preservice teachers' and inservice teachers' mathematical understanding of the concept of slope. Teachers exhibit difficulties with algebraic, functional, and trigonometric representations of slope. The problems with which they have trouble are not typical high school textbook problems, but the level of mathematics involved is not beyond high school mathematics. They are tasks involving the recognition of parameters and the interpretation of graphs. They require the utilization of connections among various representations of slope. They are problems that, as the Curriculum and Evaluation Standards (NCTM, 1989) suggest, "move away from a tight focus on manipulative facility to include a greater emphasis on conceptual understanding" (p. 150). However, they are problems that many preservice and inservice mathematics teachers did not solve.

Pedagogical Content Knowledge

Although teachers think of slope as a geometric concept, they consider physical representations to be most useful for instruction. They use ramps, ski slopes, and mountain roads as analogies for slope. They seem to assume that their students are making the connections between real-world situations and the geometric concept of slope. However, none of the teachers mentioned physical representations as a prerequisite for understanding the concept of slope. Instead they named geometric, arithmetic, algebraic, and functional concepts. According to Simon and Blume (1994), students may have trouble making the connection between real-world situations and the mathematical expressions used to represent quantitative relationships. The preservice elementary teachers in their study did not fully comprehend the notion of ratio in relation to a physical model of slope.

Teachers do, of course, include geometric representations in their instruction. Most teachers include at least one other representation of slope; some include three or four. Algebraic, functional, ratio, and trigonometric representations, however, are most notably absent from most teachers' repertoires. It is especially troublesome that functional representations of slope are missing from the descriptions of so many teachers' instructional practices. Functions are an important topic in the high school curriculum (NCTM, 1989).

Teachers did not include the trigonometric representation of slope in their descriptions of classroom instruction. Perhaps that is because they consider slope a concept belonging to an algebra course, and the tangent function a concept belonging to a trigonometry course. Thus, they may not think they are teaching slope when they teach trigonometry. In any case, there is little evidence that they emphasize the connection between the slope of a line and the angle of inclination in their classrooms.

Students' difficulties with slope as identified by teachers reflect the emphasis on the geometric representation of slope. According to inservice teachers, students have difficulties with graphing lines and computing slope. Preservice teachers guessed that students might have trouble understanding what slope represents, but inservice teachers did not mention that particular

difficulty. Perhaps that is because their students do not have trouble understanding what slope represents. However, they may never ask their students what slope represents, and thus they are not aware of any difficulties their students might have.

The curriculum resources available to them may affect the pedagogical content knowledge of both preservice teachers and inservice teachers. If their textbooks present slope, linear equations, rate of change, trigonometric functions, and ratio as unrelated concepts, then teachers may teach them as unrelated concepts. This study did not include an analysis of textbooks. Thus, some important questions to investigate are the following: What connections do textbooks make between the various representations of slope? When textbooks do a good job connecting various representations of slope, do teachers do a good job of emphasizing those connections for their students? Can teachers learn to make connections even if textbooks do not emphasize them? Can they learn to make connections from their experiences in formal teacher education?

Conclusion

A lack of connections among various representations of slope exists within many preservice teachers' and inservice teachers' knowledge of slope. Teachers demonstrate this lack of connections in their own mathematical understanding and in their descriptions of classroom instruction. Rizzuti (1991) found that instruction that included multiple representations of functions allowed students to develop comprehensive and multi-faceted conceptions of functions. Based on the results of this investigation, it is questionable whether students are being allowed to develop comprehensive and multi-faceted conceptions of slope.

We cannot assume that teachers will make connections among various representations of slope on their own. Furthermore, teaching experience alone does not guarantee development of mathematical understanding for a teacher. Although teaching experience may provide teachers with more pedagogical content knowledge, some other kind of experience may be necessary for growth in mathematical understanding. The education of secondary mathematics teachers may

need to include opportunities for them to revisit the "big ideas" of the high school mathematics curriculum. Teachers may need to investigate various representations of fundamental mathematical concepts such as slope, either in their preservice preparation or through inservice enhancement.

The results of this investigation suggest that secondary mathematics teachers have a limited understanding of the concept of slope. Their knowledge of slope lacks connections as indicated by the representations of slope that are included in their concept images and concept definitions and in their descriptions of classroom instruction. Geometric representations dominate their mathematical understanding of slope; algebraic, trigonometric, and functional representations are less understood. These results suggest that the education of secondary mathematics teachers needs to specifically address slope and perhaps other fundamental mathematical concepts contained in the secondary mathematics curriculum.

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1.	slope $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{\text{difference in } y\text{-coordinates}}{\text{difference in } x\text{-coordinates}}$. [Geometric]
2.	Rise over run, change in *y* over change in *x*. [Geometric]
3.	Slope is the mathematical concept relating the angle of a line to the axes of a graph when moving from left to right. It is $\frac{\Delta y}{\Delta x}$. [Geometric]
4.	Slope is a measure of the steepness of a line or curve. [Geometric]
5.	Constant relationship between two variables. [Functional]
6.	It can also be thought of as a rate of change. [Functional]
7.	You can find the slope of a curve at a point by finding the slope of the tangent line which involves finding the derivative of the curve at the point. [Functional]
8.	How much y changes with respect to x. [Functional]
9.	For the line $y = mx + b$, m is the slope. [Algebraic]
10.	Slope can be considered as the steepness of an incline. For example: pitch of a roof, grade of a road, ski slope, a mountain slope, bridge of the nose. [Physical]
11.	Also, calculated by taking the tangent of the angle of inclination. [Trigonometric]
12.	Slope is a ratio. [Ratio]

Figure 1. Examples of representations included in teachers' concept definitions of slope.

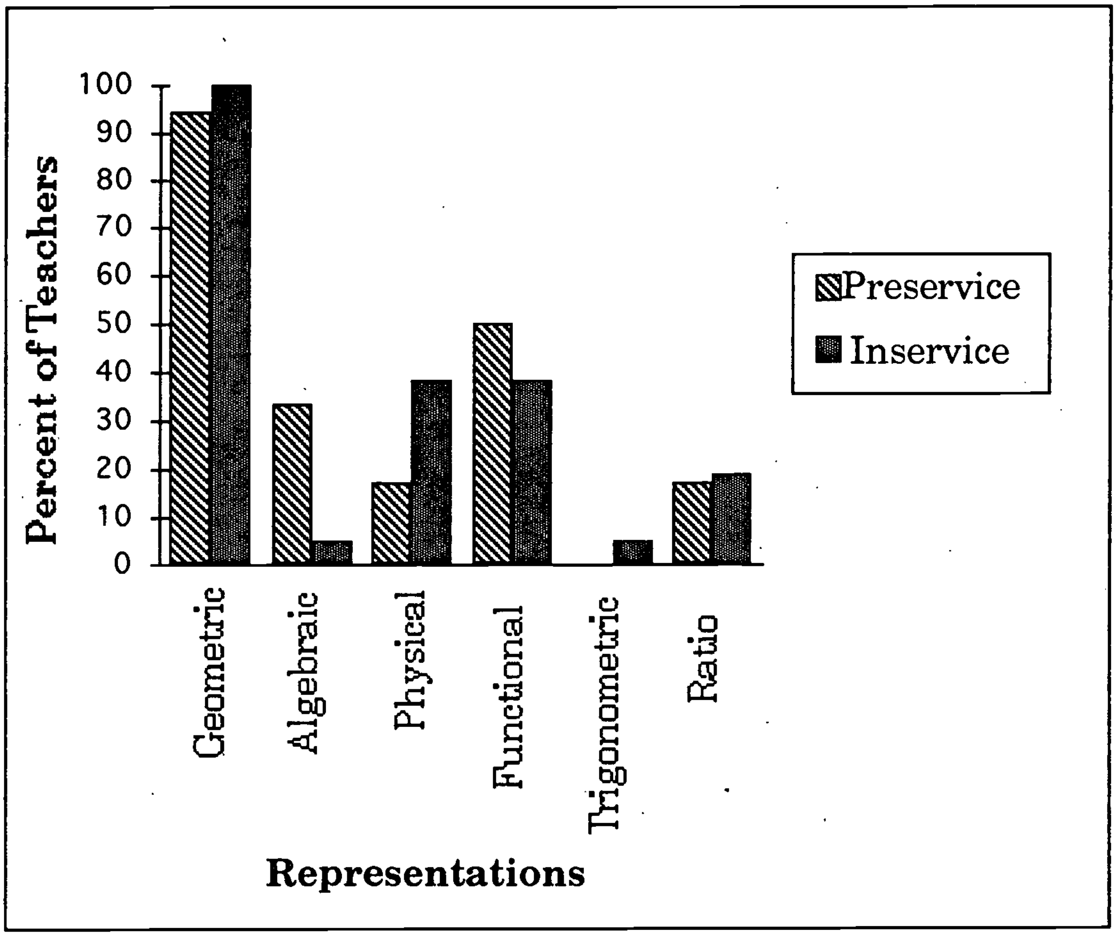


Figure 2. Percent of teachers including various representations of slope in their concept definitions. (The total is greater than 100% because some teachers gave multiple responses.)

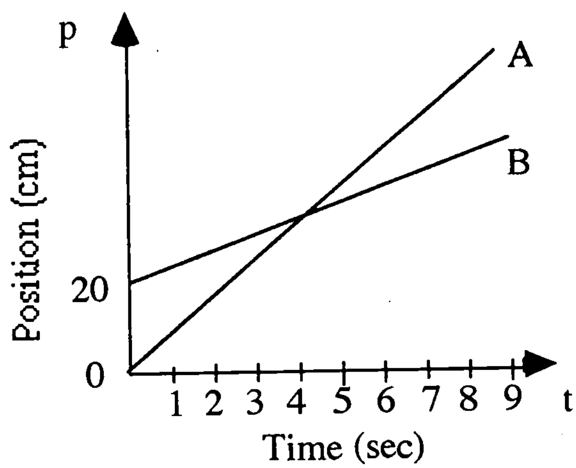
1. At age 9 Karen was 4'3" tall. At age 11 she was 4'9" tall. How fast did she grow from age 9 to age 11? [Functional]

2. A student takes a test and gets a score of 50. She gets a chance to take the test again. It is estimated that every hour of studying will increase her score by 3 points. Let x be the number of hours studied and y be her score. Find an equation which represents this situation. [Algebraic]

3. The following two points give information about an overseas telephone call: (5 minutes, \$5.30), (10 minutes, \$10.80).

   A. Calculate the slope. [Geometric]
   B. Describe what the slope stands for in the context of this situation. [Functional]

4. The figure below shows a position versus time graph for the motions of two objects A and B that are moving along the same meter stick.



   A. At the instant when $t = 2$ sec, is the speed of the object A greater than, less than, or equal to the speed of object B? Explain your reasoning.

   B. Do objects A and B ever have the same speed? If so, at what times? Explain your reasoning. [Functional]

5. For small changes in temperature, the formula for the expansion of a metal rod under a change in temperature is: $l - l_0 = a\, l_0\,(t - t_0)$ where l is the length of the object at a temperature t, and l_0 is the initial length at temperature t_0, and a is a constant which depends on the type of metal.

   A. Express l as a linear function of t.
   B. Find the slope and the y-intercept. [Algebraic]

6. A line makes an angle of 60° with the x-axis and passes through the point (3, 1). Is it possible to find the slope of this line? If yes, what is the slope? If no, why is it impossible to find? [Trigonometric]

Figure 3. Problems on the mathematics survey.

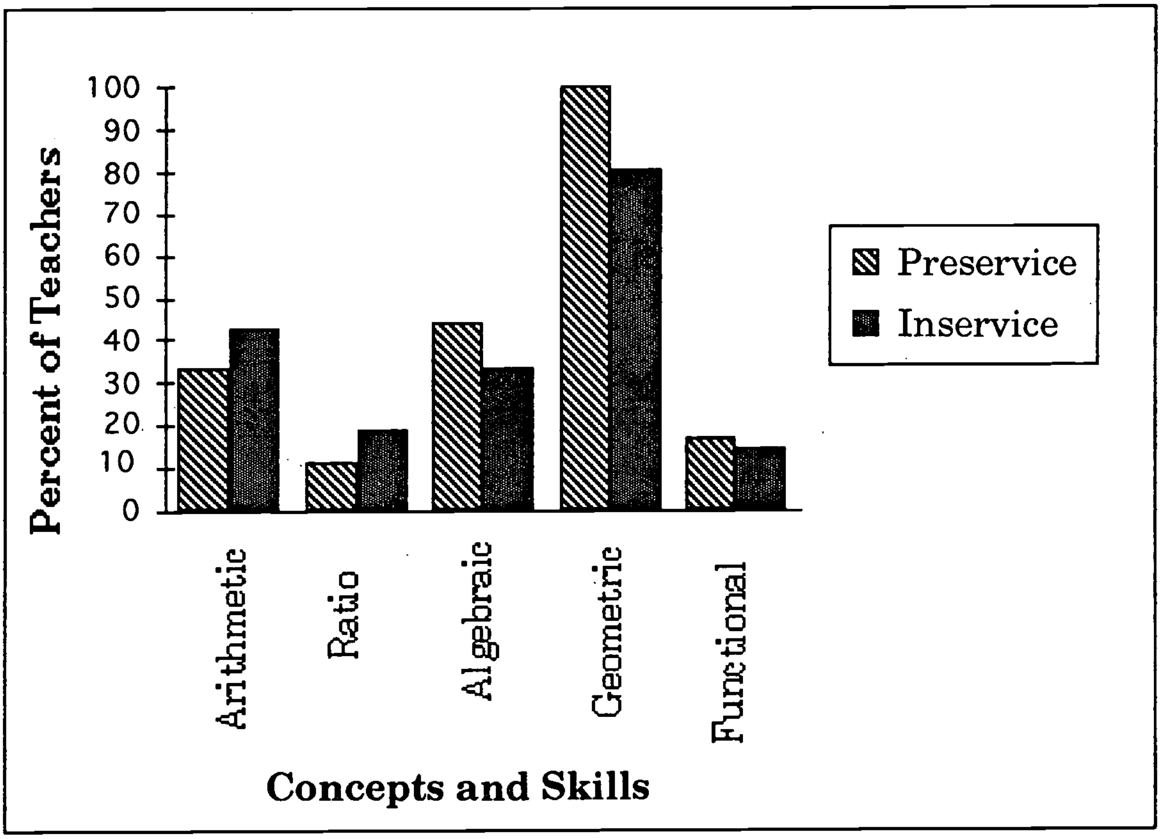


Figure 4. Percent of teachers identifying various types of mathematical concepts and skills as prerequisites for students' understanding of slope.

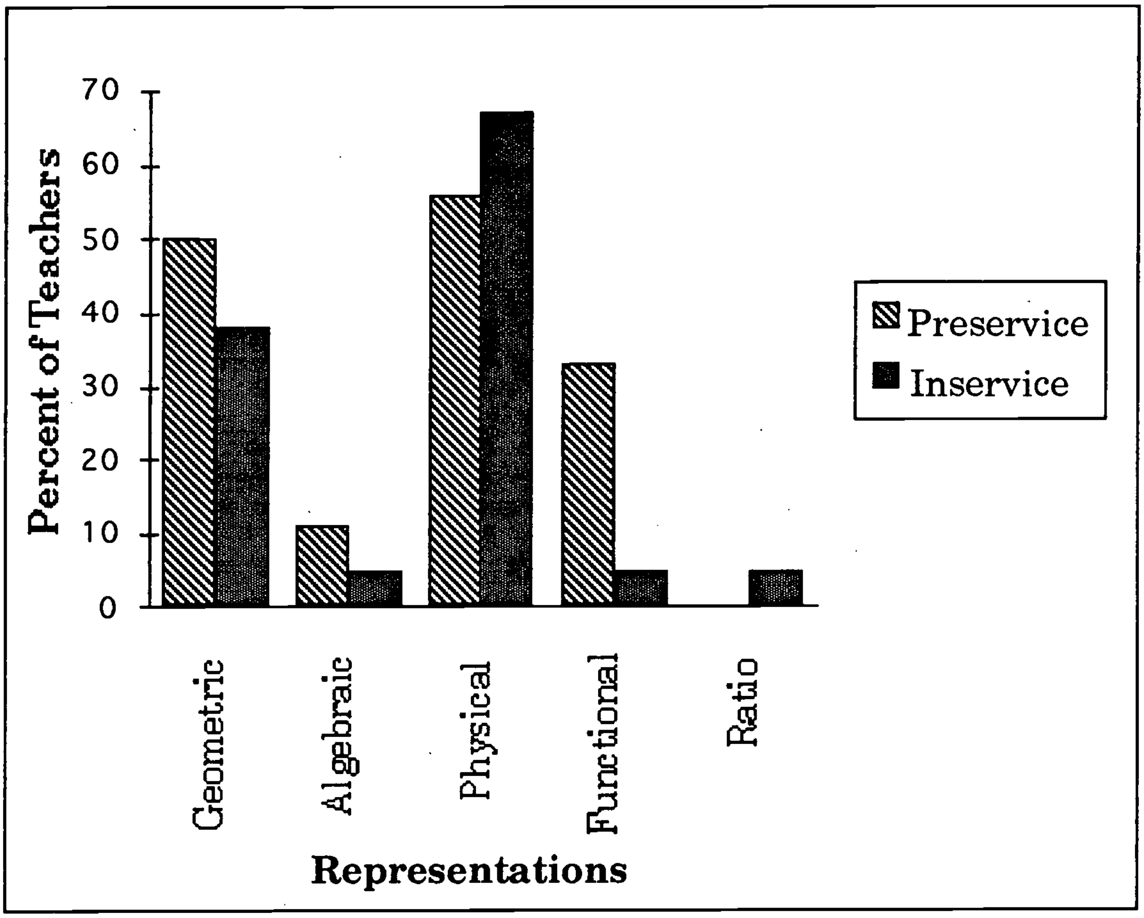


Figure 5. Percent of teachers identifying various representations of slope as most useful for instruction.

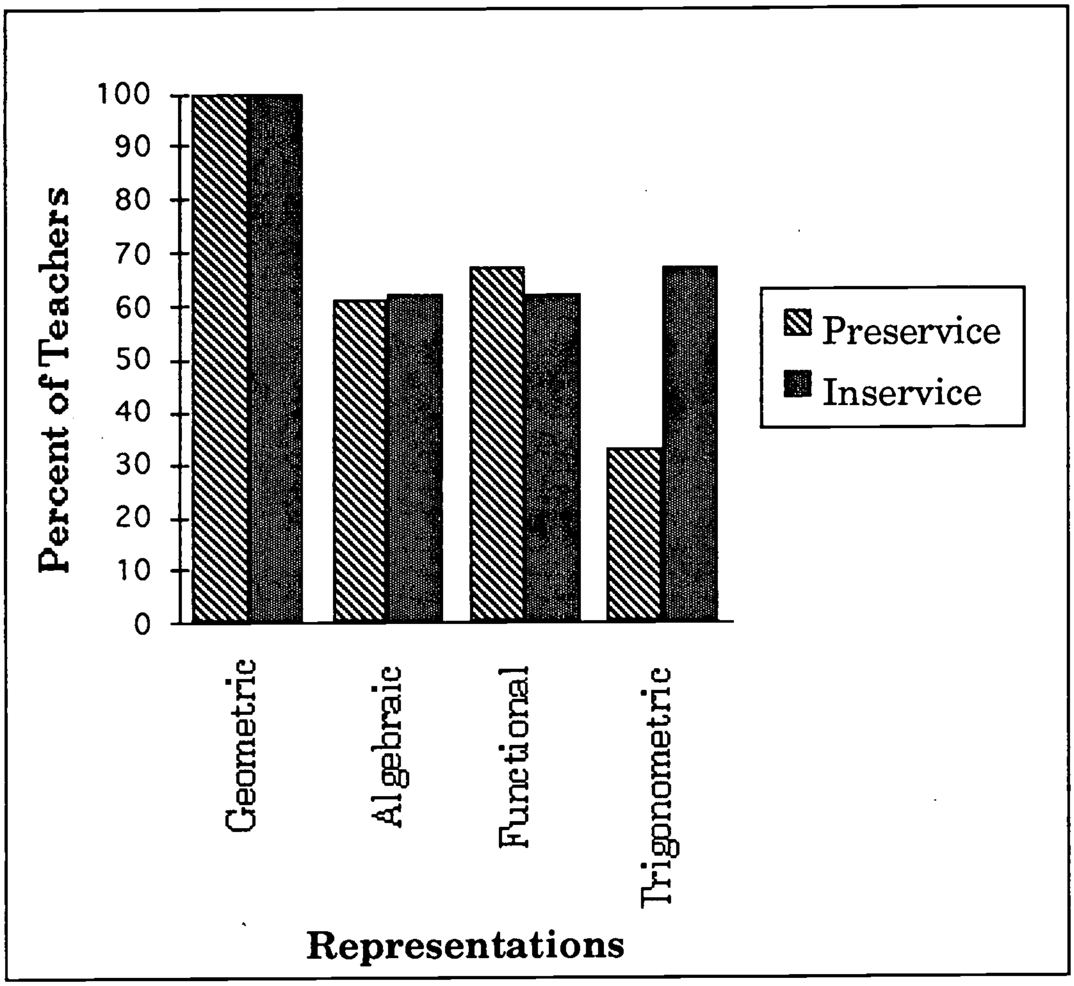


Figure 6. Percent of teachers successful with various representations of slope.

Table 1

Representations of Slope Included in Teachers' Concept Images and Concept Definitions and in Their Descriptions of Classroom Instruction

Teacher	Concept Images and Concept Definitions	Descriptions of Classroom Instruction
	Preservice Teachers	
Calvin	Geometric, Physical	Geometric, Physical
Diana	Geometric, Algebraic	Geometric, Algebraic, Physical, Functional, Ratio
George	Geometric, Physical, Ratio	Geometric, Physical, Functional, Ratio
Holly	Geometric, Functional	Geometric, Algebraic, Physical
	Inservice Teachers	
Ms. Ball	Geometric, Physical	Geometric, Algebraic, Physical
Mr. Car	Geometric, Physical, Functional, Ratio	Geometric, Physical
Ms. Day	Geometric, Algebraic, Physical, Functional	Geometric, Physical, Functional
Mr. Eno	Geometric, Physical, Functional, Trigonometric	Geometric, Physical, Functional

Suppose that using a meter stick, we measure the diameters and the circumferences of several aluminum disks. If we plot a graph of our data with diameter on the x-axis (in cm), and circumference on the y-axis (in cm), will the graph be a line? What is the slope of this line?

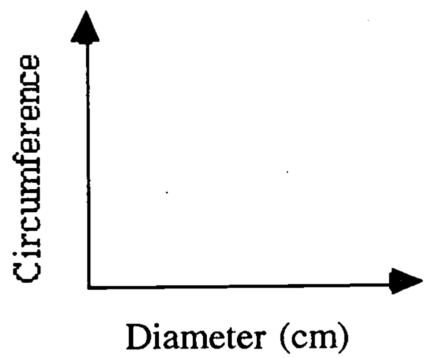
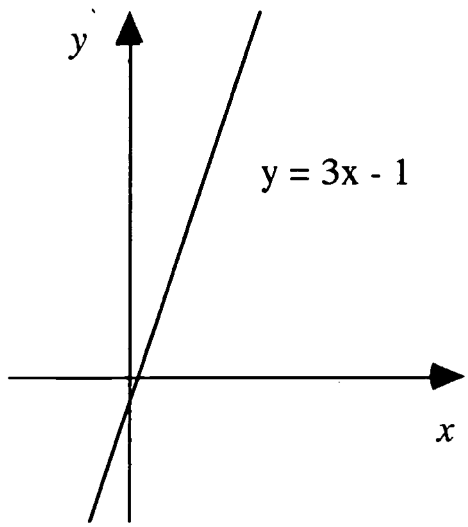


Figure 7. A problem connecting geometric, algebraic, and functional representations of slope.



A. What is the value of y when $x = a$?

B. What is the value of y when $x = a + h$?

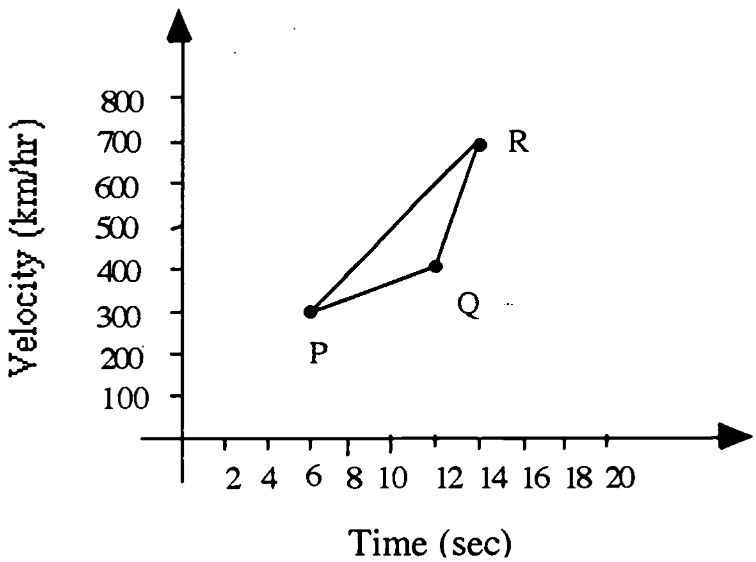
C. What is the increase in y as x increases from a to $a + h$?

D. What is the rate of increase of y as x increases from a to $a + h$?

E. What is the rate of increase of y at $x = 2\frac{1}{2}$?

Figure 8. Determining rate of increase.

A spaceship has three different rocket engines, each of which gives the ship a uniform acceleration when it is turned on. In the graph below, point P represents the velocity of the rocket at a particular time. At point P, the captain turns on the #1 engine. At point Q, the #1 engine is turned off and the #2 engine turned on. At point R, the #2 engine is turned off and the #3 engine turned on. We lose all information about the ship after point R.



Find the acceleration produced by each engine listed below, if this information is represented on the graph. Explain your reasoning.

A. #1 engine.

B. #2 engine.

C. #3 engine.

Figure 9. Determining acceleration from a graph of velocity versus time.

1. A student states that the slope of a line is related to the angle formed by the line and the x-axis. How would you respond?

2. Line l forms an angle of measure θ with the x-axis, and line k forms an angle of measure 2θ with the x-axis. A student states that the slope of line k is twice the slope of line l. How would you respond?

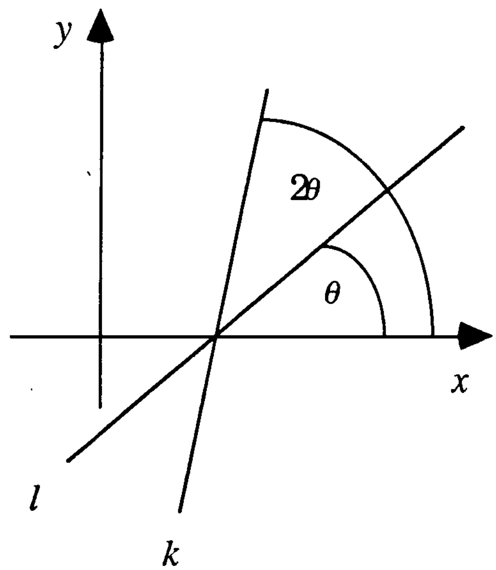


Figure 10. Two questions involving the trigonometric representation of slope.

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