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ABSTRACT

In these two companion papers, a learning activity is introduced that teaches students how mathematics works through the visual aid of picture grids. The picture grids are composed of 60 different acrylic stained-glass window overlays. Each fractional part is represented by a different color: fifths are green, quarters are yellow, etc. The square grids can be easily maneuvered by both students and teachers. The papers report on a teacher demonstrating how students can visualize math strategies. The teacher also shows that manipulatives can help clarify processes as students learn various ways to find solutions. Part 1 introduces picture grids and shows how they work with fractions and decimals. Part 2 tackles squares and square roots using Cuisenaire rods. Games are also included. Suggested games are representative of the many ways in which teachers can simplify the concepts essential for understanding fractions. Because the grids can be maneuvered, students can actually see the sections as they are divided, thus learning the reasons for their answers. When students are encouraged to develop a visual understanding of how numbers are used, numbers are no longer a mystifying body of incomprehensible symbols. (PVD)

Teaching Fractions and Decimals: Fun with Picture Grids, Part 1 and Part 2

by
Andi Stix

TEACHING FRACTIONS & DECIMALS:

Fun with Picture Grids

PART 1

Why is it that mathematics often conjures up images of desperate students trying to solve seemingly impossible problems? The subject of mathematics often compels otherwise well adjusted adults to grimace with the memory of futile hours spent poring over numbers that seem at once both meaningless and threatening. Fortunately, there is an alternative. *Yes, Virginia, there is a Santa Claus*. The impossible really is possible, and we can do more than tell our students to *clap your hands and believe!*

What can we do? I introduce to you Henry Goodman. He is a mathematics teacher. Perhaps, some might even consider him a visionary with an eye towards the coming millennium. Henry Goodman understands that mathematics can be raised to the same level of respect and importance as green leafy vegetables. After all, the children of previous generations balked at the idea of nutrition. Yet, today we see children reading food labels with the same interest they reserve for studying new Nintendo games. The future *is* really upon us.

Henry Goodman can offer activities that are appropriate for all levels of students. The materials transcend both developmental age and degree of complexity because they make it possible to *understand* how mathematics works.

Picture Grids is comprised of sixty different acrylic stained glass window overlays. Each fractional part is represented by a different color: fifths are green, quarters are yellow, thirds are hot pink, and halves are blue. The square grids can be easily maneuvered by both students and teachers. The students need to become familiar with the fractional parts represented by the grids before participating in teacher developed activities.

We are going to follow Henry Goodman as he shows us how students can visualize math strategies. In addition, he will show us that manipulatives can help clarify process as students learn various ways to find solutions. The final outcome is that students will then be able to compute answers by substituting abstract numbers and symbols for the manipulatives.

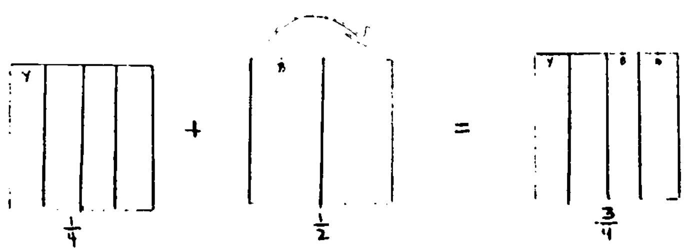
Henry Goodman reminds us that students will always be working cooperatively in pairs. This gives them an opportunity to discuss the problems between themselves and share the application of the manipulatives. In addition, after each problem is solved, students are expected to write or draw the solution in their math journals (Stix, 1992). This process bridges the gap between the abstract numbers and the materials. The writing process also aids in long-term memory (Shubert, 1991) and encourages communication between the teacher and the student. This is especially practical once the inevitability of learning differences is accepted and the need for different modes of instruction recognized (Stix, 1994).

Addition

Henry Goodman's challenge is to teach students how to comprehend fractions. How is two-fourths the same as one-half? And why does one-half and one-quarter equal three-fourths? Is it possible to explain these concepts without the use of a common denominator? An emphatic nod from Henry tells us that not only is a common denominator *not* always necessary to do fractions, but students can have the advantage of actually seeing how fractions work.

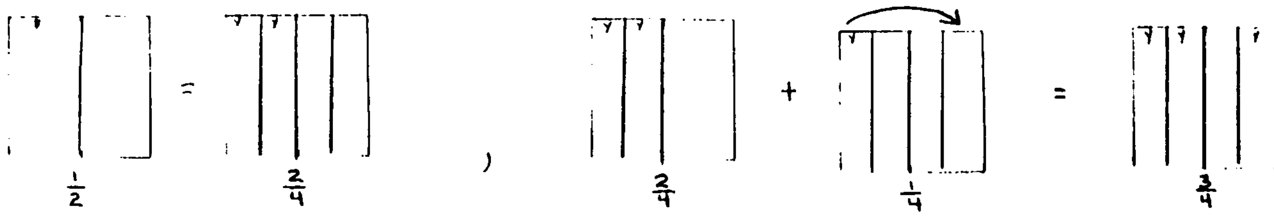
Because of the variety of grids, it is always wise to remove the grids that will be used and place the others aside. Today, Henry pulls out the following grids: halves, quarters, thirds, tenths, twelfths, and hundreds.

1. Henry uses a quarters picture grid and adds it to a halves picture grid. He asks the class, "How much is one-quarter plus one-half?" One group solved this problem by flipping the halves grid on top of the quarters grid and counting the number of quarters created by this combination. It is clear now that one-half plus one-quarter equals three-fourths.

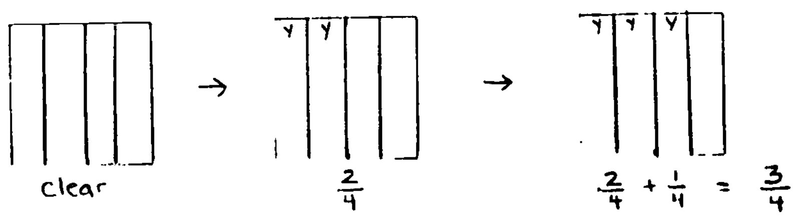


2. He asks another group to return to the original problem of one-quarter plus one-half. He asks the students, "Which of the two grids would you exchange for another grid so you can add them together?" When the students placed the quarter grid on top of the halves grid, they can *see* the exchange of one-half for two-

quarters. They flipped the grids over to prove that they can get three-quarters.



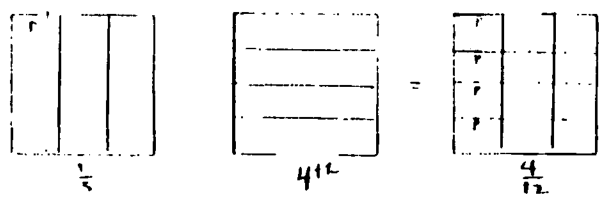
3. As the students continue to use the grids, Henry listens to them talk about how grids were exchanged. Henry clarifies how this "exchange" is recognized as the common denominator. A third group stares at the colored pencils. Mr. Goodman asks them how they could use the clear grid and pencils to solve the exercise. The group filled in two-quarters with a colored wax pencil, and then added another quarter.



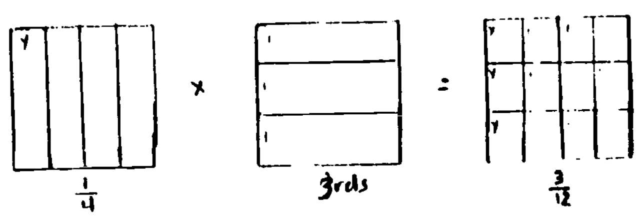
4. The following day, Henry takes out the grid that represents thirds and the clear grids that represent fourths and twelfths. At this point, Henry asks each group to use the overhead projector to explain its findings to the rest of the class. He asks the class to

"play along" with each teaching group. He gives the grids to the students and asks, "How can you use the clear acetates on top of the colored grids so that each colored section is divided into even parts?" The trick is that each grid must be divided into the same amount of parts. Through discovery learning, some students conclude that the clear grids will have to be rotated sideways in the opposite direction of the colored grids to show the equal parts. Henry asks, "What is the result?" (The students show him the division of twelfths.

One group solved it like this:

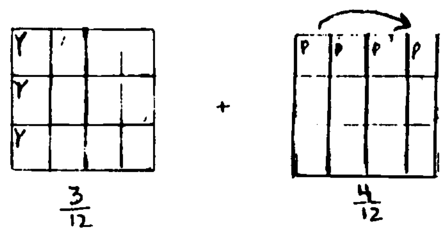


"Now," Henry says, "the fourths grid must also be divided by twelfths." Once again, through discovery learning, the students see that by rotating the clear thirds grid, twelfths are produced.



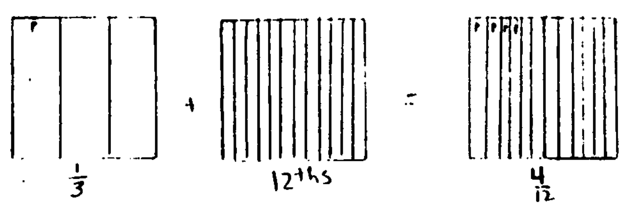
"Look," Henry says, "both acetates have an equal number of squares. This is the origin of the common denominator. If we would add them together, how many twelfths would we have?"

One girl says, "If we rotate 4/12ths, the inside parts will match each other."

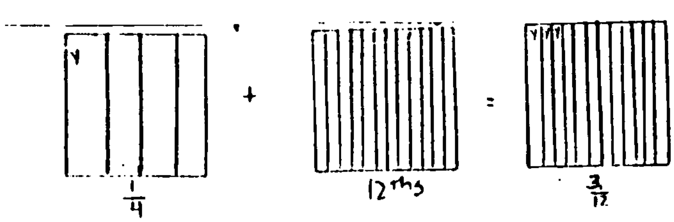


The students count to verify that there are 7/12 all together.

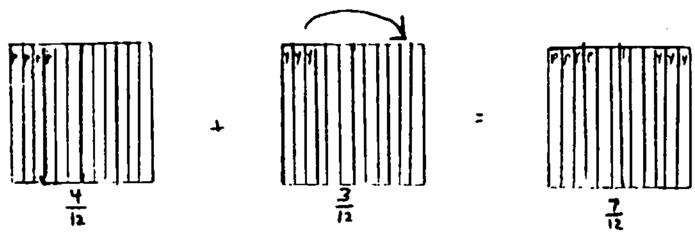
Another group of students develop a completely different system. First, they place the clear twelfths grid on top of the one-thirds grid.



They then show Mr. Goodman that if they place the twelfths grid on top of the quarters grid, it will equally divide into twelfths.



Next they rotate the 3/12 grid and place it on top of the 4/12 grid.



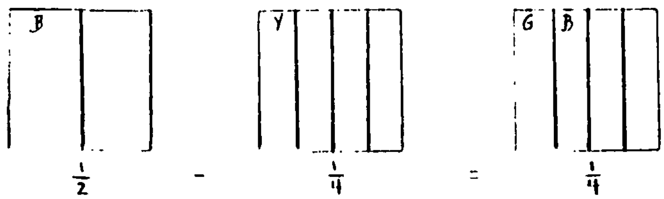
"Now," one student says, "we have 7/12!"

Henry asks, "What do you think is the meaning of the common denominator?" A student raises his hand and says, "When two fractions have the same size parts." Henry nods. "What is another way of saying this?" As students answer, he charts their definitions.

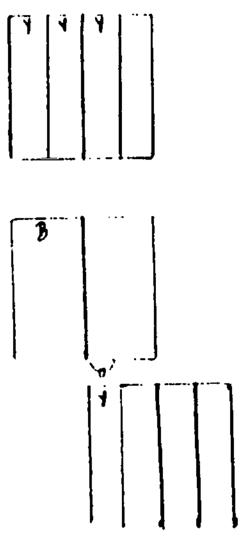
Subtraction

1. "The grids can also be used to do subtraction," Henry says. "Let's take the problem of one-half minus one-fourth." He hands out the halves grid, the fourths grid, the clear grid, and a bunch of colored pencils. He tells the students to solve the problem. One group takes the quarters grid and overlays it directly on top of the halves. "Good!" Henry says. "Which is the part being taken away?" The students are quick to respond, "The darker area!" Henry nods. "And the result?" he asks. "The lighter shade," they chorus. They can easily see from this example that the final result is one

quarter.



2. Henry knows that some examples are not quite so obvious, but he is encouraged by the students' enthusiasm. He leads a group to solve subtraction in another way. He asks them to take different grids and place them beneath each other to determine the amount left. He says, "This is called comparative subtraction. We can compare two fractions without taking things away. Let's say you have a 3/4 pizza pie, and I have a 1/2 pizza pie. Who has more and by how much? Try to figure this out:



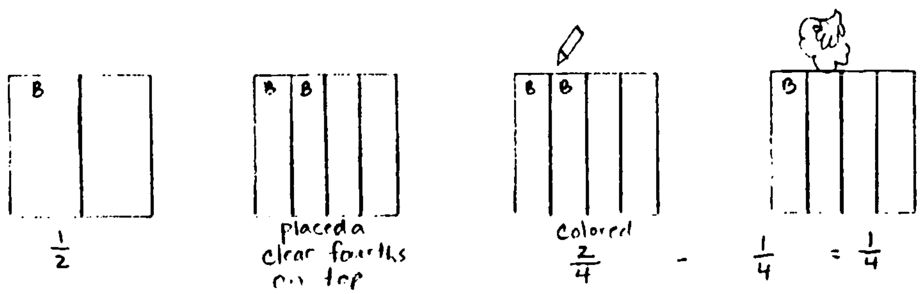
3/4

1/2

This is the amount that is different = 1/4

A few students lined up the halves grid under the 3/4 grid. One student explained, "I measured the difference and used another grid to figure out what the size of the piece left was equal to."

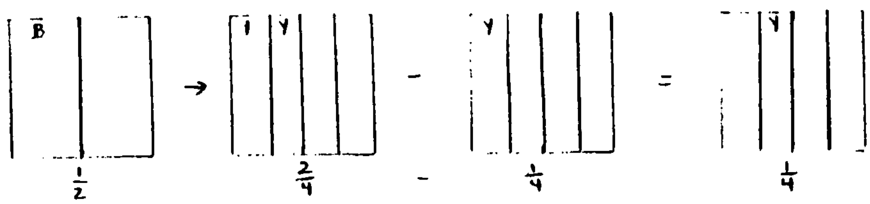
3. Henry takes another group and pulls out colored grids, clear grids, and the colored wax pencils. He asks the students to use these items to solve the problem. The group fills in two quarters and wipes away one quarter. "The answer?" he asks. "One quarter!" the students answer.



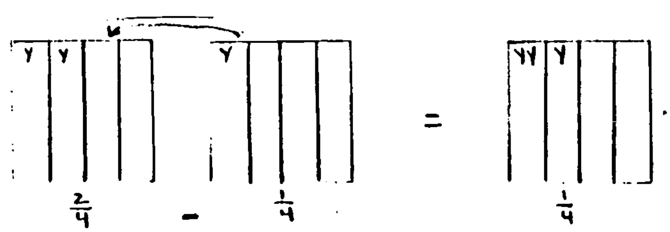
At this point, Henry brings the class together and has each group teach the other. Both Henry and the students prompt the teaching group with questions. Henry marvels at how easily they teach each other.

4. The next week Henry says, "There is still another way of using the picture grids to do subtraction. It's called the system of exchange. Let's use the example of one half minus one quarter. Let's see what you can figure out in your cooperative learning

group." One group says, "If we exchange the one half for two quarters, then we can easily subtract the one quarter from the two quarters."



He watches as another group places a quarters grid on top of the two quarters grid. "Good!" he encourages them. "Which is the area being taken away?" The students shout in unison, "The darker area!" Henry smiles. "Yes," he says. "And the area that remains?" The students are too excited to wait to be called on. "The lighter area!" they shout. Henry smiles again. Success is very sweet.



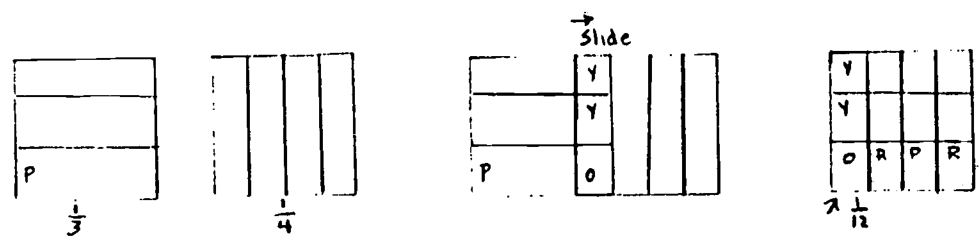
Once again the groups teach each other!

Multiplication

A few weeks later Henry asks the class, "How do you feel about multiplication?" The students groan. Multiplication is even harder than addition and subtraction. They're not ready to believe that these grids hold magic that can ease the discomfort of multiplication. After all, students rarely find "magic" in the classroom. "How many of you believe in magic?" Henry asks. Only a few hands go up. Henry knows that these few hands belong to courageous students--those brave enough to confess they still believe in magic are most are likely to be the ones in the greatest need of help. "Well," he says, "I hope more of you will feel differently once we've explored multiplication with the grids. And remember, magic means different things to different people. Each one of you would probably have a unique answer to the question. In math, however, there's only one right answer. The *magic* is that there are different approaches possible to reach that answer." The students lean forward in their seats.

Henry takes out the thirds grid and the fourths grid. He holds them up for the students to see. "All right," he says, "let's say that we want to take a third of a quarter. What does that mean?" A student replies, "If we divide the quarter into thirds, we could get three equal parts." Henry answers, "Yes, but what is each of those parts called?"

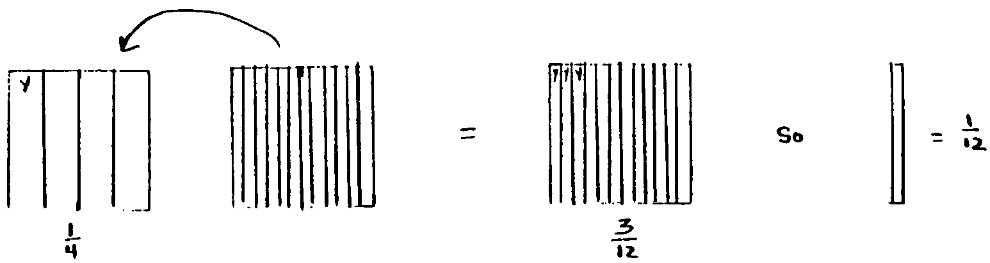
He asks the students to break into their cooperative learning groups and to then begin to manipulate the grids. Eventually, one student rotates the thirds grid and slides it across the quarters grid. "I think I'm on to something!" she calls out. Henry smiles. She is the student who groaned the loudest when he announced they would be working on multiplication. "Let's have a look," he says. The student has slid the thirds grid entirely across the quarters grid and divided the whole section into twelfths.



"What have we here?" Henry asks. "There are twelve squares now," she says. "And what do we call those squares?" he asks. "Twelfths," the students chorus. Henry turns to another student. "Can you describe what just took place?" he says. Several other students are eagerly waving their hands. "Just give him another moment," Henry says. The boys studies the grids and smiles. "Well, we took one part out of the three parts that make up the quarter." He thinks for another moment. "I guess now we know that one third of one quarter is equal to one twelfth!" Henry applauds, and the students join in. "Excellent work," Henry says. "I'm impressed."

Henry walks over to another group. "Try using the grids to divide a quarters grid into three equal parts." The students experiment with the different clear grids. "How about the tenths grid?" one girl asks. "Does that give you three equal parts?" Henry says. "I guess not," she says. "Try another one," Henry encourages.

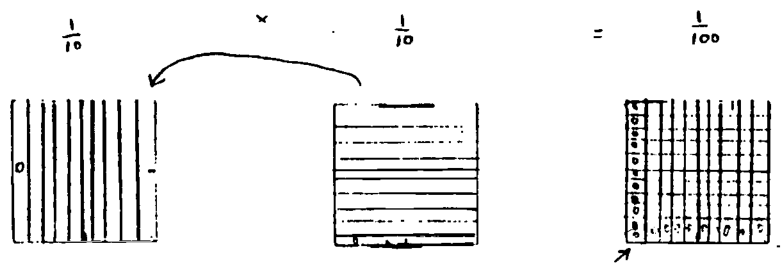
"The twelfths grid," another student says. He and his partner have slid the clear twelfths grid over the quarters grid and divided it into three equal sections.



"What are each of these three divisions called?" Henry says. "A twelfth," the students respond. "Good," he says. "Now state the problem." In unison the students say, "One third of a quarter equals one twelfth." Finally, they prove their conclusion by placing the clear twelfths grid on top of the quarters grid. At the end of the lesson, Henry brings the class together to share their findings

Decimals

"We can also use the tenths grids to examine decimals," Henry says. "Let's say we want to figure out what a tenth of a tenth is equal to. Now that you've had the opportunity to work out fractions, let's see what you can do with decimals. Henry actively listens as he roams from group to group. One student says, "First take a tenths grid. Now take another tenths grid and place it on top of the first tenths grid in the opposite direction." Another student jumps in with her conclusion. "This divides the bottom tenths grid into ten equal pieces." Henry smiles. "Well, does that mean we should we call each section a hundredth?" The class choruses their approval. "Looks like we have an answer!" Henry says.



Another group discusses the same concept: "If we slide the top grid completely over the bottom grid, we divide the entire area of the square into equal sections. Since there are one hundred divisions, each division is considered a hundredth. So what do we call the fractional part of a tenth?" The students take a few moments to adjust their grids and to consider the question. The answer is right in front of them. "A tenth of a tenth is equal to a hundredth," one student says, and everyone agrees.

Whether it's addition, subtraction, multiplication, or decimals, the grids are an invaluable teaching tool. The grids allow teachers to simplify these concepts. The students can actually maneuver the grids and really see the reasons for their answers. In this way, the students develop a visual understanding of how numbers are used. Then, and only then, do numbers become real and tangible, rather than a meaningless collection of symbols on a page.

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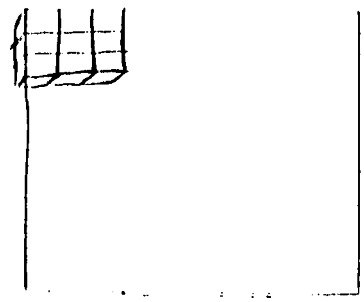
MORE FUN WITH PICTURE GRIDS

In a previous article, Henry Goodman took you on a journey that proved it was not only possible to teach students mathematics, but possible to help them *understand* how mathematics works. Through the use of both rods and grids, Henry was able to help students visualize the mathematics problems that had previously remained a mystery.

The good news is that Henry Goodman is back. The *other* news--which may be either good or bad--is that he is determined to teach square roots. *Ugh! The students recoil in horror! The teacher, desperate and without any means of support, wrings her hands and wonders, "What can I do? Oh, what can I do?"* At this point in the script, our hero, Henry Goodman, enters. He brandishes his trusty tools. He is back, and he has brought **Picture Grids** with him. Once again, everyone is safe. The enemy, square roots, *can* and *will* be overcome! All breathe a deep and steady sigh of relief!

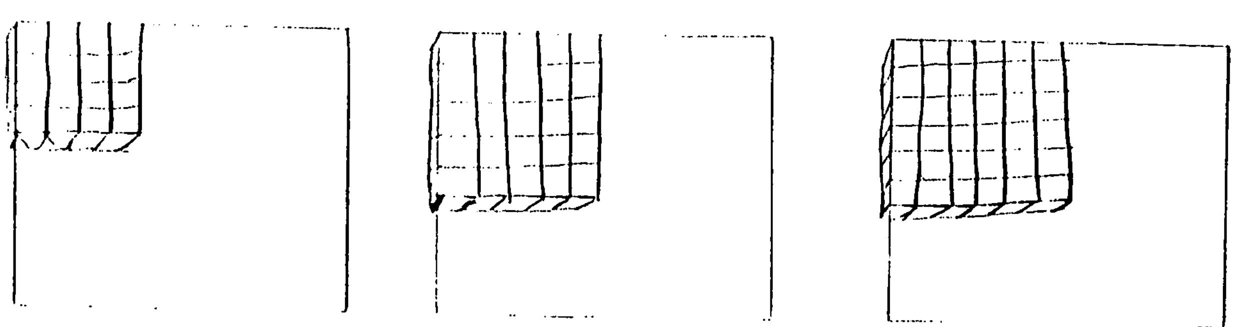
"All right now," Henry says. "I think we're ready for squares and square roots." He removes the units from the Cuisenaire rods to use in combination with the grids. "Take out the hundredths clear grid and place it in front of you." (It is sometimes best to use a white piece of paper under the grid. Colors will show up better and make the problem easier to visualize.) "Now, in the corner of the grid, create a square with nine units." He gives the groups sets of blocks and waits until they finish.

Then he holds up the blocks.



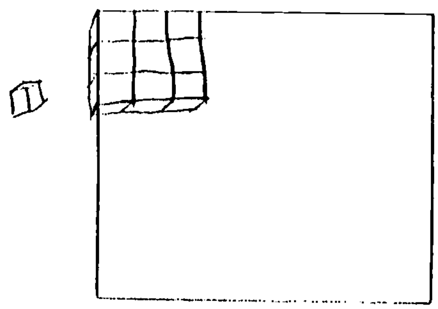
"A square root is equal to one side of the square. Rotate the grid and count the number of blocks on each side. What's your answer?" The students all say, "three." They understand that all the sides are equal because the figure is a square.

Next, Henry has the class try the same exercise with sixteen. They come up with the square root of four. Henry continues to give them more problems. They use twenty-five units for a square root of five, and thirty-six units for a square root of six.

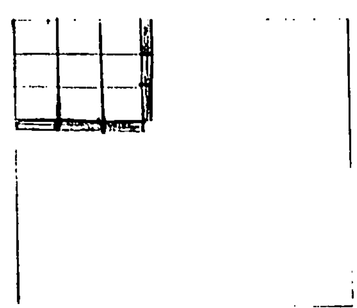


For middle school students, the following exercise is a bit

For middle school students, the following exercise is a bit more challenging and sophisticated. First, ask students to figure out the square root of ten. Tell them to build the largest square they can. The result will be a three by three square that equals nine with one leftover.

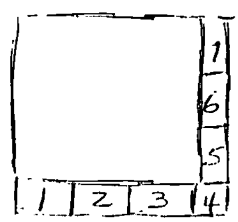


Now ask the students to pretend for a moment. What can we do to this leftover unit so that it will conform to our concept of a square? Some students suggest that the unit be cut into thin slabs and placed around the sides of the existing square. Other students realize that it's only necessary to place the slabs on one length and one width. They place three on the bottom and three slabs on the side.



Another student determines that while this is not exactly the answer, it is quite close. Henry asks the students to draw it, so they can visualize it. One students says, "We're missing the corner piece."

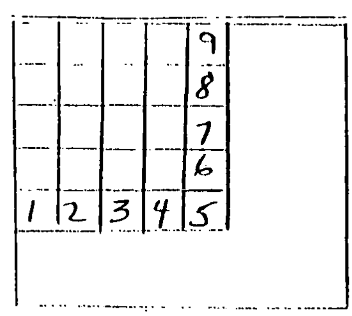
The students says, "So, we need to cut the leftover unit into seven pieces to create a square."



Henry encourages the students to determine the size of the square that follows the one already created. One student says, "We've already created a three by three, so then the next size up is a four by four." Henry says, "How many units will we need to add to the outside of the three by three in order to create a four by four?" A student says, "Seven." Henry nods and continues. "Therefore," he says, "the one Cuisenaire unit would have to be cut into seven "very" thin slabs. The corner slab would then have to be cut more than two times in order to fit into the corner."

Henry tells the students, "The approximation of the square root of ten would be three whole squares plus what?" One students answers, "Approximately one seventh?" Henry asks the students to use a calculator to compare their results of the square root of ten. They see that the answer is about three and one-seventh. Now students can making larger squares and determine the square roots. Henry gives one group the number 19:

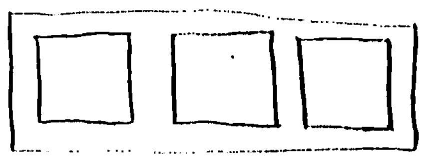
19 = One four by four + the three left
over from the next outside square of
nine. So...the answer is
4 3/9.



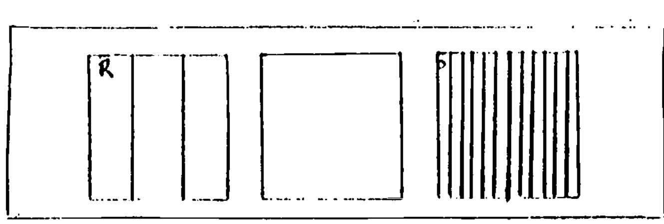
Once the students are familiar with the materials, they will become familiar with the square roots and be able to automatically calculate the roots in their heads.

Games

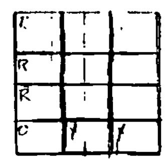
Students can also play games with the fraction grids. On a large sheet of paper, create three areas that students can use for their grids.



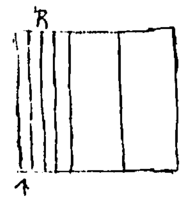
Students can place the original picture grid on the left side, keep the middle free, and use the right side for the answer. Students can play **Guess My Operation** or **Operation.** Each student can attempt to solve the mystery of his partner's solution to the problem by using the grids. Students can copy the problem, draw the results, and place a box around the answer. These sheets can be turned in to the teacher for an outcome based assessment. For example, begin with one-third. The end could display a large box divided into twelve equal parts with one area shaded darker to indicate one-twelfth. What operation was used? What fraction is missing?



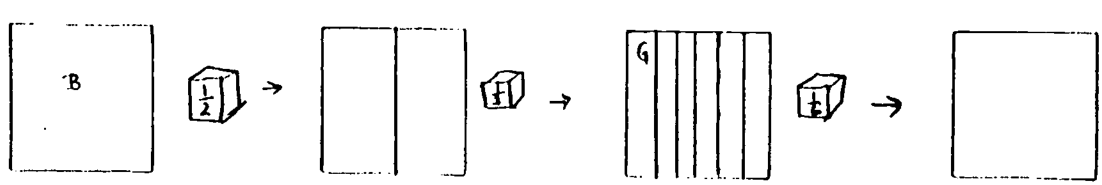
One answer could be determined by placing the quarters grid over the thirds grid until it is evident that one-third is multiplied by one-fourth.



Or, a student could take the clear grids & lay each one over the $\frac{1}{3}$rd to determine twelfths. He would discover that $\frac{1}{4}$ of $\frac{1}{3}$rd = $\frac{1}{12}$th.



Shrinking the Square is another game that students can play. Some students prefer to play with partners or against an opponent. The purpose of the game is to begin with a whole square and to shrink it. Take blank die and write one-half, one-third, one-quarter, one-sixth, and one-twelfth on its sides. Take a whole grid and place it in front of you. Roll the die and subtract the amount that is rolled. As the game continues, the students will need to keep rolling until the fraction needed is rolled.



In a previous article we saw that the grids are an invaluable tool for teaching addition, subtraction, multiplication, or decimals. Now it is clear that **Picture Grids** help students visualize squares and square roots so that *understanding* is as much a given as the right answer.

The suggested games are representative of the many ways that teachers can simplify the concepts essential for understanding fractions. Because the grids can be maneuvered, students can actually see the sections as they are divided and learn the reasons for their answers. When students are encouraged to develop a visual understanding of how numbers are used, numbers are no longer a mystifying body of incomprehensible symbols. Together with a visual understanding of numbers comes a new reality that is born of comprehension.

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