DOCUMENT RESUME

ED 407 260 SE 060 197

AUTHOR	Vahey, Phil
TITLE	Toward an Understanding of Productive Student Conceptions of Probability: The Probability Inquiry Environment.
PUB DATE	27 Mar 97
NOTE	17p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, March 24-28, 1997).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Computer Uses in Education; Educational Strategies; Educational Technology; Grade 7; *Inquiry; Junior High Schools; Learning Activities; *Mathematical Concepts; *Mathematics Instruction; Middle Schools; *Misconceptions; *Probability; Teaching Methods
IDENTIFIERS	Middle School Students

ABSTRACT

The Probability Inquiry Environment (PIE) is being designed as a computer-mediated collaborative inquiry environment to aid middle school students in learning elementary probability. This paper reports on a study in which seventh grade students engaged in probabilistic reasoning while interacting with a preliminary version of PIE. By analyzing the reasoning used by students, it was found that the findings from the standard "misconceptions" literature do not do justice to the wide range of viewpoints voiced by the students. In particular, the students did not consistently invoke such well-documented misconceptions as representativeness and the law of small numbers. Instead, the students invoked a great variety of intuitions, some of which approach normative reasoning in probability, and others which interfere with normative reasoning. The paper then discusses how probability instruction can be improved by introducing students to a progression of inquiry activities that build from the students' existing intuitions. Contains 28 references. (Author/JRH)

Toward an Understanding of Productive Student Conceptions of Probability: The Probability Inquiry Environment

by

Phil Vahey

Toward an Understanding of Productive Student Conceptions of Probability: The Probability Inquiry Environment

Phil Vahey
Education in Mathematics, Science and Technology
4533 Tolman Hall
University of California, Berkeley
Berkeley, CA 94720-1670
pvahey@violet.berkeley.edu

The Probability Inquiry Environment (PIE) is being designed as a computer-mediated collaborative inquiry environment to aid middle school students in learning elementary probability. This paper will report on a study in which seventh grade students engaged in probabilistic reasoning while interacting with a preliminary version of PIE. By analyzing the reasoning used by students, it was found that the findings from the standard "misconceptions" literature do not do justice to the wide range of viewpoints voiced by the students. In particular, the students did not consistently invoke such well documented misconceptions as representativeness and the law of small numbers. Instead, the students invoked a great variety of intuitions, some of which approach normative reasoning in probability, and others of which interfere with normative reasoning. This paper will then discuss how probability instruction can be improved by introducing students to a progression of inquiry activities that build from the students' existing intuitions (Vahey, 1996; Vahey, Enyedy, and Gifford, in press; White 1993a, 1993b).

Prior Findings in Probabilistic Reasoning

There is a rich literature based on the many misconceptions people display when asked to reason probabilistically[1]. By far the most influential work has been by Tversky and Kahneman (1982), who showed that much of people's probabilistic reasoning could be described by the heuristics of representativeness and availability. The *representativeness* heuristic is characterized by making judgments based on the degree to which *A* is representative of, or resembles, *B* (Tversky and Kahneman, 1982). This representativeness heuristic has been used to explain insensitivity to sample size, the gambler's fallacy, the base-rate fallacy, incorrect judgments about the output of random processes, and other non-normative judgments. The *availability* heuristic is characterized by making judgments based on the ease with which instances of a certain event can be brought to mind. This heuristic has been used to explain biases due to the retrievability of instances (such as thinking that car accidents are more likely in a town because you had a car accident in that town), biases due to the effectiveness of a search set (for instance, claiming that there are more words that end in "ing" than have an "n" as the second to last letter), and biases of imaginability (things that are hard to construct a mental image of are determined to be not likely).

In addition to those found by Kahneman and Tversky, many other misconceptions have been identified (for an overview, see Shaughnessy, 1992). For example, people may believe that there is a lack of variability in the world, people have too much confidence in small samples, people do not see the importance of small differences in large samples, and people seem unaware of regression to the mean in their lives (Shaughnessy, 1992).

Although this misconceptions literature has been fruitful in identifying some of the problems people have in reasoning probabilistically, several objections have been made to the misconceptions literature. General attacks point out that much of the misconceptions literature is based on the paradigm of expecting students to "replace" their current knowledge after being "confronted" with counter-evidence to their "misconceptions". This is an overly simplistic account of learning, and misconceptions studies do not provide an account of ways people can change their intuitions (which are often plagued by misconceptions) into more expert knowledge (Smith, et al., 1993), although such an account would be perhaps the single most important factor in designing learning

[1] Although many terms have been used to denote student ideas in formal domains, such as *naive conceptions*, *preconceptions*, and *alternative frameworks*, to name a few, much of the literature cited here is consistent with a view of misconceptions in which student ideas are determined to be in conflict with accepted meanings in formal domains (cf. Confrey, 1990; Smith et al., 1993).

environments. Specific attacks on much of the literature in probabilistic reasoning point out that much of the misconceptions literature is based on questionnaire and survey data, providing no account of the actual thought processes people use (cf. Konold et al., 1993; Shaughnessy, 1992), leaving researchers to infer people's reasoning. Additionally, there is no explanation for the variability of subjects' responses—for instance, there is no explanation as to why some subjects use representativeness, but others do not (Tversky and Kahneman, 1982). Finally, documenting such misconceptions in counterintuitive situations does not provide insight into correct reasoning people may employ when reasoning probabilistically (Hawkins and Kapadia, 1988; Lajoie, 1995).

Consistent with many of these objections, this study found that students did not consistently fall prey to misconceptions such as representativeness or the law of small numbers, although there was evidence that, for some students in some situations, such misconceptions did describe their behavior. Instead there was great variability as to the justifications and explanations provided, and this variability was found both across groups of students and within individual students. In short, if we are to design learning environments that are informed by our understanding of student conceptions of probability, we need to achieve a better understanding of student conceptions.

The goals of this paper are to advance our understanding of middle school students' probabilistic knowledge by analyzing seventh grade students' reasoning as they used the Probability Inquiry Environment; determine the ways in which such reasoning can be mapped to normative reasoning in probability; and begin to investigate how we can use this mapping to create authentic and collaborative learning environments that exploit students' existing conceptions as resources for cognitive growth.

Theoretical Framework

This study is situated in a constructivist view of learning that is based upon three areas of research:

(i) existing student conceptions play a productive role in the development of expertise, and we should not view misconceptions as deficiencies that must be replaced, but instead we should work to build from these conceptions (for a thorough treatment of this perspective, see Smith, et al., 1993; see also diSessa, 1988; Minstrell, 1989; Lampert, 1986; White, 1993a, 1993b)

(ii) students construct meaning and understanding while participating in a social context (Brown and Campione, 1996; Collins et al. 1989; Scardemalia and Bereiter, 1993) and student learning is facilitated by making their existing ideas explicit in order to evaluate these ideas with respect to findings in the domain and the ideas of others

(iii) learning environments should situate students in activities they consider to be legitimate and authentic. A productive learning environment will support activities that maintain fidelity with the relevant aspects of authentic practice, and not ignore the contribution of social practices and representational resources. That is, we should design intermediate tools and models that maintain fidelity with the relevant aspects of expert analysis while students participate in authentic tasks (Gordin et al. 1994; White, 1993a, 1993b)

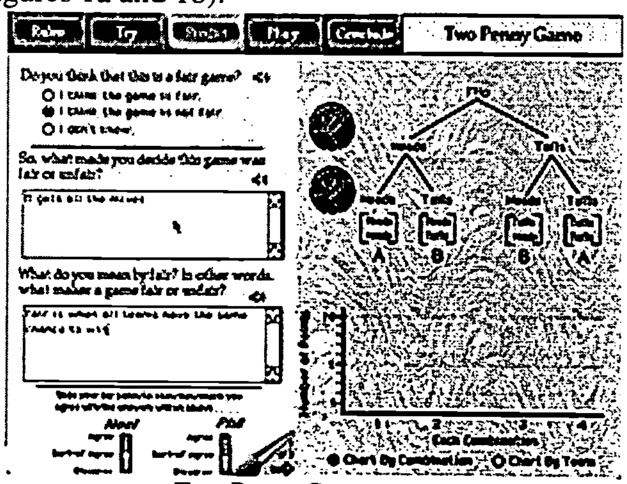
The Environment

The Probability Inquiry Environment (PIE) is being created as a collaborative, guided-inquiry learning environment in which seventh grade students are asked to evaluate the fairness of games of chance. PIE was orignially based on a provisional set of conjectures that was informed by the existing literature on students' conceptions in probability. These conjectures led to PIE consisting of a set of inquiry activities designed to address specific "misconceptions" in probability, such as representativeness and the law of small numbers. Because of the prototype nature of this version of PIE, and because of the short time the students interacted with PIE in this study, we did not expect to make a significant gain in terms of students' understanding of probability during this study. Instead, because PIE was designed as an open-ended collaborative inquiry environment, the purpose of this study was to discover the specific probabilistic intuitions that students included in their explanations and discussions[2].

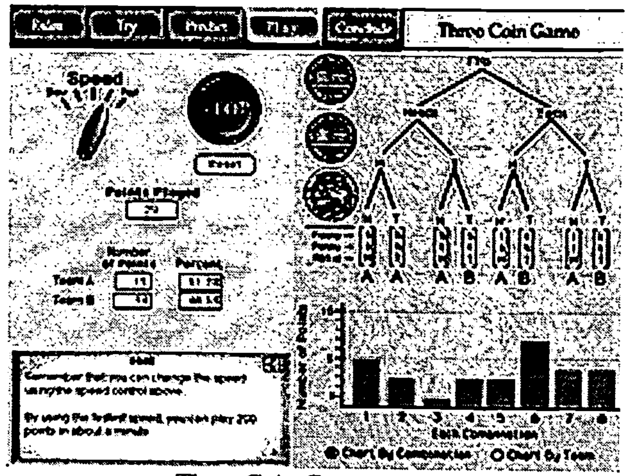
[2]It should be noted that another purpose of this study was to test the interface of PIE with seventh grade students, and not all students understood all aspects of the interface. Several of the students interpreted specific questions in ways that were unexpected, and so there is no in-depth analysis of responses to specific questions. Instead student justifications are analyzed (for whatever question they thought they were

We chose analyzing the fairness of games of chance because we have found that students are interested in the notion of fairness as it applies to games of chance (Vahey, 1996). Additionally, the literature on moral development shows that middle school children are interested in notions of fairness, and students of this age have developed sophisticated notions of fairness (Damon, 1988; Thorkildsen, 1995).

In this study students were asked to analyze two games of chance to determine if they were fair. The first game is called the Two-Penny game, where Team A scored a point whenever both coins come up the same (heads-heads or tails-tails), and Team B scored a point whenever both coins come up differently (heads-tails or tails-heads). This game is fair, as all outcomes are equally likely, and each team scores on two out of the four possible outcomes. The second game is called the Three-Coin game, where Team A scored a point on five of the eight possible outcomes, and Team B scored a point for three of the possible outcomes. Because each outcome is equally likely and Team A scores on more outcomes than Team B, this game is unfair in favor of Team A. An event tree that enumerated all the possible outcomes and that visually presented the scoring combinations for each team was on the screen at all times, as was a dynamic histogram that showed scoring either by each combination of coins, or by each team (see figures 1a and 1b).



Two Penny Game
Figure 1a: the PIE software interface



Three Coin Game
Figure 1b: the PIE software interface

An inquiry approach was chosen because cognitive science research in the field of mathematics has demonstrated that the inquiry approach to mathematics instruction can provide a rich, engaging and meaningful context which facilitates students' acquisition of mathematical concepts (Balachef, 1987, cited in Schoenfeld, 1991; Lampert, 1995; Richards, 1991), and investigating complex concepts in this manner scaffolds the development of metacognitive knowledge and skills (White & Frederiksen, 1995). In PIE students collaborate in the authentic tasks of: creating conjectures to explain genuinely puzzling situations; running a computer simulation that generates data that can be used to test their conjectures; analyzing this data to determine the appropriateness of their conjectures; and creating and negotiating an understanding that accounts for their findings, which is then agreed upon and typed into the computer environment. During this inquiry cycle students use authentic representations and tools such as event trees, simulations, and real-time graphs and histograms.

The inquiry cycle used in this version of PIE consisted of five steps: Rules, Try, Predict, Play, and Conclude. Throughout each of these five steps the right side of the screen remained relatively constant, and always showed a probability tree that denoted all the possible outcomes of the coins flips, with each outcome labeled according to which team, A or B, scored for that outcome. That is, the partitioning of all outcomes remained on the screen at all times. In *Rules* the students were shown an animated introduction to the current game. In this introduction the students were shown

---

answering) and the probabilistic reasoning used in these justifications is analyzed.

that the coins are flipped, and the outcomes that score a point for each team were explicitly shown to the students. In *Try* the students were introduced to the coins, probability tree, histogram, and controls by allowing them to manipulate these items by clicking them with the mouse. This was done to allow the students some amount of familiarity with the environment before asking them to make predictions. In *Predict* and *Conclude* the students were asked to make predictions and conclusions, respectively, and these predictions will be discussed in more detail in the following paragraphs. In *Play* the students started and stopped the game by clicking on a button that toggled between "Go" and "Stop", were able to control the speed of the coin flips with the fastest speed flipping the coins at a rate of about 100 points a minute, could switch the histogram between viewing by combination or viewing by teams, and had a set of hints that changed based on the status of the game (recommending, for instance, that students play at the fastest speed if after many points they had not already done so). As the coins flipped in *Play* the probability tree highlighted the current state of the coins.

The Prediction and Conclusion questions were similar across both activities (the two coin game and the three coin game). There were three sets of prediction questions (figure 2). The first set of questions asked students about fairness: was this game fair, what do they mean by fair, and why is this game fair or unfair. The second set of questions had students manipulate histograms to make predictions about what they might expect after 10 points and after 200 points. The third set of prediction questions asked students to predict the relative frequency of each combination. The first three conclusion questions paralleled the prediction questions—students were shown their predictions and were allowed to keep their predictions as conclusions, or enter new conclusions. One additional set of conclusion questions was asked in which students specified the most important thing in deciding the fairness of this particular game, and were explicitly asked if the number of combinations that scored for teach team were important.

Do you think that this is a fair game?
___ I think the game is fair.
___ I think the game is not fair.
___ I don't know.

How many points do you predict that Team A and Team B will have scored after playing for 10 turns? After 200 turns? Show how many points you think each team will score by pulling up and down the bars above.

Do you think any combination of coins is going to happen more often than any other? Pull each Yellow bar to determine how often you think that combination of coins will occur.

Figure 2: Prediction Questions

These games and questions were designed to elicit specific misconceptions. For example, we expected that the first game would address some aspects of the representativeness misconception. Because Team B scored a point whenever the coins where "mixed", we expected most students to answer that this game was unfair, in favor of Team B. We expected that the three different prediction questions would allow the students to look at the same situation from three different perspectives, providing a way for them to integrate their understanding of different aspects of probability. This expectation was too simplistic: asking what we considered to be similar questions from several perspectives did not provide an opportunity for students to integrate their knowledge, but instead provided students with an opportunity to invoke different reasoning strategies.

The Study

The research team recruited eight single-sex pairs of students from a local urban middle school. All of the sessions took place in the summer, between the students' sixth grade and seventh grade school years. Four of the pairs were boys, and four of the pairs were girls. The students represented a wide range of ethnicities. The students were paid $5 and hour, and spent about an hour and a half interacting with the computer during one two hour session. During this session they were videotaped, and PIE recorded their actions on the computer.

The data used in this study consists of student discussions as they participated in the activities, and their responses to the predict and conclude questions. The videotape data of the students using PIE were transcribed, and these transcripts were combined with the data recorded by PIE to create a record of all student discussions and student interactions with PIE during the session.

These transcripts were then analyzed, and all instances of students' reasoning about the games was found. Although, by the end of the study, seven of the eight pairs of students were able to reason normatively about the games, this paper will not concentrate on the events that led to this normative reasoning. Instead, the students' reasoning throughout the entire session will be compared to the way in which normative probabilistic reasoning could have been employed in those situations. The characteristics of normative probabilistic reasoning discussed in this paper corresponds to four interrelated conceptual areas in probability. These areas are randomness (understanding that the game is based on a non-deterministic mechanism), the outcome space (the possible outcomes for each point played), the probability distribution (the probability of each outcome), and the validity of evidence (the law of small/large numbers). A justification for this characterization will be presented after the next section. First, to acquaint ourselves with an example of student reasoning in PIE, a brief case study will be presented.

A Case Study of Q and T Using PIE

When Q and T were assured that they understood the two-penny game (Figure 3), they made their predictions. When predicting that the Two-Penny game is fair, they explicitly assigned a 50% chance to the combinations of coins that score a point for each team, resulting in a final answer that is perfectly aligned with normative reasoning (Figure 4). Note however, that the students never explicitly justified this 50% chance, and we will not attempt to make claims about the students' probabilistic reasoning in this instance.

T: I get it, if they're both heads team A gets a point, if they're both tails team A gets a point, and if they're one heads and one tails Team B gets a point.

Figure 3: Understanding the two-penny game.

Q: we think the game is fair because you have a 50% chance of getting both heads and both tails.

Typed: We think that the game is fair because you have a 50% chance of getting both heads and both tails. You also have a 50% chance of getting one tails and one heads.

Figure 4: Fair, based on a 50% chance for each team.

Q and T next answered what they meant by fair. Q and T decided that a game is fair if all teams have an equal chance of winning, a normative view of fairness. Student ideas of fairness will be further discussed in the next section.

In the next question, Q and T were asked to manipulate histograms to make predictions about what would happen after 10 points, and after 200 points. T stated that heads would occur more, so she would expect Team A to win more. Q countered this by saying that coins usually come up differently (note that this is consistent with representativeness), so she would expect Team B to win more. They then decided that the game will most probably be tied (Figure 5). After further discussion, the students decided that luck would be an important factor in the game (Figure 6), and this meant that Team A might win sometimes, and Team B might win other times. Again, the students provided answers that were close to normative, but looking solely at their final answer misses their reasoning process, which was quite rich and invoked more intuitions than are found in the final answer.

T: OK, what about this, I think Team A will win. Because imagine all of the heads we are gonna get

Q: I think that Team B would win it, because when you throw it it's like real luck when you get both of them the same. When you throw it, most of the time they land differently. [pause] I think they'd probably be tied, but if I had to choose one, see, this looks right to me [pointing to even histogram bars]

T: that's what I think too

Figure 5: Who will win more?

T: I think that this is just a game of luck...this is like a game of guessing

Q: it's really like someone has to win, because it's like you win some you lose some, it's not like a permanent game.

[T moves histograms so A is winning after 10, and B is winning after 200]

Q: now how can purple [Team A] be winning on this one and green [Team B] be winning on that?

T: well, once you win you don't always win

Figure 6: A game of luck

For the final prediction question in the two-penny game, the students were asked if any of the combinations (outcomes) would happen more than any other. The students were again asked to manipulate a series of histograms, and type in a justification. Although Q and T started to reason about the different combinations, they then switched to talking about the probability of heads or tails. They decided that because this is not a game of skill, but a game of luck, they would expect heads and tails to come up the same amount (Figure 7).

Q: we don't think that any combination should happen more than any other, because it's luck

T: it's how you throw it

Q: plus, it's a game of luck

T: we don't think that, it's not a game of skill

Q: that one penny will come up more than the other

T: it's all equal...wait a minute, we don't think that heads or tails is more likely to come up than the other. We don't think that heads or tails will come up more

typed: We don't think that heads would come up more than tails. We think that this game is a game of luck.

Figure 7: Will any combination happen more?

Q and T then started the game simulation. After only four points they noted that tail-head was happening more than the other combinations. The game then stopped after ten points, and told the students that they could either look at the results or continue playing. At this point Q wanted to go back into predictions to see if the predictions agreed with the results (note that this behavior is consistent with the well-documented law of small numbers). The researcher asked them to continue playing, telling them that they would be able to modify their predictions later, so Q and T continued in Play.

After playing for several more points, Q and T then decided to run the simulation at the fastest speed. At this speed the game runs ten points at a time, and individual coin flips can not be perceived. After about 20 seconds the game reached 200 points, and gave the students the option to continue playing or stop and analyze the results. Q and T chose to stop playing and go immediately into Conclude.

The first three conclusion screens asked the students to evaluate their predictions. For each of these conclusions, the students stated that, although their predictions did not exactly match the actual data, the results were close enough for them to still agree with their predictions (Figure 8). Additionally, in Figure 9, Q came back to their earlier statement that being a game of luck is an important aspect of the game. This becomes the single most important factor for these students for the remainder of the session.

T: even, look, almost even

Q: yeah, so it's pretty fair because no one is like way, way more than the other

Figure 8: Conclusion—is the game fair?

Q: No, OK, why? We keep saying the same thing over. Like this evidence-no because this is just a game of luck. And they're all equal anyway.

Figure 9: Conclusion—are any combinations more likely?

For the last set of conclusions the students were asked if the number of combinations that score a point for each team is an important factor in determining fairness. However, Q and T understood this question to be asking about the data already collected. Although they first stated that this data is important, they then decided that luck is more important, and one doesn't need to analyze the data (Figure 10). Then, when asked to state the most important thing in determining if the game is fair, they again stated the importance of luck (Figure 11).

Q: very important, don't you think...why is it very important?

T: it's data

Q: it's important data, and um

T: you need the data to play the game

Q: you need to know the number of combinations...But it's not that important though, as a matter of fact, it's not important at all, cause it's a game of luck. Yeah, it's not important

T: it's a little important...

Q: ...but the game is just a game of luck anyway. So if you didn't have the data, it wouldn't matter anyway.

Typed: It is important because it's data, but on the other hand it is not that important because it's just a game of luck.

Figure 10: It's just a game of luck

Q: The most important thing is that you understand that the game is just luck

Figure 11: The most important thing in determining fairness

Q and T were then introduced to the Three-Coin game, and Q once more decided that the game was fair because it was a game of luck, although T was hesitant to agree. However, T could not state why she thought the game was unfair, and finally determined that, since all the outcomes were possible, the game must be fair. Note that, although the partitioning of the outcomes into points for A and points for B were on the screen at all times, the students never considered counting the outcomes to determine if each team had an equal number of outcomes. Although it is dangerous to make inferences based on the *absence* of an action, the fact that it never occurred to these (or most) students to simply count up the number of outcomes, especially when something about the game seemed troubling, may point to a lack of an understanding of the importance of the outcome space in determining probabilities.

Then, consistent with their predictions for the Two-Penny game, and consistent with their idea that luck means that a game is fair, for the remainder of the predictions Q and T stated that the teams will score an approximately equal number of points, and each of the combinations should occur equally. They then put the game on the fastest speed, and quickly played up to two hundred points. When the game reached two hundred points, they went into conclude and simply agreed with all their predictions, without comparing their predictions to the actual results, even stating that the data is not important (Figure 12). Such reasoning is based, presumably, on the statements made at the end of the Two-Penny game, that a game of luck must be fair, and data is not an important factor.

```
Typed: THE COINS IS JUST DATA AND THE DATA IN THIS GAME
  IS NOT THAT IMPORTANT.
```

Figure 12: Conclusion—are the combinations in the Three-Coin game important?

At this point the researcher asked them to play some more, reminding them of the Reset button that sets the points back to zero. After playing several more rounds up to two hundred points, T decided that, because Team B kept losing, the game must be unfair. Q, however, kept stating that the game is just luck, and so must be fair (Figure 13) (note the contrast between this behavior and her earlier desire to check their predictions after only ten points). T did not accept this answer, however, and kept looking for an explanation. T finally noticed the difference in the number of outcomes that scored a point for each team, and then resorted to a strategy of making the game fair, contrasting a partitioning of points that would make the game fair with the actual partitioning (Figure 14). Q then understood how this partitioning was relevant and agreed that the game was unfair.

```
T: See, Team B is losing by a lot.  Told you it was
  unfair
...
Q: this game is just luck, it's just a penny game, it's
  just luck
```

Figure 13: Unfair versus luck

```
T: why is it fair, Q?
Q: because it's a game of luck, it's jut throwing
  pennies
T: I'm not talking about whose got the penny, I'm
  talking about right here [pointing to the tree]. They
  keep losing, and I'm trying to figure out why...wait a
  minute! See how this is AA right here? and this is AB
  AB AB
R: mmm-hmmm
T: but shouldn't it be BB?
R: what do you mean, shouldn't it be?
T: right here it says AA AB AB AB
```

Figure 14: Making the game fair

Summary: Q and T's analysis of the Two-Penny game began with them stating several different, often competing or conflicting, intuitions about probability, few of which seemed to carry any deep commitment. And, although representativeness could be used to describe some of their reasoning, it is a far from adequate account, as much of their reasoning is inconsistent with representativeness. Q and T then began to consider luck the single most important aspect of the game, even stating that they did not need data to determine the fairness of the games. Their commitment to this position was shown in the Three-Coin game, when Q explicitly denied the importance of data that showed that this game was unfair. Note that she did not fall prey to the law of small numbers, nor did she dismiss the game as "cheating", nor did she suffer from confirmation bias and misinterpret the data as showing that the games were tied. Instead, she acknowledged the results of the simulation and simply said that these results were not relevant. It was not until T was able to determine that the partitioning of the outcome space was unequal that they were able to confirm that the game was unfair, and it took a notably long time until this counting strategy was employed by the students, lending credence to the supposition that the outcome space was not a salient feature of this situation. Note that most of this behavior is not consistent with the heuristics and biases view of probabilistic reasoning. It seems as though we need another view of students' conceptions of probability that is different from that offered by the traditional literature.

Results: a framework for understanding probabilistic reasoning

As mentioned earlier, PIE was specifically designed to elicit students' ideas about representativeness and the law of small numbers. However, as the case study of Q and T shows, such a simple explanation does not account for much of the reasoning employed by the students. In the spirit of the constructivist approach outlined earlier, we must analyze this reasoning to determine the ways in which student intuitions can be expanded to include the concepts in the domain of probability if we are to design instruction that helps students bridge from their existing knowledge to the formal domain knowledge of probability. This section begins to do this by analyzing the probabilistic statements of all the students in this study.

The primary activity the students were engaged in was determining if the two games of chance were fair. This activity, situated in PIE, provided the students with an environment where they could investigate an activity that was, to them, authentic and meaningful, and all pairs of students invoked reasoning that is easily recognized as probabilistic. In order to understand how this intuitive reasoning compares to normative reasoning, we must first have an understanding of what we mean by normative reasoning in this activity. The version of "normative" reasoning presented here is that of the idealized reasoning process used by someone with an understanding of elementary probability who is faced with a novel situation. The novel situation in this case is determining if the games of chance described previously are fair.

This normative reasoning will first determine what is meant by "fair". After this any of the several different reasoning processes considered "normative" will have the characteristics that they will (i) determine that the game is based on a non-deterministic mechanism (i.e. understand some aspects of randomness); (ii) determine the outcomes that score points for each team (i.e. understand the outcome space); (iii) determine the probabilities of the outcomes that score a point for each team (i.e. understand the probability distribution); and (iv) compare the expected fairness of the games to the actual fairness after playing for some large number of points to determine if the theoretical expectations are accurate[3].

The main finding is that students displayed a wide variety of ideas, some of which approach normative reasoning in probability, and others of which interfere with normative reasoning, and there is great difficulty in attempting to characterize students as exhibiting specific misconceptions. Many of the students' statements were situation specific, and students were willing to make contradictory claims when discussing the same basic circumstance, such as the expected results after many flips of a coin. Instead of attempting to characterize each student as believing some specific misconception, it is perhaps more instructive to consider ways in which we can elicit their different ideas, and attempt to build from those ideas that seem most productive.

Fairness

Several studies have determined that fairness is a productive study of inquiry for students investigating probability (Hirst, 1977; Scheinok, 1988), and PIE is based on this premise. The findings from this study will show that, although fairness is a highly motivating and productive area of investigation for middle school students, we should be aware of different versions of fairness that students consider relevant in different situations.

---

[3]Note that this description of normative reasoning can remain agnostic with respect to specific beliefs about the probability of the elementary events involved. For instance, one reasoning strategy that fits this description of normative is as follows. The students believe that heads occurs systematically more than tails. The students then notice all the outcomes that score a point for the different teams, and then determine how the increased probability of heads compared to tails changes the probability of each outcome, and hence, the probability of scoring for each team (as an aside, the increased probability of heads-heads more than offsets the increased probability of one head and one tail, making this game unfair in favor of Team A in this scenario). The students then run a few simulations of the game and notice that, at a few hundred points, the teams are only a few points apart, and no team systematically wins more than the other team. At this point the students should either not believe the computer simulation, or revisit the assumptions of their theory. There are obviously many other reasoning strategies that could be considered in this discussion, but the point here is that any such reasoning strategy will take into account characteristics (i) through (iv) mentioned previously.

Each pair of students discussed the normative conception of fairness at least once during the session (see figure 15), stating that for a game to be fair all teams should be equally likely to win. Although Metz (1995) has found that younger students may understand fair to mean that the teams must score in lock-step fashion, students in this study exhibited no such ideas, and some explicitly stated that fairness means that there may be variation in outcomes, but that variation should not systematically favor one team (figure 16).

UV: (typed): FAIR IS IF THE TEAMS ALL HAVE A EVEN CHANCE OF WINNING.
O: It's not easier for one player to win than the others.
T: or you could say it would be fair if both teams got the same chance.

Figure 15: normative ideas of fairness

S: yeah, so it's pretty fair because no one is like way, way more than the other
----
K: I think it might be fair, because sometimes one is losing and one is winning

Figure 16: fair means that there will be variation

These were not the only notion of fairness invoked by the students. Two pairs of students stated that a game was fair if no one cheated (figure 17). Other students stated that a game was fair if it was possible to win (figure 18). Although such reasoning could be used to pronounce the Three-Coin game (an unfair game) as a fair game, none of the students in this study did so. However, some students who predicted that the Three-Coin game would be fair, after running the simulation and seeing that Team A won almost all of the games, did use different versions of fairness to justify why their predictions may still be correct. Two students stated that a game could be fair for one team, but not fair for the other. That is, if a team is winning, they could perceive the game as fair, whereas a losing team could perceive the game as unfair (figure 19). Although some of these ideas allowed students to maintain that the Three-Coin game was fair, even in the face of contrary evidence, students tended to hold more strongly to the normative notion of fairness, which states that the expected outcome should be equal for both teams. That is, students were typically willing to give up on non-normative notions of fairness after they saw that the Three-Coin game was unfair because Team A had more outcomes than Team B.

EN (typed): "BECAUSE ITS NOT A CHEATING GAME."

S: yeah, equal, no cheating, no one has a better ability of winning, it's up to you to win.
T: no one can cheat.

Figure 17: why is this game fair (part 1: no cheating)

L: yeah, it's not like impossible to win
V: [it is fair if] if every team has a chance
O: I think it's fair because everybody has a chance.

Figure 18: why is this game fair (part 2: everyone has a chance)

M: [the game is fair] because it lets me win.
C: I think it's fair because I'm winning

Figure 19: Game is fair for one team

Given this variability in notions of fairness, it is important that we ask how can we use fairness to help students learn probability. The answer offered here is that the context of fairness of these games was engaging and motivating to these students, and was a productive method of getting them to talk about probability theories in a context they considered authentic. There is caveat in that, although most students may offer normative versions of fairness in some contexts, this does not mean that students will continue to use those ideas of fairness in other contexts. Fairness is an issue that should be discussed and revisited throughout any unit of probability that depends upon fairness, with the different definitions of fairness compared and contrasted. Because most students do embrace the normative definition of fairness, it seems as though we can be optimistic in terms of expecting students to expand this definition of fairness to apply to most situations involving probabilistic reasoning.

Randomness:

During the course of the study, every pair of students made reference to the fact that non-determinism was an important factor in analyzing the outcomes of coin flips. This reference typically came through students talking about the game being based on "luck" or "chance", and also by contrasting these games with games of skill. Additionally, students stated that the random process of coin flips would result in variation between trials (figure 20). Expecting variation from a random process is consistent with the normative view of randomness.

K: It won't be totally even

S: it's really like someone has to win, because it's like you win some you lose some, it's not really like a permanent game.

Q: the tides can change

Figure 20: Randomness has variability

However, as seen in the case study of Q and T, some students also stated a conception about randomness that led to their believing that the Three-Coin game was a fair game. Three of the eight pairs of students stated that "luck" or randomness meant that nothing could be predicted about the games. When these students discussed the game being based on luck, they were stating that no predictions could be made about how the flips would turn out, and since anything could happen, the games must be fair (see figure 21). This result may be consistent with the "outcome approach" as described by Konold (1991, Konold et al. 1993), as the students were replying to a question about a series of events as if the answer depended upon being able to predict any given event. As mentioned in the previous section on fairness, and as will be discussed in the later section on data validity, these pairs of students did not accept that the Three-Coin game was unfair until they noticed the difference in the number of paths, and then were able to create a new understanding that could explain the data.

KL (typed): I THINK THIS GAME IS FAIR BECAUSE IT'S JUST LUCK OR FATE WHAT IT LANDS ON!!!!!!!

...

K: because it's luck is the answer for the whole thing

U: the most important thing is that you understand that the game is just luck

----

O: anything can happen, so that's why I think it's fair

Figure 21: Luck means you can't make any predictions

In attempting to apply these findings on randomness to the creation of learning environments, we can see that students do have productive and normative ideas about randomness. Specifically, flipping coins is understood to be a non-deterministic event, and many of the students explicitly stated that for such an event variation is to be expected from trial to trial. However, some students also believe that such a non-deterministic event means that nothing at all can be predicted about future events, even to the point of excluding data (as will be discussed in the next section on data validity). It is important to note, though, that this can be viewed as a normative understanding of randomness that has been over-extended. That is, a frequentist understanding of probability does maintain that it is meaningless to apply probabilities to predict the outcome of specific events, so these students are in many ways "correct" when the apply this intuition to short-run data. It is only when extending this idea to long-run data that this reasoning becomes non-normative.

The Outcome Space and Probability Distribution: In formal probability theory, when determining the probability of an event, one first determines the relevant outcomes (in the games discussed in this study, these are all the possible combinations of two or three coins). Then, creating or using a probability distribution, which assigns probabilities to each outcome in the outcome space, one determines the likelihood of each of the outcomes, and then combines probabilities to determine the probability of specific events (such as Team A scoring a point). Using such a theoretical model, one can clearly differentiate between the outcome space and the probability distribution, and indeed, these two entities are often introduced at different times in probability and statistics textbooks (e.g. Pitman, 1993).

Although the students in this study did invoke ideas similar to the outcome space and the probability distribution in this study, they often reasoned in a way that made it difficult to distinguish between the two, especially when discussing the three-coin game. A working hypothesis is that it was easy for students to understand and verbalize the outcome space for the Two-Penny game, whereas the outcome space of the Three-Coin game was more complex, making it harder to discuss. As a result, even though the outcome space was always on the screen for the both games, students who referred to the outcome space when making predictions about the fairness of the Two-Penny game did not refer to the outcome space making predictions about the fairness of the Three-Coin Game (figure 22).

Two Penny Game:

O: because it's heads and tails.

F: and it's two heads and two tails

Three Coin Game:

F: You will get more because you start out first

Figure 22: Outcome space referenced in Two-Penny Game

A further problem students had when discussing the outcome space of Three-Coin game was in differentiating the individual outcomes (such as head-tail-head) from the combination of all outcomes that could score a point for a team. So, for instance, when asked the probability of the specific outcomes occurring, many of the students would state that those outcomes that score a point for Team A were more probable, because they expected Team A to win. Although such statements can be interpreted as students simply not understanding the environment, it is more consistent with their other statements to posit that the outcome space is not a well-formed concept with these students, and they have a difficult time understanding how to relate the outcome space to a complex event such as the probability of a team scoring a point.

When students did discuss the outcome space, some of the students explicitly stated that order did not matter when differentiating between outcomes (figure 23), resulting in a behavior that could easily be seen as representativeness. However, this version of representativeness is based on not fully understanding how to properly enumerate and partition the outcome space. This can be contrasted with students who don't expect "patterns" in data (figure 24). The latter seems closer to the traditional definition of representativeness, and is based upon applying a non-normative probability distribution to a normative enumeration of the outcome space.

M: ... see, like, there's three of HHT, and 3 of TTH, but only one of HHH and one of TTT, so it's hard to get HHH, it's easier to get HHT

Figure 23: not differentiating outcomes based on order

D: umm, I chose those I guess cause it's like too much of a pattern, like Tails tails tail uh...yeah, seems like less of a chance for it to go on that same one all three times, and like tails heads tails, like, in a pattern like that, and like heads tails heads

Figure 24: representativeness based on a non-normative probability distribution

At other times students did not make a clear distinction between the outcome space and the probability distribution. These students made statements that were not only ambiguous as to whether they were talking about the number of outcomes or the probability of certain outcomes, but they would often switch between talking about numbers of outcomes and probabilities of outcomes (figure 25). In fact, these students sometimes seemed to be reasoning normatively, discussing the different outcomes that score a point for each team (although without explicitly counting the number of paths), but were then surprised when they later discovered that the outcomes were not equally divided between Team A and Team B.

N: I think the game isn't fair, because there is no way for team b to get their own, it's like an advantage for team a because all they have to do is just get 2 heads or 1 head with the nickel.

E: and it's easy to just get two heads

N: I know, it's easy to just get a head with the nickel, so I don't think it's fair

----

D: Just look at that and think about it. Look at how Team A scores: HHH, HHT, HTH

C: that's most of them right there

D: it's easier to get all that

Figure 25: ambiguous statements

As discussed in the sections on randomness and data validity, one set of students who ignored the outcome space when determining the fairness of the games did so for very principled reasons: they believed that, because the games were luck, nothing could be predicted about the outcomes, and so the games had to be fair. These students rarely made reference to the outcome space or to the probability distribution, as they explicitly stated that these were irrelevant to the fairness of the games.

Due to the short time scale of this study (each student used PIE for less than 2 hours), as well as the prototype quality of PIE, this study was not concerned with any gains made by the students in terms of probability concepts. However, student use of the outcome space is one area in which we observed a marked difference between students predictions for the Three-Coin game and conclusions for the Three-Coin game. In the predictions for the Three-Coin game only two pairs of students determined that the game was unfair, and only one of these pairs explicitly used the number of outcomes in their reasoning. In contrast, after playing the simulation and creating their conclusions, seven of the eight pairs of students realized that the game was unfair, and all seven pairs determined that the number of outcomes was the deciding factor in the game being unfair.

In summary, students exhibited many different ideas when discussing the outcome space and the probability distribution, making any generalizations difficult. We can say, however, that many students have difficulty in distinguishing between similar outcomes, and many students have difficulty in distinguishing between the outcome space and the probability distribution.

The Validity of Evidence:

It is in documenting people's beliefs about the validity of data that the misconceptions literature in probability is least controversial: it is well known that people, including trained statisticians, often fall prey to the law of small numbers (Tversky and Kahneman, 1982). However, the data from this study presents a picture that is not as clear-cut as the existing literature would lead one to believe. Although many of the students did exhibit behavior consistent with the law of small numbers at some times, students also expected variability between different trials of a random process (as discussed previously in randomness), and the students who have previously been described as believing that nothing can be predicted about a random process did not fall prey to the law of small numbers: in fact, they explicitly denied the relevance of data that was in contradiction to their theories, not accepting the data as relevant until they had created a scenario that could fit the data, a behavior that is consistent with findings from the science education literature (Chinn and Brewer, 1993).

These different behaviors can be roughly characterized as being either data-driven or theory-driven[4]. Data-driven behaviors were characterized either by students' unwillingness to create a theory in the absence of data, or by their willingness to give up their theory after only a small number of points had been played (typically 10 points or less) (see figure 26). Theory-driven behaviors were characterized by the students' unwillingness to believe the data when it was in conflict with their theory. When students behaving in a theory-driven manner first saw that the data was inconsistent with their theory, they explicitly denied the relevance of the data (figure 27), decided that the computer is cheating, or just ignored the data and claimed the truth of the original predictions. Two pairs of these students finally abandoned their contention that the Three-Coin game was fair, but only after running many simulations, noting the unfair score, and finally realizing the importance of the number of outcomes that scored for each team.

Reasoning when predicting for Three-Coin Game

U: Because we tried it.

Reasoning when concluding for Three-Coin Game

U: this is not right. We are kind of right on this one, but we did this point higher

...

UV typed: BECAUSE WE'RE WRONG.

Figure 26: data-driven discussions of evidence

K: I think it might be unfair, because A is winning

L: although in real life it would be fair

K: you reset it 5 times, and every time A is always winning

L: it is fair, but the computer is cheating

Figure 27: theory-driven discussions of evidence (K is in the process of abandoning his theory, and L is maintaining his theory in the face of counter evidence)

The research literature shows that many people fall prey to the law of small numbers, and this study was not an exception. However, this study did illustrate that the story may not be as simple as people always believing data that is based on a small sample. In particular, some of the subjects in this study did not believe the data when the data was in conflict with a theory that they had proposed. This suggests that perhaps there is an interaction between people's expectations and the validity that they are willing to attribute to data. We posit that this has not been appreciated in the past due to the artificial nature of the tasks that subjects were given, whereas in our tasks students were engaged and felt ownership of their predictions and conclusions.

[4]Note that different students may be considered data- or theory-driven at different times, as three pairs of students were characterized as data-driven and three as theory-driven, two were neither completely data- nor theory-driven; and students may switch between being data-driven and theory-driven, depending on such factors as if they have yet formulated a theory.

Summary

As the results from this study show, students invoked many intuitions about probability when reasoning about the activities in PIE. And these intuitions do not seem to be easily characterized by standard misconceptions in probability. Instead, these intuitions may better be described by noting their similarities and dissimilarities to specific concepts in probability, notably randomness, the outcome space, the probability distribution, and the role of data in determining probabilities.

From Theory into Action: Designing a Learning Environment

Although the original intent of PIE, to have students confront their misconceptions, turned out to be ill-conceived, we do feel that valuable lessons were learned. For one, this study allowed us to create a framework of probabilistic reasoning that will inform future versions of PIE. Additionally, we saw that a collaborative inquiry environment can lead to students expressing many intuitions about probability. This is important in at least two ways. First, as researchers, we can use such environments as a way to help us understand student intuitions about probability. That is, as the students collaborate, their reasoning can be recorded and then later analyzed, giving us an understanding of how students reason probabilistically in situations that are perhaps more authentic then can be replicated in either a survey or in a one-on-one interview. Second, as educational designers, it is important that we help students build models from their existing ideas about probability. To do this we must have students make these ideas explicit, and realize the strength and weaknesses of these ideas.

By developing a framework that looks at student understandings with an eye to the probabilist's understanding of randomness, the outcome space, the probability distribution over the outcome space, and the role of data in analyzing expectations, we can create activities that bridge the gap between what students know before probability instruction, and what probability instruction aims to teach. The creation of such activities differs radically from the traditional approach to instruction in which one starts by teaching Kolmogorov's axioms or the multiplication rule. Instead, axioms and rules are derived through the learning activities. To create such learning activities, we should be guided by theoretical frameworks that provide a coherent and consistent approach to instruction.

As mentioned at the beginning of this paper, PIE is based on the premises that learning environments should situate students in activities they consider to be legitimate and authentic, and not ignore the contribution of social practices and representational resources (Gordin et al. 1994; White, 1993a, 1993b). We cannot be so naive, however, to believe that by simply exposing the students to activities that highlight the importance the four previously mentioned features of probabilistic reasoning that they will learn to apply such reasoning in all appropriate instances. Students have formed many competing intuitions, and it is not easy to coax students into developing a systematic and coherent way of viewing probability. Instead, a principled approach is required in which activities are created that will lead students to construct a systematic and coherent view of probability. The principled approach recommended here is to have the students participate in a progression of activities, where each activity is interesting and highlights an important aspect of the domain to be studied, and each activity extends the findings from the previous activities (White, 1993a, 1993b). By incorporating such a series of activities, students are engaged in authentic activities from the beginning, they can create simple models that are consistent with some aspects of their intuitive reasoning, and, as they are systematically exposed to more complex situations, the models (and their intuitions) can be revised, and need not be completely overhauled. By internalizing these models students will expand the intuitions that are most appropriate for the domain, leading the students to a normative view of the domain of probability.

This study can help to identify the progression of activities that should be designed to help students build from their ideas to normative probabilistic reasoning. One such series of activities will be touched on briefly here, and future research will be required to determine if such a series of activities can be useful in helping students to understand normative probability.

Such a series of activities can start off with an extensive discussion of fairness, and the different ideas students may have about fairness. In such a discussion it is expected that students will easily converge on the normative definition of fair meaning that each team is equally likely to win. Note that this idea should continue to be raised throughout the activities, in a manner similar to that proposed by a "spiral" curriculum. In the next activity students can be given a simple scenario in which they investigate the law of small and large numbers. By seeing the relationship between the random event of a coin flipping, the outcome of this simple event, and the accumulation of many instances of this event, students can collaboratively come to see that there will be much variability after a small number of flips, but regularity after many hundreds of flips. It should also be noted that each of these ideas, variability and regularity, are ideas that have been expressed by the students in this study. This aspect of the model should be revisited throughout the activities. The next set of activities can highlight the role of the outcome space in determining if games are fair. In this set of activities the elementary events are linked to the compound events, which are linked to the cumulative effect of many hundreds of events. The first model the students will investigate is that of equally likely outcomes, allowing the students to derive a model that is based on counting strategies. The next model will be based on outcomes that are not equiprobable (such as spinners with unequal areas, which are not yet in this prototype version of PIE), allowing students to derive a model that is based on the multiplication rule. Although this is the extent of the activities that we expect to implement for a seventh grade probability unit, it seems fairly obvious how we can extend this model to introduce more complex concepts in probability such as conditional probability, and more complex probability distributions.

Conclusion and need for further study

Students have many ideas about probability, and these ideas are not adequately described by simply stating that students are using heuristics such as representativeness. Instead, students invoke a large number of intuitions about probability, and these intuitions can be seen to roughly correspond to the concepts of randomness, the outcome space, probability distribution, and the role of data. By viewing students' probabilistic intuitions in this way we expect that, although many of the misconceptions found in the literature are adequate ways of describing the behavior of some students some of the time, students will exhibit great variation in behaviors, based on their understanding of these four related areas. In fact, this variation is exactly what is observed in this study as well as in the research literature. We feel that by viewing student ideas about probability as consisting of four interrelated sets of intuitions, we can come to a more thorough understanding of probabilistic reasoning.

The question is then raised as how we can best harness such student intuitions to improve understanding of probability. It is posited that by creating inquiry-based learning environments we can help students to examine and expand their ideas in creating an understanding of probability that is consistent with normative probability theory, and future research will be required to determine if this is a productive method of teaching probability.

Acknowledgments

PIE was conceived and developed by Bernard Gifford, Noel Enyedy and myself as members of the InterActiveMedia Study Group (ISG). I would also like to thank Mary Williamson, Bernard and Noel, as well as the other members of ISG, for their comments on earlier versions of this paper, as well as Jesse Ragent for his help in recent designs of PIE. This research is funded in part by a grant from the UC Urban Community-School Collaborative.

References

Balacheff, N. (1987). The Evolution of a problem of construction and of a conjecture: The case of the sum of the angles of a triangle." *Cahier de didactique des mathematics No 39*. IREM Universite Paris VII.

Chinn, C. and Brewer, W. (1993). Students' Responses to Anomalous Data: A theoretical framework and implications for science instruction. *Review of Educational Research*, 63(1), 1-49.

Damon, W. (1988). The Moral Child: Nurturing Children's Natural Moral Growth. The Free Press, NY.

diSessa, A. (1988) Knowledge in pieces. In Constructivism in the computer age, The Jean Piaget symposium series, G. Forman, P. Pufall (Eds.), pp. 49-70. Lawrence Erlbaum Associates: Hillsdale, NJ.

Enyedy, N. (1996). Visual Representations in interaction: case studies of now non-native speakers of English use visual representations to reason probabilistically about games of chance. Unpublished Manuscript: Second Year Project.

Gordin, D.N., Polman, J.L., & Pea, R.D. (1994). The Climate Visualizer: sense-making through scientific visualization. *Journal of Science Education and Technology*. Vol. 3, No. 4, pp. 203-226.

Hawkins, A. and Kapadia, R. (1984). Childrens' conceptions of probability—a psychological and pedagogical review. *Educational Studies in Mathematics*, 15, 349-377.

Hirst, H. (1977). Probability. *Mathematics Teaching*, v80.

Konold, Clifford. (1991) Understanding Students' Beliefs about Probability. In Radical Constructivism in

Mathematics Education, E. von Glaserfield (ed) 139-156. Kluwer Academic Publishers

Konold, C., Pollatsek, A., Well, A., Lohmeier, J., and Lipson, D. (1993). Inconsistencies in students' reasoning about probability. *Journal for Research in Mathematics education*. 1993, v24, pp. 392-414.

Lajoie, S. Jacobs, V. and Lavigne, N. (1995). Empowering Students in the use of statistics. *Journal of Mathematical Behavior*, 14, 401-425.

Lampert, M. (1986). Knowing, Doing, and Teaching Multiplication. *Cognition and Instruction*, 3(4), 305-342.

Lampert, M. (1995). Managing the tensions in connecting students' inquiry with learning Mathematics in school. In Perkins, Schwarts, West, and Wiske (Eds.), Software goes to school: teaching for understanding with new technologies. New York: Oxford University Press.

Metz, K. (1995). Semantics of Chance and Fair in Childrens' and Adults' Problem Solving. Presented at the American Education Researchers Association annual conference, San Francisco, CA.

Minstrell (1989). Teaching Science for Understanding. In Toward the Thinking Curriculum: Current Cognitive Research, (Resnick and Klopfer, Eds). 1989 Yearbook of the Association for Supervision and Curriculum Development.

Pitman, J. (1993). Probability. Springer-Verlag, New York.

Richards, J. (1991). Mathematical Discussions. In E. von Glaserfeld (Ed.), Radical Constructivism in Mathematics Education. Boston, MA: Kluwer Academic.

Scheinok, A. (1988). A Summer Program in Probability and Statistics for Inner-City Seventh Graders. *Mathematics Teacher*, April 1988, pp. 310-314.

Schoenfeld, A. (1991). On Mathematics as Sensemaking. In Perkins, Segal and Voss (Eds.), Informal Reasoning in Education. Hillsdale, NJ: Lawrence Erlbaum Associates

Smith, J.P., diSessa, A., and Roschelle, J. (1993). Misconceptions Reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences*, 3(2), 115-163.

Shaughnessy, J.M. (1992). Research in Probability and Statistics: Reflections and Directions. In Handbook of Research on Mathematics Teaching and Learning, D. Grouws (Ed), pp. 465-493.

Thorkildsen, T. (1995). Conceptions of Social Justice. In W. Kurtines and J. Gewirtz (Eds.), Moral Development: an introduction. Allyn & Bacon, Needhan Heights, MA.

Tversky, A. and Kahnemann, D. (1982). Judgment under uncertainty: Heuristics and biases. In Judgment under uncertainty: Heuristics and biases, Kahnemann, Slovic, and Tversky (Eds.), pp. 3-22. Cambridge University Press: New York, NY.

Vahey, P. (1996) How Students' Conceptions of Fairness Informed the Design of an Interactive Instructional Probability Environment. Presented at the American Education Researchers Association annual conference, New York, NY.

Vahey, P., Enyedy, N., and Gifford, B. (in press). Designing Student-Computer Interactions for Guided Inquiry Learning Environments. To be published in proceedings from the Human Computer Interaction Conference, August, 1997, San Francisco, CA.

White, B. (1993a). ThinkerTools: Causal Models, Conceptual Change, and Science Education. *Cognition and Instruction* 10 (1), 1-100.

White, B. (1993b). Intermediate Causal Models: A missing link for successful science education? In Advances in Instructional Psychology, R. Glaser (Ed.), 177-252, LEA.

White, B. and Frederiksen, J. (1995). The ThinkerTools Inquiry Project: making scientific inquiry accessible to students and teachers. Causal Models Report 95-02.

AREA 1997

SE060997

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement (OERI)
Educational Resources Information Center (ERIC)

ERIC

REPRODUCTION RELEASE

(Specific Document)

I. DOCUMENT IDENTIFICATION:

Title: Toward an Understanding of Productive Student Conceptions of Probability: The Probability Inquiry Environment

Author(s): Phil Vahey

Corporate Source:

Publication Date: 3/27/97

II. REPRODUCTION RELEASE:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, *Resources in Education* (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic/optical media, and sold through the ERIC Document Reproduction Service (EDRS) or other ERIC vendors. Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce the identified document, please CHECK ONE of the following options and sign the release below.

☑ Sample sticker to be affixed to document	Sample sticker to be affixed to document ☐
Check here Permitting microfiche (4"x 6" film), paper copy, electronic, and optical media reproduction	**or here** Permitting reproduction in other than paper copy.
"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY ____ *Sample* ____ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."	"PERMISSION TO REPRODUCE THIS MATERIAL IN OTHER THAN PAPER COPY HAS BEEN GRANTED BY ____ *Sample* ____ TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."
Level 1	**Level 2**

Sign Here, Please

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but neither box is checked, documents will be processed at Level 1.

"I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce this document as indicated above. Reproduction from the ERIC microfiche or electronic/optical media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Signature: Phil Vahey	Position:
Printed Name: PHIL VAHEY	Organization: UC Berkeley School of Education
Address: 4155 C. Chavez #12 San Francisco CA 94131	Telephone Number: (415) 647-9371
	Date: 3/27/97

OVER

CUA

THE CATHOLIC UNIVERSITY OF AMERICA
Department of Education, O'Boyle Hall
Washington, DC 20064
202 319-5120

February 21, 1997

Dear AERA Presenter,

Congratulations on being a presenter at AERA[1]. The ERIC Clearinghouse on Assessment and Evaluation invites you to contribute to the ERIC database by providing us with a printed copy of your presentation.

Abstracts of papers accepted by ERIC appear in *Resources in Education (RIE)* and are announced to over 5,000 organizations. The inclusion of your work makes it readily available to other researchers, provides a permanent archive, and enhances the quality of *RIE*. Abstracts of your contribution will be accessible through the printed and electronic versions of *RIE*. The paper will be available through the microfiche collections that are housed at libraries around the world and through the ERIC Document Reproduction Service.

We are gathering all the papers from the AERA Conference. We will route your paper to the appropriate clearinghouse. You will be notified if your paper meets ERIC's criteria for inclusion in *RIE*: contribution to education, timeliness, relevance, methodology, effectiveness of presentation, and reproduction quality. You can track our processing of your paper at http://ericae2.educ.cua.edu.

Please sign the Reproduction Release Form on the back of this letter and include it with **two** copies of your paper. The Release Form gives ERIC permission to make and distribute copies of your paper. It does not preclude you from publishing your work. You can drop off the copies of your paper and Reproduction Release Form at the **ERIC booth (523)** or mail to our attention at the address below. Please feel free to copy the form for future or additional submissions.

Mail to: AERA 1997/ERIC Acquisitions
The Catholic University of America
O'Boyle Hall, Room 210
Washington, DC 20064

This year ERIC/AE is making a **Searchable Conference Program** available on the AERA web page (http://aera.net). Check it out!

Sincerely,

Lawrence M. Rudner, Ph.D.
Director, ERIC/AE

---

[1]If you are an AERA chair or discussant, please save this form for future use.