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ABSTRACT

This study investigated the dimensionality of mathematics subscale scores from the National Assessment of Educational Progress for the assessment's Technical Review Panel, specifically for the data from the 1990 and 1992 main assessment in relation to students' instructional and noninstructional background variables. Discriminant analysis was applied to the math subscale scores using the background variables (questions) as grouping variables. For data from 1990 and 1992 it was hypothesized that: (1) five math subscales measure a general underlying mathematics ability; (2) five distinct subscales measure five different areas in math; and (3) in addition to the five subscales, there is a general math ability underlying part of the subscale scores. Confirmatory factor analysis tested the hypothesis of multidimensionality versus unidimensionality. Results, which are consistent across the data, indicated that when analyses were performed on the total group of students, almost perfect correlations between the subscale scores were obtained. When analyses were conducted on groups based on the background variables, lower correlations that were indicative of multidimensional subscale scores were obtained. Results indicate that models with subscale scores as latent variables exhibit a better fit to the data than models that had only one general mathematics score. An appendix presents detailed tables of findings. (Contains 70 appendix tables and 103 references.) (SLD)

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NAEP TRP Task 3e: Achievement Dimensionality, Section A

Study Director: Jamal Abedi

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Executive Summary

The dimensionality of NAEP subject area subscale scores is a controversial issue in NAEP because it could effect the administration, scoring, and reporting of the NAEP data and even impact school curriculum planning. Because of the importance of subscale dimensionality in NAEP, different studies have been conducted on this topic and numerous results have been published in journal articles, NCES/ETS technical reports and/or other various forms of relevant and related literature. These studies, in few cases, have found evidence of multidimensionality. However, in general, the results of these studies in the areas of math, science, reading, and writing have indicated that the subscale scores are highly correlated. Thus, evidence of multidimensionality in past studies has not been strong enough to warrant any definitive statements on the deviation from a single dimension ability in the subject areas mentioned above.

Recently however, it has been argued that analyses of dimensionality conducted on the "total group of subjects" may indicate that subscale scores in a given subject area measure the same underlying ability, but if analyses in dimensionality are conducted in relation to the students'/subjects' background variables (e.g., instructional and personal) then evidence of multidimensionality can be observed.

This study investigates the issue of dimensionality in the NAEP math subscale scores, specifically on the data from the 1990 and 1992 main assessment in relation with students' instructional and non-instructional background variables. In this study, discriminant analysis was applied to the math subscale scores using the background variables (questions) as grouping variables. Based on this analysis, those background variables that had a significant impact on the math subscale scores were identified. Those variables were then used as a basis for grouping students into subgroups. Dimensionality analyses were performed on these subgroups.

For the 1990 and 1992 data, three different hypotheses regarding the dimensionality of the math NAEP items were examined: (1) there are five math subscales (Numbers & Operations, Measurement, Algebra, Geometry, and Statistics) that measure a general underlying math ability, (2) there are five distinct subscales that measure five different areas in math, and (3) in addition to the five subscales, there is a general math ability underlying part of the five subscale scores. Confirmatory factor analysis techniques were employed to test the hypotheses of the multidimensionality versus unidimensionality models.

The results of this study which were consistent across the different data sets (1990 and 1992, grades 4, 8 and 12) indicated that when analyses were performed on the total group of students, almost perfect correlations between the subscale scores were obtained. However, when analyses were conducted on the student subgroups that were formed based on the background variables, lower correlations which were indicative of multidimensional subscale scores were obtained. One must, however, be cautious in interpreting these low correlations as indicators of multidimensional subscale scores because several factors (such as the varying numbers of students in the individual subgroups, the different characteristics of the items in the various item parcels etc.) could have caused these correlations.

Conclusively, the results of this study indicated that models with subscale scores as separate latent variables exhibited a better fit to the data than models that had only one general math score. Furthermore, evidence of multidimensional math subscale scores was found when analyses were conducted in relation to students' background variables.

Introduction and Purpose

The issue of dimensionality is an important consideration in the National Assessment of Educational Progress (NAEP) because it affects the administration, scoring, data analyses and reporting of the results. The subject matter areas assessed by NAEP are usually analyzed by content and process. For example, the reading assessment consists of three content areas; Information Text, Literary Text, and Documents, and two process areas; Constructs Meaning and Extends Meaning (see the NAEP 1990 Technical Report, 1990, pp. 33-37). Similarly, the mathematics assessment framework consists of five content areas: Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics and Probability; and Algebra and Functions; and three process areas: Problem Solving, Procedural Knowledge, and Conceptual Understanding (see the NAEP 1990 Technical Report, 1990, p. 40). The scores in math, reading and science are reported at the subscale levels for Grades 4, 8, and 12.

Recent studies performed on the dimensionality of math, science and reading in NAEP (see, for example, Allen, 1990; Carlson & Jirele, 1992; Rock, 1991; and Zwick, 1987) have shown that the subscale scores in the three curricular subject areas are highly correlated. These high correlations between the subscales could have implications on curriculum planning, teaching, and the reporting of the students' achievement scores in these subject areas.

For example, Rock (1991), in his study on NAEP math subscale dimensionality, found very high correlations between the five subscales in math, which indicated a unidimensional trend at the subscale level. Based on his results, he concluded that "we are doing little damage in using composite scores in mathematics and science" (p. 2). Zwick (1987), in her study on NAEP reading items, concluded that "Although categorization of the NAEP reading items is useful for test development and reading research, the dimensionality analyses did not provide strong empirical evidence for the existence of multiple dimensions" (p.306).

However, what Rock and many others do not take into account is that in educational settings the environments consist of heterogeneous groups of individuals. Students' performance in math, science, reading, writing and other subject matter areas may be affected by instructional and background variables (see, for example, Muthén, 1988a; Muthén, 1989a, 1989b; Muthén, Kao, & Burstein, 1991).

The Research Question

The main question in testing, scoring, and reporting mathematics test results in NAEP is whether the five subscales measure the same underlying math ability or whether they measure five different subject areas in mathematics. If the subscale items are measuring a general mathematics factor, as suggested by some of the studies done in this area, then one may not need to test and report by subscales; a test with general math questions may serve the purpose. However, if students' performance varies across instructional and background variables, then one should pay more attention to such variables when dealing with math test scores.

The Study

The dimensionality of math subscale scores was examined in two concurrent studies conducted at the Center for Research on Evaluation, Standards, and Student Testing (CRESST), UCLA. This paper reports the findings of the first section of the study. The data for this part of the study were obtained from the NAEP main administration in 1990 and 1992. The analyses were done for the 1990 and 1992 data sets separately.

For the 1990 data, multiple discriminant analysis technique was used to identify those background variables that significantly discriminated groups of subjects on their math subscale scores. The background variables from the 1990 main administration were used as grouping variables and the math subscale scores as discriminating variables. The background variables that had significant effects on the math subscale scores were identified, and subgroups were formed based on the level of those background variables. Simple-structure confirmatory factor analysis was performed on the math subscale scores, and the correlations between the subscale latent variables were compared across subgroups. To create subscale latent variables, item parcels were prepared. These item parcels consisted of items that were homogeneous with respect to their difficulty level and intercorrelations (see Cattell, 1956a, 1956b; Cattell & Burdsal, 1975, Cook, Dorans, Eignor, & Petersen, 1983). Two item parcels were constructed for each of the subscales and were used to create the subscale latent variable. The results of these analyses, which included correlations between the latent variables and indices of goodness of fit, were reported separately for each of the subgroups that were formed based on the level of the selected background variables.

In addition to structural models with five latent variables (for the five math subscales), the assumption of one general math factor was also tested. In a series of analyses, the math subscale item parcels were used to create one general factor based on the assumption that all items under different math subscales measure a general math ability. The indices of fit obtained from these analyses were then compared across subgroups. Furthermore, models with five subscale factors and one general math factor were created. These models' indices of fit were compared with models that assume only one general factor and with those models that assume five subscale factors and no general factor. The comparison of factor mean and factor variances across subgroups of students formed by levels of background variables using multiple group factor analysis was also to be examined. However, the sister dimensionality study that was conducted at CRESST by Dr. Bengt Muthén and his team of researchers was going to include this type of analysis, so it was not covered in this study's methodology.

All of the analyses discussed above were performed on all of the math items in each subscale. Some of the math items were also selected based on their relationships with the background variables. A computer program, (Multi-Approach Correlation System, [MACS], Abedi, 1993) was specifically developed for this purpose. The correlation between each of the background variables with each of the math test items was computed using the appropriate techniques. Item parcels were then created from these selected few items. The same analyses that were conducted on the parcels with the complete sets of items were performed on the parcels consisting of selected items, and the results were compared across the subgroups.

Because of the balanced incomplete block (BIB) spiraling nature of the NAEP data, analyses were conducted at the booklet level (see Beaton, Johnson, & Ferris, 1987; Carlson & Jirele, 1992; Zwick, 1987). For the 1990 data, there were 10 booklets. Analyses were performed on booklets 8, 9 and 10, and because the test booklets were spiraled, the results of the analyses conducted on booklets 8, 9 and 10 were considered replications and were used to cross validate the results.

For the 1992 data, however, the structure of the test items in the booklets were different. There were more booklets in the 1992 administration than in the 1990 administration, with 26 booklets for the 1992 administration as compared with 10 for the 1990 administration (see the 1992 NAEP

Mathematics Report Card for the Nation and the States). Consequently, in the 1992 administration there were fewer students answering items from the same booklet than in the 1990 administration. As a result, the students could not be divided into subgroups based on the background variables, especially those variables with more than two levels.

For the 1992 data, analyses were performed on the item parcels that were formulated based on the items that manifested higher correlations with the background variables. The same models that were used for the 1990 data were created for the 1992 data. Models with five subscale latent variables were created as well as models with one general math latent variable and models with five subscales and one general math latent variable. Indices of fit were compared across all of these models.

Literature Review

This section summarizes the related literature in dimensionality. It also discusses the issue of dimensionality in NAEP curricular subject matters and describes a summary of studies conducted in an effort to clarify this issue. The purpose of this literature review is to provide the rationale for undertaking the present study.

The related literature is summarized in four different sections. In the first section, the concepts of dimensionality in educational testing are discussed. The second section describes the techniques and procedures used for assessing test items and subscale dimensionality in general. The third section summarizes the techniques and procedures used for assessing the dimensionality of NAEP test items in particular, and the last section summarizes the of results of the dimensionality studies in NAEP.

Assessing Dimensionality of Achievement Tests

The dimensionality of achievement tests is an important issue to educational assessment because it is the underlying assumption that many measurement techniques are based on (Cook et al., 1983; Hambleton & Rovinelli, 1986; Zwick, 1985). Zwick (1985), in her report summarizing the results of a dimensionality study conducted on the NAEP reading items, indicated that "it was important to investigate the dimensionality issue because the validity of the item response theory (IRT) model used to estimate reading proficiency in the 1983-1984 NAEP survey rests on the assumption of unidimensionality" (p. 1).

A set of items is considered unidimensional if a single latent trait underlies the data. Hattie (1984) explains that unidimensionality

> is not defined in terms of unit rank, percentage of variance explained by the first component or factor, deviations from a perfect scale, the type of correlation, or the number of common factors. Although these have been used as methods to determine unidimensionality, they do not define it. A unidimensional test is not necessarily reliable, internally consistent, or homogeneous. Indeed a unidimensional test may be factorially complex in terms of the linear common-factor model. While the principle of local independence is fundamental to the definition of latent traits and therefore to the definition of dimensionality, it is not synonymous with dimensionality. (p. 50)

Perhaps the concept of unidimensionality has become the most apparent in item response theory. Unidimensionality has been identified as one of the most important assumptions underlying IRT (Cook et al., 1983; Zwick, 1985). Cook et al. (1983) have indicated that if the first-order factor variance (which is an indication of a general factor) is large, the data are unidimensional. On the other hand, they have also attested that a relatively large group factor would indicate a violation of unidimensionality. Zwick (1985) points out that in practice, the unidimensionality assumption in IRT is always violated to some degree, and she stresses the need for more studies on the robustness of IRT estimation procedures to violations of the unidimensionality assumption.

Bejar (1980) discusses the issue of dimensionality based on content-area. He contends that when a measure is based on the total-test concept, then the entire latent space is unidimensional, and any other sources of variability are considered by definition "error." However, if the measure is constructed based on content-area, then the latent space is multidimensional.

Hambleton and Rovinelli (1986) maintain that despite the importance of the unidimensionality assumption in the currently popular item response model, there is confusion and controversy regarding the definition of dimensionality and the method for assessing the dimensionality of a set of test items. They cite, for example, a typical definition of dimensionality as it appears today in current psychometric literature. This particular definition--"a set of test items is unidimensional when a single trait can explain or account for examinee test performance" --is then referred to as "abstract and non-operational" (p. 287).

Procedures for Assessing Dimensionality in General

The concept of dimensionality in educational assessment has not been defined clearly in the literature. This lack of definitive clarity has caused problems in assessing indices of dimensionality. Jones, Sabera, and Trosset (1987), discuss the issues involved in the dimensionality of tests and conclude that there is not a single nor a group of the most appropriate techniques for assessing the dimensionality of a set of items. They refer to Lord (1980), who emphasized the need for a commonly acceptable test(s) for assessing dimensionality. Hattie (1985) pointed out that "unidimensionality has been confused and used interchangeably with other terms such as reliability, internal consistency, and homogeneity" (p.157). Due to the complex nature of this concept, and because of the lack of a commonly acceptable definition of dimensionality, many different procedures have been suggested in the literature for assessing the dimensionality of a set of achievement test items. Hattie (1984) has identified 87 indices suggested for determining the unidimensionality of a measure (see Hattie, 1984, pp. 51-54, Table 1, for complete data). In addition, he provides rationale for each of the procedures and discusses their weaknesses and strengths. Hattie (1985) grouped these procedures into several categories and, whenever possible, gave references to the studies that employed these approaches. At this point in the review, the different procedures suggested for assessing dimensionality will be described. This description will be based on the format and categorization used by Hattie (1985).

Indices based on answer patterns

Under this category, Hattie briefly describes Guttman's reproducibility coefficient which provides a method for testing a series of qualitative items for unidimensionality (Guttman 1944, 1950; Hattie, 1985). This index may be affected by the level of item difficulty. There are some approximations to Guttman's approach in the literature. For example, Jackson (1949) proposed the Plus Percentage Ratio (PPR) coefficient which is not affected by item difficulties, and Green (1956) suggested a formula that is less time consuming to compute.

Loevinger (1944) suggested an approach known as the index of homogeneity, in which the coefficient is 1 for a perfectly homogeneous test and departs from unity as the items within a test become more heterogeneous. Hattie (1985) comments on these indices and discusses the major criticisms of these approaches. Among the criticisms is Lumsden's (1959) remark, "these methods can only achieve their upper bounds if the strong assumption of scalability (i.e., a perfect scale) is made" (Hattie, 1985, p.143).

Indices based on reliability

The coefficient alpha is referred to in this category as one of the most widely used indexes of unidimensionality. However, the literature points out the various problems that can arise when one interprets Cronbach's Alpha as an index of internal consistency or as an indication of the unidimensionality of a set of items. Cortina (1993) demonstrated that alpha is affected by (a) the number of items in the test, (b) the average item intercorrelation, and (c) the dimensionality of the items. He suggests using factor analysis techniques to assure that there is no large departure from unidimensionality. McDonald (1981) discusses the use of coefficient alpha as an index of reliability or dimensionality; he believes that this coefficient cannot be used as a reliability coefficient or as a coefficient of generalizability, nor as a criterion for assessing dimensionality. He indicated that "indeed, one might almost put the extreme view that alpha has not been shown to be a quantitative measure of any intelligible and useful psychometric concept, except when computed from items with equal covariances" (p. 111). Green, Lissitz, and Mulaik (1977) observed that a high internal consistency indexed by a high alpha maybe due to a general factor underlying the items but that it may not always be an indication of a general factor (see Hattie, 1985). In their Monte Carlo simulation study, Green et al. (1977) concluded that "the chief defect of alpha as an index of dimensionality is its tendency to increase as the number of items increase" (Hattie, 1985, p.144). Green et al. (1977) also found that the average inter-item correlation suggested by Cronbach to overcome the problem of effects of test length is influenced by the commonalties of the items and by negative inter-item correlations. Hattie (1985) concludes that "despite its common usage as an index of unidimensionality, alpha is extremely suspect" (p. 145).

Also under this category of reliability indices, Hattie discusses "Index/Index-Max Formulas." The formulas are basically modifications of alpha. Among these formulas are Loevinger's (1944) H ratio and Horst's (1953) reconceptualization of Loevinger's index.

Indices based on correcting for the number of items

Since alpha is dependent on the length of a test, other indices of inter-item correlation or test homogeneity have been developed that claim to be independent of test length. Cronbach (1951) suggested estimating the mean correlation between items by applying the Spearman Brown formula to the alpha for the total test (Hattie, 1985). Armor (1974) suggested assessing the number of intercorrelations close to zero as a possibility for determining the number of dimensions in a test.

Indices based on principal components

Some researchers have used principal components analysis to assess the dimensionality of a set of items or subscale scores. The idea is that if a large amount of variance is explained by the first component, then the set of items or subscales could be considered unidimensional. There are many questions and concerns regarding the appropriateness of the principal components technique as a tool for assessing the dimensionality of a set of items. Some of these concerns are in regard to the applicability of principal components analysis (PCA) to dichotomously scored right or wrong (R/W) items; other questions query the clearness of the criteria for judging unidimensionality based on the results of components analysis.

Questions on the applicability of PCA to R/W items. Principal components and factor analysis can be applied to a set of conditions in which each of the variables is a score on a multi-item test. However, when the principal components technique is applied to dichotomously scored items (phi or tetrachoric correlations), it may not produce valid information that could be used for judging the dimensionality of a set of items (see, for example, Bejar, 1980; Carroll, 1983; Cook & Eignor, 1984; Cook, Dorans, & Eignor, 1988; Hambleton & Rovinelli, 1986; Hulin, Drasgow, & Parsons, 1983; Jones et al., 1987; McDonald & Ahlawat, 1974; Mislevy, 1986; Zwick, 1985).

When computing phi-coefficient, one generally assumes that the items are truly dichotomous. However, this assumption causes major problems in computing component analysis and factor analysis for the matrices of phi-coefficients. For example, the PCA or FA on the phi-coefficients often produces a second factor that is related to item difficulty but has no relationship to any of the properties of the items (see, for example, Hambleton & Rovinelli, 1983; Lord & Novick, 1968; McDonald & Ahlawat, 1974; McDonald, 1967; and McDonald, 1981). For example, Jones et al. (1987) found that "the magnitude of the phi-coefficients is affected not only by the item difficulty but also by the strength of the relationships among the variables" (p. 3). Muthén and Christofferson (1981) indicated that factor analyses of phi-coefficients produce "inconsistent and attenuated estimates in addition to incorrect standard errors of estimates and incorrect chi-square test(s) of model fit" (p. 407, also see Olsson, 1979). Hence, because these problems occur when factoring the phi-coefficient, the literature has suggested the use of tetrachoric correlations as an alternative method.

Factor analysis on tetrachoric correlation matrices may solve some of these problems, but it does, however, create a whole new set of problems. For example, tetrachoric correlations are computed based on the assumption that the item responses are functions of underlying continuous variables that have a bivariate normal distribution (Zwick, 1985). If the assumption of bivariate normality is not met, then the tetrachoric correlations may not be good estimates of the relationships between the items (Jones et al., 1987).

Another problem with tetrachoric correlations is that these correlations can not be directly estimated. Simple approximation formulas may be accurate only in the neighborhood of $\underline{r} = 0.5$ (Jones et al., 1987). More complex estimations (Castellan, 1966; Divgi, 1979; Kirk, 1973) "can become unstable when one or more cell proportion[s] of the pairwise item response table is extremely small" (Jones et al., 1987; p. 7). Hattie (1984) refers to this problem with tetrachoric correlation matrices as not being positive-definite and discusses the procedures for calculating tetrachoric correlations in order to overcome the problem of obtaining non-positive-definite correlation matrices. Muthén (1978) indicated that the sample tetrachoric correlations have a larger covariance matrix than the Pearson correlation. He also observed (1981) that ordinary factor analysis of tetrachoric correlations may produce incorrect standard errors of estimates and chi-square test(s) of fit (see also Bock & Lieberman, 1970).

Furthermore, the factor analysis of tetrachoric correlations is problematic when guessing occurs. The factor analysis of tetrachoric correlations often produces spurious factors when respondents guess on most of their test items (see for example, Carroll, 1983; Hulin et al., 1983; Zwick, 1985). In such cases, the magnitude of tetrachoric correlations is affected by item difficulty. There are, however, procedures reported in the literature for adjusting for this kind of guessing phenomenon. Carroll (1945), for instance, suggests that the effects of guessing be removed from the table of item responses. Zwick (1985) applied Carroll's proposed solution to the item responses for the NAEP reading items in grade/age (grage) 13/VIII and found unsatisfactory results (Zwick, 1985). She indicated that "16 percent of the tetrachoric coefficients were rendered incomputable because of negative adjusted cell frequencies" (p. 19). Despite these problems, Zwick (1985) applied the principal components of phi and tetrachoric correlations to her analyses of the three grades/age (grage) and found the results to be of the "worst case" (p. 15). She (1985) also reported the use of alternative procedures to correct for guessed items and found that her results revealed nothing markedly significant. In addition, Reckase (1981) noted in his research that "over-or-under-correcting yields undesirable results" (see Jones et al., 1987, p. 4).

Another alternative to factoring phi or tetrachoric correlation matrices that has often been mentioned in the literature is the factor analysis of image correlation. (For a definition of image correlation see Zwick's [1985] description of Guttman's [1953] version of image theory.) Kaiser (1970) and Kaiser and Cerny (1979) indicated that principal components and factor analysis on the image correlation matrix would be a more appropriate overall analytical approach to dichotomous data. Zwick (1985) also noted that "for the three within-grade analyses, the first roots are between 14 and 47 percent larger than those for the Pearson matrix" (p. 23).

Criteria for judging unidimensionality of PCA. There are many different views in the literature on how small or large the variance explained by the first component should be in order to determine unidimensionality. Carmines and Zeller (1979), with no apparent rationale, postulated that if 40% of the total variance is explained by the first component, then the set of items is measuring a single dimension. Reckase (1979) believes that if the first order component explains only 20% of the variance of a set of items then that set is unidimensional. One of the problems with the "percent of variance explained by the first component" is that this percent of variance depends on many different factors including the type of correlation coefficients. Zwick (1985) for example, found that when PCA was performed on phi matrices, between 17% to 25% of the variance was explained by the first component as compared to 30% to 40% of the variance that was explained by the first component when the analyses were performed on tetrachoric correlations on the same matrices. On another set of simulated data, Zwick (1985) found 25% of the variance explained by the first component as compared to 80% of the variance explained by the first component for the image correlation matrix. Clearly, there is not a set criterion for how small or large the variance explained by the first component should be to conclude that a set of items is in fact unidimensional.

Similarly, there is not set rule for determining how many factors should be extracted in a principal components analysis. Some researchers have suggested that the assessment of dimensionality be done based on the eigenvalues of the components. For example, Lumsden (1957, 1961) suggested the use of the ratio of the first to the second eigenvalue as an index of dimensionality. Hutten (1980) also suggested the use of the ratio of the first to the second eigenvalue of tetrachoric correlations as an index of dimensionality. Lord (1980) concurred that if the ratio of the first to second eigenvalues is large and the second eigenvalue is not larger than any of the others, then one can say the set of items may be unidimensional.

However, the eigenvalues of the components may not be a very reliable index to be used for assessing dimensionality. Jones et al. (1987) with their simulated data found that "the magnitudes of the eigenvalues were affected by the number of dimensions in the data, the amount of random error present in the data, the difficulty level of the items, when an item loaded on multiple factors, and the discrepancies between the loadings" (p. 15). Hattie (1984) concluded that "on theoretical grounds some indices must fail (e.g., ratio of eigenvalues)" (p. 55).

Finally, the analysis of residuals has been suggested as a test for assessing dimensionality. Hattie (1985) proposed that either the sum of the residuals or the sum of the absolute values of the residuals after the first component was removed can be used as an index of dimensionality. McDonald (1981, 1982) proposed the analysis of the residual items' covariances after fitting a one-factor IRT model as an index of departure from unidimensionality (see also Cook, et al., 1983; Zwick, 1985). Hambleton and Rovinelli (1983) based on McDonald's (1981, 1982) procedure suggested analyzing the residual covariances after fitting a nonlinear single factor model (see also Hambleton & Swaminathan, 1985). Hattie (1981) applied McDonald's procedure on large-scale simulation data and found this approach to provide the best results. However, Hambleton and Rovinelli (1986) found the residual analysis to be "of limited value in addressing item dimensionality because large residuals may be due to the violations of several model assumptions, including unidimensionality" (p. 300).

Indices based on factor analysis

Common factor analysis has also been used for assessing the dimensionality of a set of items or subscale scores. Some researchers found principal components and common factor analysis to produce very similar results while others may disagree on this finding (Jones et al., 1987). Velicer and Jackson (1990) compared component analysis with common factor analysis. They concluded on the basis of their review that

> only small differences existed in the numeric results produced by the two methods. In particular, we noted that numeric differences typically occurred only in the second decimal place and that decisions based on the patterns produced by alternative methods would be identical (p. 99).

Hattie (1985) however, indicated that there are major differences between the two methods. He stated that he had "clearly demonstrated that contrary conclusions can result from using the two methods" (p. 147).

The same problems and limitations that were discussed for the application of PCA to dichotomously scored items arise in the application of common FA to test items. The hypothesis of unidimensionality can also be examined in a large sample by a chi-square test when using the maximum likelihood estimation method, assuming normality (see, for example, Bollen & Long, 1992; Cook et al., 1988; Gerbing & Anderson, 1992; Gold, 1990, Gold & Muthén, 1991; Marsh, Balla, & McDonald, 1988). A test of a fit of one factor versus two factors can be done also by testing the difference in chi-squares of the two models (i.e., a one-factor model versus a two-factor model). Jorskog (1978) indicated that the chi-square from the models are independently distributed as a chi-squares with (df_2 - df_1) degrees of freedom. Hattie (1985) cited McDonald's (1982) recommendation to use the residual covariance matrix as a very reasonable basis for the misfit of the model to the data. Hattie (1985) also offers Tucker and Lewis' (1973) suggestion of a "goodness-of-fit" test based on the ratio of variance explained by one factor to the total test variance.

Hattie (1985) briefly discusses the two coefficients based on factor analysis, (a) the maximized-alpha (Armor, 1974; Lord, 1958) and (b) Omega

(Heise & Bohrnstedt, 1970; McDonald, 1970); the latter is a lower bound to reliability. Hattie (1985) also discusses the indices based on communality. In his discussion he refers to Green et al. (1977) who proposed two indices for assessing dimensionality. The first index which was called u is the sum of the absolute values of the correlation between all the possible pairs of items divided by the square root of the product of their respective commonalties from a principal component analysis. The second index is one for which the correlations are first corrected for communality by dividing the correlations by the product of the square roots of their commonalties. Such an index would also range from 0 to 1, 1 showing unidimensionality. Hattie points out that the main problem with these two approaches is that they lack the commonalties of the items which are contingent upon the knowledge of correct dimensionality.

Nonlinear factor analysis. The literature has clearly shown the problems involved with using linear factor analysis on phi or tetrachoric matrices (see Hambleton & Rovinelli, 1983; Lord & Novick, 1968; McDonald & Ahlawat, 1974; McDonald, 1967, 1981). A linear factor analysis on phi or tetrachoric correlations could produce artifactual factors because of nonlinear relationships between the observed responses and the underlying trait. There are two possible alternatives to the use of linear factor analysis on dichotomously scored responses for assessing the dimensionality of test items. One approach is to use the random regressor factor analysis model which evaluates residual covariances after fitting a nonlinear single-factor model. However, the literature is ambiguous on the effectiveness of nonlinear factor analysis as a method for assessing dimensionality.

For example, Hambleton and Rovinelli (1986) found nonlinear factor analysis to be the most promising tool for the assessment of dimensionality of dichotomous data. Hattie (1984), however, concluded that "nonlinear factor analysis cannot be recommended as there [is] too much overlap between the cases with one and more than one factor" (p. 74).

Item parcel. Another alternative to using linear factor analysis on phi or tetrachoric matrices is to use item parcels, that is, the collections of items measuring underlying dimensions. To linearize the nonlinear relationship between observed responses and underlying trait (which could happen because of the effects of item difficulty), one may use a small collection of non-overlapping items. These items are usually referred to as item parcels. Linear factor analysis could then be performed on the parcel scores. However, a serious problem may arise in factor analyzing item parcel scores when the items for a parcel are not selected carefully. For example, a parcel of items with different levels of difficulty could produce artifactual difficulty factors which could then introduce bias into the dimensionality assessment (see Cook et al., 1983, 1988).

Cook et al. (1988), in their study on the dimensionality of reading items, grouped their items based on four item types. They created parcels of three to seven items with approximately equal mean difficulty based on the items' delta difficulty indices. They then used the correlations among the parcels in a linear factor analysis. Cattell (1956) suggested defining the general structure of underlying factors by items and then precisioning this structure with the parcels. There are, however, several different views on this strategy (Cattell & Burdsal, 1975). Evidently, some psychometricians believe that item-level factor analysis is the only method for factoring (see, for example, Eysenck & Eysenck, 1969; Howarth & Browne, 1971). There are others who maintain that item-level data may not be stable enough to be used for factoring. In these cases, homogeneous parcels of items are more desirable for factoring (see, for example, Nunnally, 1978).

Cattell and Burdsal (1975), for example, state:

> [Items are unsatisfactory in several ways]: (a) their repeat reliability (dependability coefficient) is poor, due to the effect of incidental events on a judgment of very short duration; (b) any one item, defining a specific situation, is more vulnerable to cultural localism (low transferability coefficient) than the means of a set of items; and (c) the rotation of items is less definitive because it presents relatively blurred hyperplanes. (pp. 165-166)

Cattell and Burdsal (1975), however, conclude that "regardless of whether the first explorations are made by items or parcels, the best procedure for the ultimate, reliable, and precise definition of source traits seems to be the use of radial parceling" (p. 167). To do item parceling according to Cattell's radial procedure, two factor analyses should be performed, one on the items and the other on the parcel scores. The first factor analysis provides information for constructing the parcels, and the second factor analysis helps define the dimensionality of the parcels.

Other factor analytic approaches. Recently, a procedure for assessing the dimensionality of dichotomous data called full-information factor analysis (Bock, Gibbons, & Muraki, 1985) has been used (see also Jones et al., 1987; Zwick, 1985). This procedure uses the marginal maximum likelihood method (Bock & Aitkin, 1981) for estimating the parameters of the common factor model (Zwick, 1985).

Christofferson (1975) and Muthén (1978) proposed the generalized least square method. This technique provides a fit statistic that is asymptotically distributed as chi-square. But Jones et al. (1987) explained that

> the statistical test is based on distributional assumptions which may be too restrictive for the variables. Furthermore, for tests of moderate size, very large samples are required to insure the accuracy of the asymptotic approximation. In addition, restrictions are placed on the number of items which may be factor analyzed (according to Mislevy, (1986), 25 is an upper limit for the GLS procedure) or the number of factors in the solution (1-3 for tests with 60 items in the ML technique) (p. 4).

Other approaches. Multidimensional scaling (MDS) models has also been suggested as a technique for assessing the dimensionality of items. MDS does not need a full correlation or covariance matrix; rather, it only requires that the similarity of measures be ordered (see Jones et al., 1987, for more information). MDS is not a more general version of principal components analysis; it is an alternative approach for assessing the dimensionality of items. Reckase (1981) used principal components analysis, factor analysis, MDS, item response theory, and cluster analysis to assess the dimensionality of a set of simulated data. He found the MDS technique more effective in assessing dimensionality. On real data however, Reckase found that the MDS technique was not satisfactory. Zwick (1985) applied MDS for assessing the dimensionality of a set of items and found the MDS analysis of the actual data not very clear.

Indices based on latent trait models

The most fundamental assumption underlying the latent model is the assumption of local independence (for a definition of local independence see Anderson, 1959; Hambleton, Swaminathan, & Rogers, 1991, pp. 10-12; McDonald, 1962, 1981). Since the assumption of local independence may not hold in many educational test environments, Stout (1990) proposed a less restrictive assumption of "essential independence." Under this assumption, major latent dimensions are considered while minor dimensions are ignored. Even though the assumption of essential independence is less restrictive, the dimensionality of the items is an important issue in item response theory. Therefore, different techniques have been suggested for assessing the dimensionality of items for different IRT models.

The one-parameter model, often referred to as the Rasch model, involves only the estimation of difficulty parameters. For this model, different indices for testing unidimensionality have been suggested (see Hattie, 1985, Table 2, for complete data). Hattie (1985) concludes that most of these tests for assessing unidimensionality based on the Rasch model are insensitive to the violation of the unidimensionality assumption (see, for example, Gustafsson & Lindblad, 1978; Rogers, 1984; Van den Wollenberg, 1982)

With the two-parameter model that assumes no guessing, one can estimate difficulty and discrimination. The fit statistics summarized by Hattie (1985, Table 2) can be applied to the two- and three-parameter models with some modifications. Hattie (1985) reported that Rogers (1984) has proposed appropriate formulas for each of the indices listed by Hattie.

After reviewing the literature extensively and discussing the indices of dimensionality, Hattie (1985) concludes that "there are still no known satisfactory indices. None of the attempts to investigate unidimensionality has provided clear decision criteria for determining it" (p. 158).

A Summary of Techniques for Assessing Dimensionality in NAEP

This part of the literature review will briefly describe the statistical techniques used to assess the dimensionality of NAEP items. In the next section of the review, the results obtained by these studies will be summarized.

Very few studies on the dimensionality of NAEP test items have been reported in the literature. One of the most comprehensive studies conducted in the assessment of the dimensionality of NAEP items is a study conducted by Zwick (1985, 1987), which assessed the dimensionality of NAEP reading items. Zwick (1985) used the following methods: (a) principal components analysis (PCA) of the phi and tetrachoric correlations, (b) principal components analysis of the image correlation matrix, (c) Bock's full-information factor analysis implemented in the TESTFACT program, and (d) Rosenbaum's test of unidimensionality, monotonicity, and conditional independence using the Mantel-Haenszel procedure. Because guessing affected the results of the PCA on the tetrachoric correlations, Zwick (1985) used several different procedures for correcting the "guessing phenomenon," including Carroll's (1945) procedure and modification of the guessing phenomenon. Zwick also applied principal components analysis of image correlation to simulated data sets.

Others who studied the assessment of the dimensionality of NAEP items include Rock (1991), who used simple-structure confirmatory factor analysis on item parcels to assess the dimensionality of the NAEP math items. He reported intercorrelations between five math latent variable subscales. Cook and Eignor (1984) also used the latent variable modeling approach to assess the dimensionality of the NAEP reading items. They created item parcels of two to five items each with approximately the same mean difficulties and then applied simple-structure confirmatory factor analysis to the matrix of parcel correlations. First-order and second-order models were also applied to the data.

Carlson and Jirele (1992) also conducted a study on the dimensionality of the NAEP 1990 math items. They used item responses from four NAEP math booklets, two at Grade 4 and two at Grade 8. In addition to the NAEP math data, they analyzed four simulated one-dimensional data sets. Two different procedures, full information item factor analysis as implemented in the TESTFACT computer program and the normal harmonic factor analysis as implemented in the NOHARM program, were used. They reported the number of factors, chi-squares and AIC indices for each of the data sets. Yamamoto and Jenkins (1990) conducted a study to examine the dimensionality of the NAEP math tests for the 1990 main assessment. They performed comfirmatory factor analysis on the item parcels formed within each booklet at each grade level. They considered the results obtained from the different booklets as replications. They estimated the correlations between the five factors corresponding to the five math subscale latent variables, and the reported interfactor correlations were very high.

Allen (1990) conducted a study on the dimensionality of science test items for the 1990 main assessment. She used a three-latent-variable model that corresponded with the three science subscales. She then estimated the correlations between the three subscale latent variables which were averaged over the booklets and used as indices of dimensionality (see the NAEP 1990 Technical Report).

A Summary of the Results of the Dimensionality Studies in NAEP

The results of Zwick's (1985) extensive study of the NAEP reading items, which used several procedures and examined subscale dimensionality from different views, indicated that the reading items in different subscales were measuring the same underlying dimensions. She reported sizable first roots obtained by principal components analyses of phi and tetrachoric correlations. She also reported roots that ranged from 17% to 25% of the trace for the phi matrices and 30% to 40% for the tetrachoric matrices. The first roots of PCA to image correlation matrices, as she reported, were even larger than those obtained from the application of PCA to phi and tetrachoric correlations. When Bock's full-information factor analysis was applied to a subset of the grage 13/VIII, she found that the first factor accounted for 29% of the total variance. In her report, she stated that "overall, the four dimensionality analyses of the NAEP reading items indicate that it is not unreasonable to treat the data as unidimensional" (p. 39).

Rock in his (1991) study of the dimensionality of math subscales found that there was little discriminant validity in the math subscales except for the geometry subscale at the 8th-grade level. He found near perfect correlations between the subscale latent variables in math and science. For math subscales the average correlations were .94 for Grade 4, .91 for Grade 8, and .93 for Grade 12. The intercorrelations between the science subscales were high and very similar to the intercorrelations between the math subscales. The average intercorrelations for science subscales were .94 for Grade 4, .95 for Grade 8, and .95 for Grade 12. Based on these results, Rock (1991) concluded that "we are doing little damage in using a composite score in mathematics and science" (p. 2).

Due to the very large intercorrelations between the math subscales, Yamamoto and Jenkins (1990) concluded that there is a general math factor that accounts for a large amount of variability in the math subscale scores. The average intercorrelations between the math subscale latent variables in their study were .94 for Grade 4, .91 for Grade 8, and .93 for Grade 12.

Finally, Allen (1990) found high correlations between the three latent variables corresponding to the three science subscales indicating a unidimensional science test. In her study, the average intercorrelations between the subscale latent variables formed by item parcel scores were .95 for Grade 4, .95 for Grade 8, and .94 for Grade 12.

Method

The literature section of this paper summarizes the most commonly used techniques for assessing the dimensionality of a set of items and/or subscale scores. No examinee had a complete set of items in any one subject area or in any one subscale of a subject area because NAEP uses a BIB spiraling design (Beaton et al., 1987; Zwick, 1987). Due to this limitation and because the previous studies on the dimensionality of NAEP test items (Zwick, 1987) have revealed that the results of BIB spiraling were very similar to the results obtained from the complete data sets, we decided to conduct our dimensionality analysis at the booklet level.

It was possible to perform the dimensionality analysis at either the item-level or the subscale level within each booklet. However, because the literature revealed that the results of principal components and (linear) factor analysis done on dichotomously scored items (phi or tetrachoric correlations) were much affected by factors such as item difficulty, item homogeneity, and guessing (see, for example, Bejar, 1980; Carroll, 1983: Hambleton & Rovinelli, 1983, 1986; Hulin et al., 1983; Jones et al., 1987; McDonald & Ahlawat, 1974; Mislevy, 1986; Zwick, 1985), linear factor analysis was not performed on the item-level data. To linearize the nonlinear relationship between the observed responses and the underlying trait, we decided to use parcels of items and study the math subscale dimensionality based on the item parcels.

It must be noted, however, that the literature cautioned that serious problems could arise when factor analyzing parcel scores if the selection of the items for a parcel is not done properly (see Cook et al., 1983). For example, item parcels with different levels of difficulty could produce difficulty factors which introduce bias into the dimensionality assessment process (see, for example, Swinton & Powers, 1980). In response to this potential problem, Cattell and Burdsal (1975) suggested the use of radial parceling. To do item parceling according to Cattell's radial procedure, two factor analyses should be performed, one on the items and another on the parcel scores. The first factor analysis would provide information for constructing the parcels, and the second factor analysis would help define the dimensionality of the parcels.

We took Cattell's suggestion into consideration and tried to construct item parcels in such a way that the items within the parcels were homogeneous and the parcels had approximately equal variance and equal means. Because background variables could have an impact on subscale math dimensionality (see, for example, Muthén et al., 1991), we used these item parcels in factor analytic models to study the effects of background variables on math dimensionality.

NAEP collects a substantial number of instructional and non-instructional (cognitive and noncognitive) background variables. Because it was very difficult and time consuming to use all of these variables in our dimensionality analysis, we decided to select a small set of the background variables that had a significant impact on the math subscale scores. We ultimately selected all of the cognitive (instructional) variables and some of the noncognitive background variables that seemed to be related to student mathematics ability. Then these variables were used as grouping variables in a multiple discriminant analysis in which the math subscale scores were used as discriminating variables. Based on the results of the discriminant analysis, the cognitive and noncognitive variables that had a significant impact on the math subscale scores were further identified and selected to be used as a basis for forming student subgroups. We had planned on comparing the factor means and factor variances across the subgroups of students that were formed based on the levels of background variables using the multiple group factor analysis procedure. However, because of the small number of subjects in the subgroups, we decided to use the Multiple Indicators and Multiple Causes (MIMIC) approach.

We applied the MIMIC model to the 1990 and 1992 data. The application of the MIMIC model to the 1992 data was particularly useful because of the small number of subjects per booklet for the 1992 data. However, because the sister dimensionality study that was conducted at CRESST/UCLA by Bengt Muthén and his team performed the same analysis on the same data file, we decided to report only the simple-structure factor analysis results.

For the 1990 data, all the students who answered the items in a given booklet were divided into subgroups based on their responses to the selected background variables (questions). Once the subgroups were formed, item parcels were created for each of the subscales within each subgroup. These parcels were used in three different latent-variable models: (Model 1) a one-factor model which assumes one general mathematics factor; (Model 2) a five-subscale-factor model which assumes five mathematics subscales; and (Model 3, a hierarchical model) a five-subscale-one-general-factor model which assumes five math subscales plus one general math factor. Indices of fit for the three models were obtained and were compared within each subgroup as well as across subgroups. For Model 2 (five subscale factors with no general math factor), correlations between the five subscales or five latent variables were estimated and were compared across the subgroups. We did this to see if more discrimination would appear (i.e., lower correlations) between the five math subscale latent variables for the subgroups that were formed based on the variables with more discriminating power. All of these analyses were performed on all of the items within each of the booklets.

For the second phase of analyses on the 1990 data, correlations between each individual item and the background variables were computed using the Multi-Approach Correlation System (MACS) (Abedi, 1993). Items that had high and low correlations with the background variables were selected and two different sets of item parcels were created. One set consisted of items that had higher correlations with the background variables, and the other set consisted of items that had lower correlations with the background variables. The item parcels were then used in three different latent variable models (i.e., the one-factor model, the five-subscale-factor model, and the five-subscale-one-general-factor model or hierarchical model). The second-phase analyses were performed on all of the subjects and the results of the three models were compared. It must be noted, however, that for some of the booklets and for some of the subscales, there were not enough items to create item parcels of high and low correlations.

The 1992 math test item data were different from the 1990 data in at least two aspects. First there were more open-ended questions in the 1992 administration than in the 1990 administration, and second, the math items were distributed into more booklets in the 1992 administration than in the 1990 administration (there were approximately 2 and half times more booklets in 1992 than in 1990). Because of these differences, the analyses performed on the 1992 data were different from those performed on the 1990 data.

Because we did our 1992 data analyses at the booklet level, we found that there were not enough subjects per booklet to form into subgroups based on the subjects' responses to the background variables (questions), especially for those variables (questions) that had more than two responses/categories. Therefore, for the 1992 data, we selected items based on their relationships with the cognitive and noncognitive background variables. Once these items were selected, we compared the intercorrelations between the five math subscale latent variables of the parcels that contained items having high correlations with the background variables and the parcels that contained items having low correlations with the background variables.

Results

As mentioned earlier, we selected a small set of cognitive and non-cognitive background variables that had a significant effect on the math subscale scores. This was accomplished by using a multiple discriminant analysis. Discriminant analysis was employed because of "(1) parsimony of description; and, (2) clarity of interpretation" (Stevens, 1992). A discriminant analysis is parsimonious because out of the five subscale scores the subgroups may differ on only one or two subscales. For example, some of our analyses performed on the total group of students have revealed that even though the subscales were highly correlated, the geometry subscale was more distinct than the others (see also Rock, 1991). Discriminant analysis (DA) has clarity of interpretation because the discrimination of groups on one function is quite independent from the discrimination of groups on the other functions.

The discriminant analyses that yielded the cognitive and noncognitive background variables that were going to be used in our study satisfied the following conditions (Pedhazur, 1982):

1. At least one discriminant function was significant with the following statistics:
    a. The canonical correlation was .250 or greater.
    b. χ^2 was significant at the .050 level.
2. The univariate F-ratios for the subscales were significant at the .050 level.

3. The structure coefficients of all or most of the subscales on the discriminant functions were greater than .300.

We will now discuss the results of the multiple discriminant analysis to show how the background variables were selected for our analyses. We will only present the results of those analyses which yielded significant results. Thus, if some of the background variables (such as SES, gender, and ethnicity) were not used that is because they did not show much discriminating power when used on the math subscale scores. We will first discuss the results for the 1990 data and then for the 1992 data.

Results of the Discriminant Analysis for the 1990 Data

1. Results for Grade 4 - DA 1990

For Grade 4, three different groups of subjects who answered items from booklets 11, 12, and 14 were used. Tables A1 through A3 present the results of the DA for Booklet 11.

Students were divided up into subgroups based on what answer choice/ response they selected to the posed background variable (question). For example, for the background variable question "Home Environment-Reading Materials" there were three answer choices available for selection: (1) 0-2 Types, (2) 3 Types, and (3) 4 Types. Thus, there was a total of three student subgroups that were formed from the selected responses to this background variable (question).

Table A1 summarizes the results of the DA for the subgroups that responded to the background variable (question) "Home Environment-Reading Materials." As Table A1 indicates, only one function was statistically significant (χ^2=121.180, df=10, p=.000). For this function, the F-ratios showing significant differences in the subscales across subgroups were all significant beyond the .01 nominal level.

Table A2 presents the results of the DA for the subgroups that responded to the background variable (question) "How Do You Feel About This Statement: I Am Good In Math." In this analysis there was also only one function that was significant (χ^2=129.530, df=10, p=.000). In addition, the F-ratios for all the subscale scores were significant beyond the .01 nominal level indicating that the subgroups performed differently on the five subscales.

Table A3 presents similar DA results for the subgroups that responded to the background variable (question) "In Math Class How Often Do You Work With Rulers, Blocks, Shapes." In this analysis, one significant function (χ^2=82.340, df=20, p=.000) emerged as well. The F-ratios that demonstrate the significant discriminating power of the five subscale scores were all significant.

As we explained earlier, the analyses were done on the booklet level due to NAEP's BIB spiraling design, and the results obtained from the different booklets were used as cross validation data. Tables A4, A5, and A6 present the results of the DA on booklet 12. These results are similar to the results that are presented in Tables A1, A2, and A3 respectively. Tables A7, A8, and A9 also present similar results for booklet 14.

The results of the DA that was conducted on the three independent groups of subjects were consistent across booklets. For instance, compare Table A1 with Tables A4 and A7. Then compare Table A2 with Tables A5 and A8, and Table A3 with Tables A6 and A9 to see how consistent the results are across the three different groups of students who answered items in booklets 11, 12, and 14.

2. Results for Grade 8 - DA 1990

For Grade 8, subjects who answered math items in booklets 8, 9, and 10 were chosen for the DA. Table A10 presents the results of the DA for the subgroups that responded to the background variable (question) "Home Environment-Reading Materials." As Table 10 indicates, only one significant function discriminated between the groups that were formed based on the above background variable (question) (χ^2=123.050, df=10, p=.000). The five subscale scores all had significant mean differences across the subgroups and the F-ratios corresponding to the five subscale mean differences were all significant beyond the .01 nominal level.

Table A11 summarizes the results of the DA for the subgroups that responded to the background variable (question) "Do You Agree: I Am Good In Math" for booklet 8. Once again, only one significant function emerged (χ^2=156.310, df=20, p=.000). The F-ratios for the subscale scores were all significant above the .01 nominal level which indicated that there were differences in all of the subscale scores across the subgroups.

Table A12 presents similar results for the DA for the subgroups that responded to the background variable (question) "What Kind of Math Class Are You Taking This Year." This analysis yielded two significant functions: (χ^2=303.280, df=20, and p=.000) for function one; and (χ^2=33.230, df=12, and p=.000) for function two. The F-ratios for all of the subscale scores were significant.

Tables A13, A14, and A15 present the results of the DA that was conducted on the data from booklet 9. These results are comparable with those in Tables A10, A11, and A12 respectively and cross validate each other. Tables A16, A17, and A18 present results that are comparable with the results in Tables A10, A11, and A12. Results in Tables A10, A11, and A12 are also comparable with the results in Tables A13, A14, and A15 respectively.

After comparing the results presented in these tables it is quite evident that there is a consistent trend in the results of the DA of the different groups of subjects that answered different sets of math items. The only major difference between the results obtained from the data in the three booklets is that the DA for the subgroups that responded to the background variable (question) "What Kind of Math Class Are You Taking This Year" in booklet 8, yielded two significant discriminant functions. In this analysis, some of the subscale scores had significant structure coefficients on two of the functions. For example, the Numbers & Operations subscale had a structure coefficient of .790 on function 1, and .580 on function 2. The same analysis on booklet 10 (including the same background question/response series) also yielded two functions. But for booklet 10, all of the subscale scores had high structure coefficients on the first function and a few moderate structure coefficients on the second function. The analysis of booklet 9, however, resulted in only one significant function; therefore, the subscale scores had only high structure coefficients on the first function.

3. Results for Grade 12 - DA 1990

For Grade 12, DAs were performed on the data obtained from booklets 8, 9, and 10. Tables A19 through A21 present the DA results for booklet 8. Tables A22 through A24 present the DA results for booklet 9, and Tables A25 through A27 present the DA results for booklet 10.

Table A19 summarizes the DA results for the subgroups that responded to the background variable (question) "Home Environment-Reading Materials." As the analysis indicates, the first function was significant (χ^2=79.850, df=10, p=.000) and all of the five subscale scores were significantly different across the subgroups.

Table A20 presents similar DA results for the subgroups that responded to the background variable (question) "Do You Agree: I Am Good In Math." This analysis yielded two significant functions: (χ^2=196.360; df=20; p=.000) for the first function; and (χ^2=25.090; df=12; p<.01) for the second function. The subscale scores were significantly different across the subgroups with F-ratios that were significant above the .01 nominal level.

Table A21 presents the DA results for the subgroups that responded to the background variable (question) "In Math Class How Often Do You Do Problems On Worksheet." One function was significant in this analysis (χ^2=77.120, df=20, p=.000). The significant F-ratios indicated that all the subscale scores were different across the subgroups.

Tables A22, A23, and A24 present the results of the DA for booklet 9. These results are comparable to the results in Tables A19, A20, and A21. Tables A25, A26, and A27 present the results of the DA for booklet 10. These results are comparable with the results in Tables A19, A20, and A21 and the results in Tables A22, A23, and A24 respectively.

When these results were compared, consistencies were found across the three different groups that answered different sets of items from the three separate booklets. There was only one disparity in the results across the three groups. The results of the DA on booklet 8 and 9 yielded two significant functions for the subgroups that were formed based on the selected response choices to the background variable (question) "Do You Agree: I Am Good In Math." The results of the analyses performed on booklet 10 revealed only one significant function.

Based on the series of DAs that were run on the 1990 math data, the following set of cognitive and noncognitive background variables were found to have significant effects on the math subscale scores:

Grade 4:

Home Environment-Reading Materials
How Do You Feel About This Statement: I Am Good In Math
In Math Class How Often Do You Work With Rulers, Blocks, Shapes

Grade 8:

Home Environment-Reading Materials

Do You Agree: I Am Good In Math

What Kind Of Math Class Are You Taking This Year

Grade 12:

Home Environment-Reading Materials

Do You Agree: I Am Good In Math

In Math Class How Often Do You Do Problems On Worksheet

Results of the Discriminant Analysis for the 1992 Data

As mentioned earlier, the 1992 math items were distributed into 26 booklets as compared to the 1990 items which were placed into 10 booklets. Thus, there were fewer students per booklet in the 1992 administration than in the 1990 administration. Because of this limitation, for the 1992 analyses, we had some difficulty grouping the students based on the background variables, especially for those variables that had a large number of responses/choices. We will discuss the results of discriminant analysis on the 1992 data set for each grade separately.

1. Results for Grade 4 - DA 1992

Tables B1 through B3 summarize the results of the DAs for the three selected background questions that were used on the data in booklet 15. As indicated earlier, we performed DA on almost all of the background variables but we only reported those results which were significant. Table B1 presents the results of the DA for all of the subjects that responded to the background variable (question) "Agree/Disagree: I Am Good In Math" for Grade 4, booklet 15. As Table B1 indicates, only one function (χ^2=19.570, df=10, p<.030) significantly discriminated between the subgroups that were formed based on the range of responses to the above background variable (question). All of the subscale scores except the Measurement and Algebra subscales yielded different means across the subgroups.

Table B2 presents the DA results for the subjects that responded to the background variable (question) "Agree/Disagree: I Like Math." In this analysis, one function also emerged (χ^2=20.090, df=10, p<.030). The subscale scores were all different with the exception of the Measurement and Algebra subscales across all of the subgroups.

Table B3 presents the DA results for the subjects that responded to the background variable (question) "How Much Time Spent Each Day On Math Homework." One significant function emerged (χ^2=45.800, df=30, p<.030). The Numbers & Operations and the Statistics subscales yielded different means across the subgroups.

Tables B4, B5, and B6 present the results for the DA analyses that were performed on the data from booklet 17. These results are parallel to the results that were reported in Tables B1, B2, and B3 respectively. When comparing these two sets of tables one can see the consistency in the results obtained based on from these two different groups of students.

2. Results for Grade 8 - DA 1992

The results of the DA that was performed on the Grade 8, 1992 data, in which two background variables (questions) were selected are reported here. Tables B7 and B8 present the results of the DA for booklet 1. Tables B9 and B10 present the results of the DA for booklet 2, and Tables B11 and B12 summarize the results of the DA for booklet 15.

Table B7 presents the DA results for the subjects that responded to the background variable (question) "Do You Agree: I Am Good In Math" for Booklet 1. There was only one significant function that emerged in this analysis (χ^2=60.100, df=20, p=.000). As the F-ratios indicate, all of the subscale scores yielded significant differences across the subgroups except for the Statistics subscale.

Table B8 summarizes the results of the DA for the subjects that responded to the background variable (question) "Agree/Disagree: Math Is Mostly Memorizing Facts." This analysis also yielded one function that was significant (χ^2=31.870, df=20, p<.050). The F-ratios in this table indicated that all of the subscale scores had different means across the subgroups.

Table B9 reports the results of the DA of the first background variable, "Do You Agree: I Am Good In Math" for booklet 2 and Table B10 reports similar results for the second background variable, "Agree/Disagree: Math Is Mostly Memorizing Facts" for the same booklet. Similarly, Table B11 summarizes the results of the DA for the first background variable that was applied to the data in booklet 15, and Table B12 summarizes the results of the DA for the second background variable for the same booklet.

The results presented in Tables B7 and B8 are comparable with the results presented in Tables B9 and B10. These two sets of tables are also comparable with Tables B11 and B12 respectively. When comparing the sets of tables that display the first background variable, "Do You Agree: I Am Good In Math," there is only one disparity in the results of the three different groups of students that answered the math items in the three different booklets. The dissimilarity appears in the F-ratio that is reported for the Statistics subscale. The results for booklet 1 yielded no significant differences in the mean score for this subscale across the subgroups, whereas, for the other two booklets there were significant mean differences in this subscale across the subgroups. Except for this one instance, the results of the analyses for the three booklets were consistent.

3. Results for Grade 12 - DA 1992

We applied DA on the data from the three selected booklets (1, 15, and 17) for Grade 12. We will only report the summary results of the DA for two background variables that were applied to the data in each of the three booklets. These two variables were: (1) "Do You Agree: I Am Good In Math" and (2) "Agree/Disagree: Math Is Mostly Memorizing Facts."

Table B13 presents the results of the DA for the first background variable, "Do You Agree: I Am Good In Math" for booklet 1. As Table B13 indicates, only one function significantly discriminated between the subgroups (χ^2=97.800, df=20, p=.000). The F-ratios in this table were all significant which indicated that the subscale scores were all different across the subgroups.

Table B14 presents the DA results for the second background variable "Agree/Disagree: Math Is Mostly Memorizing Facts" for booklet 1. Consistent with the data presented in Table B13 and prior DA analyses, one significant function emerged (χ^2=97.400, df=20, p=.000). All of the F-ratios that corresponded to the subscale scores were also significant.

Similar results were obtained when the two background variables were used in analyzing the data from booklets 15 and 17. Tables B15 and B16 report the results of the DA for booklet 15 and Tables B17 and B18 present the results of the DA for booklet 17. Tables B13 and B14 can be compared with Tables B15 and B16 and Tables B17 and B18 respectively. In all of these analyses, one significant function emerged, and all of the subscale scores were significantly different across the subgroups.

Based on the series of DAs that were run on the 1992 math data, the following set of cognitive and noncognitive background variables were found to have significant effects on the math subscale scores:

Grade 4:

Agree/Disagree: I Am Good In Math
Agree/Disagree: I Like Math
How Much Time Spent Each Day On Math Homework

Grade 8:

Do You Agree: I Am Good In Math

Agree/Disagree: Math Is Mostly Memorizing Facts

Grade 12:

Do You Agree: I Am Good In Math

Agree/Disagree: Math Is Mostly Memorizing Facts

Results of the Factor Analysis (FA)

Discriminant analysis helped us identify those background variables that had a greater impact on student math performance. After the background variables that had a higher discriminating power were identified and selected, we then performed confirmatory factor analysis on the subgroups that were formed based on the level of responses to the selected background variables.

In approaching confirmatory factor analysis, we set out to test the following three hypotheses: (Model 1) the five math subscales are highly correlated because they are actually measuring one general math factor; (Model 2) the test items in the five math subscales are measuring five distinct math abilities; and, (Model 3) while each subscale may measure a distinct math ability, the five subscales are measuring a common math ability. From these three hypotheses, three entirely separate latent variable models were created. Each latent variable model was derived from and corresponded with one of the above hypotheses. We then applied these three new models to the data sets that were obtained from the different booklets and different subgroups. Indices of fit from the three different models were compared to see which model exhibited the best fit to the data.

We once again ran into the problem of not having enough subjects per booklet to group into fairly equable subgroups. The BIB spiraling design of NAEP and the presence of background variables (questions) that had a higher number of responses/categories caused many of the subgroups to have far too few subjects. Due to this complication, in some of our analyses, rather than grouping the subjects into subgroups, we decided to use the items that had high correlations with the background variables. In other words, in any given booklet, we used all of the subjects' responses to the items that had high correlations with the background variables. We then applied the three models to the composite scores of the selected items (item parcels).

As mentioned earlier in the literature section of this report, several studies have been conducted on the NAEP math items that used the total group of students. These studies mainly revealed that the math items were generally unidimensional. In our study, we examined the dimensionality of the math items on the total group of students as well as on the subgroups of students that were formed based on the range of responses to the selected background variables (questions) that we found (in our DA) to affect math performance. We will first report our findings for the total group of students and then for the subgroups of students. We will discuss these results individually for each of the three grades (4th, 8th, and 12th).

Results of the factor analysis for the 1990 data

1. Results for Grade 4 - FA 1990

Table TA1 summarizes the results of the confirmatory factor analysis that was conducted on the data from booklet 11. The upper half of the table lists the indices of fit including chi-square, degrees of freedom, chi-square ratio, normed fit index (NFI), non-normed fit index (NNFI), and comparative fit index (CFI) for the following three models: (Model 1) assumes that a general math factor is underlying all of the five math subscale scores and that there is no subscale variation; (Model 2) assumes that the five subscales are measuring five different areas in math, and thus, the math items could be categorized under five different factors (or subscales); and, (Model 3) assumes that in addition to a general math factor, there are also some subscale factors that cannot be explained by the general math factor. The lower half of Table TA1 presents the estimated correlations between the five subscale latent variables. As Table TA1 indicates, all three of the models fit the data based on the fit indices. However, Models 2 and 3 (which assume more than a general math factor) seem to display a better fit to the data. The chi-square ratio for Model 1 was 2.170 whereas the chi-square ratios for Models 2 and 3 were 1.790 and 1.900 respectively. The correlations between the five subscale latent variables (as shown in the lower half of the table) were all extremely high except for the Geometry subscale. These high correlations were all indicative of unidimensionality of the five subscales.

Table TA2 presents the results of the confirmatory factor analysis for the data from booklet 12. These results are very similar to those presented in Table TA1 for booklet 11. The three models fit the data, but Models 2 and 3 once again seem to exhibit a better fit. The chi-square ratio for Model 1 (which assumes only one general math factor) was 1.460 whereas the chi-square ratios for Models 2 and 3 (which assume subscale factors as well) were .810 and .710 respectively. In this table, the correlations between the subscale latent variables were also extremely high with the exception of the Geometry subscale. This once again indicates unidimensionality of the math subscale scores.

Table TA3 reports the results of the analyses for booklet 14. Consistent with the results presented in Tables TA1 and TA2, the indices of fit in this table indicate that the data fit the three models. However, Models 2 and 3 exhibit a better fit. The chi-square ratio for Model 1 was 2.640. For Model 2 the chi-square ratio was 1.890, and for Model 3 it was 1.850. The subscale latent variable correlations were all extremely high and indicative of unidimensional subscale scores.

Results for Grade 4-FA 1990 based on the levels of selected background variables. We now turn our discussion to the results of the analyses that were conducted separately on each of the subgroups that were formed based on the levels of the selected background variables. Table 4A1 presents the results of the confirmatory factor analysis on the group of students who indicated that they were Undecided (2) when responding to the background variable (question) "How Do You Feel About This Statement: I Am Good In Math." The fit indices in this table indicate that the three models had a satisfactory fit. However, based on the chi-square ratios, one may conclude that Model 2 had a slightly better fit than the other two models. The lower half of the table lists the correlations between the five math subscale latent variables. When comparing the subscale correlations that were obtained from the total group analyses to the subscale correlations from the subgroup analyses, the subscale correlations were smaller for the subgroups. These smaller correlations indicate that there is evidence of multidimensionality in the subscale scores for the subgroups. For example, in the analyses that were performed on the subgroups, the correlation between the Geometry and Numbers & Operations subscales was .680, and between the Statistics and Geometry subscales the correlation was .810 as compared with the respective correlations of .830 and .990 for the total group analyses.

Table 4A2 presents the results of the analyses for those students who responded to the background variable (question) "How Do You Feel About This Statement: I Am Good In Math" by selecting either Agree (1) or Disagree (3). The indices of fit for both of these subgroups (displayed in the top and middle portions of the table) indicate that the three models fit the data at approximately the same level. However, the bottom portion of the table reveals a different trend in the correlations between the subscale latent variables. For example, for the Disagree (3) subgroup, the subscale correlations were smaller than for the Agree (1) subgroup, which is indicative of multidimensionality in the math subscales. The correlations for the Disagree (3) subgroup between the Statistics subscale with the Numbers & Operations, Measurement, and Geometry subscales were .910, .770, and .870 respectively, whereas the correlations for the Agree (1) subgroup between the Statistics subscale with the Numbers & Operations, Measurement, and Geometry subscales were .950, 1.000, and .980 respectively.

Table 4A3 summarizes similar results for booklet 12 to those that were presented in Table 4A2 for booklet 11. The top portion of this table indicates that all of the three models fit the data for the subgroup that indicated Agree (1) to the background variable (question) "How Do You Feel About This Statement: I Am Good In Math." But Models 2 and 3 exhibited a slightly better fit than Model 1. The chi-square ratio for Model 1 was 1.730, for Model 2 it was 1.250, and for Model 3 it was 1.080. The middle section of the table lists the indices of fit for the subgroup that selected the Disagree (3) response to the background variable (question) listed above. For this group the three models fit the data at approximately the same level. However, the bottom portion of the table, which lists the correlations between the subscale latent variables, indicates that the there is more evidence of dimensionality for the subgroup that indicated Disagree (3) for the background variable (question) "How Do You Feel About This Statement: I Am Good In Math" than for the Agree (1) subgroup. For example, the correlations between the Geometry subscale with the Numbers & Operations and Measurement subscales were .790 and .840 respectively for the Disagree (3) subgroup whereas the correlations for the

Agree (1) subgroup between the same subscales were .860 and .870 respectively.

These results were consistent with those results that were obtained for booklet 11. Table 4A4 summarizes the results for those subjects that selected the Undecided (2) response/category to the "How Do You Feel About This Statement: I Am Good In Math" background variable (question). In this analysis, the three models fit the data at about the same level. The chi-square ratios were .630 for Model 1, .530 for Model 2, and, .570 for Model 3. There were high correlations between the subscale latent variables but some of these correlations were not as high as the correlations that were found for the same background variable/response choice in the total group analyses.

Table 4A5 presents the results of the analyses for the "How Do You Feel About This Statement: I Am Good In Math" background variable (question) for booklet 14. For this analysis, all three models fit the data at approximately the same rate. In Model 1, the students who chose Agree (1) to the above background variable (question) had a chi-square ratio of 1.730. In Model 2 the chi-square ratio was 1.310, and for Model 3, it was 1.320. For the Disagree (3) subgroup, the chi-square ratios were .750, .710, and .680 for the three models respectively.

Table 4A6 presents the results for those subjects that selected the Undecided (2) response/choice to the background variable (question) "How Do You Feel About This Statement: I Am Good In Math." Again, the three models fit the data at about the same rate. There were, however, lower correlations in the subscale latent variables' correlation matrix which indicates evidence of multidimensionality in the subscales.

Tables 4A7 and 4A8 summarize the results of the confirmatory factor analyses on the three subgroups of students that selected one of the three response choices to the background variable question "Home Environment-Reading Materials" for booklet 11. Tables 4A9 and 4A10 present the results for the same background variable (question) and set of answer response choices to booklet 12, and Tables 4A11 and 4A12 report the results for the same background variable/answer choice sequence for booklet 14.

Table 4A13 presents the results for the two subgroups that selected either one of the two responses (1=Almost Every Day; or 5=Never) to the background variable (question) "In Math Class How Often Do You Work With Rulers, Blocks, Shapes" for booklet 11. There were originally 5 selection responses (1=Almost Every Day; 2=Several Times A Week; 3=About Once A Week; 4=Less Than Once A Week; and 5=Never) for this background variable (question). However, we only used the data from those subjects who selected response (1) or (5) because we wanted to examine the differences in subscale scores between the two extreme range choices. Table 4A14 reports the results for the same background variable (question) and answer choices for booklet 12, and Table 4A15 summarizes the results for the same background variable (question) sequence for booklet 14.

The results that were reported for the "How Do You Feel About This Statement: I Am Good In Math" background variable (question) were very similar to those found for the "Home Environment-Reading Materials" and "In Math Class How Often Do You Work With Rulers, Blocks, Shapes" background variables (questions). For instance, the three models that were created for the "Home Environment-Reading Materials" and the "In Math Class How Often Do You Work With Rulers, Blocks, Shapes" background variables (questions) fit the data at approximately the same levels as the three models that were created for the "How Do You Feel About This Statement: I Am Good In Math" background variable (question) sequence. There was a trend of a better fit for

Models 2 and 3 in the "Home Environment-Reading Materials" and the "In Math Class How Often Do You Work With Rulers, Blocks, Shapes" background variables (questions). In these tables (4A13, 4A14, and 4A15) one can see that there were also lower correlations between the subscale latent variables than those that were observed for the same two background variable (question) sequences in the total group analyses. Refer to Tables TA1, TA2, and TA3 for the appropriate comparisons.

Results for Grade 4 - 1990 FA on item-background correlations. As mentioned earlier, we also computed the correlations between the individual math test items and the individual background variables using a special program that was prepared solely for this purpose. Based on the item-background correlation results, we selected certain items, and then we applied our latent variable approach to these selected items.

Table I1 presents the results of the factor analysis of the selected items based on the item-background correlations for Grade 4, booklet 11. As the top half of Table I1 indicates, the three models fit the data, but Models 2 and 3 once again displayed a slightly better fit. The bottom half of the table presents the correlations between the subscale latent variables. In some cases the correlations between the subscale latent variables were considerably lower than those correlations that were obtained from the total group analyses. For example, the correlations between the Numbers & Operations subscale with the Measurement, Geometry and Statistics subscales were .571, .725, and .727 respectively as compared to the total group correlations for booklet 11 which were .960, .830, and 1.000 respectively (see Table TA1).

Table I2 presents the results for the data for Grade 4, booklet 12. Again, in this table, the three models exhibited approximately the same level of fit. The chi-square ratio for Model 1 was 1.700, for Model 2 it was 1.990, and for Model 3 it was 1.850. The subscale latent variable correlations were lower in comparison to the correlations in the total group analyses (Tables TA1, TA2, and TA3).

Table I3 summarizes the results of the analysis for Grade 4, booklet 14. The top half of the table indicates that the three models displayed a fit to the data similar to the previous model fits. The bottom half of the table lists the subscale latent variable correlations, which seem to be lower, mainly for the Numbers & Operations subscale, than those that were obtained from the total group analyses (Tables TA1, TA2, and TA3).

2. Results for Grade 8 - FA 1990

We will report some of the results of the latent variable analyses that were conducted on the total group of subjects using all of the items and then we will report the results of the analysis by subgroups that were formed based on the response choices to the selected background variables (questions). Finally, we will report the results of the analyses that were conducted on the total group of subjects using the selected items that were chosen based on the item-background correlation analyses.

Table TA4 summarizes the results of the analyses for Grade 8 for the subjects that answered all of the items in booklet 8. The top half of this table lists the indices of fit for the three models (Model 1 which assumes one general factor, Model 2 which assumes five subscale factors, and Model 3 which assumes five subscale factors plus one general factor). The indices of fit indicate a relatively good fit of the data to Models 2 and 3 but a poor fit for Model 1. For example, the chi-square ratio was 3.470 for Model 1, 1.410 for Model 2, and 1.860 for Model 3. The correlations between the five subscale latent variables were all high and indicative of unidimensional subscales.

Table TA5 presents the same series of results for Grade 8, booklet 9. Models 2 and 3 displayed a better fit to the data and were similar to the model fits that were reported for booklet 8. The chi-square ratios for booklet 9 were 3.610, 1.820, and 2.290 for the three models respectively. The subscale latent variables were also relatively high for booklet 9.

Table TA6 summarizes the results of the analyses conducted on booklet 10. The indices of fit at the top half of the table indicate that Models 2 and 3 once again exhibited a much better fit to the data than Model 1. The chi-square ratio for the Model 1 was 4.900, for Model 2 it was 1.180, and for Model 3 it was 1.690. The size of the correlations between the subscale latent variables, however, indicated that the subscales were unidimensional.

Results for Grade 8-FA 1990 on the levels of selected background variables. We will now discuss the results of the latent variable analysis that was conducted on the subgroups that were formed based on the levels of responses/choices to the background variables. For each subgroup, we will present the indices of fit for the three models and discuss the correlations between the subscale latent variables.

Table 8A1 presents the results (fit indices and subscale correlations) for the subjects who indicated Strongly Agree (1) or Disagree (4) as a response to the background variable (question) "Do You Agree: I Am Good In Math" for booklet 8, grade 8. For the first subgroup (those subjects who selected Strongly Agree[1] to the background variable [question]), Models 2 and 3 exhibited a much better fit than Model 1. Thus, multidimensional models seem to fit the data better than unidimensional models. For the second subgroup (those subjects who selected Disagree [4] to the background variable [question]), all three models displayed a good fit, but Models 2 and 3 exhibited a slightly better fit. The chi-square ratios for Models 1, 2, and 3 were .800, .370, and .450 respectively. The subscale correlations, located in the bottom portion of the table, were relatively high which indicated unidimensionality of the math items.

Table 8A2 reports results that are similar to Table 8A1 for the subjects who selected 0-2 Types (1) or 4 Types (3) as their response to the background variable (question) "Home Environment-Reading Materials." For the first subgroup (those students who selected a 0-2 Types [1] as their answer choice to the background variable [question]), the indices of fit indicated that all three models fit the data at approximately the same level. For example, the chi-square ratios for the first subgroup's three models were 1.700, 1.370, and 1.220 respectively. For the second subgroup (those students who selected 4 Types [3] as their answer choice), the indices of fit indicated that all three models had a relatively good fit to the data, but again Models 2 and 3 exhibited a slightly better fit. The chi-square ratios were 2.310, 1.530, and 1.570 for the three models respectively.

The lower part of this table does however, indicate that the correlations between the subscale latent variables for the first subgroup (1) were lower than the parallel correlations that were obtained from the analyses on the total group(s) of subjects for booklet 8 (see Table TA4).

The upper section of Table 8A3 presents the results for the analyses conducted on the subgroup of students who selected Algebra (4) to the background variable (question) "What Kind Of Math Class Are You Taking This Year" for Grade 8, booklet 8. As table 8A3 indicates, Models 2 and 3 (which include subscale factors) exhibited a better fit to the data than Model 1 (which assumes only one general math factor). The chi-square ratios were 1.910, 1.440 and 1.540 for Models 1, 2, and 3 respectively. The middle section of Table 8A3 presents the results for the those students who selected Eighth

Grade Math (2) to the same background variable (question). The chi-square ratios for this subgroup were 1.630, 1.270 and 1.130 respectively for the three models. The correlations between the subscale latent variables, as shown in the lower section of Table 8A3, are relatively lower than the corresponding correlations that were obtained in the total group analyses.

Table 8A4 presents the results of the analyses conducted on the subjects who responded by selecting Pre-Algebra (3) to the "What Kind Of Math Class Are You Taking This Year" background variable (question) for Grade 8, booklet 8. The fit indices in this table indicate that Models 2 and 3 exhibited a better fit to the data than Model 1. The chi-square ratios were 2.000, 1.540, and 1.660 for the three models respectively. The subscale correlations, which are reported at the bottom section of the table, reveal relatively lower correlations than those that were reported for the corresponding total subject analyses.

Table 8A5 summarizes the results for those students who selected Strongly Agree (1) or Disagree (4) to the "Do You Agree: I Am Good In Math" background variable (question) for booklet 9. For the first subgroup (those students who chose Strongly Agree [1] as their response to the background variable (question) mentioned above), the indices of fit indicated that Models 2 and 3 exhibited a better fit to the data. The chi-square ratios were 3.200, 1.960, and 2.030 for the three models respectively. For the second subgroup (those students who chose Disagree [4] as their response to the background variable [question]), the indices of fit indicated that the three models fit the data as well. The chi-square ratios were 1.390, 1.310, and 1.290 for the three models respectively. The subscale latent variable correlations for the two subgroups (reported in the bottom section of the table) were all relatively high but are slightly lower than the corresponding correlations that were reported for the total group analyses (see Table TA5).

The results of the analyses reported in this table are comparable with Table 8A1 which summarizes the results of the analysis that was conducted on the same background variable (questions/answer choices) for booklet 8. The comparison of these two tables reveals a consistency in the results across independent samples of students. Likewise, the results of the analyses reported in Table 8A6 are comparable with the results that are reported in Table 8A2, and the results in Table 8A7 are comparable with the results presented in Table 8A3. In addition, the data in Table 8A8 is comparable to the data in Table 8A4.

Tables 8A9, 8A10, 8A11, and 8A12 report the results of the analyses that were conducted on the data from booklet 10. Table 8A9 can be compared with Tables 8A1, and 8A5; Table 8A10 is comparable with Tables 8A2, and 8A6; Table 8A11 can be compared with Tables 8A3, and 8A7; and, Table 8A12 can be compared with Tables 8A4 and 8A8. These comparisons also reveal a consistency in the findings between the different groups of 8th-grade students who answered math items in booklets 8, 9, and 10.

Results for Grade 8 - 1990 FA on item-background correlations. We can now turn to the results of the analyses on the items that were selected based on their correlations with the background variables. Table I4 presents the results of the latent variable analysis that was conducted on the selected items for Grade 8, booklet 8. As this table indicates, all three of the models fit the data at approximately the same level, but Models 2 and 3 exhibited a slightly better fit. The chi-square ratio was 1.650 for Model 1, 1.170 for Model 2, and 1.150 for Model 3. The lower half of the table presents the correlations between the subscale latent variables. These correlations were considerably lower than the correlations that were obtained from the corresponding analyses on the total group (see Table TA4).

Table I5 summarizes the results of the analyses that were conducted on the selected items for Grade 8, booklet 9. The three models in this analysis fit the data at about the same level. The chi-square ratios were 2.650 for the first model, 2.650 for the second model, and 2.330 for the third model. The correlations between the subscale latent variables were lower in comparison to the corresponding correlations that were obtained from the analyses conducted on all of the items/the total group (see Table TA5).

Table I6 presents the results of the analyses for the selected items for Grade 8, booklet 10. These results were consistent with the results that were presented in Table I4 for booklet 8, and Table I5 for booklet 9.

3. Results for Grade 12 - FA 1990

The results that were obtained for Grade 12 were similar to those that were obtained for Grades 4 and 8. Latent variable modeling was performed on the data from booklets 8, 9 and 10. The three hypotheses: (Models 1,2 and 3) were examined. This was accomplished by applying the three different models (each representing one of the above hypotheses) to the data which included all of the students' answers to all of the items in any of the given booklets. We also applied the three models to data that consisted of the subjects who were grouped into subgroups based on their chosen responses to the selected background variables. The three models were also applied to the items that were selected based on their correlations with the background variables subscale scores.

First, we will discuss the results of the analyses that were performed on the total groups of subjects who answered all the items in a given booklet. Table TA7 summarizes the results of the analyses for Grade 12, booklet 8.

The upper half of this table presents the indices of fit for the three models. The indices of fit indicated that Model 1 does not fit the data as well as Models 2 and 3. The chi-square ratio was 3.830 for Model 1, 1.560 for Model 2, and 1.950 for Model 3. The correlations between the subscale latent variables (listed in the lower section of the table) were, however, all high indicating unidimensionality of the math items.

Table TA8 presents similar results for Grade 12, booklet 9. Consistent with the results presented in Table TA7, Models 2 and 3 exhibit a slightly better fit than Model 1. The chi-square ratios were 5.310, 3.190 and 3.970 for the three models respectively. The correlations between the subscale latent variables (listed in the lower half of the table) were all extremely high and indicated unidimensionality of the math items.

Table TA9 summarizes the results of the analyses conducted on the data from booklet 10. The results in this table are very consistent with those results that were reported in Table A7 and Table A8. The indices of fit in this analysis reveal that Models 2 and 3 clearly fit the data better than Model 1. The chi-square ratios were 4.620 for Model 1, 2.080 for Model 2, and 2.770 for Model 3. The very high correlations between the five subscale latent variables, however, indicated unidimensionality of the math items.

We then applied latent variable analyses to the subgroups that were formed based on the responses to the selected background variables (questions) for Grade 12. The same background variables that were used in the previous 4th- and 8th-grade analyses were applied to the data for Grade 12 in booklets 8, 9 and 10.

Results for Grade 12-FA 1990 on the levels of selected background variables. Table 12A1 presents the results of the latent variable modeling analysis that was conducted on the subjects' who indicated that they Strongly Agree (1) or Agree (2) to the background variable (question) "Do You Agree: I Am Good In Math" for booklet 8, Grade 12. The top section of the table lists the indices of fit for the first subgroup (those subjects who selected Strongly Agree [1] to the above background variable [question]). The middle section of the table lists the indices of fit for the second subgroup (those subjects who selected Agree [2] to the above background variable [question]). Finally, the bottom section of the table lists the correlations between the subscale latent variables for the two subgroups that are defined above.

The indices of fit for the first subgroup (Strongly Agree [1]) indicated that all three models fit the data, but Models 2 and 3 exhibited a much better fit to the data than Model 1. The chi-square ratios for subgroup 1 (Strongly Agree [1]) were 2.080, .840, and 1.060 and the chi-square ratios for subgroup 2 (Agree [2]) were .810, .330, and .410. The correlations between the subscale latent variables were relatively high, but they were not as high as those obtained for the corresponding total group analysis (see Table TA7).

Table 12A2 summarizes the results of the analyses that were performed on the subgroups that were formed based on the response choices to the selected background variable "Home Environment-Reading Materials." Two subgroups were formed based on the subjects who selected either 0-2 Types (1) or 3 Types (2) as their response to the above background variable (question). The top section of the table reports the fit indices for the first subgroup (those subjects who selected 0-2 Types [1] to the above background variable [question]). The middle section of the table reports the fit indices for the second subgroup (those subjects who selected 3 Types [2] to the above background variable [question]). Finally, the bottom section lists the correlations between the subscale latent variables for the two subgroups.

For the first subgroup (0-2 Types [1]), the indices of fit indicated that the three models fit the data at about approximately the same level. The chi-square ratios were 1.510, 1.530, and 1.550 for the three models. In the second subgroup (3 Types [2]), the three models also fit the data at approximately the same level, but Models 2 and 3 exhibited a better fit. The ratios for the second subgroup were 2.540, 1.440, and 1.290. Consistent with the results that were presented in Table 12A1, the correlations between the subscale latent variables were all high but not as high as the correlations that were obtained for the corresponding analysis on the total group (compare this table with Table TA7).

Table 12A3 presents the results of the latent variable modeling for the subgroups that were formed based on the selected answer choices to the "In Math Class How Often Do You Do Problems On Worksheet" background variable (question) for booklet 8. Two subgroups were formed based on the students who chose either Almost Every Day (1) or Several Times A Week (2) as their response the above background variable (question). For the first subgroup (those students who selected Almost Every Day [1] to the above background variable [question]), the fit indices indicated that all three models performed about the same but Models 2 and 3 did slightly better. The chi-square ratios were 1.980, 1.560, and 1.560 for the three models respectively. Similar fit indices were obtained for the second subgroup (those students who selected Several Times A Week [2] as their response to the above background variable [question]). For example, the three models in the second subgroup fit the data at about the same level, and again, Models 2 and 3 did slightly better. The chi-square ratios for the three models were: 1.550, 1.190, and 1.080 respectively. The correlations between the subscale latent variables were all high, but they were slightly lower than the corresponding correlations that were obtained for the total group analysis (see Table TA7).

Tables 12A4, 12A5, and 12A6 present the results of the analyses for booklet 9 and are similar to the results that were reported in Tables 12A1, 12A2, and 12A3 for booklet 8. Likewise, Tables 12A7, 12A8, and 12A9 summarize concordant results for booklet 10. Thus, one could compare the data in Table 12A1 with the data in Table 12A4 and 12A7 or compare the data in Tables 12A2, 12A5, and 12A8. Also Tables 12A3, 12A6, and 12A9 can be compared. These highly comparable results can be used to cross validate the results that have been obtained throughout the study.

Results for Grade 12 - 1990 FA on item-background correlations. The results of the analyses on the item parcels that consisted of the items that were selected based on their item-background correlations are reported in Tables I7, I8, and I9. Table I7 summarizes the results of the analysis that was conducted on the selected items for Grade 12, booklet 8. The upper section of the table lists the fit indices for the three models, and the lower section lists the correlations between the subscale latent variables. The indices of fit indicate that the three models performed very similarly. For instance, the chi-square ratios of the three models were 2.560, 2.680, and 2.840, and the correlations between the subscale scores ranged from a low of .506 to a high of 1.000. These correlations were lower than those that were reported in the total group's corresponding analysis (Tables TA7, TA8, and TA9).

Table I8 presents the results for Grade 12, booklet 9. The fit indices in this table indicate that the three models performed about the same. The chi-square ratios were 3.000, 2.540, and 2.370 for the three models respectively. Consistent with the data reported for booklet 8, the correlations between the subscale latent variables were considerably lower than those that were observed in the corresponding analysis on the total group.

Finally, Table I9 summarizes the results for Grade 12, booklet 10. When comparing Tables I7, I8, and I9 (which all report the results of the analyses that were conducted on the same background variable [question/answer] sequence for the three different booklets) a consistency in the results can be clearly determined.

Results of the factor analysis for the 1992 data

Because there were fewer subjects per booklet in the 1992 administration than in the 1990 administration, we could not successfully divide the subjects into equitable subgroups based on their response patterns to the background variables (questions). Therefore, we performed our analyses on the items that displayed different levels of correlations with the background variables.

For the 1992 analyses, we used booklets 15 and 17 for Grade 4; booklets 1, 2, and 15 for Grade 8; and booklets 1, 15, and 17 for Grade 12. We applied the same three models that we used in the 1990 data to the 1992 data which were: (Model 1) a one-general-factor model; (Model 2) a five-subscale-factor model; and, (Model 3) a five-subscale plus one-general-factor model. We performed our structural equation modeling on the total groups of subjects that answered all of the items in one of the given booklets. We also applied our structural models to the item parcels that consisted of the items that were selected based on the item-background correlations. We will discuss the results of our analyses separately for each of the three grades. For each grade, we will discuss the results of the analyses that were conducted on all of the items and on the selected items.

1. Results for Grade 4 - FA 1992

Table TB1 summarizes the results of the analyses that were conducted on the total group of 4th-grade students who answered the math items in booklet 15. The upper section of the table lists the indices of fit for the three models, and the lower section of the table lists the correlations between the subscale latent variables. The indices of fit for the three models indicated that all of the models fit the data at approximately the same level. For example, the chi-square ratios for the three models were 1.450, 1.770, and 1.670 respectively, and the correlations between the subscale latent variables were all very high indicating unidimensionality of the math items.

Table TB2 presents the results of the analyses done on booklet 17. Consistent with the results presented in Table TB1, the indices of fit indicated that all three models equally fit the data. The chi-square ratios were 1.320, 1.360, and 1.400 for the three models and the intercorrelations between the subscales were also high and indicative of unidimensionality.

Results for Grade 4 - 1992 FA on item-background correlations. Table IB1 presents the results of the analyses that were conducted on the selected items for Grade 4, booklet 15. The indices of fit for the three models indicated that all they all fit the data equally, but the subscale latent variable correlations were lower than those reported for the total items (see Table TB1).

Table IB2 summarizes the results of the analyses that were conducted on the selected items from booklet 17. The indices of fit in this table also indicate that the three models performed about the same level. The chi-square ratios were 1.340, 1.080, and 1.230 for the three models respectively, and the subscale correlations were also all high.

2. Results for Grade 8 - FA 1992

Table TB3 reports the results of the analyses on the total group of 8th-grade students who answered the math items in booklet 1. The indices of fit in this table indicated that Models 2 and 3 performed much better than Model 1. The chi-square ratios were 3.300, 1.200, and 1.100 for the three models respectively. The correlations between the subscale latent variables also indicated that some of the subscales were not highly related. For example, the correlations between the Statistics subscale with the Numbers & Operations, Measurement, and Geometry subscales were .780, .780, and .820. These correlations were relatively lower than those that were reported in the 1990 data.

Table TB4 reports similar results for booklet 2, Grade 8. Consistent with the results that were reported in Table TB3, the indices of fit in this table indicate that Models 2 and 3 performed better than Model 1. The chi-square ratios were 1.150, .610, and .650 for the three models respectively.

Similarly, Table TB5 reports the results for Grade 8, booklet 15. Consistent with the results obtained for booklets 1 and 2 (reported in Tables TA3 and TA4), models 2 and 3 showed a better fit to the data. The chi-square ratios were 1.140, .970 and .950 for the three models respectively.

Results for Grade 8 - 1992 FA on item-background correlations. Table IB3 summarizes the results of the structural modeling that was conducted on the selected items from booklet 1, Grade 8. The indices of fit, which are reported in the upper section of the table, indicate that all of the three models fit the data but that Models 2 and 3 fit the data better than Model 1. The subscale correlations, which are reported in the lower half of the table, are considerably lower than those that were reported for the parcels containing all the items (see Table TB3). For example, the correlations between the Number & Operations subscale with the other four subscales were .820, .570, .530, and .640 and were lower than the corresponding subscale correlations (.930, .970, .780, and .950) in the total item analysis.

Table IB4 presents similar results for the selected items from booklet 4, Grade 8. The indices of fit in this table indicate that all three models performed at about the same level. The chi-square ratios were 1.270, 1.700, and 1.460 for the three models respectively and the subscale correlations were relatively high as well.

Table IB5 presents the results of the analyses done on booklet 5 and the results are similar to those that are reported in Tables IB3 and IB4. The indices of fit in this analysis indicated that all three of the models fit the data, but Models 2 and 3 performed slightly better. The chi-square ratios were 2.250, 1.230, and 1.240 for the three models respectively. The subscale correlations were considerably lower than those that were reported for the cases in which all of the items were used in the parcels (see Table TB5).

3. Results for Grade 12 - FA 1992

Table TB6 presents the results of the analyses that were conducted on all of the items from booklet 1. The models appeared to perform at about the same level based on the reported indices of fit. The chi-square ratios were 1.640, 1.360, and 1.220 for the three models respectively. The subscale correlations were relatively high, though not as high as those that were reported in the 1990 data.

Table TB7 presents the results of the analyses that were conducted on booklet 15. The three models performed about the same in this analysis, and these findings were consistent with the results that were reported in Table TB6. The subscale correlations were all very high indicating unidimensionality of the math items.

Table TB8 reports the results of the analyses conducted on booklet 17. All of the three models in this analysis fit the data, but Models 2 and 3 performed better once again. The correlations between the subscale latent variables were all relatively high but not as high as those that were reported in the 1990 data.

Results for Grade 12 - 1992 FA on item-background correlations. Table IB6 summarizes the results of the analyses that were conducted on the selected items from booklet 1. The indices of fit indicated that the three models fit the data very well. However, the correlations between the subscale latent variables were drastically lower than those that were reported for the item parcels that were constructed from the total items (see Table TB6). For example, the correlations between the Number & Operations subscale with the other four subscales were .510, .313, .346 and .557 whereas the same series of comparative correlations for the total item parcels were .910, .810, .790, and .870.

Tables IB7 and IB8 present the results of the analyses that were conducted on the selected items for booklets 15 and 17 respectively. These results are comparable and almost identical with those reported for booklet 1 (Table IB6). The indices of fit in both of these tables (IB7 and IB8) indicate that all three models performed equally well. The correlations between the subscales that were reported in both tables were considerably lower than those correlations that were obtained from the item parcels created from the total item set.

Discussion

To test the dimensionality of the math items, we examined the following three hypotheses for the 1990 and 1992 NAEP math data: (a) All NAEP math items measure the same underlying math ability; (b) the NAEP math items measure five different subscales (i.e., Numbers & Operations, Measurement, Algebra, Geometry, and Statistics); and (c) in addition to the five different subscales, a general math ability underlies the math items. We created three different latent variable models that corresponded to each of the three hypotheses. The results of our analyses that were performed on the data for the total group of subjects that answered the math items in the given booklets supported the first hypothesis. The math subscale scores were highly correlated and were primarily unidimensional.

The results were consistent across all three grade levels and for both the 1990 and 1992 data sets. However, when the background variables that had significant relationships with the students' math performance were used in connection with the math subscale scores, more discrimination among the math subscale scores was found. The results of the analyses that were performed on the subgroups (that were formed based on the responses/choices to the selected background variables) supported hypotheses two and three (see prior page). These results indicate that there is some evidence of multidimensionality in the NAEP math items. For example, for the 1990 data, in all three grade levels, when the analyses were performed on the subgroups, lower correlations were obtained and more evidence of multidimensionality was observed.

Similarly, the results of the analyses conducted on the item parcels consisting of items that were correlated with the selected background variables for both the 1990 and 1992 data indicated evidence of multidimensionality in the NAEP math items. Evidence of multidimensionality was more visible in Grades 8 and 12 than in Grade 4.

Caution should be used when comparing the results of the analyses obtained on the total group of subjects for each of the booklets versus the results that were obtained from the subgroups. Several factors could have contributed to the differences between the statistics that were obtained for the total group and the statistics that were obtained for the subgroups. Number of subjects, within-group homogeneity, homogeneity of the items within parcels, and student ability/level are just a few of the factors that could produce some of these differences.

Discretion should also be applied when comparing the results of the analyses that were performed on the item parcels that included all the items versus the results that were obtained from the item parcels that were comprised of only the selected items (based on the item-background correlations). Factors such as the number and characteristics of the items, differences in the homogeneity of the items within the parcels, and differences in the parcel means and variances across the parcels could account for some of the differences in the statistics across parcel group analyses.

The findings in this study also reveal that a one-general-math-factor model may not be the most effective way to describe the NAEP math subscale scores. The statistics that were obtained across the three grade levels/ages indicate that the five-factor model and the five-subscale-one-general-factor model exhibited better fits to the data than the unidimensional one-factor model.

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Table A1

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 4, Booklet 11, 1990)
N = 1255

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.92142	.92091	53.030	.000
Measurement	.77099	.94320	37.190	.000
Geometry	.51570	.97376	16.640	.000
Statistics	.78546	.94127	38.530	.000
Algebra	.59410	.96463	22.640	.000
Function 1	%Var = 98.120	Canon R = .300		
After Function 0	Λ = .9064	χ^2 = 121.180	df = 10	p = .000

Table A2

Results of Discriminant Analysis on: "How Do You Feel About This Statement: I Am Good In Math"
(Grade 4, Booklet 11, 1990)
N = 1255

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.99283	.90241	65.160	.000
Measurement	.71850	.94634	34.160	.000
Geometry	.51275	.97156	17.630	.000
Statistics	.59753	.96144	24.160	.000
Algebra	.56537	.96512	21.780	.000
Function 1	%Var = 96.840	Canon R = .310		
After Function 0	Λ = .8979	$\chi 2$ = 129.530	df = 10	p = .000

Table A3

Results of Discriminant Analysis on: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 11, 1990)
N = 1255

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.88425	.95606	14.100	.000
Measurement	.76097	.96595	10.810	.000
Geometry	.73046	.96910	9.780	.000
Statistics	.66029	.97517	7.810	.000
Algebra	.78369	.96571	10.890	.000

Function 1	%Var = 83.520	Canon R = .230		
After Function 0	Λ = .935	$\chi 2$ = 82.340	df = 20	p = .000

Table A4

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)
N = 1250

Variable	Structural Coeff. Func 1	Univariate Test: Wilks' Lambda	F-ratio	Significance Level
Numbers	.86356	.95144	31.470	.000
Measurement	.80552	.95729	27.510	.000
Geometry	.66005	.96959	19.330	.000
Statistics	.80546	.95706	27.660	.000
Algebra	.77767	.95995	25.720	.000

Function 1	%Var = 93.270	Canon R =.250		
After Function 0	Λ =.9314	$\chi 2$ = 87.530	df = 10	p = .000

Table A5

Results of Discriminant Analysis on: "How Do You Feel About This Statement: I Am Good In Math"
(Grade 4, Booklet 12, 1990)
N = 1250

Variable	Structural Coeff. Func 1	Univariate Test: Wilks' Lambda	F-ratio	Significance Level
Numbers	.88445	.94604	34.110	.000
Measurement	.88773	.94552	34.460	.000
Geometry	.64649	.97017	18.390	.000
Statistics	.59363	.97417	15.860	.000
Algebra	.65519	.96935	18.910	.000

Function 1	%Var = 95.500	Canon R = .260		
After Function 0	Λ = .9289	$\chi 2$ = 87.990	df = 10	p = .000

Table A6

Results of Discriminant Analysis on: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 12, 1990)
N = 1250

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.87713	.94187	18.890	.000
Measurement	.61246	.96835	10.000	.000
Geometry	.74186	.95607	14.060	.000
Statistics	.72039	.95887	13.130	.000
Algebra	.78317	.95098	15.770	.000

Function 1	%Var = 83.320	Canon R = .270		
After Function 0	Λ = .9116	$\chi2$ = 113.240	df = 20	p = .000

Table A7

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 4, Booklet 14, 1990)
N = 1242

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.91475	.93152	44.850	.000
Measurement	.83640	.94191	37.620	.000
Geometry	.68310	.95917	25.960	.000
Statistics	.73941	.95337	29.840	.000
Algebra	.72215	.95568	28.290	.000
Function 1	%Var = 93.360	Canon R = .280		
After Function 0	Λ = .9136	χ2 = 110.070	df = 10	p = .000

Table A8

Results of Discriminant Analysis on: "How Do You Feel About This Statement:
I Am Good In Math"
(Grade 4, Booklet 14, 1990)
N = 1242

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.93878	.92254	49.410	.000
Measurement	.84450	.93631	40.030	.000
Geometry	.51278	.97433	15.500	.000
Statistics	.56274	.97033	18.000	.000
Algebra	.57943	.96790	19.510	.000
Function 1	%Var = 97.030	Canon R = .290		
After Function 0	Λ = .9104	χ2 = 110.320	df = 10	p = .000

Table A9

Results of Discriminant Analysis on: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 14, 1990)
N = 1240

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.90005	.97909	6.550	.0000
Measurement	.86675	.98071	6.030	.0001
Geometry	.67952	.98431	4.890	.0006
Statistics	.59714	.98854	3.550	.0068
Algebra	.79814	.98325	5.230	.0004
Function 1	%Var = 66.810	Canon R = .160		
After Function 0	L = .9619	c2 = 47.520	df = 20	p < .001

Table A10

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 8, Booklet 8, 1990)
N = 1234

		Univariate Test		
	Structural Coeff.			Significance
Variable	Func 1	Wilks' Lambda	F-ratio	Level
Numbers	.95963	.91341	58.060	.000
Measurement	.78242	.94067	38.630	.000
Geometry	.76156	.94277	37.180	.000
Statistics	.84091	.93215	44.580	.000
Algebra	.82414	.93466	42.820	.000
Function 1	%Var. = 97.470	Canon R = .310		
After Function 0	Λ = .9043	χ2 = 123.050	df = 10	p = .000

Table A11

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 8, 1990)
N = 1234

		Univariate Test		
	Structural Coeff.			Significance
Variable	Func 1	Wilks' Lambda	F-ratio	Level
Numbers	.96642	.88661	37.600	.000
Measurement	.76513	.92550	23.670	.000
Geometry	.57612	.95583	13.590	.000
Statistics	.68958	.93821	19.360	.000
Algebra	.82390	.91461	27.450	.000
Function 1	%Var = 96.640	Canon R = .350		
After Function 0	Λ = .8754	χ2 = 156.310	df = 20	p = .000

Table A12

Results of Discriminant Analysis on: "What Kind Of Math Class Are You Taking This Year" (Grade 8, Booklet 8, 1990)
N = 1234

Variable	Structural Coeff. Func 1	Structural Coeff. Func 2	Univariate Test		
			Wilks' Lambda	F-ratio	Significance Level
Numbers	.79019	.58101	.85323	49.670	.000
Measurement	.66102	.15475	.89499	33.880	.000
Geometry	.67805	.22291	.89075	35.420	.000
Statistics	.71981	.00059	.87804	40.110	.000
Algebra	.98956	−.06623	.79470	74.600	.000

Function 1	%Var = 90.080	Canon R = .460		
After Function 0	Λ =.7689	χ^2 = 303.280	df = 20	p = .000
Function 2	%Var = 7.260	Canon R = .140		
After Function 1	Λ = .9716	χ^2 = 33.230	df = 12	p < .0009

Table A13

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 8, Booklet 9, 1990)
N = 1234

	Structural Coeff.	Univariate Test		
Variable	Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.85788	.93530	42.780	.000
Measurement	.74839	.95003	32.530	.000
Geometry	.74801	.95002	32.540	.000
Statistics	.91205	.92753	48.330	.000
Algebra	.79514	.94346	37.060	.000
Function 1	%Var = 97.870	Canon R = .290		
After Function 0	Λ = .9123	χ^2 = 113.380	df = 10	p = .000

Table A14

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 9, 1990)
N = 1234

	Structural Coeff.	Univariate Test		
Variable	Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.92604	.92580	23.720	.000
Measurement	.72626	.95208	14.900	.000
Geometry	.73849	.95065	15.370	.000
Statistics	.77313	.94626	16.810	.000
Algebra	.86510	.93432	20.810	.000
Function 1	%Var = 89.250	Canon R = .290		
After Function 0	Λ = .9046	χ^2 = 118.610	df = 20	p = .000

Table A15

Results of Discriminant Analysis on: "What Kind Of Math Class Are You Taking This Year" (Grade 8, Booklet 9, 1990)
N = 1234

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.78329	.86801	43.680	.000
Measurement	.58020	.92198	24.310	.000
Geometry	.64085	.90673	29.550	.000
Statistics	.66936	.89735	32.860	.000
Algebra	.96820	.81447	65.430	.000
Function 1	%Var = 92.080	Canon R = .440		
After Function 0	Λ = .7883	χ^2 = 273.090	df = 20	p = .000

Table A16

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 8, Booklet 10, 1990)
N = 1230

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.84858	.94288	36.960	.000
Measurement	.86936	.93999	38.940	.000
Geometry	.78795	.95029	31.910	.000
Statistics	.93932	.93085	45.310	.000
Algebra	.70604	.95974	25.590	.000
Function 1	%Var = 98.580	Canon R = .280		
After Function 0	Λ = .9213	χ^2 = 99.850	df = 10	p = .000

Table A17

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 10, 1990)
N = 1230

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.96113	.92868	22.690	.000
Measurement	.78409	.95039	15.420	.000
Geometry	.82657	.94564	16.990	.000
Statistics	.79663	.94945	15.730	.000
Algebra	.76291	.95365	14.360	.000
Function 1	%Var = 90.960	Canon R = .280		
After Function 0	Λ = .9158	χ^2 = 103.850	df = 20	p = .000

Table A18

Results of Discriminant Analysis on: "What Kind Of Math Class Are You Taking This Year" (Grade 8, Booklet 10, 1990)
N = 1230

Variable	Structural Coeff. Func 1	Structural Coeff. Func 2	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.76769	.19552	.91338	27.360	.000
Measurement	.63357	.33094	.93743	19.260	.000
Geometry	.74784	−.16270	.91767	25.880	.000
Statistics	.90733	−.24769	.88381	37.930	.000
Algebra	.87190	.41695	.89025	35.570	.000

Function 1	%Var = 85.950	Canon R = .370		
After Function 0	Λ = .8415	χ^2 = 198.910	df = 20	p = .000
Function 2	%Var = 8.960	Canon R = .130		
After Function 1	Λ = .9746	χ^2 = 26.610	df = 12	p < .003

Table A19

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 12, Booklet 8, 1990)
N = 1201

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.77337	.96160	23.800	.000
Measurement	.59208	.97711	13.960	.000
Geometry	.75161	.96327	22.730	.000
Statistics	.81826	.95708	26.730	.000
Algebra	.86699	.95198	30.070	.000

Function 1	%Var = 96.380	Canon R = .250		
After Function 0	Λ = .9351	χ^2 = 79.850	df = 10	p = .000

Table A20

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 8, 1990)
N = 1201

			Univariate Test		
Variable	Structural Coeff. Func 1	Structural Coeff. Func 2	Wilks' Lambda	F-ratio	Significance Level
Numbers	.92301	.29961	.87478	39.860	.000
Measurement	.86726	–.16915	.88820	35.050	.000
Geometry	.70518	–.18771	.92023	24.140	.000
Statistics	.55848	.43055	.94741	15.460	.000
Algebra	.82897	.26179	.89695	32.000	.000

Function 1	%Var = 88.010	Canon R = .380		
After Function 0	Λ = .8383	χ^2 = 196.360	df = 20	p = .000
Function 2	%Var = 7.010	Canon R = .110		
After Function 1	Λ = .9777	χ^2 = 25.090	df = 12	p < .0144

Table A21

Results of Discriminant Analysis on: "In Math Class How Often Do You Do Problems On Worksheet"
(Grade 12, Booklet 8, 1990)
N = 1201

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.89435	.95960	12.480	.000
Measurement	.65637	.97446	7.770	.000
Geometry	.71724	.97087	8.900	.000
Statistics	.57640	.98211	5.400	.000
Algebra	.92926	.95628	13.560	.000
Function 1	%Var = 78.860	Canon R = .220		
After Function 0	Λ = .937	χ^2 = 77.120	df = 20	p = .000

Table A22

Results of Discriminant Analysis on: "Home Environment-Reading Materials" (Grade 12, Booklet 9, 1990)
N = 1201

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.90034	.97291	16.610	.000
Measurement	.89537	.97662	14.280	.000
Geometry	.85703	.97094	17.850	.000
Statistics	.82678	.96803	19.700	.000
Algebra	.76423	.96834	19.500	.000
Function 1	%Var = 98.750	Canon R = .200		
After Function 0	Λ = .9604	χ^2 = 48.160	df = 10	p = .000

Table A23

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math" (Grade 12, Booklet 9, 1990)
N = 1201

			Univariate Test		
Variable	Structural Coeff. Func 1	Structural Coeff. Func 2	Wilks' Lambda	F-ratio	Significance Level
Numbers	.85173	.52293	.89467	32.350	.000
Measurement	.58027	.20856	.94881	14.820	.000
Geometry	.80299	.00331	.90898	27.510	.000
Statistics	.55085	.38588	.95127	14.070	.000
Algebra	.93702	−.14325	.87949	37.650	.000
Function 1	%Var = 87.550	Canon R = .370			
After Function 0	Λ = .8468	χ^2 = 182.530	df = 20		p = .000
Function 2	%Var = 10.510	Canon R = .140			
After Function 1	Λ = .983	χ^2 = 24.060	df = 12		p < .020

Table A24

Results of Discriminant Analysis on: "In Math Class How Often Do You Do Problems On Worksheet"
(Grade 12, Booklet 9, 1990)
N = 1201

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.91728	.95682	13.350	.000
Measurement	.90297	.96788	9.810	.000
Geometry	.87612	.958346	14.440	.000
Statistics	.81060	.96211	11.650	.000
Algebra	.74155	.95511	13.900	.000
Function 1	%Var = 82.860	Canon R = .230		
After Function 0	Λ = .9344	χ^2 = 80.190	df = 20	p = .000

Table A25

Results of Discriminant Analysis on: "Home Environment-Reading Materials"
(Grade 12, Booklet 10, 1990)
N = 1193

		Univariate Test		
	Structural Coeff.			Significance
Variable	Func 1	Wilks' Lambda	F-ratio	Level
Numbers	.92852	.95432	28.290	.000
Measurement	.86542	.96907	18.870	.000
Geometry	.81223	.96222	23.200	.000
Statistics	.78342	.94784	32.530	.000
Algebra	.70617	.95964	24.860	.000
Function 1	%Var = 98.500	Canon R = .240		
After Function 0	Λ = .9392	χ^2 = 74.070	df = 10	p = .000

Table A26

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 10, 1990)
N = 1193

		Univariate Test		
	Structural Coeff.			Significance
Variable	Func 1	Wilks' Lambda	F-ratio	Level
Numbers	.86963	.90685	27.990	.000
Measurement	.83835	.91249	26.130	.000
Geometry	.83438	.91360	25.770	.000
Statistics	.68814	.93642	18.500	.000
Algebra	.89011	.90304	29.260	.000
Function 1	%Var = 89.330	Canon R = .340		
After Function 0	Λ = .8671	χ^2 = 155.280	df = 20	p = .000

Table A27

Results of Discriminant Analysis on: "In Math Class How Often Do You Do Problems On Worksheet"
(Grade 12, Booklet 10, 1990)
N = 1193

	Structural Coeff.	Univariate Test		
Variable	Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.76175	.96756	9.840	.000
Measurement	.86833	.95942	12.410	.000
Geometry	.94488	.95223	14.720	.000
Statistics	.74851	.96904	9.380	.000
Algebra	.85332	.96024	12.150	.000
Function 1	%Var = 88.540	Canon R = .230		
After Function 0	Λ = .9403	χ^2 = 72.220	df = 20	p = .000

Table B1

Results of Discriminant Analysis on: "Agree/Disagree: I Am Good In Math"
(Grade, 4, Booklet 15, 1992)
N = 349

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.86099	.96002	7.204	.0009
Measurement	.24375	.99536	.8058	.4476
Geometry	.65240	.97681	4.108	.0172
Statistics	.64654	.97637	4.186	.0160
Algebra	.17043	.99813	.3234	.7239
Function 1	%Var = 95.400	Canon R = .230		
After Function 0	Λ =.945	χ^2 = 19.570	df = 10	p < .030

Table B2

Results of Discriminant Analysis on: "Agree/Disagree: I Like Math"
(Grade 4, Booklet 15, 1992)
N = 352

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.59811	.97936	3.678	.0263
Measurement	-.03866	.99991	.1534e-01	.9848
Geometry	.73789	.96943	5.503	.0044
Statistics	.60857	.97912	3.722	.0252
Algebra	.30960	.99440	.9829	.3753
Function 1	%Var = 96.770	Canon R = .230		
After Function 0	Λ = .944	χ^2 = 20.090	df = 10	p < .030

Table B3

Results of Discriminant Analysis on: "How Much Time Spent Each Day On Math Homework" (Grade 4, Booklet 15, 1992)
N = 356

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.94514	.90995	5.756	.0000
Measurement	.41061	.97796	1.311	.2515
Geometry	.53189	.96784	1.933	.0748
Statistics	.81474	.92986	4.388	.0003
Algebra	.48986	.96683	1.996	.0656

Function 1	%Var = 80.100	Canon R = .310		
After Function 0	Λ = .877	χ^2 = 45.800	df = 30	$p < .030$

Table B4

Results of Discriminant Analysis on: "Agree/Disagree: I Am Good In Math "
(Grade 4, Booklet 17, 1992)
N = 348

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.86335	.93115	12.760	.000
Measurement	.72548	.95341	8.429	.000
Geometry	.70891	.95627	7.889	.000
Statistics	.84305	.93948	11.110	.000
Algebra	.80478	.94330	10.370	.000
Function 1	%Var = 77.000	Canon R = .290		
After Function 0	Λ = .890	χ^2 = 38.900	df = 10	p = .000

Note.

Table B5 (Results of Discriminant Analysis on: Agree/Disagree: I Like Math," Grade 4, Booklet 17, 1992, N = 348) has been omitted because of incomplete data.

Table B6

Results of Discriminant Analysis on: "How Much Time Spent Each Day On Math Homework "
(Grade 4, Booklet 17, 1992)
N = 358

		Univariate Test		
Variable	Structural Coeff. Func 1	Wilks' Lambda	F-ratio	Significance Level
Numbers	.79374	.94962	3.103	.0056
Measurement	.59386	.96028	2.420	.0264
Geometry	.42830	.96161	2.335	.0317
Statistics	.60726	.96354	2.214	.0413
Algebra	.94701	.93733	3.911	.0009
Function 1	%Var = 50.740	Canon R = .260		
After Function 0	$\Lambda = .867$	$\chi^2 = 49.900$	df = 30	$p < .010$

Table B7

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 1, 1992)
N = 384

Variable	Structural Coeff. Func 1	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.82099	.92690	7.472	.000
Measurement	.76098	.93639	6.437	.000
Geometry	.43525	.97590	2.340	.055
Statistics	.33349	.98181	1.755	.137
Algebra	.75858	.92515	7.666	.000
Function 1	%Var = 69.400	Canon R = .320		
After Function 0	L = .853	$\chi^2 = 60.100$	df = 20	p = .000

Table B8

Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 8, Booklet 1, 1992)
N = 369

Variable	Structural Coeff. Func 1	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.77676	.95392	4.396	.0018
Measurement	.90284	.93916	5.895	.0001
Geometry	.75494	.95653	4.135	.0027
Statistics	.65000	.96608	3.196	.0134
Algebra	.78563	.95140	4.648	.0011
Function 1	%Var = 86.530	Canon R = .270		
After Function 0	L = .916	$\chi^2 = 31.870$	df = 20	p < .050

Table B9

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 2, 1992)
N = 368

Variable	Structural Coeff. Func 1	Univariate Test		
		Wilks' Lambda	F-ratio	Significance Level
Numbers	.92811	.91784	8.124	.0000
Measurement	.87596	.92731	7.114	.0000
Geometry	.66933	.95676	4.101	.0029
Statistics	.80626	.93663	6.140	.0001
Algebra	.77434	.94082	5.708	.0002

Function 1	%Var = 78.950	Canon R = .300		
After Function 0	L = .885	$\chi^2 = 44.400$	df = 20	$p < .001$

Table B10

Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 8, Booklet 2, 1992)
N = 359

Variable	Structural Coeff. Func 1	Univariate Test		
		Wilks' Lambda	F-ratio	Significance Level
Numbers	.96364	.90222	9.591	.0000
Measurement	.62237	.94669	4.984	.0006
Geometry	.56198	.95684	3.992	.0035
Statistics	.77975	.92498	7.178	.0000
Algebra	.69021	.94654	4.998	.0006

Function 1	%Var = 70.590	Canon R = .320		
After Function 0	L = .854	$\chi^2 = 55.590$	df = 20	$p = .000$

Table B11

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 15, 1992)
N = 380

Variable	Structural Coeff. Func 1	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.82615	.91284	8.952	.0000
Measurement	.87615	.91248	8.992	.0000
Geometry	.61950	.95206	4.721	.0010
Statistics	.54393	.95981	3.926	.0039
Algebra	.76203	.93075	6.976	.0000
Function 1	%Var = 69.670	Canon R = .330		
After Function 0	L = .845	$\chi^2 = 63.070$	df = 20	p = .000

Table B12

Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 8, Booklet 15, 1992)
N = 369

Variable	Structural Coeff. Func 1	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.68145	.93759	6.058	.0001
Measurement	.90227	.89836	10.300	.0000
Geometry	.66814	.93692	6.127	.0001
Statistics	.85111	.90259	9.821	.0000
Algebra	.48114	.95681	4.107	.0029
Function 1	% Var = 74.110	Canon R = .350		
After Function 0	L = .838	$\chi^2 = 64.040$	df = 20	p = .000

Table B13

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 1, 1992)
N = 368

Variable	Structural Coeff. Func 1	Univariate Test: Wilks' Lambda	F-ratio	Significance Level
Numbers	.55906	.92187	7.692	.000
Measurement	.74575	.87705	12.720	.000
Geometry	.60306	.91342	8.602	.000
Statistics	.63598	.90125	9.943	.000
Algebra	.94818	.81630	20.420	.000
Function 1	%Var = 82.780	Canon R = .440		
After Function 0	Λ = .763	χ^2 = 97.800	df = 20	p = .000

Table B14

Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Booklet 1, 1992)
N = 368

Variable	Structural Coeff. Func 1	Univariate Test: Wilks' Lambda	F-ratio	Significance Level
Numbers	.73453	.87300	13.200	.000
Measurement	.82546	.84670	16.430	.000
Geometry	.58755	.91134	8.830	.000
Statistics	.54652	.92429	7.430	.000
Algebra	.89256	.82685	19.000	.000
Function 1	% Var = 87.370	Canon R = .450		
After Function 0	Λ = .764	χ^2 = 97.400	df = 20	p = .000

Table B15

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 15, 1992)
N = 359

Variable	Structural Coeff. Func 1	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.65618	.94029	5.620	.0002
Measurement	.64444	.93904	5.745	.0002
Geometry	.69825	.92939	6.745	.0000
Statistics	.40508	.97235	2.517	.0412
Algebra	.94793	.88545	11.450	.0000
Function 1	%Var = 78.300	Canon R = .350		
After Function 0	L = .840	χ^2 = 60.900	df = 20	p = .000

Table B16

Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Booklet 15, 1992)
N = 359

Variable	Structural Coeff. Func 1	Univariate Test Wilks' Lambda	F-ratio	Significance Level
Numbers	.80529	.85825	14.620	.000
Measurement	.64338	.90418	9.379	.000
Geometry	.82403	.85082	15.520	.000
Statistics	.73157	.88180	11.860	.000
Algebra	.93265	.82050	19.360	.000
Function 1	%Var = 90.190	Canon R = .450		
After Function 0	L = .778	χ^2 = 88.400	df = 20	p = .000

Table B17

Results of Discriminant Analysis on: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 17, 1992)
N = 348

Variable	Structural Coeff. Func 1	Univariate Test		
		Wilks' Lambda	F-ratio	Significance Level
Numbers	.87213	.88151	11.530	.000
Measurement	.84175	.87297	12.480	.000
Geometry	.81331	.88781	10.840	.000
Statistics	.65493	.91795	7.664	.000
Algebra	.67827	.91720	7.741	.000
Function 1	%Var = 84.300	Canon R = .400		
After Function 0	L = .813	$\chi^2 = 70.930$	df = 20	p = .000

Table B18

Results of Discriminant Analysis on: "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Booklet 17, 1992)
N = 347

Variable	Structural Coeff. Func 1	Univariate Test		
		Wilks' Lambda	F-ratio	Significance Level
Numbers	.90143	.90139	9.353	.0000
Measurement	.63952	.94726	4.760	.0009
Geometry	.84905	.91162	8.289	.0000
Statistics	.74637	.93104	6.332	.0001
Algebra	.86036	.90981	8.476	.0000
Function 1	%Var = 90.940	Canon R = .340		
After Function 0	L = .870	$\chi^2 = 46.970$	df = 20	$p < .001$

Table TA1

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 11, 1990)
N = 1255

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	75.930	35	2.170	.980	.986	.989
Five Factor	44.780	25	1.790	.988	.991	.995
5 + 1 Factor	57.020	30	1.900	.985	.989	.993

$\Delta_{5, 5+1}\chi^2 = 12.240$ Δ df = 5 $p < .030$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 11, 1990)
N = 1255

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.960	—			
Geometry	.830	.890	—		
Statistics	1.000	1.000	.990	—	
Algebra	1.000	.990	.900	.990	—
Factor Loading	.970	1.000	.880	1.000	1.000

Table TA2

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 12, 1990)
N = 1250

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	51.020	35	1.460	.987	.995	.996
Five Factor	20.200	25	.810	.995	1.002	1.000
5+1 Factor	21.210	30	.710	.995	1.003	1.000

$\Delta_{5, 5+1}\chi^2 = 1.010$ Δ df = 5 $p < .975$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 12, 1990)
N = 1250

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.900	—			
Geometry	.870	.880	—		
Statistics	.960	.960	.920	—	
Algebra	.950	.960	.890	1.000	—
Factor Loading	.950	.960	.910	1.000	1.000

Table TA3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 14, 1990)
N = 1242

	χ^2	df	$\chi^2/_{df}$	NFI	NNFI	CFI
One Factor	92.280	35	2.640	.979	.983	.987
Five Factor	47.270	25	1.890	.989	.991	.995
5+1 Factor	55.540	30	1.850	.987	.991	.994
$\Delta_{5, 5+1}\chi^2 = 8.270$	Δ df = 5	$p < .100$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 14, 1990)
N = 1242

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.900	—			
Geometry	.920	.930	—		
Statistics	1.000	.960	.960	—	
Algebra	.920	.880	.890	1.000	—
Factor Loading	.960	.940	.960	1.000	.960

Table 4A1

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Undecided (2)
(Grade 4, Booklet 11, 1990)
N = 279

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	40.910	35	1.170	.934	.987	.990
Five Factor	20.930	25	.840	.966	1.013	1.000
5+1 Factor	36.490	30	1.220	.941	.983	.989
$\Delta_{5,\,5+1}\chi^2 = 15.560$	Δ df = 5	$p < .010$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good In Math"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.980	—			
Geometry	.680	.900	—		
Statistics	1.000	.840	.810	—	
Algebra	.910	.970	1.000	1.000	—
Factor Loading	.950	1.000	.850	1.000	1.000

Note. Range for this background variable question was: (1) = Agree, (2) = Undecided, and (3) = Disagree. Lower half of the triangle reports for the selected response: Undecided (2) for the above background variable (question).

Table 4A2

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Agree (1)
(Grade 4, Booklet 11, 1990)
N = 763

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	60.660	35	1.730	.974	.986	.989
Five Factor	37.440	25	1.500	.984	.990	.995
5+1 Factor	47.520	30	1.580	.980	.989	.992
$\Delta_{5, 5+1}\chi^2 = 10.080$	Δ df = 5	$p < .075$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Disagree (3)
(Grade 4, Booklet 11, 1990)
N = 166

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	42.890	35	1.220	.886	.969	.976
Five Factor	38.130	25	1.200	.898	.928	.960
5+1 Factor	40.090	30	1.340	.893	.954	.969
$\Delta_{5, 5+1}\chi^2 = 9.960$	Δ df = 5	$p < .080$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good In Math"
(Grade 4, Booklets 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	FactorLoading
Numbers	—	.940	.850	.950	.990	.950
Measurement	.970	—	.900	1.000	1.000	1.000
Geometry	.850	.870	—	.980	.900	.910
Statistics	.910	.770	.870	—	.900	1.000
Algebra	1.000	.910	.790	.880	—	1.000
Factor Loading	1.000	.960	.860	.900	1.000	—

Note. Range for this background variable question was: (1) = Agree, (2) = Undecided, and (3) = Disagree. Upper half of the triangle reports for the selected response: Agree (1) and the lower half of the triangle reports for the selected response: Disagree (3) for the above background variable (question).

Table 4A3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Agree (1)
(Grade 4, Booklet 12, 1990)
N = 745

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	60.410	35	1.730	.976	.987	.990
Five Factor	31.280	25	1.250	.988	.996	.998
5+1 Factor	32.480	30	1.080	.987	.999	.999
Δ 5, 5+1χ^2 = 1.200	Δ df = 5	p < .900				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Disagree (3)
(Grade 4, Booklet 12, 1990)
N = 155

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	39.350	35	1.120	.896	.983	.987
Five Factor	32.050	25	1.280	.916	.962	.979
5+1 Factor	35.430	30	1.180	.907	.976	.984
Δ 5, 5+1χ^2 = 3.380	Δ df = 5	p < .700				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good In Math
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	FactorLoading
Numbers	—	.890	.860	.920	.900	.940
Measurement	.970	—	.870	.940	.930	.950
Geometry	.790	.840	—	.890	.850	.910
Statistics	1.000	.970	.770	—	.980	.990
Algebra	.980	.960	.990	1.000	—	.970
Factor Loading	.990	.970	.860	1.000	1.000	—

Note. Range for this background variable question was: (1) = Agree, (2) = Undecided, and (3) = Disagree. Upper half of the triangle reports for the selected response: Agree (1) and the lower half of the triangle reports for the selected response: Disagree (3) for the above background variable (question).

Table 4A4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Undecided (2)
(Grade 4, Booklet 12, 1990)
N = 299

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	22.220	35	.630	.966	1.027	1.000
Five Factor	13.210	25	.530	.980	1.035	1.000
5+1 Factor	17.200	30	.570	.974	1.032	1.000
$\Delta_{5, 5+1}\chi^2 = 3.990$	Δ df = 5	$p < .550$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good In Math
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.870	—			
Geometry	.970	.860	—		
Statistics	.990	.910	.940	—	
Algebra	1.000	.860	.770	.910	—
Factor Loading	1.000	.890	.940	.990	.990

Note. Range for this background variable question was: (1) = Agree, (2) = Undecided, and (3) = Disagree. Lower half of the triangle reports for the selected response: Undecided (2) for the above background variable (question).

Table 4A5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Agree (1)
(Grade 4, Booklet 14, 1990)
N = 732

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	60.540	35	1.730	.978	.988	.990
Five Factor	32.840	25	1.310	.988	.995	.997
5+1 Factor	39.750	30	1.320	.985	.995	.996
$\Delta_{5, 5+1}\chi^2 = 6.910$	Δ df = 5	$p < .250$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Disagree (3)
(Grade 4, Booklet 14, 1990)
N = 161

	χ^2	df	χ^2/df	NFI	NNFI
One Factor	26.130	35	.750	.927	1.036
Five Factor	17.770	25	.710	.951	1.041
5+1 Factor	20.560	30	.680	.943	1.045
$\Delta_{5, 5+1}\chi^2 = 2.790$	Δ df = 5	$p < .700$			

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good In Math"
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.980	1.000	.970	.950	.890
Measurement	.980	—	1.000	.900	1.000	.910
Geometry	.890	.880	—	.960	.920	1.000
Statistics	.900	.820	1.000	—	.950	.940
Algebra	1.000	.880	.980	.880	—	.950
Factor Loading	.920	.860	.960	.720	.860	—

Note. Range for this background variable question was: (1) = Agree, (2) = Undecided, and (3) = Disagree. Upper half of the triangle reports for the selected response: Agree (1) and the lower half of the triangle reports for the selected response: Disagree (3) for the above background variable (question).

Table 4A6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"How Do You Feel About This Statement: I Am Good In Math"
Selected Response = Undecided (2)
(Grade 4, Booklet 14, 1990)
N = 287

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	57.670	35	1.650	.931	.963	.971
Five Factor	37.030	25	1.480	.956	.973	.985
5+1 Factor	48.020	30	1.600	.943	.966	.977
$\Delta_{5, 5+1}\chi^2 = 10.990$	Δ df = 5	$p < .055$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "How Do You Feel About This Statement: I Am Good In Math"
(Grade 4, Booklets 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.880	—			
Geometry	1.000	.990	—		
Statistics	1.000	.850	.820	—	
Algebra	.860	.760	.850	1.000	—
Factor Loading	.990	.910	1.000	1.000	.890

Note. Range for this background variable question was: (1) = Agree, (2) = Undecided, and (3) = Disagree. Lower half of the triangle reports for the selected response: Undecided (2) for the above background variable (question).

Table 4A7

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 4, Booklet 11, 1990)
N = 395

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	36.770	35	1.050	.959	.997	.998
Five Factor	26.780	25	1.070	.970	.996	.998
5+1 Factor	32.360	30	1.080	.964	.996	.997
$\Delta_{5, 5+1}\chi^2 = 5.580$	Δ df = 5	$p < .400$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 4 Types (3)
(Grade 4, Booklet 11, 1990)
N = 405

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	31.590	35	.900	.977	1.003	1.000
Five Factor	25.330	25	1.010	.981	1.000	1.000
5+1 Factor	26.280	30	.880	.981	1.004	1.000
$\Delta_{5, 5+1}\chi^2 = .950$	Δ df = 5	$p < .975$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.930	.760	.980	1.080	.960
Measurement	.980	—	.900	.990	.990	.980
Geometry	.870	.880	—	.950	.920	.850
Statistics	.990	1.000	.940	—	1.000	1.000
Algebra	.930	.940	.870	.990	—	1.000
Factor Loading	.980	1.000	.890	1.000	.950	—

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, and (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 4 Types (3) for the above background variable (question).

Table 4A8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types (2)
(Grade 4, Booklet 11, 1990)
N = 438

	χ^2	df	$\chi^2/_{df}$	NFI	NNFI	CFI
One Factor	71.020	35	2.030	.943	.962	.970
Five Factor	46.640	25	1.870	.963	.968	.982
5+1 Factor	58.850	30	1.960	.953	.964	.976

$\Delta_{5, 5+1}\chi^2 = 12.210$ Δ df = 5 $p < .050$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.920	—			
Geometry	.790	.890	—		
Statistics	.990	1.000	1.000	—	
Algebra	.960	.950	.870	.840	—
Factor Loading	.940	.990	.890	1.000	.980

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, and (3) = 4 Types. Lower half of the triangle reports for the selected response: 3 Types (2) for the above background variable (question).

Table 4A9

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 4, Booklet 12, 1990)
N = 408

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	32.940	35	.940	.970	1.003	1.000
Five Factor	20.790	25	.830	.981	1.007	1.000
5+1 Factor	25.020	30	.830	.977	1.007	1.000
$\Delta_{5,5+1}\chi^2 = 4.230$	Δ df = 5	$p < .550$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)
Selected Response: 4 Types (3)
N = 404

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	49.250	35	1.410	.968	.988	.990
Five Factor	30.520	25	1.220	.980	.993	.996
5+1 Factor	32.420	30	1.080	.979	.998	.998
$\Delta_{5,5+1}\chi^2 = 1.900$	Δ df = 5	$p < .800$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.870	.870	.960	.930	.930
Measurement	.900	—	.920	.910	1.000	.950
Geometry	.860	.830	—	.890	.940	.940
Statistics	.920	.920	.920	—	1.000	1.000
Algebra	.920	.900	.880	.920	—	1.000
Factor Loading	.950	.940	.910	.970	.960	—

<u>Note</u>. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, and (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 4 Types (3) for the above variable (question).

Table 4A10

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types (2)
(Grade 4, Booklet 12, 1990)
N = 424

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	35.770	35	1.020	.968	.999	.999
Five Factor	24.390	25	.980	.978	1.001	1.000
5+1 Factor	27.450	30	.910	.975	1.004	1.000
$\Delta_{5, 5+1}\chi^2 = 3.060$	Δ df = 5	$p < .800$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.900	—			
Geometry	.840	.880	—		
Statistics	.980	1.000	.870	—	
Algebra	.910	.910	.780	1.000	—
Factor Loading	.950	.980	.870	1.000	.950

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, and (3) = 4 Types. Lower half of the triangle reports for the selected response: 3 Types (2) for the above background variable (question).

Table 4A11

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 4, Booklet 14, 1990)
N = 409

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	49.250	35	1.410	.956	.983	.987
Five Factor	27.220	25	1.090	.976	.996	.998
5+1 Factor	30.240	30	1.010	.973	1.000	1.000
$\Delta_{5, 5+1}\chi^2 = 3.020$	Δ df = 5	$p < .700$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 4 Types (3)
(Grade 4, Booklet 14, 1990)
N = 383

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	44.550	35	1.270	.966	.990	.993
Five Factor	27.180	25	1.090	.979	.997	.998
5+1 Factor	31.810	30	1.060	.976	.998	.999
$\Delta_{5, 5+1}\chi^2 = 4.630$	Δ df = 5	$p < .500$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.860	.880	1.000	.940	.980
Measurement	.940	—	.860	.880	.790	.880
Geometry	.810	.900	—	.920	.870	.920
Statistics	1.000	1.000	.830	—	.950	1.000
Algebra	.960	.990	.900	1.000	—	.940
Factor Loading	.950	1.000	.870	1.000	1.000	—

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, and (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 4 Types (3) for the above background variable (question).

Table 4A12

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "Home Environment-Reading Materials"
Selected Response: 3 Types (2)
(Grade 4, Booklet 14, 1990)
N = 431

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	63.140	35	1.800	.960	.976	.982
Five Factor	34.420	25	1.380	.978	.989	.994
5+1 Factor	40.990	30	1.370	.974	.989	.993

$\Delta_{5, 5+1}\chi^2 = 6.570$ Δ df = 5 p < .275

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 4, Book 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.860	—			
Geometry	1.000	.970	—		
Statistics	.970	.920	1.000	—	
Algebra	.890	.850	.900	1.000	—
Math	.960	.910	1.000	1.000	.930

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, and (3) = 4 Types. Lower half of the triangle reports for the selected response: 3 Types (2) for the above background variable (question).

Table 4A13

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: Almost Every Day (1)
(Grade 4, Booklet 11, 1990)
N = 337

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	37.910	35	1.080	.953	.995	.996
Five Factor	27.160	25	1.090	.966	.995	.997
5+1 Factor	29.960	30	1.000	.963	1.000	1.000
$\Delta_{5, 5+1}\chi^2 = 2.800$	Δ df = 5	$p < .750$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: Never (5)
(Grade 4, Booklet 11, 1990)
N=382

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	55.840	35	1.590	.948	.974	.980
Five Factor	31.220	25	1.250	.971	.989	.994
5+1 Factor	43.650	30	1.450	.959	.980	.987
$\Delta_{5, 5+1}\chi^2 = 12.430$	Δ df = 5	$p < .030$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.880	.840	.980	.980	.930
Measurement	1.000	—	.860	.990	1.000	.950
Geometry	.720	.790	—	1.000	1.000	.970
Statistics	1.000	.940	.930	—	1.000	1.000
Algebra	.980	1.000	.700	.830	—	1.000
Factor Loading	1.000	1.000	.770	.990	.970	—

Note. Range for this background variable question was: (1) = Almost Every Day, (2) = Several Times A Week, (3) = About Once A Week, (4) = Less Than Once A Week, and (5) = Never. Upper half of the triangle reports for the selected response: Almost Every Day (1) and the lower half of the triangle reports for the selected response: Never (5) for the above background variable (question).

Table 4A14

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: Almost Every Day (1)
(Grade 4, Booklet 12, 1990)
N = 318

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	51.020	35	1.460	.949	.978	.983
Five Factor	17.830	25	.710	.982	1.014	1.000
5+1 Factor	21.510	30	.720	.978	1.013	1.000

$\Delta_{5, 5+1}\chi^2 = 3.680$ Δ df = 5 $p < .600$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: Never (5)
(Grade 4, Booklet 12, 1990)
N = 377

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	24.870	35	.710	.978	1.012	1.000
Five Factor	16.340	25	.650	.985	1.015	1.000
5+1 Factor	18.340	30	.610	.983	1.016	1.000

$\Delta_{5, 5+1}\chi^2 = 2.000$ Δ df = 5 $p < .875$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.770	.780	.970	.850	.890
Measurement	.950	—	.770	.880	.880	.870
Geometry	.840	.820	—	.920	.920	.900
Statistics	1.000	1.000	.830	—	1.000	1.000
Algebra	1.000	.920	.830	.990	—	.990
Factor Loading	1.000	.970	.840	1.000	1.000	—

Note. Range for this background variable question was: (1) = Almost Every Day, (2) = Several Times A Week, (3) = About Once A Week, (4) = Less Than Once A Week, and (5) = Never. Upper half of the triangle reports for the selected response: Almost Every Day (1) and the lower half of the triangle reports for the selected response: Never (5) for the above background variable (question).

Table 4A15

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: Almost Every Day (1)
(Grade 4, Booklet 14, 1990)
N = 329

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	67.370	35	1.920	.944	.964	.972
Five Factor	55.580	25	2.220	.954	.953	.974
5+1 Factor	62.550	30	2.080	.948	.958	.972
$\Delta_{5,\,5+1}\chi^2 = 6.970$	Δ df = 5	$p < .250$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
Selected Response: Never (5)
(Grade 4, Booklet 14, 1990)
N = 368

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	55.680	35	1.590	.950	.975	.981
Five Factor	19.850	25	.790	.982	1.009	1.000
5+1 Factor	25.590	30	.850	.977	1.006	1.000
$\Delta_{5,\,5+1}\chi^2 = 5.740$	Δ df = 5	$p < .300$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Work With Rulers, Blocks, Shapes"
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.910	.930	.890	1.000	.960
Measurement	.820	—	.950	.980	.940	.960
Geometry	.890	.840	—	1.000	.920	.970
Statistics	1.000	.850	.830	—	1.000	1.000
Algebra	.880	.760	.800	.960	—	1.000
Factor Loading	.980	.860	.910	1.000	.900	—

Note. Range for this background variable question was: (1) = Almost Every Day, (2) = Several Times A Week, (3) = About Once A Week, (4) = Less Than Once A Week, and (5) = Never. Upper half of the triangle reports for the selected response: Almost Every Day (1), and the lower half of the triangle reports for the selected response: Never (5) for the above background variable (question).

Table IB6

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good In Math," and "Agree/Disagree: Math Is Mostly Memorizing Facts" (Grade 12, Book 1, 1992)
N = 370

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	35.140	35	1.000	.849	.999	.999
Five Factor	21.060	25	.840	.910	1.038	1.000
5+1 Factor	25.770	30	.860	.889	.034	1.000
$\Delta_{5, 5+1}\chi^2 = 4.750$	Δ df = 5	$p < .500$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good In Math," and "Agree/Disagree: Math Is Mostly Memorizing Facts" (Grade 12, Book 1, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.510	—			
Geometry	.313	.746	—		
Statistics	.346	1.000	.765	—	
Algebra	.557	.764	.893	.866	—
Loading on Second Order Factor	.450	1.000	.848	.972	.969

Table IB7
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "How Much Time Spent Each Day On Math Homework"
(Grade 12, Book 15, 1992)
N = 360

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	37.420	35	1.070	.900	.991	.993
Five Factor	33.780	25	1.350	.910	.952	.973
5+1 Factor	36.960	30	1.230	.901	.968	.979
$\Delta_{5,\ 5+1}\chi^2$ = 3.180	Δ df = 5	$p < .700$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "How Much Time Spent Each Day On Math Homework"
(Grade 12, Book 15, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.929	.964	—		
Statistics	.878	.925	1.000	—	
Algebra	.512	.291	.335	.323	—
Loading on Second Order Factor	.978	1.000	1.000	1.000	.468

Table IB8

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math"
(Grade 12, Book 17, 1992)
N = 35

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	64.800	35	1.850	.885	.926	.943
Five Factor	38.240	25	1.530	.932	.954	.974
5+1 Factor	50.470	30	1.680	.911	.941	.961

$\Delta_{5, 5+1}\chi^2 = 12.230$ Δ df = 5 $p < .030$

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math "
(Grade 12, Book 17, 1992)
N = 35

	Numbers	Measurement	Geometry	Statisitcs	Algebra
Numbers	—				
Measurement	.652	—			
Geometry	.649	.881	—		
Statistics	.735	.820	.909	—	
Algebra	.639	.638	1.000	.592	—
Loading on Second Order Factor	.713	.865	1.000	.892	.886

Table I1
Indices of Fit of One-Factor, Five- Factor and Six Factor Models Based on the Level of Item Correlations With: "What Kind Of Math Class Are You Taking This Year"
(Grade 4, Book 11, 1990)
N = 1255

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	65.750	35	1.880	.917	.947	.959
Five Factor	29.100	25	1.160	.963	.990	.995
5+1 Factor	35.570	30	1.190	.955	.989	.993
$\Delta_{5,\,5+1}\chi^2 = 6.470$	Δ df = 5	$p < .275$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind Of Math Class Are You Taking This Year"
(Grade 4, Booklet 11, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.571	—			
Geometry	.725	.625	—		
Statistics	.727	.668	1.000	—	
Algebra	1.000	.742	1.000	1.000	—
Factor Loading	.919	.671	1.000	1.000	1.000

Table I2

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 12, 1990)
N = 1240

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	59.350	35	1.700	.956	.976	.981
Five Factor	49.820	25	1.990	.963	.966	.981
5+1 Factor	55.420	30	1.850	.959	.971	.980

$\Delta_{5,\,5+1}\chi^2 = 5.670$ Δ df = 5 $p < .350$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 12, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.891	.778	—		
Statistics	.895	.966	.736	—	
Algebra	.723	.937	.738	1.000	—
Factor Loading	1.000	.976	.779	1.000	1.000

Table I3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Book 14, 1990)
N = 1220

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	73.950	35	2.110	.940	.958	.967
Five Factor	66.890	25	2.680	.945	.936	.965
5+1 Factor	68.820	30	2.290	.944	.951	.967
$\Delta_{5,\ 5+1}\chi^2 = 1.930$	Δ df = 5	$p < .900$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "In Math Class How Often Do You Take Math Tests"
(Grade 4, Booklet 14, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.614	—			
Geometry	.678	.982	—		
Statistics	.707	1.000	.962	—	
Algebra	.745	1.000	1.000	1.000	—
Factor Loading	.699	1.000	.999	1.000	1.000

Table TA4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 8, 1990)
N = 1234

	χ^2	df	$\chi^2/_{df}$	NFI	NNFI	CFI
One Factor	121.560	35	3.470	.983	.984	.988
Five Factor	35.150	25	1.410	.995	.997	.999
5+1 Factor	55.940	30	1.860	.992	.994	.996
$\Delta_{5, 5+1}\chi^2 = 20.790$	Δ df = 5	$p < .001$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 8, 1990)
N = 1234

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.900	—			
Geometry	.900	.920	—		
Statistics	.930	.910	.900	—	
Algebra	.970	.900	.960	.930	—
Factor Loading	.970	.940	.950	.960	.990

Table TA5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 9, 1990)
N = 1234

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	126.390	35	3.610	.979	.980	.985
Five Factor	45.420	25	1.820	.992	.994	.997
5+1 Factor	68.790	30	2.290	.989	.990	.993

$\Delta_{5,\,5+1}\chi^2 = 23.370$ Δ df = 5 p < .001

Correlations Eetween and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 9, 1990)
N = 1234

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.880	.980	—		
Statistics	.950	1.000	.950	—	
Algebra	.920	.930	.900	.930	—
Factor Loading	.970	1.000	.940	.990	.950

Table TA6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 10, 1990)
N = 1230

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	171.440	35	4.900	.976	.975	.981
Five Factor	29.450	25	1.180	.996	.999	.999
5+1 Factor	50.600	30	1.690	.993	.996	.997
$\Delta_{5,\,5+1}\chi^2 = 21.150$	Δ df = 5	$p < .001$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 10, 1990)
N = 1230

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.930	—			
Geometry	.850	.880	—		
Statistics	.950	.920	.910	—	
Algebra	.940	.910	.910	.970	—
Factor Loading	.960	.950	.910	.990	.980

Table 8A1

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Strongly Agree (1)
(Grade 8, Booklet 8, 1990)
N = 710

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	92.260	35	2.640	.977	.981	.985
Five Factor	30.610	25	1.220	.992	.997	.999
5+1 Factor	69.770	30	2.330	.922	.930	.953
$\Delta_{5, 5+1}\chi^2$ = 13.910	Δ df = 5	p < .020				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Disagree (4)
(Grade 8, Booklet 8, 1990)
N = 217

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	28.110	35	.800	.997	1.088	1.000
Five Factor	9.330	25	.370	.992	1.024	1.000
5+1 Factor	13.560	30	.450	.989	1.021	1.000
$\Delta_{5, 5+1}\chi^2$ = 4.230	Δ df = 5	p < .575				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.880	.920	.920	.960	.960
Measurement	.880	—	.910	.900	.890	.920
Geometry	.920	.910	—	.900	.990	.970
Statistics	.920	.900	.900	—	.910	.950
Algebra	.960	.880	.980	.910	—	.990
Factor Loading	.960	.920	.970	.940	.990	—

Note. Range for this background variable question was: (1) = Strongly Agree, (2) = Agree, (3) = Undecided, (4) = Disagree, and (5) = Strongly Disagree. Upper half of the triangle reports for the selected response: Strongly Agree (1) and the lower half of the triangle reports for the selected response: Disagree (4) for the above background variable (question).

Table 8A2

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 8, Booklet 8, 1990)
N = 268

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	59.470	35	1.700	.954	.973	.979
Five Factor	34.180	25	1.370	.972	.986	.922
5+1 Factor	36.510	30	1.220	.970	.992	.994
$\Delta_{5, 5+1}\chi^2 = 2.330$	Δ df = 5	$p < .700$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 4 Types (3)
(Grade 8, Booklet 8, 1990)
N = 594

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	80.970	35	2.310	.974	.981	.985
Five Factor	38.280	25	1.530	.988	.992	.996
5+1 Factor	47.220	30	1.570	.985	.992	.994
$\Delta_{5, 5+1}\chi^2 = 8.940$	Δ df = 5	$p < .750$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.880	.890	.890	.880	.950
Measurement	.890	—	.860	.880	.810	.910
Geometry	.910	.940	—	.900	.910	.950
Statistics	.920	.870	.890	—	.880	.940
Algebra	.960	.900	.970	.910	—	.920
Factor Loading	.960	.930	.970	.940	.980	—

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 4 Types (3) for the above background variable (question).

Table 8A3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Algebra (4)
(Grade 8, Booklet 8, 1990)
N = 183

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	66.780	35	1.910	.951	.969	.976
Five Factor	35.910	25	1.440	.974	.985	.992
5+1 Factor	46.160	30	1.540	.966	.982	.988
$\Delta_{5, 5+1}\chi^2 = 10.250$	Δ df = 5	$p < .075$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind of Math Class Are You Taking This Year"
Selected Response: Eighth Grade Math (2)
(Grade 8, Booklet 8, 1990)
N = 556

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	57.460	35	1.630	.973	.986	.989
Five Factor	31.710	25	1.270	.985	.994	.997
5+1 Factor	34.030	30	1.130	.984	.997	.998
$\Delta_{5, 5+1}\chi^2 = 2.320$	Δ df = 5	$p < .800$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind Of Math Class Are You Taking This Year"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.880	.880	.940	1.000	.980
Measurement	.890	—	.920	.890	.910	.920
Geometry	.870	.890	—	.890	.950	.930
Statistics	.890	.900	.870	—	.950	.960
Algebra	.980	.960	.990	.950	—	1.000
Factor Loading	.940	.950	.940	.940	1.000	—

Note. Range for this background variable question was: (1) = No Math This Year, (2) = Eighth-Grade Math, (3) = Pre-Algebra, (4) = Algebra, and (5) = Other. Upper half of the triangle reports for the selected response: Algebra (4) and the lower half of the triangle reports for the selected response: Eighth-Grade Math (2) for the above background variable (question).

Table 8A4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Pre-Algebra (3)
(Grade 8, Booklet 8, 1990)
N = 235

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	70.050	35	2.000	.935	.956	.966
Five Factor	38.580	25	1.540	.964	.976	.987
5+1 Factor	49.780	30	1.660	.954	.971	.981

$\Delta_{5, 5+1}\chi^2 = 11.200$ Δ df = 5 $p < .050$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind Of Math Class Are You Taking This Year" (Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.830	—			
Geometry	.910	.930	—		
Statistics	.920	.840	.870	—	
Algebra	.950	.750	.860	.830	—
Factor Loading	.990	.870	.950	.920	.930

Note. Range for this background variable question was: (1) = No Math This Year, (2) = Eighth-Grade Math, (3) = Pre-Algebra, (4) = Algebra, and (5) = Other. Lower half of the triangle reports for the selected response: Pre-Algebra (3) for the above background variable (question).

Table 8A5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Strongly Agree (1)
(Grade 8, Booklet 9, 1990)
N = 746

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	112.250	35	3.200	.969	.972	.978
Five Factor	48.980	25	1.960	.987	.990	.994
5+1 Factor	60.850	30	2.030	.983	.987	.991

$\Delta_{5, 5+1}\chi^2 = 11.870$ Δ df = 5 $p < .050$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Disagree (4)
(Grade 8, Booklet 9, 1990)
N = 189

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	48.480	35	1.390	.933	.974	.980
Five Factor	32.700	25	1.310	.955	.980	.989
5+1 Factor	38.640	30	1.290	.946	.981	.987

$\Delta_{5, 5+1}\chi^2 = 5.950$ Δ df = 5 $p < .450$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.940	.880	.910	.990	1.000
Measurement	.940	—	.950	.890	1.000	.870
Geometry	1.000	.840	—	.940	.930	.900
Statistics	.920	.890	.980	—	1.000	.920
Algebra	1.000	.860	1.000	.830	—	.960
Factor Loading	.920	1.000	.950	.990	1.000	—

Note. Range for this background variable question was: (1) = Strongly Agree, (2) = Agree, (3) = Undecided, (4) = Disagree, and (5) = Strongly Disagree. Upper half of the triangle reports for the selected response: Strongly Agree (1) and the lower half of the triangle reports for the selected response: Disagree (4) for the above background variable (question).

Table 8A6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 8, Booklet 9, 1990)
N = 263

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	49.230	35	1.410	.958	.984	.987
Five Factor	35.730	25	1.430	.968	.983	.990
5+1 Factor	43.040	30	1.430	.963	.983	.988
$\Delta_{5,5+1}\chi^2 = 7.350$	Δ df = 5	$p < .200$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 4 Types (3)
(Grade 8, Booklet 9, 1990)
N = 606

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	95.930	35	2.740	.965	.971	.780
Five Factor	48.820	25	1.950	.982	.984	.991
5+1 Factor	61.050	30	2.040	.978	.983	.989
$\Delta_{5,5+1}\chi^2 = 12.230$	Δ df = 5	$p < .050$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.950	.990	1.000	1.000	1.000
Measurement	.950	—	1.000	.910	1.000	.890
Geometry	1.000	.880	—	.960	.910	.960
Statistics	.950	.950	.930	—	1.000	.940
Algebra	1.000	.850	1.000	.860	—	.940
Factor Loading	.920	.920	.960	1.000	1.000	—

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 4 Types (3) for the above background variable (question).

Table 8A7

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Algebra (4)
(Grade 8, Booklet 9, 1990)
N = 242

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	48.740	35	1.390	.959	.985	.988
Five Factor	36.190	25	1.450	.970	.982	.990
5+1 Factor	39.490	30	1.320	.967	.988	.992

$\Delta_{5,\,5+1}\chi^2 = 3.300$ Δ df = 5 $p < .650$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Eighth Grade Math (2)
(Grade 8, Booklet 9, 1990)
N = 176

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	58.500	35	1.670	.940	.970	.980
Five Factor	30.870	25	1.230	.970	.990	.990
5+1 Factor	39.890	30	1.330	.960	.980	.990

$\Delta_{5,\,5+1}\chi^2 = 90.200$ Δ df = 5 $p < .150$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	1.000	.920	1.000	.960	1.000
Measurement	1.000	—	.920	1.000	.950	1.000
Geometry	.860	.980	—	.990	.900	.940
Statistics	.900	.970	.910	—	.920	1.000
Algebra	.890	.900	.910	.940	—	.950
Factor Loading	.940	1.000	.940	.970	.950	—

Note. Range for this background variable question was: (1) = No Math This Year, (2) = Eighth-Grade Math, (3) = Pre-Algebra, (4) = Algebra, and (5) = Other. Upper half of the triangle reports for the selected response: Algebra (4) and the lower half of the triangle reports for the selected response: Eighth Grade Math (2) for the above background variable (question).

Table 8A8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Pre-Algebra (3)
(Grade 8, Booklet 9, 1990)
N = 558

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	65.550	35	1.870	.967	.980	.984
Five Factor	38.350	25	1.530	.981	.988	.993
5+1 Factor	48.620	30	1.620	.976	.986	.990

$\Delta_{5,5+1}\chi^2 = 10.270$ Δ df = 5 p < .075

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind Of Math Class Are You Taking This Year" (Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.980	—			
Geometry	.830	.960	—		
Statistics	.900	1.000	.950	—	
Algebra	.920	.930	.890	.880	—
Factor Loading	.930	1.000	.940	.990	.940

Note. Range for this background variable question was: (1) = No Math This Year, (2) = Eighth-Grade Math, (3) = Pre-Algebra, (4) = Algebra, and (5) = Other. Lower half of the triangle reports for the selected response: Pre-Algebra (3) for the above background variable (question).

Table 8A9

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Strongly Agree (1)
(Grade 8, Booklet 10, 1990)
N = 742

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	116.370	35	3.320	.973	.976	.981
Five Factor	37.410	25	1.500	.991	.995	.997
5+1 Factor	47.980	30	1.600	.989	.994	.996
$\Delta_{5,\,5+1}\chi^2 = 10.570$	Δ df = 5	$p < .075$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Disagree (4)
(Grade 8, Booklet 10, 1990)
N = 196

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	81.550	35	2.330	.914	.934	.948
Five Factor	32.990	25	1.320	.965	.984	.991
5+1 Factor	47.080	30	1.570	.950	.972	.981
$\Delta_{5,\,5+1}\chi^2 = 14.090$	Δ df = 5	$p < .025$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.920	.850	.940	.960	.950
Measurement	.870	—	.920	.930	.950	.970
Geometry	.780	.740	—	.900	.940	.920
Statistics	.970	.770	.900	—	1.000	.980
Algebra	.870	.710	.830	.890	—	1.000
Factor Loading	.970	.840	.860	.990	.890	—

Note. Range for this background variable question was: (1) = Strongly Agree, (2) = Agree, (3) = Undecided, (4) = Disagree, and (5) = Strongly Disagree. Upper half of the triangle reports for the selected response: Strongly Agree (1) and the lower half of the triangle reports for the selected response: Disagree (4) for the above background variable (question).

Table 8A10

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 8, Booklet 10, 1990)
N = 285

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	75.850	35	2.170	.949	.964	.972
Five Factor	33.420	25	1.340	.978	.990	.994
5+1 Factor	44.860	30	1.490	.970	.985	.990

$\Delta_{5,\,5+1}\chi^2 = 11.440$ Δ df = 5 $p < .050$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 4 Types (3)
(Grade 8, Booklet 10, 1990)
N = 576

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	125.980	35	3.600	.960	.963	.971
Five Factor	26.410	25	1.060	.992	.999	1.000
5+1 Factor	38.000	30	1.270	.988	.996	.997

$\Delta_{5,\,5+1}\chi^2 = 11.590$ Δ df = 5 $p < .050$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.890	.810	.960	.940	.950
Measurement	.920	—	.850	.860	.880	.910
Geometry	.800	.850	—	.910	.930	.900
Statistics	.940	.950	.910	—	1.000	1.000
Algebra	.900	.860	.860	.960	—	1.000
Factor Loading	.940	.960	.890	1.000	.940	—

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 4 Types (3) for the above background variable (question).

Table 8A11

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Algebra (4)
(Grade 8, Booklet 10, 1990)
N = 165

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	79.310	35	2.270	.935	.952	.962
Five Factor	50.530	25	2.020	.959	.961	.978
5+1 Factor	57.580	30	1.920	.953	.965	.977
$\Delta_{5, 5+1}\chi^2 = 7.050$	Δ df = 5	$p < .200$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
Selected Response: Eighth-Grade Math (2)
(Grade 8, Booklet 10, 1990)
N = 561

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	95.660	35	2.730	.962	.968	.975
Five Factor	23.030	25	.920	.991	1.001	1.000
5+1 Factor	31.770	30	1.060	.987	.999	.999
$\Delta_{5, 5+1}\chi^2 = 8.740$	Δ df = 5	$p < .150$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of:
"What Kind Of Math Class Are You Taking This Year"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.910	.910	.920	.870	.920
Measurement	.920	—	.980	.970	.870	.970
Geometry	.780	.830	—	.990	.920	.990
Statistics	.940	.900	.860	—	1.000	1.000
Algebra	.910	.920	.840	.960	—	.950
Factor Loading	.950	.950	.850	.980	.970	—

Note. Range for this background variable question was: (1) = No Math This Year, (2) = Eighth-Grade Math, (3) = Pre-Algebra, (4) = Algebra, and (5) = Other. Upper half of the triangle reports for the selected response: Algebra (4) and the lower half of the triangle reports for the selected response: Eighth-Grade Math (2) for the above background variable (question).

Table 8A12

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of: "What Kind Of Math Class Are You Taking This Year"
Selected Response: Pre-Algebra (3)
(Grade 8, Booklet 10, 1990)
N = 248

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	55.090	35	1.570	.954	.977	.982
Five Factor	25.530	25	1.020	.979	.999	1.000
5+1 Factor	33.160	30	1.110	.972	.996	.997
$\Delta_{5, 5+1}\chi^2 = 7.630$	Δ df = 5	$p < .150$				

Correlations Between Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "What Kind Of Math Class Are You Taking This Year"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.900	—			
Geometry	.810	.880	—		
Statistics	.930	.900	.900	—	
Algebra	.960	.870	.900	1.000	—
Factor Loading	.940	.940	.900	.990	1.000

Note. Range for this background variable question was: (1) = No Math This Year, (2) = Eighth-Grade Math, (3) = Pre-Algebra, (4) = Algebra, and (5) = Other. Lower half of the triangle reports for the selected response: Pre-Algebra (3) for the above background variable (question).

Table I4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "What Kind Of Math Class Are You Taking This Year," and "Parents' Education Level"
(Grade 8, Book 8, 1990)
N = 1234

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	57.840	35	1.650	.941	.968	.975
Five Factor	29.220	25	1.170	.970	.992	.995
5+1 Factor	34.440	30	1.150	.965	.993	.995
$\Delta_{5, 5+1}\chi^2 = 5.220$	Δ df = 5	$p < .500$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind Of Math Class Are You Taking This Year," and "Parents' Education Level"
(Grade 8, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.597	—			
Geometry	.601	.740	—		
Statistics	.584	.817	.836	—	
Algebra	.672	.744	.939	.790	—
Factor Loading	.681	.827	.960	.866	.942

Table I5
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "What Kind Of Math Class Are You Taking This Year," and "Do You Agree: I Am Good In Math"
(Grade 8, Book 9, 1990)
N = 1244

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	92.810	35	2.650	.897	.913	.932
Five Factor	66.270	25	2.650	.926	.913	.952
5+1 Factor	69.770	30	2.330	.922	.930	.953
$\Delta_{5,\,5+1}\chi^2 = 3.500$	Δ df = 5	$p < .750$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind Of Math Class Are You Taking This Year," and "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.968	.964	—		
Statistics	.834	.971	1.000	—	
Algebra	.778	.837	.733	.710	—
Factor Loading	1.000	1.000	.994	.931	.777

Table I6
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "What Kind Of Math Class Are You Taking This Year," Fathers' Education Level," and "Do You Agree: I Am Good In Math"
(Grade 8, Book 10, 1990)
N = 1230

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	129.250	35	3.690	.917	.920	.938
Five Factor	84.560	25	3.380	.946	.929	.966
5+1 Factor	97.530	30	3.250	.938	.933	.955
$\Delta_{5,\ 5+1}\chi^2 = 12.970$	Δ df = 5	$p < .025$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "What Kind Of Math Class Are You Taking This Year," Fathers' Education Level," and "Do You Agree: I Am Good In Math"
(Grade 8, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.993	—			
Geometry	.760	1.000	—		
Statistics	.767	.838	.702	—	
Algebra	.863	1.000	.738	.755	—
Factor Loading	.923	1.000	.924	.806	1.000

Table TA7

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 8, 1990)
N = 1201

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	134.200	35	3.830	.978	.979	.984
Five Factor	39.000	25	1.560	.994	.996	.998
5+1 Factor	58.440	30	1.950	.990	.993	.995
$\Delta_{5, 5+1}\chi^2 = 19.440$	Δ df = 5	$p < .005$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 8, 1990)
N = 1201

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.900	—			
Geometry	.920	.960	—		
Statistics	.940	.880	.880	—	
Algebra	.910	.890	.910	.890	—
Factor Loading	.960	.950	.980	.940	.940

Table TA8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 9, 1990)
N = 1201

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	185.720	35	5.310	.973	.972	.978
Five Factor	79.850	25	3.190	.989	.986	.992
5+1 Factor	118.990	30	3.970	.983	.981	.987
$\Delta_{5,\,5+1}\chi^2 = 39.140$	Δ df = 5	$p < .001$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 9, 1990)
N = 1201

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.960	—			
Geometry	.940	.940	—		
Statistics	.930	.980	.910	—	
Algebra	.930	.890	.970	.880	—
Factor Loading	.970	.970	.990	.940	.950

Table TA9

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 10, 1990)
N = 1193

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	161.680	35	4.620	.976	.976	.981
Five Factor	52.020	25	2.080	.992	.993	.996
5+1 Factor	83.050	30	2.770	.988	.988	.992

$\Delta_{5, 5+1}\chi^2 = 31.030$ Δ df = 5 $p < .001$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 10, 1990)
N = 1193

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.930	—			
Geometry	.890	.990	—		
Statistics	.920	.900	.880	—	
Algebra	.910	.950	.960	.890	—
Factor Loading	.940	.990	.980	.920	.970

Table 12A1

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Strongly Agree (1)
(Grade 12, Booklet 8, 1990)
N = 651

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	72.690	35	2.080	.978	.985	.988
Five Factor	21.120	25	.840	.994	1.002	1.000
5+1 Factor	31.650	30	1.060	.990	.999	.999
$\Delta_{5, 5+1}\chi^2$ = 10.530	Δ df = 5	$p < .075$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Agree (2)
(Grade 12, Booklet 8, 1990)
N = 254

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	28.290	35	.810	.978	1.007	1.000
Five Factor	8.220	25	.033	.994	1.024	1.000
5+1 Factor	12.320	30	.410	.990	1.021	1.000
$\Delta_{5, 5+1}\chi^2$ = 4.100	Δ df = 5	$p < .550$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.900	.920	.940	.910	.960
Measurement	.900	—	.960	.880	.890	.950
Geometry	.900	.960	—	.880	.910	.980
Statistics	.940	.880	.880	—	.890	.940
Algebra	.910	.890	.910	.890	—	.940
Factor Loading	.960	.950	.980	.940	.940	—

Note. Range for this background variable question was: (1) = Strongly Agree, (2) = Agree, (3) = Undecided, (4) = Disagree, (5) = Strongly Disagree. Upper half of the triangle reports for the selected response: Strongly Agree (1) and the lower half of the triangle reports for the selected response: Agree (2) for the above background variable (question).

Table 12A2

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 12, Booklet 8, 1990)
N = 176

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	52.680	35	1.510	.937	.971	.978
Five Factor	38.280	25	1.530	.955	.970	.983
5+1 Factor	46.530	30	1.550	.945	.969	.979

$\Delta_{5,\,5+1}\chi^2 = 8.250$ $\quad \Delta$ df = 5 $\quad p < .150$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 3 Types (2)
Grade 12, Booklet 8, 1990)
N = 718

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	88.940	35	2.540	.975	.980	.985
Five Factor	36.000	25	1.440	.990	.994	.997
5+1 Factor	38.650	30	1.290	.989	.996	.998

$\Delta_{5,\,5+1}\chi^2 = 2.650$ $\quad \Delta$ df = 5 $\quad p < .750$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	——	.910	.950	1.000	.920	1.000
Measurement	.890	——	.990	.870	.880	.950
Geometry	.900	.930	——	.830	.890	.970
Statistics	.880	.870	.870	——	.930	1.000
Algebra	.900	.920	.900	.900	——	.930
Factor Loading	.940	.960	.960	.920	.960	——

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 3 Types (2) for the above background variable (question).

Table 12A3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
Selected Response: Almost Everyday (1)
(Grade 12, Booklet 8, 1990)
N = 347

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	69.130	35	1.980	.960	.974	.980
Five Factor	39.090	25	1.560	.978	.985	.992
5+1 Factor	46.850	30	1.560	.973	.985	.990
$\Delta_{5, 5+1}\chi^2 = 7.760$	Δ df = 5	$p < .200$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
Selected Response: Several Times A Week (2)
(Grade 12, Booklet 8, 1990)
N = 292

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	54.320	35	1.550	.960	.981	.985
Five Factor	29.760	25	1.190	.978	.994	.996
5+1 Factor	32.310	30	1.080	.976	.997	.998
$\Delta_{5, 5+1}\chi^2 = 2.550$	Δ df = 5	$p < .900$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
(Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	.910	.950	.950	.890	.970
Measurement	.880	—	.960	.860	.850	.940
Geometry	.880	.980	—	.870	.930	.990
Statistics	.910	.950	.930	—	.910	.950
Algebra	.890	.930	.910	.940	—	.930
Factor Loading	.920	.980	.970	.980	.950	—

Note. Range for this background variable question was: (1) = Almost Every Day, (2) = Several Times A Week, (3) = About Once A Week, (4) = Less Than Once A Week, and (5) = Never. Upper half of the triangle reports for the selected response: Almost Every Day (1) and the lower half of the triangle reports for the selected response: Several Times A Week (2) for the above background variable (question).

Table 12A4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Strongly Agree (1)
(Grade 12, Booklet 9, 1990)
N = 601

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	78.890	35	2.250	.926	.945	.957
Five Factor	37.090	25	1.480	.965	.979	.988
5+1 Factor	48.840	30	1.630	.954	.972	.982

$\Delta_{5,\,5+1}\chi^2 = 11.750$ Δ df = 5 $p < .050$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Agree (2)
(Grade 12, Booklet 9, 1990)
N = 250

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	100.150	35	2.860	.974	.978	.983
Five Factor	59.790	25	2.390	.984	.983	.991
5+1 Factor	76.590	30	2.550	.980	.981	.988

$\Delta_{5,\,5+1}\chi^2 = 16.800$ Δ df = 5 $p < .005$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	——	.980	.940	.920	.920	.960
Measurement	.940	——	.960	1.000	.930	1.000
Geometry	.930	.960	——	.930	.970	.990
Statistics	.940	1.000	.890	——	.880	.950
Algebra	.940	.910	.980	.900	——	.950
Factor Loading	.970	.990	.970	.950	.960	——

Note. Range for this background variable question was: (1) = Strongly Agree, (2) = Agree, (3) = Undecided, (4) = Disagree, (5) = Strongly Disagree. Upper half of the triangle reports for the selected response: Strongly Agree (1) and the lower half of the triangle reports for the selected response: Agree (2) for the above background variable (question).

Table 12A5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 12, Booklet 9, 1990)
N = 176

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	78.890	35	2.250	.926	.945	.957
Five Factor	37.090	25	1.480	.965	.979	.988
5+1 Factor	48.840	30	1.630	.954	.972	.982

$\Delta_{5,5+1}\chi^2 = 11.750$ Δ df = 5 $p < .050$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 3 Types (2)
(Grade 12, Booklet 9, 1990)
N = 693

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	100.150	35	2.860	.974	.978	.983
Five Factor	59.790	25	2.390	.984	.983	.991
5+1 Factor	76.590	30	2.550	.980	.981	.988

$\Delta_{5,5+1}\chi^2 = 16.800$ Δ df = 5 $p < .005$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 12, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	—	1.000	.900	.910	.860	.960
Measurement	.970	—	.900	1.000	.880	1.000
Geometry	.960	.970	—	.850	.930	.950
Statistics	.950	1.000	.940	—	.840	.930
Algebra	.940	.900	.970	.890	—	.910
Factor Loading	.980	.980	1.000	.960	.960	—

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1), and the lower half of the triangle reports for the selected response: 3 Types (2) for the above background variable (question).

Table 12A6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
Selected Response: Almost Every Day (1)
(Grade 12, Booklet 9, 1990)
N = 386

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	91.630	35	2.620	.958	.966	.974
Five Factor	49.590	25	1.980	.977	.979	.989
5+1 Factor	58.360	30	1.950	.973	.980	.987

$\Delta_{5, 5+1}\chi^2 = 8.770$ Δ df = 5 $p < .150$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
Selected Response: Several Times A Week (2)
(Grade 12, Booklet 9, 1990)
N = 299

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	68.590	35	2.000	.959	.973	.979
Five Factor	34.860	25	1.390	.979	.989	.994
5+1 Factor	42.260	30	1.410	.975	.989	.992

$\Delta_{5, 5+1}\chi^2 = 7.400$ Δ df = 5 $p < .200$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Do Problems On Worksheet"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	___	.940	.940	.900	.880	.950
Measurement	.970	___	.950	1.000	.900	1.000
Geometry	.940	.910	___	.920	.940	.980
Statistics	.910	.910	.910	___	.870	.950
Algebra	.910	.850	.950	.850	___	.930
Factor Loading	.970	.950	.980	.920	.940	___

Note. Range for this background variable question was: (1) = Almost Every Day, (2) = Several Times A Week, (3) = About Once A Week, (4) = Less Than Once A Week, and (5) = Never. Upper half of the triangle reports for the selected response: Almost Every Day (1) and the lower half of the triangle reports for the selected response: Several Times A Week (2) for the above background variable (question).

Table 12A7

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Strongly Agree (1)
(Grade 12, Booklet 10, 1990)
N = 629

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	93.270	35	2.660	.976	.981	.985
Five Factor	35.550	25	1.420	.991	.995	.997
5+1 Factor	47.940	30	1.500	.988	.993	.995

$\Delta_{5,\,5+1}\chi^2 = 12.390$ Δ df = 5 $p < .05$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Do You Agree: I Am Good In Math"
Selected Response: Agree (2)
(Grade 12, Booklet 10, 1990)
N = 273

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	43.300	35	1.240	.960	.990	.992
Five Factor	30.290	25	1.210	.972	.991	.995
5+1 Factor	37.120	30	1.240	.966	.990	.993

$\Delta_{5,\,5+1}\chi^2 = 15.640$ Δ df = 5 $p < .01$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Do You Agree: I Am Good In Math"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	___	.920	.910	.910	.910	.940
Measurement	.820	___	.990	.910	.980	.990
Geometry	.650	.900	___	.880	.970	.980
Statistics	.870	.860	.810	___	.900	.920
Algebra	.790	.770	.870	.840	___	.980
Factor Loading	.870	.930	.880	.940	.900	___

Note. Range for this background variable question was: (1) = Strongly Agree, (2) = Agree, (3) = Undecided, (4) = Disagree, (5) = Strongly Disagree. Upper half of the triangle reports for the selected response: Strongly Agree (1) and the lower half of the triangle reports for the selected response: Agree (2) for the above background variable (question).

Table 12A8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"Home Environment-Reading Materials"
Selected Response: 0-2 Types (1)
(Grade 12, Booklet 10, 1990)
N = 182

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	50.950	35	1.460	.932	.971	.977
Five Factor	18.110	25	.720	.976	1.018	1.000
5+1 Factor	26.260	30	.880	.965	1.008	1.000

$\Delta_{5,5+1}\chi^2 = 8.150$ Δ df = 5 $p < .150$

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
Selected Response: 3 Types (2)
(Grade 12, Booklet 10, 1990)
N = 680

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	83.320	35	2.380	.978	.984	.987
Five Factor	28.430	25	1.140	.993	.998	.999
5+1 Factor	48.780	30	1.630	.987	.993	.995

$\Delta_{5,5+1}\chi^2 = 20.350$ Δ df = 5 $p < .005$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "Home Environment-Reading Materials"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	——	.750	.770	.850	.830	.870
Measurement	.960	——	.920	.790	.870	.920
Geometry	.890	1.000	——	.780	.880	.920
Statistics	.930	.900	.880	——	.810	.880
Algebra	.930	.970	.970	.890	——	.940
Factor Loading	.960	1.000	.980	.920	.980	——

Note. Range for this background variable question was: (1) = 0-2 Types, (2) = 3 Types, (3) = 4 Types. Upper half of the triangle reports for the selected response: 0-2 Types (1) and the lower half of the triangle reports for the selected response: 3 Types (2) for the above background variable (question).

Table 12A9

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
Selected Response: Almost Everyday (1)
(Grade 12, Booklet 10, 1990)
N = 366

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	70.180	35	2.010	.963	.975	.981
Five Factor	40.390	25	1.620	.979	.985	.992
5+1 Factor	55.510	30	1.850	.971	.979	.986
$\Delta_{5, 5+1}\chi^2 = 15.120$	Δ df = 5	$p < .010$				

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Levels of:
"In Math Class How Often Do You Do Problems On Worksheet"
Selected Response: Several Times A Week (2)
(Grade 12, Booklet 10, 1990)
N = 306

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	60.420	35	1.730	.963	.979	.984
Five Factor	32.740	25	1.310	.980	.991	.995
5+1 Factor	43.280	30	1.440	.974	.987	.992
$\Delta_{5, 5+1}\chi^2 = 10.540$	Δ df = 5	$p < .100$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of Item Correlations With: "In Math Class How Often Do You Do Problems On Worksheet"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra	Factor Loading
Numbers	——	.930	.900	.950	.910	.950
Measurement	.900	——	.970	.900	.980	.980
Geometry	.840	1.000	——	.880	1.000	.980
Statistics	.890	.900	.880	——	.890	.930
Algebra	.910	.940	.940	.920	——	.990
Factor Loading	.920	.990	.970	.930	.980	——

Note. Range for this background variable question was: (1) = Almost Every Day, (2) = Several Times A Week, (3) = About Once A Week, (4) = Less Than Once A Week, and (5) = Never. Upper half of the triangle reports for the selected response: Almost Every Day (1) and the lower half of the triangle reports for the selected response: Several Times A Week (2) to the above background variable (question).

Table I7
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Algebra And Calculus Course Taking," and "Do You Agree: I Am Good In Math"
(Grade 12, Book 8, 1990).
N = 1197

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	89.680	35	2.560	.856	.878	.905
Five Factor	67.070	25	2.680	.892	.869	.927
5+1 Factor	85.210	30	2.840	.863	.857	.905

$\Delta_{5, 5+1}\chi^2 = 18.140$ $\Delta df = 5$ $p < .003$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "Algebra And Calculus Course Taking," and "Do You Agree: I Am Good In Math" (Grade 12, Booklet 8, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.695	—			
Geometry	.880	.895	—		
Statistics	.982	1.000	.935	—	
Algebra	1.000	.506	.601	1.000	—
Factor Loading	.975	.884	.928	1.000	.833

Table I8
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Algebra And Calculus Course Taking," "Geometry-Trigonometry Course Taking," and "Prior Knowledge Before The Test "
(Grade 12, Book 9, 1990)
N = 1176

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	104.830	35	3.000	.899	.909	.930
Five Factor	63.550	25	2.540	.939	.930	.961
5+1 Factor	71.210	30	2.370	.931	.938	.958
$\Delta_{5,\,5+1}\chi^2 = 7.660$	Δ df = 5	$p < .200$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "Algebra And Calculus Course Taking," "Geometry-Trigonometry Course Taking," and "Prior Knowledge Before The Test"
(Grade 12, Booklet 9, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.916	—			
Geometry	.772	.869	—		
Statistics	.966	1.000	.930	—	
Algebra	.501	.543	.683	6.000	—
Factor Loading	.909	1.000	.907	1.000	.601

Table I9
Indices of Fit of One-Factor, Five-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Algebra And Calculus Course Taking," and "Geometry-Trigonometry Course Taking"
(Grade 12, Book 10, 1990)
N = 1190

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	108.220	35	3.090	.887	.897	.920
Five Factor	77.220	25	3.090	.919	.897	.943
5+1 Factor	91.440	30	1.050	.905	.899	.933
$\Delta_{5,\,5+1}\chi^2 = 3.050$	Δ df = 5	$p < .700$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables Based on the Levels of: "Algebra And Calculus Course Taking," and "Geometry-Trigonometry Course Taking"
(Grade 12, Booklet 10, 1990)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.955	—			
Geometry	.636	.793	—		
Statistics	1.000	.997	.901	—	
Algebra	.944	1.000	.996	.930	—
Factor Loading	.868	1.000	.840	1.000	1.000

Table TB1

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 15, 1992)
N = 361

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	50.600	35	1.450	.995	.981	.985
Five Factor	44.290	25	1.770	.960	.968	.982
5+1 Factor	49.960	30	1.670	.955	.972	.981
$\Delta_{5, 5+1}\chi^2 = 5.670$	Δ df = 5	$p < .500$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 15, 1992)
N = 361

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.940	—			
Geometry	.940	.920	—		
Statistics	1.000	.940	1.000	—	
Algebra	.890	.950	1.000	.970	—
Factor Loading	.990	.950	1.000	1.000	1.000

Table TB2

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 4, Booklet 17, 1992)
N = 367

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	46.120	35	1.320	.965	.989	.991
Five Factor	33.990	25	1.360	.974	.987	.993
5+1 Factor	42.110	30	1.400	.968	.986	.991
$\Delta_{5,\,5+1}\chi^2 = 8.120$	Δ df = 5	$p < .250$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 4, Booklet 17, 1992)
N = 367

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.910	—			
Geometry	.890	.880	—		
Statistics	1.000	.880	1.000	—	
Algebra	.970	.940	.900	.950	—
Factor Loading	.990	.930	.950	1.000	.980

Table IB1
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," and "Agree/Disagree: I Am Good In Math"
(Grade 4, Book 15 , 1992)
N = 361

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	27.280	35	.780	.940	1.024	1.000
Five Factor	20.570	25	.820	.955	1.019	1.000
5+1 Factor	26.800	30	.890	.941	1.012	1.000
$\Delta_{5,\ 5+1}\chi^2 = 6.230$	$\Delta df = 5$	$p < .300$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," and "Agree/Disagree: I Am Good In Math"
(Grade 4, Book 15 , 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.986	—			
Geometry	.807	.754	—		
Statistics	1.000	1.000	.790	—	
Algebra	.730	.978	.956	1.000	—
Loading on Second Order Factor	1.000	1.000	.834	1.000	1.000

Table IB2
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," and "How Much Time Spent Each Day On Math Homework"
(Grade 4 , Book 17, 1992)
N= 367

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	33.470	35	1.340	.956	.978	.988
Five Factor	37.870	25	1.080	.950	.995	.996
5+1 Factor	36.970	30	1.230	.951	.985	.990
$\Delta_{5,\,5+1}\chi^2 = 3.500$	Δ df = 5	$p < .600$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "I Like Math," and "How Much Time Spent Each Day On Math Homework"
(Grade 4 , Book 17, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.820	—			
Geometry	.917	.937	—		
Statistics	1.000	.915	1.000	—	
Algebra	.962	.963	.927	.957	—
Loading on Second Order Factor	.994	.939	1.000	1.000	.977

Table TB3

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 1, 1992)
N = 395

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	112.950	35	3.300	.934	.940	.953
Five Factor	30.400	25	1.200	.982	.994	.997
5+1 Factor	31.520	30	1.100	.982	.999	.999

$\Delta_{5,\,5+1}\chi^2 = 1.120$ Δ df = 5 p < .950

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 1, 1992)
N = 395

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.930	—			
Geometry	.970	1.000	—		
Statistics	.780	.780	.820	—	
Algebra	.950	.970	.970	.810	—
Factor Loading	.960	.980	1.000	.810	.990

Table TB4

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 2, 1992)
N = 382

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	40.320	35	1.150	.980	.997	.997
Five Factor	15.180	25	.610	.993	1.009	1.000
5+1 Factor	19.560	30	.650	.990	1.008	1.000
$\Delta_{5, 5+1}\chi^2 = 4.380$	Δ df = 5	$p < .500$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 2, 1992)
N = 382

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.920	—			
Geometry	.900	.880	—		
Statistics	.990	.990	1.000	—	
Algebra	.950	.890	.910	.990	—
Factor Loading	.970	.940	.940	1.000	.960

Table TB5

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 8, Booklet 15, 1992)
N = 389

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	39.860	35	1.140	.980	.997	.997
Five Factor	24.240	25	.970	.988	1.001	1.000
5+1 Factor	28.600	30	.950	.985	1.001	1.000

$\Delta_{5,\,5+1}\chi^2 = 4.360$ Δ df = 5 $p < .500$

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 8, Booklet 15, 1992)
N = 389

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.950	—			
Geometry	.940	.970	—		
Statistics	.940	.970	.960	—	
Algebra	.890	.890	.960	.960	—
Factor Loading	.950	.980	.990	.990	.950

Table IB3
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math"
(Grade 8, Book 1, 1992)
N = 395

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	85.390	35	2.440	.815	.845	.879
Five Factor	34.350	25	1.370	.926	.960	.978
5+1 Factor	38.400	30	1.280	.917	.970	.980
$\Delta_{5,\,5+1}\chi^2 = 4.050$	$\Delta df = 5$	$p < .550$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math"
(Grade 8, Book 1, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.816	—			
Geometry	.574	.890	—		
Statistics	.526	.532	.599	—	
Algebra	.635	.953	.831	.653	—
Loading on Second Order Factor	.726	.983	.871	.671	.950

Table IB4
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Agree/Disagree: Math Is Mostly Memorizing Facts," and "Do You Agree: I Am Good in Math"
(Grade 8, Book 2, 1992)
N = 382

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	44.620	35	1.270	.884	.964	.972
Five Factor	42.380	25	1.700	.890	.908	.949
5+1 Factor	43.860	30	1.460	.886	.939	.959
$\Delta_{5, 5+1}\chi^2 = 1.480$	Δ df = 5	$p < .975$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Agree/Disagree: Math Is Mostly Memorizing Facts," and "Do You Agree: I Am Good In Math"
(Grade 8, Book 2, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.871	—			
Geometry	1.000	.888	—		
Statistics	.916	1.000	1.000	—	
Algebra	.790	.847	.933	.878	—
Loading on Second Order Factor	.970	.979	1.000	1.000	.885

Table IB5
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math"
(Grade 8, Book 5, 1992)
N = 394

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	78.800	35	2.250	.834	.869	.898
Five Factor	30.430	25	1.230	.936	.977	.987
5+1 Factor	37.240	30	1.240	.921	.975	.983
$\Delta_{5, 5+1}\chi^2 = 6.810$	Δ df = 5	$p < .250$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good in Math"
(Grade 8, Book 5, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.682	—			
Geometry	.893	.560	—		
Statistics	.806	.692	.709	—	
Algebra	.483	.468	.713	.701	—
Loading on Second Order Factor	.903	.721	.845	.931	.702

Table TB6

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 1, 1992)
N = 270

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	57.380	35	1.640	.957	.978	.983
Five Factor	33.970	25	1.360	.975	.988	.993
5+1 Factor	36.600	30	1.220	.973	.992	.995
$\Delta_{5, 5+1}\chi^2 = 2.630$	Δ df = 5	$p < .750$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 1, 1992)
N = 370

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.910	—			
Geometry	.810	.870	—		
Statistics	.790	.870	.860	—	
Algebra	.870	.920	.880	.820	—
Factor Loading	.910	.970	.920	.890	.950

Table TB7

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 15, 1992)
N = 360

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	88.910	35	2.500	.949	.960	.969
Five Factor	55.130	25	2.200	.969	.968	.982
5+1 Factor	64.890	30	2.200	.963	.969	.980
$\Delta_{5, 5+1}\chi^2 = 9.760$	Δ df = 5	$p < .100$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 15, 1992)
N = 360

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.950	—			
Geometry	.960	.890	—		
Statistics	.950	.920	.910	—	
Algebra	.930	.910	.910	.800	—
Factor Loading	1.000	.960	.960	.930	.920

Table TB8

Indices of Fit of One-Factor, Five-Factor and Six-Factor Models
(Grade 12, Booklet 17, 1992)
N = 350

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	100.980	35	2.890	.945	.953	.963
Five Factor	35.080	25	1.400	.981	.990	.994
5+1 Factor	41.830	30	1.390	.977	.990	.993
$\Delta_{5, 5+1}\chi^2 = 6.750$	Δ df = 5	$p < .250$				

Correlations Between and Factor Loadings of the Five Subscale Latent Variables
(Grade 12, Booklet 17, 1992)
N = 350

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.950	—			
Geometry	.880	.900	—		
Statistics	.900	.820	.800	—	
Algebra	1.000	.930	.950	.870	—
Factor Loading	1.000	.950	.920	.880	1.000

Table IB6

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good In Math," and "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Book 1, 1992)
N = 370

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	35.140	35	1.000	.849	.999	.999
Five Factor	21.060	25	.840	.910	1.038	1.000
5+1 Factor	25.770	30	.860	.889	.034	1.000
$\Delta_{5,\,5+1}\chi^2 = 4.750$	Δ df = 5	$p < .500$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "Do You Agree: I Am Good In Math," and "Agree/Disagree: Math Is Mostly Memorizing Facts"
(Grade 12, Book 1, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	.510	—			
Geometry	.313	.746	—		
Statistics	.346	1.000	.765	—	
Algebra	.557	.764	.893	.866	—
Loading on Second Order Factor	.450	1.000	.848	.972	.969

Table IB7
Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "How Much Time Spent Each Day On Math Homework"
(Grade 12, Book 15, 1992)
N = 360

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	37.420	35	1.070	.900	.991	.993
Five Factor	33.780	25	1.350	.910	.952	.973
5+1 Factor	36.960	30	1.230	.901	.968	.979
$\Delta_{5,5+1}\chi^2 = 3.180$	$\Delta df = 5$	$p < .700$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With: "How Much Time Spent Each Day On Math Homework"
(Grade 12, Book 15, 1992)

	Numbers	Measurement	Geometry	Statistics	Algebra
Numbers	—				
Measurement	1.000	—			
Geometry	.929	.964	—		
Statistics	.878	.925	1.000	—	
Algebra	.512	.291	.335	.323	—
Loading on Second Order Factor	.978	1.000	1.000	1.000	.468

Table IB8

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math"
(Grade 12, Book 17, 1992)
N = 35

	χ^2	df	χ^2/df	NFI	NNFI	CFI
One Factor	64.800	35	1.850	.885	.926	.943
Five Factor	38.240	25	1.530	.932	.954	.974
5+1 Factor	50.470	30	1.680	.911	.941	.961
$\Delta_{5,\ 5+1}\chi^2 = 12.230$	Δ df = 5	$p < .030$				

Indices of Fit of One-Factor and Six-Factor Models Based on the Level of Item Correlation With:
"Do You Agree: I Am Good In Math "
(Grade 12, Book 17, 1992)
N = 35

	Numbers	Measurement	Geometry	Statisitcs	Algebra
Numbers	—				
Measurement	.652	—			
Geometry	.649	.881	—		
Statistics	.735	.820	.909	—	
Algebra	.639	.638	1.000	.592	—
Loading on Second Order Factor	.713	.865	1.000	.892	.886

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