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ABSTRACT

This paper reviews the current theory and research on algebra/mathematics word problems. First, the major research findings are identified and summarized along with the theoretical perspectives that produced the research. The findings are then interpreted from the perspectives of traditional and wholetheme approaches. Also, a sample methodology of each approach is presented to provide a concrete understanding of the differences between the two. Contains 14 references. (Author/AA)

Running Head: WORD PROBLEMS

Solving Word Problems in Mathematics: A Comparison of Traditional and Wholetheme Approaches

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Abstract

This paper reviews the current theory and research on algebra/mathematics word problems. First, the major research findings are identified and summarized along with the theoretical perspectives that produced the research. The findings are then interpreted from the perspectives of traditional and wholetheme approaches. Also, a sample methodology of each approach is presented to provide a concrete understanding of the differences between the two.

Solving Word Problems in Mathematics: A Comparison of Traditional and Wholetheme Approaches

Introduction

If you ask students what is the most difficult topic in any mathematics course they will undoubtedly say word problems (Mayer, 1982; Weaver & Kintsch, 1992). It has been known for a long time that students , even good ones, find word problems difficult and dislike them (Terry, 1921/1992; Weaver & Kintsch, 1992). The difficulty students encounter in solving word problems and the dislike for them are reflected in their performance. Improving student performance and the fundamental problem-solving skills involved, though considered to be critically important math educators, have not been easy.

This paper reviews the current theory and research on algebra/mathematics word problems. First, the major research findings are identified and summarized along with theoretical perspectives that produced the research. The findings are then interpreted from the perspectives of traditional and wholetheme approaches. Also, a sample methodology of each approach is presented to provide a concrete understanding of the differences between the two.

Traditional Models of Problem-Solving

Models of problem-solving have been developed in order to understand the problem-solving process. One such model is described in Mayer (1982). This two stage model consists of a translation stage and a solution stage. In the translation stage, the words of the problems are used to build an understanding of it in terms of an internal representation in memory. In the solution stage, the student applies the rules of algebra and arithmetic to this internal representation in order to find the answer. Students have more difficulty in the translation stage than in the solution stage (Mayer, 1982). Nevertheless, in the classroom more emphasis is given to the teaching of the solution phase. This is perhaps because of an absence of an adequate understanding of what is involved in the translation stage.

Mayer (1982) identified linguistic, factual, and schema knowledge in the translation stage. On translating the word problem the student needs to have knowledge of the English language, the mathematical terminology (e.g. factoring, squared) and any words used in special ways (e.g. product, difference) in problem solving situations. The student must also possess schematic knowledge that allows him/her to determine the information that is relevant and to transfer the understanding of one problem to other similar problems. This part of the model requires the student to have knowledge of the objects and events of the problems as well as metacognition related to the solving of word problems.

Mayer (1982) stated that strategic and algorithmic knowledge are required for the solution stage. The student here is required to break the problem into parts and store the parts in memory in such a way he/she will be able to retrieve them when needed. In this part of the model, the student is supposed to hold a lot of information in the working memory. Furthermore, the student also requires a knowledge of algebra and of algebraic procedures and operations involved in finding of the answer. In an effort to build schema knowledge for students, Mayer designed a structure for word problem formulae presentation.

Following the pattern in which word problems were presented in textbooks and taught in classrooms, Mayer (1981) divided word problems into families, categories, and templates. A family grouped all the word problems using same source formula to solve the problem. In this way the students were able to master one type of formula at a time. Each family was divided into categories based on the general form of the story line so that within a chapter problems were presented by categories. This format allowed the student to focus on one variation of the formula at a time. Furthermore, each category was divided into templates based on the propositional structure of the problem. Let's take for example, the family of "distance = rate x time". In this family of problems, stories can be dealing with motion, current, or work. These will be the categories. In the category of motion, there are different templates. Examples of these are "overtake", "round trip", and "closure". Each template consisted of a unique list of propositions. Mayer felt that if the students would become familiar enough with the template, they would automatically associate the new problem with the appropriate template. As a result they would be able to solve the problem readily.

Mayer (1982) conducted a study to determine which types of word problems presented students with the most difficulty at the template level. Even at this basic level, Mayer found that translation is influenced by the structural property of propositions in the problem and that translation is influenced by the learner's schema of the problem. However, the application of his model in textbooks did not account for this additional step within the translation stage so student learning was diminished (Mayer, 1982).

Reed (1987), another researcher interested in studying word problem, developed a model of solving word problems by analogy. One of the difficulties in solving word problems by analogy is that the story context has an enormous effect on the analogy. In the analogy, students have less difficulty with the problem when the context is the same even if the equation needed to solve the problem is different than when the context is different but the equation is the same (Catrambone & Holyak, 1989; Reed, 1987). For this reason, Reed (1987) classified pairs of word problems into four different types. Equivalent problems have the same solution procedure and story context. Similar problems have the same story context but required a modification of a solution procedure. Isomorphic problems have the same solution, but different story contexts. Unrelated problems have different story contexts and different solutions. The implication then is that the context chosen for the story must be broad enough so that many variations of the formula or many formulae can be used. In this way students develop a really broad schema that will enable them to handle many varying equation types. Using analogies in this way allowed students to assimilate the content in a more global way.

Weaver and Kintsch (1992) used Reed's classification of word problems together with a model of arithmetic word problem developed by Kintsch and Greeno (1985) to build a model for solving algebra word problems. This model consists of a set of knowledge structures and a set of strategies for using these knowledge structures in building a representation and in solving the problem. In the process of problem representation, quantities which refer to objects in the pictorial, symbolic, and verbal propositional relational structure are organized into a schema or connected to an already existing schema from long term memory. This is a set of propositional frames, used in translating sentences into propositions. Schemata also represent properties and the relations of sets. This organized set of propositions is referred as text base. Along with the set of propositions is the problem model. This reflects the knowledge of the information required to solve the problem but is not included in the text. However, it does not reflect the information in the text not required for the solution of the problem. Where as Reed used a single story context within which students were better able to handle diverse equation types, Weaver & Kintsch established a representational model to serve as the context for various equation types. In the 1985 model a conceptual problem representation must be constructed according to arithmetic schemata. Then mathematics operators are applied to this schematic problem representation. The difference between arithmetic and algebra is only one of complexity. There are more algebraic schemata than arithmetic schemata, algebra problems are interrelated in more ways than arithmetic problems, and calculation procedures are more complex. With this in mind and the fact that it appeared to be a relatively small number of algebraic schemata will be needed to deal with the majority of algebra word problems as they were classified by Mayer (1981). Mayer's model (1982) was formulated in terms of a hierarchical structure while the Weaver and Kintsch model (1992) consisted of an interconnected web of propositions.

Strengths and Limitations of the Models

American education is piecemeal in nature. Teachers simplify using simplification by isolation. This means teachers present concepts, facts, skills, procedures, routines, and definitions piece by piece. By doing this, teachers are working with the professional knowledge base (PKB) without ever tapping the intuitive knowledge base (IKB)of the students in the hope of increasing it to a complex knowledge domain (Iran-Nejad, 1994). This approach depends entirely on previously acquired professional knowledge. The teacher just adds more newer professional knowledge to the one that the students already has. There is an assumption here, the sum of the parts equals the whole.

Within this approach, information is received directly by the learner from an external source, usually the teacher. This is what Iran-Nejad (1990) calls straight internalization of external knowledge. The learner in this situation, almost always is exclusively extrinsically motivated, although there can exist some intrinsic motivation.

Teachers using this approach, present topics at the simplest level of the knowledge domain.

As the topics get more complicated teachers try to simplify the content by using different processes. More importance is given to the parts than to the whole. One such process is the "bottom-up" (Anderson, 1995). This is a data-driven processing. In the bottom-up technique, the teacher breaks down the subject matter into a continuous sequence of discrete events and then presents each discrete step until all the steps are covered. Sometimes a teacher uses "top-down" processing. This is a conceptually driven processing. The teacher tries to illustrate the specific content by giving a relevant framework decomposed into isolated components and subcomponents.

In mathematics most of the teaching is by using simplification by isolation. A problem is broken down into parts and each part is independently solved. We see that the theoretical framework stems from at least three sources, Mayer, Reed, and Weaver and Kintsch.

A strength of Mayer's model is existence and importance of the translation stage. This is in fact the most important aspect of problem solving. This stage acknowledges the need of understanding the context in order to get to the solution stage. On the other hand, in his model, he uses the broad classification of word problem just an organizational tool. Mayer uses a bottom-up when he starts at the template level to build student expertise. This can be a problem because students can get too involved in the details and forget the main problem. Mayer breaks down even further than the template after the student has dealt with the words of the story.

Reed, on the other hand, moves towards a wholetheme approach to teaching and learning by establishing a broad context which is closer to real world circumstances. In Reed's analogy model for problem solving the strength is obtained from the empirical evidence. The empirical evidence shows that content is more important than the solution equation and it makes sense to say that the closer we can get to understanding the story the faster we can translate it to mathematical language and hence to the needed solution procedure. The limitation of his model is that although the model takes in consideration the story content it might not carry that importance far enough.

Not only do Weaver and Kintsch promoted simplification by isolation but they further complicate learning for the student by introducing a rigid artificial propositional network. This can also suggest to the students that there is only one way to arrive to the solution. Students might encounter difficulty in trying to apply their knowledge to solve real-world problems. A limitation of the model is that the problem has to fit one of those artificially developed structure types. The students would have to learn those structures, select the appropriate one and be able to take the information and place it in the matching structure. Silver (1979) found that good problem solvers used more the mathematical structure while poor problem solvers used the story context. The strength of this model is restricted to the good problem solvers. The existence of the intermediate structure type as a mean to categorize the problems is done dynamically by good problem solvers. But, in the case of poor problem solvers has to be acquired by active self-regulation and not dynamically as in the other group.

In using the piecemeal approach, teachers tend to think that the delivery of knowledge is the sole objective in teaching. Hence, the student is only a passive recipient. This contributes to the gap between what is taught and students' real world experiences.

Moving from Piecemeal to Wholetheme

Reed's studies (1987) showed that students were able to differentiate what needed to be done to solve the problem when working with similar story contexts. It tapped their existing knowledge base so that they were not wrestling with the words just with the computations. Students had the greatest difficulty when trying to solve isomorphic problems. In the wholetheme approach we find a very logical reason for that. These problems are incompatible (Iran-Nejad, 1989), because they have different themes (different story contexts) but share the same elements (equation type). Knowing all of these it makes sense for math teachers to teach word problem solving techniques at the broadest context level possible in order to have opportunities to reach each student at his or her knowledge base.

The idea behind the wholetheme approach is that learning is natural, from the inside, and is holistic in essence. The wholetheme approach aims at providing the learner with a holistic way of thinking about the knowledge domain (Iran-Nejad, Marsh, & Clements, 1992). There are two kinds of knowledge, thematic and categorical knowledge (Iran-Nejad, Marsh & Clements, 1992). Knowledge is represented as thematic knowledge and categorical knowledge is created in the context of thematic knowledge. Since one of the two functions of the brain is the ongoing brain activity, it is implied in this model that human beings learn best when they are presented with thematic knowledge (Iran-Nejad, 1989). Hence, the focus in education should be thematic knowledge and not categorical knowledge as it is now.

A theme is the fundamental ground against what everything that is related to it is attracted and becomes part of it. As these things are integrated there is a rich figure-ground relationship that erupts and then the focus changes slightly. Through systematically reintegrating and reorganizing the learner's IKB the professional knowledge base of the learner expands. Learners should be presented first with the "whole picture" in the context of the living experiences.

In mathematical word problems, the figure-ground relationship is between the story context and the underlying structure, where learning occurs in authentic context instead of in isolation.. In the traditional method, the relationship is established between the story context of a given word problem and the story context of another word problem. Limiting the applicability of this knowledge to a different word problem because in this model simplification is achieved by isolation. On the other hand, from the wholetheme approach the relationship of the word problem is set in the appropriate way, this is establishing a relationship between the story context and the underlying. Where the structure of the problem allows the learner to apply that to an infinite number of word problems because in this model simplification is achieved by integration.

An alternative approach to teaching word problem could be using the wholetheme approach. The teacher could start the class by showing a map of the United States or a state in particular. Each student will then mention the name of one city they have visited or want to visit. The names will be written on the blackboard. Students will then select two cities from the list and determine which one of the two cities is at a shorter distance from their city. Then students will have to determine, of the two selected cities, which will take less time to get there and which one will take more time from their city. The students will then be asked to select a third city and do the same thing of comparing the amount of time to get to that city from their city. The students would have established by now the concepts of distance and time by tapping their IKB.

The discussion will continue within the context of distance. A student will be ask to select a city. Students will be asked how long it will take to get to that city. Different travelling times will be mentioned. Students will be asked how can this happen. Students will need to tap into their IKB to find reasons. At this moment the students will be grouped to find out the reasons and discuss them. At the end of the discussion, the group will have to report the reasons for getting the different travelling times. Ten minutes will be devoted to this discussion in groups. By then, the group would have come up with the concept of rate as related to distance and time. The students would have arrived intuitively to the conclusion that if distance is being held constant, rate and time are inversely related and that the formula is distance = rate x time.

An analogy will be presented by the teacher at this moment. This analogy being that the relationship among distance, velocity, and time is the same as the relationship among the human body, exercise, and the weight of the person. After this point being made, each group will have to write up four problems relating distance, rate, and time and to solve them for homework. This will be the first session.

Similar procedure will be used on subsequent sessions but the students will be asked to plan different things. Plans for a dream vacation, going to a rock concert, going shopping at the mall, the class trip, a fund-raising activity for the class, going to the beach, and buying something expensive will be asked the students to make. A different activity will be used everyday. As in session one, the students will have to come up with four problems that they will have to do for homework. If there is a student that has not experienced the activity or is not interested in that particular activity can make plans for any other thing. Students are going to be grouped in groups of four. Groups won't be the same every time. The students will be asked to pose possible problems they might encounter in the plans. By tapping into their IKB, all the different types of problems will emerge from the students. By consensus in the group, students will select a particular type of problem to study during that session. Students will come up with a way or ways of solving that word problem. If an answer provided by a student encounters difficulty in being accepted by the group, the student will be asked to provide compelling reasons for his/her answers. Time will be allowed for the group to get consensus in the answer. Moreover, if there is a student having difficulty tapping his/her own IKB in solving these problems, that student will be allowed to engage in whatever activity he or she wants to engage. At any time during the session a student may ask a question about a homework problem. The solution to that problem will come from the group. Continuing with this process the students will cover all the different types of word problems. This will be achieved by the students tapping their own IKB and not just because the teacher has presented all the different types of problems.

At first the process might go slowly because students will have to go through many reorganizations of their IKB. As students are tapping their IKB, students begin to reorganize their IKB so that a more elaborate IKB forms through this constant reorganization. Thus, providing the student more ground for further reorganizations and reconceptualizations.

Teachers using this approach must meet the following conditions. First of all, they should be able to recognize the very broad themes. This teacher requires a very broad professional knowledge (IKB almost = PKB) in order to see the broad themes and not restrict his/herself to the known categories of the textbooks (Mayer, 1981). The teacher must also recognize that there is also more than one way to solve the problem. In math this is not very often the case but it might happen. What it is important to notice here is that although the teacher might see the connections in one way the student might see them coming from a different source and teachers have to respect that. What is important here is what makes sense (what taps) to the student's IKB since this is the one that is being reorganized not what makes sense to the teacher.

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