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ABSTRACT

Although it takes only 2 years to attain conversational competence in a second language, it takes up to 7 years to realize sufficient language competence to achieve academically at the level of native speakers. Specific adaptations in instructional methods in mathematics for language minority students should include techniques from English as a second language or bilingual education and those of current practices in mathematics education focusing on communication. This paper describes a method for teaching mathematical problem solving for use with students with limited English proficiency. The instructional method is based on ethnographic examination of techniques used in teaching mathematics to a sixth grade Spanish/English bilingual class of 30 students with a wide range of English competence. The five key components of the method are: (1) provide a linguistic warm-up to the problem; (2) break down the problem into natural grammatical phrases; (3) students work out the problem in pairs; (4) students present their own solutions to the group; and (5) students create problems with similar structures. Study results indicated that students became more successful independent mathematical problem solvers. Appendices contain a description of classroom procedure, a sample word problem broken down into natural grammatical phrases, and scoring criteria for pretests and posttests. (PVD)

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The Effects of a Communicative Approach on the Mathematical Problem Solving Proficiency of Language Minority Students

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The Effects of a Communicative Approach on the Mathematical Problem Solving Proficiency of Language Minority Students

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Introduction

Among its position statements, the National Council of Teachers of Mathematics (NCTM, 1995) has indicated its commitment to equity in mathematics education for all students including language minority students. Specifically the statement indicates that "cultural background and language must not be a barrier to full participation in mathematics programs" (p. 20). The Council also stresses that mathematical problem solving, with its demand for linguistic competence in discerning mathematical relationships, be emphasized in curricula more than arithmetic procedures and algorithmic routines (NCTM, 1989). But what exactly do these goals mean in practical terms for students with limited English proficiency? Surely it is not enough to change textbooks or provide grade level instruction for all students.

Research on achievement of language minority students tells us that although it takes only 2 years to attain conversational competence in a second language, it takes up to 7 years to realize sufficient second language competence to achieve in academic areas at the level of native speakers (Cummins, 1986). This discrepancy in mastery is particularly acute in the area of mathematical problem solving in which words and language are used to express not only concrete facts or examples, but relationships. These relationships may be overtly stated or, as is more often the case, have to be inferred from the phrasing of the problem situation. Thus, many mainstreamed students with limited English proficiency continue to face the difficulty of learning English as a second language while studying mathematics framed in language that can be confusing even to native speakers. The difficulty these students often encounter in mathematics classes and their poor performance on subsequent assessments of their learning of mathematics, therefore, may be unrelated to their potential for learning and understanding mathematics concepts and procedures. Rather many language minority students are likely to be forced into a pattern of failure simply because they do not yet understand the language in which mathematics problems and concepts are embedded.

Logic dictates, then, that if we are to take the NCTM's recommendation for mathematics for everyone seriously, we need to have some very specific adaptations in instructional methods for language minority students, particularly for those in mainstream classes with monolingual teachers. In general, these adaptations should include techniques from the field of English as a second language or bilingual education and those of current practices in mathematics education focusing on communication. Toward this end, we have developed a method for teaching mathematical problem solving for use with students who have limited English proficiency. The implementation and assessment of the effectiveness of this method provide the basis for our study.

The Instructional and Assessment Methods

The instructional method we developed began with an ethnographic examination of the techniques Rodrigo, the co-investigator, regularly used in teaching mathematics to his sixth grade Spanish/English bilingual class of approximately 30 students. This class contained students who had been in this country anywhere from a few weeks to just under three years, from a variety of Spanish speaking countries of origin, and with a wide range of English competence from none to near mainstream levels. The more advanced students were considered Level II English users as rated on the Maculaitis inventory of language skills. However, because many of the students were not in this country long enough to take the test, they were considered Level I English users.

The goal of our collaboration was to refine Rodrigo's use of bilingual and ESL techniques so as to bring them into line with the Curriculum and Evaluation Standards for School Mathematics issued by the National Council of Teachers of Mathematics in 1989 and reinforced by its later Professional Standards (1991) and Assessment Standards (1995). Further our goal was to make these techniques accessible to mainstream mathematics classroom teachers with ESL students.

We began by selecting several problems from the district's sixth grade mathematics textbook normally used in his class. Rodrigo then taught these problems to the students in his natural way utilizing bilingual/ESL techniques while I videotaped these sessions. Each problem took a minimum of one full class period to complete. Following the videotaping, we viewed the tapes together and I recorded the sequences of activities and techniques used. From these notes I was able to abstract a generic sequence of essential activities and processes in which Rodrigo engaged across problems. This sequence became the starting point for the instructional model during the subsequent week.

Over a period of several weeks, the teaching method underwent this progressive formative evaluation until finally five essential components, each with some degree of elaboration, became defined. Several processes remained central to the method throughout, but several other components emerged as we refined Rodrigo's techniques. In particular, a focus on phrases rather than on particular word meanings became salient. The five key components of the method appear in Appendix A and included:

1. Providing a **linguistic warm-up to the problem** - This component has its roots in bilingual educational techniques. It requires that the vocabulary and the situational context of the problem be

discussed before any mathematics is addressed. The warm-up provides a cognitive-linguistic set for students and, most importantly, enables them to attach some personal meaning to the problem.

2. **Breakdown of the problem into natural grammatical phrases** - This component should occur before students attempt to solve the problem. It is a key element in our technique derived from whole language and literacy instructional philosophies and goes beyond simple vocabulary building. Instead it focuses on the breaking down of longer sentences into natural grammatical phrases - a process in which students can take on increasing responsibility as the instruction progresses. The idea here is to teach the students a technique for word problem analysis - a way of understanding the meaning of the context and mathematical relationships expressed in the problem - and then to encourage them to use that process independently.

During the natural grammar phrase analysis process of this method, the meaning of the problem is derived from the utilization of a variety of non-verbal techniques adapted from ESL/bilingual education. These enable students to derive the linguistic and mathematical meanings embedded in the problems by using **graphic representations** (diagrams, charts), gestures and **physical enactments** and or role playing of the relationships in the problem, and by **rephrasing the problem components in their own words**. A sample of a problem broken down into natural grammatical phrases appears in Appendix B. The technique for using this problem would include:

1. Discussing transportation in general, leading up to different kinds of boats

2. Put problem on overhead and read through once

3. Have teachers read through once

4. Ask for first natural phrase break - take suggestions - then show first break on overhead

5. Ask for meaning of phrase - clarify with pictures where needed - Go phrase by phrase for vocabulary and meaning until problem is completed

6. Ask teachers to restate the whole problem in their own words - including mention of facts of problem and question that problem asks

(Clarification of question and meaning of question through physical demonstration with someone who is motor boat and someone who is sailboat would be appropriate).

3. **Students work out the problem in pairs**. During this process students are expected to provide both a solution and an explanation for why they did what they did. We suggest that students be provided with calculators to be used as needed.

4. **Students present their own solutions** to the group. Problem solutions and explanations are put on overhead transparencies to be shared with the group upon completion of the work. Two or three student pairs should share their work each day.

5. **Students create problems with similar structures** which are subsequently shared with the rest of the class and solved. These too may be conveniently placed on overhead transparencies.

The Data Collection Procedure

In order to assess the effectiveness of this technique, once developed, we utilized a pretest-posttest method of validation. To do this we selected a group of 20 problems from a supplemental problem solving book (Cook, 1987). These problems were presented in fairly simple language, but varied in mathematical content (fractions, whole number operations, logic) and leant themselves to a variety of solution processes (charts, diagrams, arithmetic). From among these problems, 8 were selected to serve as pretest/posttest assessment items. The other 12 problems were utilized for instructional purposes. What we wanted to do was very pointedly to focus the students' attention on how to analyze long word problems into their meaningful units of expression so that the important facts, relationships, and questions of the problem could be understood without direct instruction from the teacher. Therefore, we deliberately selected problems that would present less of an arithmetic challenge and more of a communication challenge.

Prior to conducting the formal instruction, we had students take the 8-problem pretest, first in English and then in Spanish, to serve as a baseline for assessing their growth as a function of the instructional period. On the pretests, students were instructed to obtain solutions and to offer explanations about how and why they got their answers. At these times as during some of the instructional periods, students were provided with the calculators normally used during their mathematics lessons. Following an 8-week instructional program using the techniques described above, the same problems were again administered (with numbers changed), first in English and then in Spanish. On the assessments, two problems were given per day until all items were completed. Each administration took approximately 2 weeks to complete. After students moved or were transferred to other classes, 24 students remained who participated in the complete program of instruction and assessment, 14 of whom were at Level II in English and 10 of whom were at Level I.

Following the administration of the posttest items, performance on pretest and posttests was compared. The assessments were scored for accuracy and for quality of explanations, each on a 4-point scale (see Appendix C for scoring system).

Results

Utilizing t-tests for related samples, comparisons were made between test scores within each language. We did these analyses for the class as a whole, for the 14 members of the class who had the highest levels of English proficiency, and for the 10 students who had minimal or essentially no competence in English. Results pertaining to the 14 students with the greater English proficiency are presented here because these students are most like the mainstreamed ESL students that the majority of monolingual teachers deal with daily.

As depicted in the Table found in Appendix D, we found that:

1. Students did not demonstrate significant differences in their accuracy scores for problem solving when taken in English or Spanish although there was a strong trend for scores to be higher in Spanish both at pretest and posttest times (Pretest $t(13) = 1.77$, $p = 0.097$; Posttest $t(13) = 1.73$, $p = 0.104$).

2. Students significantly increased their accuracy scores in both English and Spanish from pretest to posttest times, (Spanish ($t(13) = 4.66$, $p < .01$; English ($t(13) = 2.53$, $p < .05$).

3. At posttest time, English accuracy scores were higher than Spanish pretest accuracy scores, though not quite significantly so ($t(13) = 1.73$, $p = 0.105$).

4. Students demonstrated virtually equivalent explanation scores in English and in Spanish. This was the case at both pretest and posttest times (Pretest mean for Spanish and English = 1.01; Posttest = 1.20).

5. Students did not significantly increase their explanation scores in either English or Spanish from pretest to posttest time, although there was a trend toward an increase in these scores in both languages (Spanish $t(13) = 1.16$, $p = 0.265$; English $t(13) = 1.04$, $p = 0.317$).

Outcomes and Implications

Using pretest-posttest assessments, we found that our method for teaching mathematical problem solving to language minority students utilizing a combination ESL/bilingual and current mathematics education teaching techniques enabled them to become more successful independent mathematical problem solvers. On the one hand the techniques enabled these students to apply those arithmetic procedures with which they were familiar to contexts that went beyond rote computation. It also provided them with tools for independent work in mathematics. On another level, the problem solving activity itself provided these students with another channel for raising their use of English in academic contexts - it acted as a language building tool.

We believe that the techniques that were applied here can be applied to other subject areas as well and that they can provide monolingual teachers with an improved method for communicating with language minority students. It does this through the introduction of a problem analysis strategy that can be incorporated into lessons at varying levels for students at different levels of linguistic competence. Further, the method should be adaptable and useful for working with native English speaking students who have difficulty with mathematics word problem analysis because of limited literacy skills or habits of avoidance that have developed around word problem analysis.

We now would like to expand the piloting of this technique and evaluate its effectiveness with monolingual teachers who have ESL students mainstreamed in their classes. In this context, we have considered systematically varying some of the components to determine which are essential and which might be further adapted, eliminated, or elaborated. We would also like to extend the range of mathematical problems used so that more mathematically challenging open-ended problems are included in further piloting and assessment of the method. Finally we would be interested in training a cadre of teachers in the school district in which the study was conducted in using this technique and assessing the extent to which it impacts on students' performance on a statewide competency test taken in eighth grade.

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APPENDIX A - PROBLEM SOLVING PROCEDURE UTILIZING NATURAL GRAMMAR PHRASES

1. **Linguistic Warm-Up: Focus discussion on special vocabulary words and problem context**

2. **Natural Grammatical Phrases Breakdown**

   A)Teacher reads the whole problem through slowly and fluently, as it is projected on the overhead.

   B)Teacher has students repeat the fluent reading exercise (meaning not addressed at this point)

   C)Teacher models phrase-by-phrase reading of the problem for meaning - (use gestures, pictures, translation, personal experience, other English words; spiral the reading of phrases until the whole problem has been discussed in terms of meaning)

   D)Students repeat phrase-by-phrase reading of the full problem

   E)Students take responsibility for phrase-by-phrase analysis

   F)Ask students to rephrase problem in their own words

   G)Ask students to record the facts and the question of the problem

3. **Have students solve problems in pairs including explanations** (have some pairs write their solutions on overhead transparencies)

4. **Have students present problem solutions on overhead transparencies and discuss**

5. **Ask students to write problems that are similar to the one done and share their created problems and solutions**

APPENDIX B - MOTORBOAT PROBLEM*: AN EXAMPLE OF "NATURAL GRAMMAR PHRASES"

A motorboat travels 30 miles in the time a sailboat travels 10 miles. At that rate, how far will the motorboat travel when the sailboat has traveled 60 miles?

A motorboat travels 30 miles

in the time

a sailboat travels 10 miles.

At this rate,

how far will the motorboat travel

when the sailboat

has traveled 60 miles?

* Problem adapted from: Cook, M.C. (1987). Numbers and words: A Problem per day. New Rochelle, NY: Cuisenaire Company of America.

APPENDIX C - SCORING CRITERIA FOR PRETEST AND POSTTEST

Computational Accuracy Score

0 Answer is missing or completely irrelevant to problem

1 Answer is completely incorrect

2 Answer is almost correct except for a small computational error

3 Answer is completely correct

Quality of Explanation **(Scored regardless of accuracy of answer and appropriateness of solution strategy)**

0 No explanation or diagram

1 Student provides only a sketchy explanation of the solution strategy

2 Student provides a detailed step-by-step account of what was done *without telling why* the strategy was used

3 Student provides an accurate, complete, and reasonably clear explanation describing step-by-step what was done *and why* the solution strategy used (even if the answer has a computational error)

APPENDIX D - RESULTS OF THE STUDY IN TABLE FORMAT

Comparison of Mean Pretest and Posttest Scores for Accuracy and Explanations on Problem Solving Assessment of Level II Students

	Accuracy			Explanation		
	English	Spanish	t-score	English	Spanish	t-score
Pretest	1.76	1.97	**1.77**	1.01	1.01	----
Posttest	2.28	2.55	**1.73**	1.20	1.20	----
t-score	**2.53***	**4.66****		**0.317**	**0.265**	

* $p < .05$

**$p < .01$

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