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ABSTRACT

The concept of Arithmetic Story Grammar is introduced. Arithmetic Story Grammar maintains that any equation containing a single operand tells, in and of itself, a complete story with a setting, plot, theme, and resolution. Multiple-operand equations differ from their single-operand siblings only in the number of actions undertaken and the number of themes. There are similarities between reading and mathematics that make extending the use of story grammar beyond the reading of graphemic text and into mathematical story problems and ideographic script. A conceptual model of single-operand equation is introduced that roots itself in literary analysis. Research has found that familiarity with syntax and semantics facilitates comprehension and mathematical problem solving. Instruction in story grammar provides positive effects on the comprehension of stories. Only one study has been located regarding story grammar and mathematics. The study's results were inconclusive, but the approach seems promising for enhancing problem solving. A taxonomy of mathematical story genres is presented, based on the analysis of addition, subtraction, multiplication, and division in single-operand stories and equations. (Contains 1 table, 4 figures, and 65 references.) (SLD)

Arithmetic Story Grammar:

Using Literary Devices to Analyze and Categorize Story Problems

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Abstract

Three purposes drive this paper: 1) to delineate the similarities that exist between reading and mathematics; 2) to extend the use of story grammar (Guthrie, 1985) beyond the reading of graphemic text as it relates to short stories and into the expanses of mathematical story problems and ideographic script; and 3) to introduce a conceptual model of single-operand equations that roots itself in the loam of literary analysis rather than in the sands of the mathematical strands. To achieve the first purpose, an overview of the similarities between mathematics and reading is provided that examines the influences of syntax, semantics, readability, and story grammar on comprehension of and facility with reading and mathematics. It was found the familiarity with syntax and semantics facilitates comprehension and problem solving. Also, instruction in story grammar generally provides positive effects on the comprehension of stories. Only one study regarding story grammar and mathematics could be found; its results were inconclusive.

The second and third purposes are severed by providing a description of Arithmetic Story Grammar, a conception of literary devises useful in analyzing arithmetic stories, and illustrations of how it is applied. A taxonomy of mathematical-story genres based upon the analyses of addition subtraction, multiplication, and division single-operand stories and equations is presented. To culminate the paper, implications are investigated and suggestions for further research are made.

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Most often, mathematics and reading are viewed, more or less, as independent fields requiring unique skills. Mathematics is frequently associated with spatial and quantitative abilities while reading is viewed as one aspect of verbal abilities (Gage and Berliner, 1988; Harris & Sipay, 1990). Seldom do researchers of mathematics intrude into the verbal domain of the scribes, and, likewise, those who question reading tread infrequently upon the ground dominated by the number masters. Most often researchers align themselves with both only when boundaries overlap naturally, in fields where words and numbers occur together, such as in the valleys where story problems thrive. The implicit pact existing between the two camps of researchers to make but few incursions into the others' kingdom could be easily understood if, indeed, mathematics and reading were truly independent of one another. This, however, is not the case.

Most psychologists support the construct of general intelligence. If an individual demonstrates a high propensity to accomplish in one field, there exists the tendency to show similar abilities in other areas, also. Mathematical and reading abilities are just so related; both are viewed as aspects of general intelligence, (Gage & Berliner, 1988), and both are associated with symbol systems and syntactic structures from which meaning must be derived.

The purposes of this paper are to establish similarities between reading and mathematics; to extend the use of story grammar (Guthrie, 1985) beyond the reading of graphemic text as it relates to short stories and tall tales and into the expanses of mathematical story problems and ideographic script; and to introduce a conceptual model of single-operand equations that roots itself in the loam of literary analysis rather than in the sands of the mathematical strands. To achieve these purposes, it is first necessary to provide an overview of the similarities between mathematics and reading and to review previous observations regarding fluency and prior knowledge. This will be followed by a description of Arithmetic Story Grammar. To culminate the paper, implications will be investigated and suggestions for research will be made.

AN OVERVIEW OF RELEVANT RESEARCH AND THEORY

To convey meaning, symbols must be read. To impart messages, symbols must be interpreted. No one questions the necessity of teaching students to read messages constructed of letters. Such instruction goes unchallenged, unquestioned; it happens as a matter of course. However, the proposition that students must be taught to read mathematical symbols, receives little consideration.

Mathematicians and practitioners pen mathematical symbols to freeze and convey meaning. However, for the symbols to convey meaning, a reader must consider and interpret them. In equational form, arithmetic symbols express stories filled with meaning particular to circumstance. Research provides much insight into the reading of the graphemic text of literature but reveals little regarding the comprehension of the ideographic text of mathematics.

The review of literature that follows investigates research related to the comprehension of text and attempts to establish a rationale for teaching students to read mathematics and understand Arithmetic Story Grammar. Further, through implication, it is hoped that Arithmetic Story Grammar may be deemed worthy of consideration when designing curriculum and planning instruction. Arithmetic Story Grammar offers a new paradigm for interpreting operational equations depicting mathematical situations. In contrast to the single-premised, action-based models currently in use, this new paradigm has its basis in four basic story elements: setting, theme, plot, and resolution (Carpenter & Moser, 1983; Greer, 1989; Guthrie, 1985; Hinsley, Hayes, & Simon,1977; Mayer, 1985).

Similarities between Mathematics and Reading

The Act of Reading

Abstract, Meaningful Symbol Systems

Mathematics and reading are both highly associated with abstract symbol systems that convey meaning (Brodie, 1989; Gallistel & Gillman, 1990; Harris & Sipay, 1990; Hegarty, Carpenter, & Just, 1991; Kane, 1970). To secure meaning from the symbols, they must first be decoded and then comprehended (Dee-Lucas & Larkin, 1991; Harris & Sipay, 1990). Decoding alone, though, provides no understanding of the text. The decoder must ascribe meaning to the symbols (Dee-Lucas & Larkin, 1991). Comprehension of the text occurs only when the reader reconciles the meanings of associated symbols with the syntactic structures in which they are found. Once this is accomplished, comprehension of text is achieved (Harris & Sipay, 1990; Romberg, 1990; Wearne, & Hiebert, 1988).

Semantics in Reading and Mathematics

In both mathematics and reading, the term semantics refers to meaning. The semantics of symbols and their various combinations in both mathematics and reading are constructed through individual reflection on interactions with others and the environment. The greater the familiarity with the meanings inherent in the symbols encountered, the greater the comprehension one achieves (Clements & Battista, 1990; Kamii & Lewis, 1990; Mtetwa & Garafalo, 1989; Mumme & Shepherd, 1990; Trafton & Bloom, 1990; Vygotsky, 1978). Semantic mastery of both the symbols and the language they represent is crucial to success in both mathematics and reading (Brodie, 1989; Gallistel & Gelman, 1990; Kane, 1970; Harris & Sipay, 1990; Hegarty, Carpenter, & Just, 1991).

To develop a high level of semantic understanding in either mathematics or reading, the student must engage in a variety of experiential activities (Brady, 1991; Clements & Battista, 1990; Crosswhite, 1990; Kamii & Lewis, 1990; Romberg, 1990; Wearne & Hiebert, 1988). Such experiences provide the student with familiar referents, enabling the bridging of gaps between concrete materials and the abstractions devised to represent them (Trafton & Zawojewski, 1990). Individuals that develop concrete, semi-concrete, or semi-abstract referents through experiences with such materials often outperform those who have not (DeCorte & Verschaffel, 1987; Pearson & Ferguson, 1989). Verification of initial understandings occurs through social interactions (Clark & Stahle, 1991; Lampert, 1988; Research Advisory Committee, 1988; Vygotsky, 1978; Yackel, Cobb, Wood, & Merkel, 1990). When semantic structures are in place, children make fewer mistakes and often invent their own methods for finding solutions (Quintero, 1985; Trafton and Bloome, 1990).

Fluency and Prior Knowledge

Vocabulary, Syntax, and Conventions

Further, in both mathematics and reading, comprehension is mediated and facilitated by prior knowledge (DeCorte & Verschaffel, 1987; Engelhardt & Usnick, 1991; Harris & Sipay, 1990; Kintsch & Greeno, 1985; Lave, 1985). If an individual possesses little, imperfect, or no prior knowledge of a given matter, accurate interpretation of the symbols and, perhaps, the syntax is doubtful. Imparting such knowledge is the job of the teacher, but, regarding mathematics, few teachers possess the requisite skills (Brown, Cooney, & Jones, 1990).

Instruction provided in vocabulary, syntax, and conventions such as punctuation and directional flow facilitates fluency (Aiken, 1972; Harris & Sipay, 1990; Nibbelink, 1990; Singer & Donlan, 1980; Stockdale, 1991). A hoped for result in both mathematics and reading is the attainment of automaticity with the most common symbols and combinations. Some claim that automaticity with a skill demonstrates mastery of the underlying concepts. However, others insist that mastery is achieved only when it is accompanied by comprehension (Harris & Sipay, 1990; Usnick, 1991; Wearne & Hieberg, 1988).

When it comes to mathematical word problems, much study has been done and many factors have been considered. Chief among these are problem length, readability, key words, clarity of discourse, and vocabulary. Problem length was found to have no effect on student success (Paul, Nibbelink, & Hoover, 1986). Further, the formal concept of readability used to evaluate narrative discourse was shown to be inapplicable for classifying mathematical text due to the use of ideographic symbols and specialized terms (Kane, 1970; Paul, Nibbelink, & Hoover, 1986). Utilizing clearer text and common language does improve problem-solving by making problems easier for students to understand and model (Aiken, 1972; Carpenter, 1985; Davis-Dorsey, Ross, & Morrison, 1991; DeCorte, & Verschaffel, 1987); however, neither factor is associated with the semantic structure of the problem. The use of clearer text and common language merely facilitate access to a problem's structure.

Other research focusing on language has shown that instruction focusing on vocabulary development also increases problem solving (Siegel, Borasi, & Smith, 1989). However, keyword strategies, often taught by teachers and employed by students, are nearly worthless in helping to find appropriate solutions. Encouragements to use keywords to solve problems unwarily lead to strategies that focus on surface-level meanings. As of result of this, deep meanings are ignored

(Adetula 1989; Stockdale, 1991). However, as reading ability and comprehension of the text increase, so does an individual's ability to solve word problems (Moyer, Moyer, Sowder, & Threadgill-Sowder, 1984).

Story Grammar

Theorists of both mathematics and reading envision users of the two systems developing and using schematic structures. In mathematics, no term has yet been firmly ascribed to differentiate one type of structure from another. In reading, however, schematic structure is known as story grammar, macrostructure, or story schema (Pearson & Camperell, 1985). Story grammar may also refer to the set of rules used to describe a story's structure or schema (Guthrie, 1985). The terms story grammar and story schema apply to most short stories equally well. Most short stories, fables, and tall tales possess the four basic elements of story grammar: 1) setting; 2) plot; 3) theme; and 4) resolution. The setting consists of several subunits, e.g., characters, location, and time. The sequence of actions associated with the main characters depicted within the story comprise the plot, whereas the theme entails the unifying idea or archetypical experience expressed within the literary work. The resolution embodies the result of the actions taken within the story that resolve the conflict implied by the theme. These four elements follow a fairly standard set of recursive rules that, when understood, facilitate increased comprehension (Guthrie, 1985; Pearson & Camperell, 1985; Singer, 1985; Singer & Donlan, 1985).

An understanding of story grammar or schemata develops through repeated exposure to a particular genre. The typical structure of a genre is learned and related to the individual's life experiences, thus facilitating comprehension. This familiarity provides a mental framework from which to hang salient portions of the text. From the textual, schematic, and personal information deemed to be relevant, meaning is construed and acted upon (van Dijk & Kintsch, 1985). In reading, the story schema enables the reader to anticipate action and make sense of the plot (Harris & Sipay, 1990).

By age six, most children begin to internalize story grammar and use this knowledge to comprehend the major elements of the story. As children grow older, their knowledge of story grammar becomes more acute, fostering expectations that are more differentiated and precise (Guthrie, 1985). The further removed the schema of a story stands from standard story grammar, though, the less the reader tends to remember. The more obtuse the theme becomes, the fewer the details the reader recalls; the same holds true for other story grammar elements, as well (Pearson & Camperell, 1985).

Providing students with direct instruction regarding the elements of story grammar has proven beneficial. Gordon (1980) found that fifth-graders could be taught to assess basal stories using a simplified story schema and that the skill was transferable to previously unread stories. Students who received instruction recalled significantly more details than students in a control group who received no instruction. This held especially true regarding certain categories of high-level information. Singer and Donlan (1982, 1985), through direct instruction, taught students to use a problem-specific schema and ask self-generated, story-specific questions that improved their comprehension of the compelling meanings within a given story.

More recently, Rekrut (1992) found that the interaction of requiring fifteen-year-old students to tutor twelve-year-old students in the elements of story structure, after they themselves had received instruction, fostered greater comprehension and learning for the tutors. Here, however, neither of the main effects provided for significant differences. Yet, Leaman (1993) found that providing direct instruction in the elements of story grammar to the learning disabled facilitated greater success in both comprehending and generating stories. Additionally, Davis (1994) provided instruction to third and fifth-grade students in story grammar during a prereading mapping of a story and followed this with a challenge to find the accuracy of their predictions. Davis found that this resulted in a 14 percent positive difference for third-graders over students having received a directed reading activity treatment. The results revealed no differences for the fifth-grade students in her sample. Davis (1994) attributes this to either differences due to individual development or to the stories with which the two levels of students worked.

To date, only one study has yet linked mathematics and story grammar. In a 2X2X2 repeated-measures investigation of the effects of an experimental problem format, Arithmetic Story Grammar, and quick writing on students' arithmetic abilities, attitudes, and levels of anxiety, Gilbert (1995) provided sixth-grade students with direct instruction in story grammar as it related to single-operand arithmetic equations. After providing fifteen-minute periods of instruction in story grammar for 27 days, only the quick writing proved significant, but only in affecting attitudes and levels of anxiety. Gilbert attributed the lack of positive cognitive results to the newness and complexity of the material presented and the limited time available for instruction. In contrast, however, much research has been conducted related to the story formats and solution strategies of word problems.

Format research divides into three distinct levels. At the apex of this research lies a classification system that crosses operational boundaries and identifies five general schemata that are based on the semantics of arithmetic story problems. Research with adults demonstrates that, once learned, individuals use schemata to facilitate correct solutions for some problem types (Marshall, Pribe, & Smith, 1987).

The intermediate level, also based on semantics, categorizes problems by the kinds of actions occurring within the stories. Carpenter & Moser (1983), devised one set for addition and subtraction; Greer (1989) developed another for multiplication and division. These taxonomies, thus far, have been used mainly to identify the strategies used by students in finding solutions and to ascertain levels of individual proficiency.

The bottom layer of investigation, problem categorization, deals with story schemata of a more superficial nature. Such schema are found within algebra and geometry problems that center on the situations described, e.g., river current, work, interest, and triangles. Such situations could be described as story-problem genres. Though the recognition of the various genres empowers the reader to perform the required tasks with little delay and few misunderstandings (Kintsch & Greeno, 1985; Mayer, 1985), several problems exist. Chief among these is that teaching problems by kind is the antithesis of problem solving. This becomes obvious when students are presented with a new format and complain that they have never seen such a problem before (Sowder, 1985). Schoenfeld (1985) expressed concern that such strategies may be used without regard to metacognitive monitoring. Another problem is that such taxonomies are never complete, making it difficult to identify the genre most efficacious to teach. In 1977, Hinsley, Hayes, and Simon identified 18 categories; in 1981, Mayer identified over 100 (Mayer, 1985).

Restricting models to only a few creates other problems when teaching students to use algorithms. This restriction hinders the students in developing correct rules. The rules they develop and internalize often fit the limited situations well; however, they interfere with complete concept attainment at later stages of learning (Carpenter, Moser, & Bebout, 1988; Thompson, 1991; Wearne & Hiebert, 1988). A limited understanding of syntax restricts facility with the symbol system. The syntax must be cognitively connected to semantic structure for generalization and abstraction to occur (Wearne & Hiebert, 1988).

Other research related to solution strategies has confirmed that even when students appear to have one-step story problems mastered, they may be dependent on inadequate or inappropriate strategies to compute answers. Such strategies may have nothing to do with semantics; instead, students may focus on irrelevant factors such as recent class activities, the sizes of the numbers involved, or they may use all four operations to provide a set of answers from which the most reasonable is chosen (Sowder, 1989). De Corte and Verschaffel (1987) demonstrated that when students did focus on the semantics, choices of solution strategies depended on the structure of the individual problems. Usiskin and Bell (1983), commenting on pedagogical practices, state that students possess little more than their own intuitions or fixed rules, such as keyword strategies, when attempting to choose appropriate operations.

In one attempt to provide students with skills that facilitate problem solving, first and second-grade students were taught to write equations in noncanonical forms, mirroring the actions the story problems presented. As a result, correct solutions increased

(Carpenter, 1985). Further, Carey (1991) found that children often write equations that model the actions depicted within story problems. Other research has shown that as aid increases in modeling story problems, correct solutions also increase (Ibarra & Lindvall, 1982).

Villaseñor and Kepner (1993) used a technique dubbed Cognitively Guided Instruction (CGI) with urban first-graders and found significant improvement in solving word problems for the CGI group when compared to a control group. The CGI technique focused on teaching the subjects to identify problem types and to apply the strategies that were most often used to solve them. The technique centered on the Carpenter taxonomy (Carpenter & Moser, 1983) and subsequent work that investigated the strategies children employ when working with word problems (Carpenter, Moser, & Bebout, 1988).

It is unfortunate that the research conducted to date has focused primarily on only one side of the problem. All but one of the studies reviewed have dealt solely with the graphemic text of word problems, focusing upon the story, its structure, the information provided within it, and how the individual interprets and acts upon the information.

A much neglected aspect of research and teaching is the highly formalized schematic representation of the numeric equation. Though voices have called for investigations of ideographs and number sentences (Siegel, Barosi, & Smith, 1989; O'Mara, 1981), only the one by Gilbert can be found. Of the models presented to date to categorize schematic representations, it is the work of Usiskin and Bell (1983) that comes closest to the construct of Arithmetic Story Grammar presented here. These authors proposed a taxonomy for operational class meanings and uses, i.e., the definitions and uses of the operations. Their definitions and descriptions approach those provided by Arithmetic Story Grammar; however, after defining class meanings and uses, the authors failed to synthesize a construct that would relate their taxonomy to the reading of ideographic text.

ARITHMETIC STORY GRAMMAR

Individuals who read mathematical story problems tend to focus on the problem rather than the story. While concentrating on creating an equation that represents a satisfactory means of achieving a solution, the problem solver generally ignores the structure of the story and its constituent elements. To develop an equation, the solver appears to restructure the story into a problem so rapidly that vital transformational steps go unrecognized (Gilbert, 1992). As an illustration of this, consider the following problem taken form Carpenter and Moser (1983):

> "Connie has 13 marbles. Five are red and the rest are blue. How many blue marbles does Connie have?"

Carpenter and Moser (1983) classify this problem as a *Combine subtraction* problem. First, the authors state that the problem is static, which is reasonable because the set of marbles remains unchanged. It appears that the problem is classified as a subtraction problem because subtraction is the expeditious means of arriving at an answer to the question. However, if subtraction is defined as the act of removing or losing, either physically or hypothetically, it is impossible to combine and subtract in one act. Since nothing in the story is being taken away or lost, the story should not be classified as a subtraction story.

Using the tenets of Arithmetic Story Grammar, defined later, the story is classified as one of addition and action becomes apparent. The story provides a resolution of 13 marbles in all. It provides a setting in which five marbles are red and the rest are blue. It provides action, implied by the question, that when the five red marbles are hypothetically combined with the remaining marbles, which are blue, all 13 marbles will be accounted for. The agent of the action may be assumed to be the problem solver. Combining the setting with the plot results in a theme that most logically translates into addition since the five red marbles must be combined with the remaining marbles, which are blue, to yield the entire set of 13. It is possible to restate this story as follows: Connie's five red marbles when considered with the remainder of her marbles, which are blue, yield the total of 13 marbles. Written as an equation, this becomes $5 + \square = 13$.

Further, it appears that most individuals lack the conceptual resources that allow for the classification of mathematical stories beyond the fundamental categories of addition , subtraction, multiplication, and division. This appears to be the result of curricular content and instructional practices, both past and present, that fail to articulate the structural nuances of mathematical stories Gilbert, 1992).

As mentioned earlier, Singer and Donlan (1982)demonstrated that after students were taught to identify story elements, story comprehension improved. It has also been demonstrated that when an individual understands the structure of an object or a set of ideas and possesses the ability to classify according to that structure, comprehension improves (Gage & Berliner, 1988). Guthrie (1985) describes a structure for classifying story elements that others have found effective in fostering comprehension of short stories (Davis, 1994; Leaman 1993). However, no such structure existed for mathematics until now.

In an effort to provide mathematical stories such a structure, the elements of story grammar have been applied. The resulting analysis yielded several interesting results: 1) A realization that all single-operand equations are not only sentences, they are stories, as well; 2) Mathematical stories readily divide into a rather limited set of categories, subcategories, genres, and scenarios; and 3) An articulated structure for describing these categories, subcategories, genres, and scenarios did not exist. The structure that resulted from this work has been dubbed Arithmetic Story Grammar and is described below.

The Basic Structure

Arithmetic Story Grammar maintains that any equation containing a single operand tells, in and of itself, a complete story consisting of four basic elements: setting, plot, theme, and resolution. This premise holds regardless of whether or not the single-operand equation stands alone or represents one small part of an intricate mathematical process. Complex equations merely express a series of mini-stories that, when combined, reveal complex tales filled with blunt acts and subtle nuances. Multiple-operand equations differ from their single-operand siblings only in the number of actions undertaken and, perhaps, by the number of themes maintained.

Arithmetic Story Grammar begins by assuming four basic categories: addition, subtraction, multiplication, and division. When analyzed under the light of setting, plot, theme, and resolution, these categories reveal eight subcategories and 18 basic story types, each forming a distinct genre. Further analysis reveals that three of the genres, all specific to division, present two different scenarios each. Two of the 18 genres, one addition, the other subtraction, differentiate no lower than their subcategories, making these, their genres, and their scenarios identical. The remaining 13 genres maintain only one scenario each. As a result, each of these 13 genres and their individual scenarios mirror each other. In all, the 18 genres manifest 21 distinct scenarios. The remainder of this paper endeavors to provide descriptions of the four basic categories, the four elements of Arithmetic Story Grammar, and the 18 genres as well as any associated scenarios. Table 1 displays the discussion above which resulted from the interplay of following descriptions.

Table 1
A Story-Grammar Based Taxonomy of Single-Operand Story Problems and Their Equations

<table>
<tr><td>I. Addition
A. Joining
1. Expository addition
2. Comparisons with one replication
3. Comparisons with two replications
B. Increases due to secondary agents</td><td>II. Subtraction
A. Removing
1. Expository subtraction
2. Comparisons with one replication
3. Comparisons with two replications
B. Decreases due to secondary agents</td></tr>
<tr><td>III. Multiplication
A. Multiplying Sets by Magnitude
1. Sets of Sets
2. Comparisons
3. Probabilities
B. Multiplying Components by Components
1. Composites
2. Combinations</td><td>IV. Division
A. Divisions Involving Magnitude
1. Sets of Sets
a. For magnitude (to find the number sets)
b. By magnitude (to find the number of units per set)
2. Comparisons
a. For magnitude (to determine the ratio)
b. By magnitude (to find size of second set)
3. Probabilities
a. For magnitude (to find the statistical rate per attempt)
b. By magnitude (to find statistical average of necessary attempts)
B. Divisions not Involving Magnitude
1. Composites
2. Combinations</td></tr>
</table>

The Four Elements of Story Grammar

All single-operand equations possess four fundamental elements: setting, plot, theme, and resolution. Sometimes these elements are manifestly clear. Occasionally, they lay hidden and are only implied by the contents of the stories and the equations that depict them.

Setting

The setting may include characters, time, place, and quantities. The characters may consist of elements that act or those that are acted upon. The equation, depicting the story in a highly abbreviated and cryptic form, seldom reveals any nonnumeric descriptive information about the quantities involved. Unless labels are ascribed, the undisclosed information about the type of units remains the mental property of the equation's author or it becomes known only by cross-referencing the equation with the actual story.

Plot

For each equation containing an operand, there exists a coinciding arithmetic plot. Each plot shares two qualities with all others: 1) Each possesses an operational condition depicting action; and 2) Each makes a statement regarding either yields or equivalencies. In subtraction, the operational condition may convey the action conducted upon the initial quantity. In addition, a single operand may reflect action taken upon two separate quantities. In multiplication, the operand might relate the relationship existing between two factors, whereas, in division, it may represent an act upon a quantity by one of its constituent factors. Statements of yield display operational outcomes and may reveal units fundamentally different from those of the setting. Statements of equivalency compare the quantities on one side of an equation with those found on the other side.

The simplest plots of single-operand equations possess only two actions, one specified by the operand, the other illustrated by the equal sign. Other arithmetic plots of single-operand equations brim with activity that remains all but hidden due to the terse nature of mathematical script. These tacit actions only become apparent when considered in tandem with the associated circumstances.

Theme

The intent behind the equation indicates the theme. Themes become apparent when the setting and plot are considered together. Personal perspectives allow many situations to be interpreted in two or more ways. For example, a balloon's growth due to an increase in temperature may be interpreted as either addition or multiplication. Themes include the following: 1) combining or separating like units; 2) increasing or decreasing; 3)comparing like units; 4) finding sets of sets or reversing the process; 5) combining similar or dissimilar units to form new units or reversing the process to reveal the components of melded units; 6) matching the individual units of one set with those of another, one at a time, to reveal the number of combinations or vice versa; and 7) finding the most likely outcome or the chance of an occurrence for a specific number of trials. Refer to Table 1 to see where a theme can be applied.

Resolution

In Arithmetic Story Grammar, the resolution is the end of the story, the consequence of the plot upon the elements of the setting. This definition is a bit looser than the one applied to literary stories. In nonarithmetic short stories, often a conflict exists that must be resolved. In arithmetic stories, however, seldom does a conflict appear; usually the situation merely requires a conclusion. As a result, the resolution may best be looked at as the yield or a statement of equivalence. In addition, yield may be exemplified as the sum resulting from the joining of two separate sets, whereas, equivalence may be demonstrated with an equation that represents a comparison of two sets on one side of the equal sign with a third set on the opposite side. Depending on individual perspective, a set that appears to be the resolution to one person may be interpreted as a part of the setting by another. Regardless of how a story is interpreted, the resolution is that quantity that results from the actions within the story. It is not the final quantity derived by a problem solver.

The Four Mathematical Categories

Addition, subtraction, multiplication, and division comprise the four mathematic categories. Below, the nature and uses of each is described. Three premises constrained the descriptions: 1) Base the descriptions on the tenets of story analysis; 2) Keep the descriptions simple; and 3) Make the descriptions as universal as possible.

By examining equations under the lens of story analysis, the focus changes from an emphasis of achieving equational balance and finding solutions to understanding plots and resolutions. Keeping definitions simple proves vital for providing efficient instruction and developing accessible curriculum. The same interests make universal descriptions essential, as well. The definitions that follow appear complete enough to accommodate all conditions involving magnitudes of both real and imaginary numbers. In some cases, vectors are also accommodated, but only when the operation affects magnitudes along a single axis. Such conditions create apparitions, providing illusions that only magnitude exists. All single-operand equations involving vectors are in actuality abbreviated expressions of a series of operations. Due to an spoken desire to maintain simplicity, often when the manipulation of vectors does not effect direction, only the magnitude is quantified.

Addition

Addition is the act of considering and joining sets of like units with similar or dissimilar quantities two at a time, or it is the act of increasing a set via a secondary agent. The often implicit condition that units possess identical monikers is germane to Arithmetic Story Grammar regarding settings and resolutions. If two or more uncommon units are to be added, a commonality must first be found. If one is not found, the units are not suited for the intended action. In practice, however, changing the classifications of unlike units to like units is achieved tacitly. For example, adding two cats to three dogs, yields five animals. Specifying a commonality, even one as nebulous as *thing*, whether before or after summing the addends, remains essential.

Addition divides into 2 subcategories: Joining and Increases Due to Secondary Agents. Examples A.1, A.2, and A.3 in Figure 1 illustrate joining, whereas, the fourth demonstrates increasing.

Figure 1
Addition

A. Joining

1. Expository Addition
   Example: John had two apples. He picked three more. How many apples does he have now? (2 + 3 = □);

2. Additions Requiring One Replication
   Example: John had two apples more the Gail. Gail picked three. How many apples does John have? (2 + 3 = □);

3. Additions Requiring Two Replications
   Example: John had two apples. Gail had three apples. Together they had as many apples as Lisa. How many apples does Lisa have? (2 + 3 = □);

B. Increases Due to Secondary Agents:
   Example: A balloon occupied three cubic feet. After being heated, the balloon's volume increased one cubic foot. What is the volume of the balloon now? (3 + 1 = □).

Joining

Joining combines the elements of one set with the elements of another and manifests three distinct genres: Expository Addition, Additions Requiring One Replication, and Additions Requiring Two Replications. The equations for the three genres look identical; the differences appear in the stories behind them. Though the three equations look the same, each represents very different conditions.

Equations may be homographic, appearing the same but expressing different conditions. Such equations may be read tersely to specify only number, operation, and outcome, or in expanded form to fill in details of comparison and kind. The goal should not be to foster expanded readings of equations. Rather, it should be to develop schema that allow consideration of the possibilities an equation may represent.

Expository Representations. Expository Representations tender story information in a forthright manner. In addition, two of the three sets indicated by the equation are clearly stated. The theme underlying expository equations is the unification of complete sets. Two of the equation's three sets are joined to yield the combined set. Consider example A.1 in Figure 1:

> John had two apples. He picked three more. How many apples does he have now? ($2 + 3 = \square$)

Though not required by the genre, this example gives two addends. A story may specify only one addend, providing the resolution is also clearly stated. Situations, regardless of the operation specified, failing to enumerate both elements of the setting yield noncanonical equations. Here, two addends comprise the setting, and the sum is the resolution. To realize Combining these addends produces the resolution.

Additions Requiring One Replication. Additions Requiring One Replication provide a complete set and, through comparison, one of two constituent subsets of another set. Consider Example A.2 of Figure 1:

Example: John had two apples more the Gail. Gail picked three. How many apples does John have? ($2 + 3 = \square$);

This example states Gail's set clearly and provides a model for John's. Replication of Gail's set produces the first addend, the number of apples John held in common with Gail. The comparison of John's set to Gail's gives the second addend. If Gail's set is not replicated, Gail's set is joined to John's, yielding an inaccurate model, containing only one set instead of two as specified in the story.

Additions Requiring Two Replications. Additions Requiring Two Replications also make comparisons, but resolution is achieved only after replicating both of the given sets. Consider Example A.3 of Figure 1:

John had two apples. Gail had three apples. Together they had as many apples as Lisa. How many apples does Lisa have? ($2 + 3 = \square$)

A model consistent with the story must contain three sets, John's, Gail's, and Lisa's. Construction of this model requires that Gail's and John's sets be replicated. Joining John's and Gail's sets produces a model of Lisa's set but fails to depict the story accurately. The second subcategory of addition is Increases Due to Secondary Agents.

Increases Due to Secondary Agents

Increases Due to Secondary Agents do not combine sets; rather, as the name implies, such additions occur due to the effects of a secondary agent. It may be argued that such conditions do not represent addition, but are instead manifestations of multiplication. In practice, however, such increases are expressed both ways. Consider Example B of Figure 1:

> A balloon occupied three cubic feet. After being heated, the balloon's volume increased one cubic foot. What is the volume of the balloon now? (3 + 1 = □).

Though the balloon's volume increased, no cubic feet were directly added. The increase in volume occurred due to addition energy effecting the balloon. Such additions are difficult to model. If this situation were described in terms of multiplication, it would be characterized as a multiplication by magnitude which will be addressed later.

Subtraction

Subtraction is the act of removing units from a set or decreasing a set by a given amount due to a secondary agent. Subtraction, like addition, also divides into two subcategories: Removing and Decreases Due to Secondary Agents. Examples A.1, A.2, A.3, and A.4 of Figure 2 show Removing; Example B illustrates Decreases Due to Secondary Agents.

Figure 2
<u>Subtraction</u>

A. Removing

1. Expository Subtractions
Example: John had five apples. He ate three. Now he has two apples (5 - 3 = □);

Example: Gail had three dollars in her checking account. After the bank removed an errant five-dollar service charge, how much did Gail have in her account? (3 - -5 = □);

2. Subtractions Requiring One Replication
Example: John had five apples, three less than Gail. How many apples did John have? (5 - 3 = □)

3. Subtractions Requiring Two Replications
Example: John had five apples. Gail had three apples. Lisa had as many apples as the difference between John's and Gail's apples. How many apples did Lisa have? (5 - 3 = □)

B. Decreases Due to Secondary Agents
Example: A balloon occupied three cubic feet. After being cooled, the balloon's volume decreased one cubic foot. How large is the balloon now? (3 - 1 = □).

<u>Removing</u>

Removing is the antithesis of Joining. For each Joining genre, a similar but opposite genre occurs in Removing. Conditions on the identity of set elements identity hold for subtraction, also. Removing consist of three subcategories: Expository Subtractions, Subtractions Requiring One Replication, and Subtractions Requiring Two Replications.

<u>Expository Subtraction</u>. Expository Subtraction stories specify the contents of two of the equation's three elements: the minuend, the subtrahend, or the difference. When the difference is specified, the

equation appears in a noncanonical form. Consider the first example in A.1 of Figure 2:

> John had five apples. He ate three. How many apples remained? ($5 - 3 = \square$).

In this example, the minuend, John's five apples, and the subtrahend, the three apples that John ate, are given. The minuend and the subtrahend are parts of the setting. The difference needs to be calculated, which, in this case, results in the resolution, the final point in the story. Again, the story is straight forward, providing full story components. The second example in A.1 does the same; the story elements are stated directly. See below:

> Gail had three dollars in her checking account. After the bank removed an errant five-dollar service charge, how much did Gail have in her account? ($3 - -5 = \square$)

This example is provided only to show that the application of Arithmetic Story Grammar is not dependent on the form of numbers that are used and that terms and conditions remain constant. Though expediency may dictate the use of addition to arrive at the correct difference, the method of achieving the difference does not alter the conditions that make this an example of subtraction. Further, by imagining a limitless reservoir of paired positive-negative dollars, the process of removing the negatives to reveal the positives, can be easily demonstrated. This, however, goes beyond the focus of this paper.

Subtractions Requiring One Replication. Subtractions Requiring One Replication allude to the minuend or the subtrahend through a process of comparison. Consider the Example A.2 in Figure 2:

> John had five apples, three less than Gail. How many apples did John have? (5 - 3 = □)

In this case, the minuend is alluded to through a comparison of Gail's apples to John's apples. John's set must be replicated so that an independent model of Gail's set may be developed. If this is not done, John's set will no longer exist as a complete set. To remain true to the story, John's set must be maintained because in the story his set of apples never changes. Subtractions Requiring two Replications are similar.

Subtractions Requiring Two Replications. Subtractions Requiring Two Replications necessitate that two parts of the story be replicated. Consider the following example:

> John had five apples. Gail had three apples. Lisa had as many apples as the difference between John's and Gail's apples. How many apples did Lisa have? (5 - 3 = □)

To arrive at an independent set of Lisa's apples, it is necessary to replicated both John's and Gail's apples. By matching the apples that these two sets have in common, via a one-to-one correspondence, and removing the matched pairs, a model of Lisa's set is achieved. Further, this leaves John's and Gail's sets intact. Decreases Due to Secondary Agents are fundamentally different from Joinings.

Decreases Due to Secondary Agents

Decreases Due to Secondary Agents involve subtraction stories were the removal of units from the minuend is not considered. Instead, the minuend is effected by a secondary agent. Similar to addition, such conditions may be described using multiplication; however, in practice, such decreases are expressed both ways. Consider Example B of Figure 2:

> A balloon occupied three cubic feet. After being cooled, the balloon's volume decreased one cubic foot. How large is the balloon now? (3 - 1 = □)

In this example, one cubic foot of volume is not removed from the original three cubic feet. Rather, the one cubic foot disappears, a result of losing energy due to environmental conditions. Again, as is similar to Increases Due to Secondary Agents, the decrease of one cubic foot may be demonstrated, but the use of subtraction, when defined as the act of removing, cannot be shown.

Multiplication

Multiplication is the act of calculating number by using magnitude, the act of creating new units by melding effects of two disparate sets, or the act of pairing all of the units of one set with all of the units of another set. Magnitude can be defined as a rate or a scalar. Examples include units per set, such as miles per hour or chances per attempt, and ratios, such as ½ (to 1) or 3 (to 1). The five stories in Figure 3 the two subcategories of multiplication: 1) Multiplying by Magnitude; and 2) Multiplying Components by Components.

Figure 3
Multiplication

A. Multiplying by Magnitude:

1. Sets of Sets
Example: John had five bags, each containing three apples. How many apples did John have in all? ($5 \times 3 = \square$);

2. Comparisons
Example: Juan stood three times taller than his baby sister, who stood two feet tall. How tall was Juan? ($3 \times 2 = \square$);

3. Probabilities
Example: Gail threw a die three times, attempting to roll a six. Her chance of getting a six stood at 1/6 for each roll she made. What chance did she have of hitting a six? ($3 \times 1/6 = \square$);

B. Multiplying Components by Components

1. Composites
Example: A weight of three pounds pulled down on a lever four feet from the fulcrum. What was the force created? ($3 \times 4 = \square$);

2. Combinations
Example: Maria owns two blouses and four pairs of pants. How many blouse-and-pants combinations could Maria make? ($2 \times 4 = \square$).

Multiplying by Magnitude

The subcategory of multiplication referred to as Multiplying by Magnitude uses magnitude to calculate the total number of units within a set. Two necessary conditions of all involved sets is that the sets under consideration equal one another in size and that all units possess identical qualifying characteristics. Implicit within the concept is the idea that all sets are regarded together. Though analogies may be drawn between this form of multiplication and addition, especially when the range of the factor representing the magnitude is restricted to positive integers, the analogy falls short, particularly when fractions are multiplied by fractions. In the interest of clarity, multiplication by magnitude, or in any other form, is never referred to as repeated addition. The factor representing the magnitude exists as a rate or a scalar. The remaining factor is the number of units within a set. The product indicates either the total number of units contained in the specified sets or it reveals the likelihood of an outcome. In each of the following three examples, the number of specified sets is multiplied by the magnitude produce different kinds of yields.

Sets of Sets. The category Sets of Sets consists of multiplications involving a specified number of sets by the number of units contained within an individual set. The number of sets involved is referred to here as the magnitude. Two tacit conditions governing the composition of the sets involved is that they all contain the same kind of units and that the sets all be of equal size. Example A.1 of Figure 3, an illustration of the genre Sets of Sets:

> John had five bags, each containing three apples. How many apples did John have in all? (5 x 3 = □)

This example displays magnitude as rate: three apples per bag.

Comparisons. Comparisons involve multiplications that compare the units of one set to those of another, rather than simply determining the total number of units involved, as in Sets of Sets multiplications.

Comparisons yield resolutions that differ from those of Sets of Sets multiplications. The resolutions of Sets of Sets merely provide total the number that exists; those of Comparisons produce the quantity of a second set through the manipulation of the initial set. In comparative multiplications, the magnitude is defined as a scalar, a unitless number based upon a ratio, rather than as a rate. Example A.2 of Figure 3, provides a representation of the genre Comparisons:

> Juan stood three times taller than his baby sister, who stood two feet tall. How tall was Juan? ($3 \times 2 = \square$)

This illustration shows magnitude as a unitless scalar, the number 3. The factor 2 depicts the size of Juan's sister, the set under consideration.

Probabilities. Probabilities use rate as do multiplications of the genre Sets of Sets. Probabilities also involve magnitudes as do Sets of Sets and Comparison multiplications. However, the resolutions of multiplications involving probabilities differ those of Sets of Sets and from those of Comparisons. In Sets of Sets and Comparisons, the resolutions represent hard and fast quantities, whereas the resolutions of probabilities represent the statistical likelihood of an occurrence. Probabilities further differ from those of Comparisons. As with Sets of Sets, Probabilities use rates, whereas Comparisons use scalars. Consider example A.3:

Gail threw a die three times, attempting to roll a six. Her chance of getting a six stood at 1/6 for each roll she made. What chance did she have of hitting a six? (3 x 1/6 = □)

In this example, the 3 represents the number of sets involved, and the 1/6 stands for the statistical rate. The product yields the most likely outcome that may or may not be realized.

Comparisons. Comparisons also use magnitude, but the magnitude is a ratio rather than a rate. Consider Example A.2 of Figure 3:

Juan stood three times taller than his baby sister, who stood two feet tall. How tall was Juan? (3 x 2 = □);

In this example, Juan's baby sister is compared to him at a ratio of three sisters to one Juan. It is this comparison and the use of ratio that make this genre unique. The ratio is the magnitude, whereas, the size of Juan's sister is the set. Both of these elements are parts of the setting. Multiplying them together resolves the resolution of the story implied by the question.

The question begs the solver to continue the action the was begun when the comparison was initiated. The story will not be concluded until Juan's height is determined. This story when resolved might read as follows: A comparison was made that established that Juan stood three times taller than his baby sister, who stood two feet tall. You used this information to calculate that Juan stood six feet tall. Probabilities differ form Comparisons.

Probabilities are similar to Sets of Sets. They differ only because the elements of the setting and the resolution are not absolute representations of the actual events that may transpire and the resolution that may be had. They represent probabilities. Consider Example A.3 of Figure 3:

> Gail threw a die three times, attempting to roll a six. Her chance of getting a six stood at 1/6 for each roll she made. What chance did she have to hit a six? ($3 \times 1/6 = \square$)

In this example, the 1/6 represents the most likely rate, not a definite rate. The resolution of 3/6 represents the most likely outcome. Neither are absolute. What Gail actually rolls could be quite different from that which is expected. Multiplications of components by components do not use magnitudes.

Multiplying Components by Components

The subcategory referred to Multiplying Components by Components consists of two genres: Composites and Combinations. This subcategory differs from Multiplying by Magnitude because of the number of magnitudes involved in the multiplication process. When multiplying by magnitude, only one of the two factors represents a magnitude. If magnitude is defined as the number of sets being multiplied, as it is here, magnitude plays no part in multiplications of components by components. However, if magnitude is defined as the number of elements in a specific set, defined here as rate, two magnitudes enter into the process.

Composites. Composites form when the elements of one set meld with the units of another set to form a new unit possessing properties distinctly different form those of either of the individual factors melded. As indicated, such products are referred to here as composites. Examples of composites include foot-pounds, square feet, and man-hours. Example B.1 of figure 3 provides an illustration of this by crossing pounds, a measure of weight, with feet, a measure of distance, to produce foot-pounds, a measure of force:

> A weight of three pounds pulled down on a lever four feet from the fulcrum. What was the force created? (3 x 4 = □)

The force resembles neither the pounds nor the feet it originated from. It possesses completely different attributes than either of the units it was created from. This, however, is not always so pronounced as it is in this example. Some Composites do bare some semblance to the units that bore them, such as the feet of square feet; however, even here considerable differences exist. Feet represent a single dimension; square feet represent two.

Combinations. Combinations differ from Composites in one distinct way. Combinations pair all of the elements of one set with all of the elements of a second set to reveal the number of possible combinations that can be produced. The properties of the elements of the factors remain unchanged; they are simply paired with another unit to form a paired unit were each element of pair retains its distinct qualities.

Further, Composite multiplications produce tangible products. Consider example B.2 of Figure 3:

> Maria owns two blouses and four pairs of pants. How many blouse-and-pants combinations could Maria make? (2 x 4 = □).

In this example, Maria may produce 8 differ outfits. However, only two of these outfits can exist at a given time. The other six remain only possibilities and may be produced only if the existing two are dissolved.

Division

Division, the antithesis of multiplication, is the act of calculating magnitude, rate of units per set, or scale; it is also the act of breaking composite units into their constituent parts or the act of separating paired combinations into sets of similar components. The eight stories in Figure 4 provide examples illustrating Divisions Involving Magnitude and Divisions not Involving Magnitude, the two subcategories of division.

Figure 4
Division

A. Divisions Involving Magnitude
   1. Set of Sets
      a. Dividing by Magnitude
         Example: John had 15 apples. He separated the apples into groups of three. How many groups of apples resulted (15 ÷ 3 = □);

      b. Dividing to Find Magnitude
         Example: John had 15 apples. He divided his apples evenly between five bags. How many apples went into each bag? (15 ÷ 5 = □);

   2. Comparisons
      a. Dividing by Magnitude
         Example: Juan stood six feet tall. His baby sister stood two feet tall. After comparing Juan to his sister. How much taller was Juan than his sister? (6 ÷ 2 = □);

      b. Dividing to Find Magnitude
         Example: Juan stood six feet tall, three times taller than his baby sister. How tall is Juan's sister? (6 ÷ 3 = □);

   3. Probabilities
      a. Dividing by Magnitude
         Example: In all, Gail had 3/6 of a chance to hit a six while throwing a die. She also had a 1/6 chance of hitting the six on any one throw. How many times would Gail throw the die? (3/6 ÷ 1/6 = □);

      b. Dividing to Find Magnitude
         Example: In all, Gail had 3/6 of a chance to hit a six, if she threw a die three times. What was the chance of hitting a six if she threw the die only once. (3/6 ÷ 3 = □);

Figure 4 continued

B. Divisions not Involving Magnitude

A. Composites
Example: A force of 12 foot-pounds was divided by three pounds. What is the distance from the fulcrum (12 ÷ 3 = □);

B. Combinations
Example: Maria can make eight blouse-and-pants combinations altogether. How many pairs of pants does Maria own? (8 ÷ 2 =□)

Divisions Involving Magnitude

Divisions Involving Magnitude merely reverse the process of Multiplication by Magnitude. The nature of each of the genres reflects its identically-named counterpart in multiplication; however, instead of containing a theme of multiplication, the stories possess a themes of division. As with multiplication, analogies may be drawn between this form of division and subtraction. But, again the analogy falls short, particularly when fractions are divided by fractions and when negative numbers are divided by negative numbers. In the interest of clarity, division is never referred to as repeated subtraction.

Dividing to find magnitude and dividing by magnitude are similar to a coin--two aspects complement each other. Dividing to determine magnitude is analogous to heads, whereas, dividing to find the quantity that a set will contain is akin to tails. Heads, in the case of magnitude, harbors two faces. The first takes a set and divides it by the size of the sets desired. Such a maneuver yields the magnitude, the number of sets producible from a set of a given size. The second face divides one set by another, both sets consisting of identical units. Such an action delivers a unitless scalar, that manifests the magnitude to which the two sets correspond in size with 1 indicating equality. The other side of the coin show magnitude's complement.

Tails represents the use of division to determine the number of units that goes into each set. The magnitude acts as the divisor and yields the quantity that goes into each set or the relative size of the second set.

Each side of the coin may be seen as a separate scenario of the same story. The story elements remain the same, only the plot changes to reveal a different aspect of the setting.

Divisions Involving Magnitude and their antitheses, Multiplications Involving Magnitude, divide into three subcategories. Since the qualities of these two subcategories and their genres are quite similar, they are not repeated here. Instead, examples are again provided. The number of examples has been increased so that two scenarios, Dividing by Magnitude and Dividing to Find Magnitude, appearing in each of the three genres involved may be illustrated.

Examples A.1.a, A.2.a, and A.3.a in Figure 4 demonstrate Division by Magnitude. Each example uses magnitude to determine the number of units that will go into any one particular set. Examples A.1.b, A.2.b, and A.3.b illustrate the Divisions to Find Magnitude. Each of these examples demonstrate stories that seek the size of the magnitude, that is, the number of sets or the ratio between sets.

Divisions not Involving Magnitude

This form of division is characterized by disjoining composite units into their constituent parts or by breaking combinations into sets of components. Such divisions do not involve magnitude.

Composites. Divisions involving composites take units composed of melded units and break those units into separate sets of similar units. Such divisions yield units with characteristics fundamentally different from those of the dividend. To do this, one factor of the composite, a part of the setting, is isolated and used to determine the other factor. Example B.1 in Figure 4 depicts dividing by Composites:

> A force of 12 foot-pounds was divided by three-pounds. What is the distance from the fulcrum? ($12 \div 3 = \square$)

The 12 foot-pounds of the setting is divided by three pounds, one of the foot-pounds constituent parts. The act of division is part of the plot of the story. This act isolates the three pounds, that when used to divide by isolates the second component of the composite, the four feet. Choosing to use the four feet as the divisor rather than the three pounds merely isolates a different component. The process remains the same.

Combinations. Combinations provide the second genre within the subcategory Divisions not Involving Magnitude. Similar to divisions of composites, one factor, a part of the setting, is isolated and used to determine the other factor. Example B.2 in Figure 4 illustrates this:

> Maria can make eight blouse-and-pants combinations altogether. How many pairs of pants does Maria own?
>
> ($8 \div 2 = \square$),

The eight blouse-and-pants combinations are divided by the two blouses to isolate the four pairs of pants. The characteristic of this genre of division that differentiates it from divisions by Composites is the nature of the setting. The dividends of the two genres differ in nature as the resolutions of multiplying Composites and multiplying Combinations differ. As with Comparisons, only one scenario exists for this genre. Reversing the process plays out in the same manner, it merely isolates a different factor.

Conclusion

Arithmetic Story Grammar offers clear definitions of addition, subtraction, multiplication, and division that appear to remain unambiguous across all situations and number types. Further, the taxonomy of one-operand equations that deal with addition, subtraction, multiplication, and division stories presented here is believed to be complete, possessing devises that allow analysis of relatively complete stories as well as those that are more cryptic along literary lines. One last example is provided here to demonstrate the power of Arithmetic

Story Grammar in dissecting and analyzing stories lacking the richness of detail contained in the stories presented above. Consider the following statement:

Simon's 3 sisters brought the total to eight.

Using the devises of Arithmetic Story Grammar to identify the setting, plot, theme, and resolution expose this terse statement to be a complete story, though not very much detail is given. The setting consists of Simon's three sisters and another unknown quantity and a place in time that takes us through the resolution. The plot, though only implied, consists of a quantity of three sisters being added to an unknown quantity of an obscure unit to yield a total of eight units. The theme is joining. The resolution is the realization of eight units. In its ideographic form, this story takes the following form: $3 + \square = 8$.

As with exclamatory sentences, much of what may appear in a mathematical story is understood by the reader even though it is not declared. The meaning of a sentence such as "Go to bed." is understood by the reader, especially when presented in context. Grammarians instruct that this sentence is complete though the subject is not directly provided. They teach that the subject, *you*, is understood (Warriner, Whitten, & Griffith, 1977). Such is the case with many arithmetic stories; elements of stories are implied.

Due to the perceived completeness and simplicity of Arithmetic Story Grammar, additional long-ranged research is recommended to answer several questions: 1) Would providing consistent definitions and models representing the definitions reduce math anxiety and enhance abilities to understand mathematics and solve problems? 2) Would a curriculum designed around the principles of Arithmetic Story Grammar facilitate greater understanding of and comfort with mathematics than a traditional curriculum or those based on the recommendations of the National Council of Teachers of Mathematics (NCTM, 1989)? 3) Would providing instruction in Arithmetic Story Grammar for a year or longer enhance abilities to interpret mathematical situations and equations? 4) Would instruction in Arithmetic Story Grammar transfer to the interpretation and comprehension of short stories and other genres of literature? 5) Which of the genres are the most difficult to comprehend? 6) Would instruction be more effective if the plots described by Arithmetic Story Grammar were used in modeling solution processes; 7) Might Arithmetic Story Grammar help reduce math anxiety by presenting an alternative, literary-based model of mathematics; and 8) Might Arithmetic Story Grammar have different effects on groups of individuals of differing intellectual ability.

These questions can only be answered if put to the test. The work of others regarding literature and story grammar offers hope. The schematic structures are already in place. Students need only learn to apply them the mathematical stories.

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