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ABSTRACT

The purpose of this research was to determine the effects of two different elementary mathematics programs on student learning. The subjects, third-grade students in two schools, were matched by socio-economic background and teacher experience. Students in each school were exposed to a different mathematics program since kindergarten--a skills based program and a concept based program. Data included observations of students' mathematical learning, journals of student writing, and class averaged standardized test data. Quantitative results indicate that both groups learned mathematics skills equally well but the concept instructed class outperformed the skill instructed class in problem solving. Qualitative results suggest that the skill instructed class liked mathematics and correctly used computation algorithms in their journal entries. The students used words and numbers to express ideas but did not use inferencing to draw conclusions. The concept instructed class often used mathematical language to express ideas that were based on prior knowledge and also consistently used inferencing to draw conclusions. Contains 17 references. (Author/JRH)

Children's Mathematical Learning

by
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Abstract

The purpose of this research was to determine the effects of two different elementary mathematics programs on student mathematics learning. School personnel invited the researchers to gather data that would help them understand the effects of their specific mathematics program on students' mathematics learning. The subjects included third grade students from two, rural Minnesota elementary schools. The subjects were matched for socio-economic background, and teacher experience. One school had adopted a skills based mathematics program when the subject class was in kindergarten, and used that program to date. The other school had adopted a concept based mathematics program when the class was in kindergarten, and used that program each year with its class.

The data collected included observations of students' mathematical learning, journals of student writing, and class averaged standardized test data. Both quantitative and qualitative data was collected. This project has provided information about how well children using different mathematics programs have learned mathematics. Quantitative results suggest that both groups learned mathematics skills equally well. Also, the concept instructed class outperformed the skill instructed class in problem solving.

Qualitative results suggest that the skill instructed class liked mathematics, and correctly used computation algorithms in their journal entries. Often, words and numbers were used to express ideas. However, they did not use inferencing to draw conclusions. Rather, they stated facts they had memorized. The concept instructed class often used mathematical language to express ideas that were based on prior knowledge. In addition, the concept instructed class consistently used inferencing to draw conclusions, which included some computation errors.

CHILDREN'S MATHEMATICAL LEARNING

Purpose and Rationale

The purpose of this research was to determine the effects of two different elementary mathematics programs on student mathematics learning. The first program was skill based, while the second program was concept based. Both instruction and curriculum were different.

As early as the late 1940's, numerous programs have been proposed and implemented for the purpose of improving the teaching and learning of mathematics. One of the earliest influences stemmed from the research of Brownell, who provided a sharp contrast to the essentially stimulus/response teaching and learning procedures used in schools at the time (Brownell 1944, 1945, 1947). He rejected the view of learning mathematics that focused more on performance than on meanings underlying performance.

Brownell proposed a meaning theory of arithmetic, based on reason. He suggested that students give reasons to justify procedures rather than memorize the procedures without meaning, which was often characteristic of mathematics teaching at the time. Brownell suggested that meaning was derived from within the connectedness of mathematics itself. For example, explaining why 4 times 5 is 20 on the basis of what is known about the meaning of those numbers and on the nature of multiplication as repeated addition would be a better school activity than memorizing $5 \times 4 = 20$. This pre-1950's idea that citizens should, and indeed deserve, to understand mathematics is perhaps the single most important idea within the new Mathematics Standards suggested by the National Council of Teachers of Mathematics (NCTM, 1989). Mathematics should be understood.

At about the time Brownell was proposing his meaning theory, progressive education was in its heyday and began to influence curriculum. John Dewey's focus upon the social context of education and upon the need for the entire curriculum to be understood as a connected, rather than a disjointed enterprise, had begun to influence educators to view mathematics as a subject best understood in relation to other subjects (Dewey, 1896, 1898; Dewey & McLellan, 1895). Another idea embedded within the new Mathematics Standards was born. Mathematics knowledge should be related to the other school subjects.

While progressive education diminished in importance within education in general, the idea that school mathematics should be understood within a context, and related to real world applications, grew in importance and was encouraged by both theoretical and applied mathematicians. Thus, another important idea embedded within the new Mathematics Standards was born and nurtured over the years: School mathematics should be connected to topics within mathematics, and to the other school subjects. Mathematics teaching and learning should be connected or integrated with other disciplines/subjects.

One of the early attempts to implement the research theories of Brownell and Dewey evolved during the post-sputnik years (1950's-1960's), and came to be known as "Modern or New Mathematics." These programs (the Madison Wisconsin Project, The Greater Cleveland Mathematics Project, etc.) were based on the notion that students would understand mathematics if mathematical structures and methods were taught. People such as Jerome Bruner supplied a psychological rationale for approaching mathematics as the study of structure. The approach was believed to be superior not only because it portrayed the nature of mathematics, but also because the emphasis on structure would aid memory, understanding, and a deep sense of transfer of understanding to other subjects (Bruner 1960, 1966).

Historically, these programs were very successful for mathematically talented students, and required teachers well prepared in mathematics. For some students these programs were very successful. However, many students failed to reach such understandings, and "Modern Math" fell from favor during the 1970's.

However, during this same time a movement within elementary school mathematics became popular and has continued to gain support. The primary focus was/is on understanding mathematics through the use of hands-on manipulatives such as Cuisenaire rods, geoboards, and multibase blocks to promote children's development of mathematical meanings. Various projects such as the Madison Project, the University of Illinois Arithmetic Project, the Greater Cleveland Mathematics Project, and the Minnesota Mathematics and Science Teaching Project contributed to a focus on the use of manipulatives that complemented the already established movements focused on understanding mathematics. These projects stressed the teaching of elementary school mathematics in a meaningful way, much as Brownell had emphasized, through exploration and use of concrete materials. This position was strengthened by the influence of Piaget and the Genevan school of psychology, which held that learning is a function of one's active experiences. Still another important aspect of learning mathematics embedded in the Mathematics Standards was popularized: Elementary school aged children need to touch, feel, and experience mathematics in concrete forms.

More recently, mathematics education has emphasized problems solving and applications. In the National Council of Teachers of Mathematics (NCTM) Agenda for Action (1980), problem solving became the first priority for school mathematics in the 1980's. Problem solving may be viewed as an extension of Brownell's ideas although its more recent roots lie with George Polya and his enormous contributions (Polya 1945, 1962, 1965). Fine tuning the reason theme once again, problem solving is viewed by many mathematics educators as an effort to learn and appropriately invoke heuristics to deal not only with what is already known (as was the case with Brownell), but also with what is unknown.

The heuristic approach focuses on efforts such as understanding what a problem is asking of the student, what students already know, and how that knowledge can be put to use in coming to understand what is not known. Polya's idea that problem solving is an important objective in learning mathematics grew in importance over the years, and gained significant support by all school interested groups. Thus, another major thrust of the NCTM Standards evolved: Problem Solving. Citizens need to be able to use mathematics in problem solving situations to solve problems of interest to business, science, the arts, social studies, economics, etc.

The most recent professional statement on the teaching and learning of mathematics appears in the Curriculum and Evaluation Standards developed by the National Council of Teachers of Mathematics (1989). These standards were written, at least in part, as a response to a growing concern about the status of school mathematics as presented in "The Underachieving Curriculum" (Mcknight et. al., 1987) and in various National Assessment of Educational Progress reports on mathematics (e.g. C.A. Brown, et al. 1988; Carpenter, Corbitt, Kepner, Lindquist & Reys, 1980). In a sense, the book on standards represents a call for reform motivated in part by a perceived loss of international economic competitive edge somewhat reminiscent of the post-Sputnik era. The school mathematics curriculum described in "The Standards" document assumes the availability of calculators for all students and a computer, for at least demonstration purposes, for every classroom. In addition to highlighting problem solving and mathematics reasoning, the Standards emphasize mathematical understanding, and connecting mathematics to other curricular areas.

Keeping all of the above in mind, the two school districts felt a strong need to evaluate their mathematic curricular practices. Very little research has been conducted since 1989 on the effects of implementing the NCTM Standards. Most of it has centered on the effects of this new curriculum on children's understanding of mathematical ideas. In general, it has been found that test scores, on average, do not change, and slight improvement is noted on test items reflecting understanding and application. However, it is too soon to look to research for definitive results. Solid research takes time. Perhaps, the place to look for results is not to school children in the schools. Rather, research efforts need to be focused on the business community. The question to ask is: What kind of mathematical understandings, skills, and knowledge is needed by citizens to effectively function and keep the scientific, mathematical, and community based businesses successful into the next millennium? The best of current collective wisdom feels that mathematics needs to be understood, related to the other school content areas, and developed to such a degree that citizens are able to use mathematics as a tool to solve real world problems.

Authors' Note: It is interesting to note that nationally, the school mathematics curriculum has changed from a skill based curriculum to a concept, understanding based curriculum. To our knowledge, every professional organization has endorsed this change, and every level of school government has endorsed this change. from the National Education Association (NEA) to the state level departments of public instruction. In Minnesota, the Mathematics Consultant (Sharon Stenglein) for the Department of Public Instruction has, since its initial publication, endorsed this new curriculum. In our experience, this is the first time in education, that nearly unanimous support has been awarded to curriculum renewal. Literally, there is no serious professional opposition to concept based school mathematics curriculum.

It is also interesting to note that school mathematics is only the first of many content areas to begin the move from a skill based curriculum to a concept based curriculum. The school mathematics community has lead the way, and others follow closely behind. Educational futurists expect that by the year 2000, the entire school curriculum will be concept, understanding based. School districts do have a very real challenge, in these new, exciting and changing times.

Description of the Programs

One program was a skill based arithmetic program; The curriculum was largely organized around the traditional arithmetic algorithms. The purpose of the program was largely to develop skill at executing the basic arithmetic algorithms with paper and pencil. Instruction was organized in such a way as to encourage the development of arithmetic skills with pencil and paper. In addition, the curriculum was hierarchically ordered; first foundation skills were mastered, followed by more complex skills. Repetition was valued, and rote memory important. Instructional strategies included both oral and written practice and drill. Assessment was generally structured, and criterion based.

The teacher often modeled the algorithm as part of the lesson, and students practiced. The teacher was very experienced, was considered a master teacher, and strongly believed in the program. The teacher had support within the school and within the community.

The second program was a concept based mathematics program. The curriculum was largely organized around themes which encourage mathematical thinking, mathematical application, problem solving, and use of language to express ideas. The purpose of the program was to teach children to think mathematically, apply mathematics in real world situations, and to ask questions. The emphasis was on the process used to determine solutions rather than the solutions themselves. Instructional strategies included oral questioning, problem posing, problem solving, oral explanations, classroom dialog, reasoning situations, and solution presentations. Assessment was both formal and informal, and involved multiple methods.

The teacher presented a mathematical problem situation, discussed a possible way to solve the problem, and then had the students experiment in their own way to solve the problem. The teacher was very experienced, was also considered a master teacher, and also strongly believed in the program. In addition, the teacher was formally trained to use the program.

Subjects

The subjects consisted of two, third grade classrooms located in rural southern central Minnesota. The classrooms were well matched for social economic status. Both resided in working rural farm towns, had similar lifestyles, and religion. The students were matched for past school achievement, and for school behavior. They were experienced in working with their respective mathematical programs since kindergarten. Both teachers were well-prepared to teach the way they did, and believed in their practices.

Methodology

Data was collected for six months (January to June) during the spring of the year. Methods of collection included four classroom observations in each classroom, 3 to 5 journal entries per student, (informal) teacher comments, and grade-averaged achievement test data on mathematics skills, mathematics problem solving, and mathematics totals for the complete mathematics section.

Analysis and Findings

Notes taken by the researchers during the classroom observation sessions, produced data that allowed the researchers to determine the type and style of classroom instruction (skill or concept). Teacher's informal comments gave the researchers confirmation about what they had observed. The student journal entries produced levels of mathematical thinking, recurring themes, and choices of words the children chose to use. Finally, since both schools used the same standardized achievement test, data were analyzed to determine differences between mathematics scores on the two mathematics subtests: Mathematics Skills and Mathematics Problem Solving, and the Mathematics Total for the Mathematics Battery.

The researchers were able to state both qualitative and quantitative findings. Classroom observations illicted the teaching style and process used by the teacher for mathematic instruction. There were repetitive observations of the same procedures and behaviors by the teachers and the students. It was clear that one school classroom was using a skill based program and that the other school classroom was using the concept based program.

Analysis of the student journals found that In specific reference to the skill based classroom: The students clearly, and repeatedly stated in their journals that mathematics was fun. Also, many stated that it was their favorite class. It was found that they enjoyed finding correct answers to well defined problems.

The children often used such phrases as "I guess the answer might be..." or "The answer to the problem is..." indicating a shallow or more complex level of thinking. The children's language included much positive mathematics usage. Students felt confident, and many described their confidence positively. This entry examples the feeling expressed in most of the student entries:"I like to get the right answer."

It was noted also that there was much use of mathematical language to express solutions to arithmetic examples, and to express the algorithms themselves. The researchers noted no computation errors in the journals, references to prior knowledge, or use of creative problem solving. (Appendix A)

On the other hand, the concept instructed class made no references in their journals that indicated that they felt mathematics was fun. They often visualized mathematics and represented their visualizations with appropriate drawings, used written explanations of steps within algorithms or composed problems stories. (Appendix B)

The journals of the concept instructed class contained many computation errors. The students did use mathematics language, and often built their reasoning on prior knowledge. Dialog was often found in the journals. One student wrote "First I did this. I time(sed) you know multiplied 30 x 6 equals 180 and then I time(sed) 80 x 12 = 192. That's how I did it." This written response was given after students were allowed worktime to think about how they would solve this problem. "If our class of 30 students had a dozen boxes of candy that would hold 6 pieces of candy in each box, how many pieces of candy would the class need to buy? How would you solve this problem?" This example had contained algorithm error, properly used mathematical language, and included the use of prior knowledge (30 x 6 =180) and knew that there were 12 things in dozen.

The researchers felt that comparitive measures of achievement would be an important supplement to the observations and journal entries. Consequently, the Iowa Test of Basic Skills that are administered to each student in the spring of third grade was used in this study.

It was noted on the Iowa Test Of Basic Skills (ITBS) Group Summary Statistics Report that the skills-based school district requested to have the group summary statistics report not include Mathematics Computation in the Mathematics Total or report grade equivalent for that subtest.

Although the concept based group used calculators in the classroom on a regular basis, they did not use calculators while taking the ITBS. Thus, the Mathematics battery gives scores that reflect the non-calculator norms for both groups.

The ITBS Group Summary Statistics Report results for the mathematics section are shown on the following chart:

IOWA TESTS OF BASIC SKILLS
Grade 3 1996 Form K Level 9/10
Group Average - Mathematics Subset

	Grade Equiv. Group		National Percentile	
	SKILLS	**CONCEPTS**	**SKILLS**	**CONCEPTS**
MATH COMPUTATION	N/A	4.3	50	53
MATH PROB SOLVING*	4.5	6.1	65	84
MATH TOTAL**	4.2	5.4	59	79

***Note: Non calculator norms were used on Prob Solving subtest.**
****Note: Math Computation not included in Math Totals.**

Students in both groups achieved scores of above grade level on all reported data. However, it was found that the concept instructed group's grade equivalent score was somewhat higher in the problem solving section of the subtest for mathematics.

It is clear that the two sites teach differently, and have achieved very different results with their mathematics instruction. The most significant finding was that children will learn what they are taught. The site that chose to emphasize thinking, problem solving and communication within their mathematics program produced children clearly strong in those areas. The site that chose to emphasize skills, memorization, and recall produced children clearly strong in those areas.

Recommendation

These researchers learned that it is important to realize that the curriculum we choose to implement has a serious impact on the learning of children served by that curriculum. The recommendation of the researchers would be to ask these questions: What kind of curriculum, what would it look like, if it would achieve both objectives equally? Are you in an either/or situation?

Summary

Both rural school districts, aware of the rapid changes in mathematics instruction and curriculum requested a study that would gather data to assist their school district in determining the effects of the mathematics program used in their school on their students' mathematical learning. The study also provided information to guide them in future mathematical curricular considerations. Data was collected for six months.

Researchers focused on the following issues in the third-grade classrooms: 1) how the teachers instructed; 2) student journal entries reflecting their learning; and 3) the individual program's impact as gauged by the Iowa Tests of Basic Skills (ITBS).

Data was obtained by classroom observations, informal comments from the teachers, journal entries of the students, and class averaged scores from the Iowa Test of Basic Skills Mathematics subtests section. The observations determined the teaching form of instruction (skills based or concept based). The student journals exposed their feelings and/or understandings about arithmetic and mathematic processes. Findings also indicated that the skill based students were happy with knowing they were finding the expected solution using an algorithm. The concept based students recorded a feeling of interest in experimenting and taking risks to find solutions. The ITBS scores found that children learn what they are taught. The scores were above grade level for both groups. However, there was a slightly higher score on the problem solving subtest by the concept group.

Future curricular changes in mathematics should be based on the school district's awareness of the NCTM (1989) Standards and how the world of mathematics has changed to become consistent with the world-class performance standards described in the state directives and national policy documents like *Goals 2000*. Based on the researchers' findings, it is recommended that each district will need to determine a mathematics program that will best meet the needs of their students in preparation for future success.

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APPENDIX A

Skills based
Entry from student journal

2-21-96 J-1

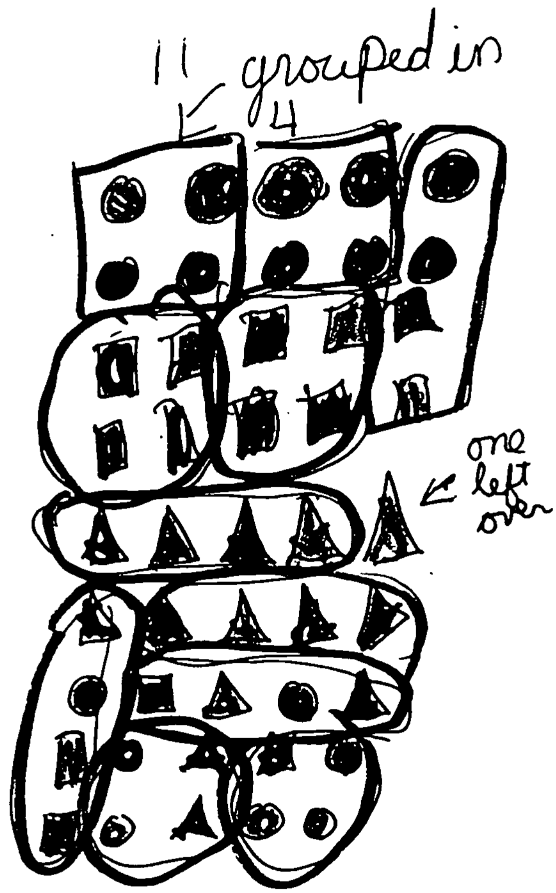
p.51 #4

First I wrote down 28, 45 and 35 and added them together. Then I had 108 chocolates. I took 120 and taked away 108 and I got 12. I am right

```
  26      120
+ 45    - 108
  35    -----
----       12
 108
```

APPENDIX B

Concepts based
Entry from student journal



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