This study continued the research on analogy problem-solving on psychometric tests pursued by I. I. Bejar, R. Chaffin, and S. Embretson (1991). Characteristics of a semantic taxonomy and a cognitively and empirically motivated intensional/pragmatic (I/P) dichotomy were explored. There were two research questions: (1) Could the results of Bejar et al. be replicated with items from the Scholastic Aptitude Test (SAT)? and (2) Would factor analyses support the bidimensional processing structure suggested by the I/P distinction? A specially constructed test of disclosed SAT analogies was administered to a group of 189 undergraduate students. Although factor analyses did not support the expected bidimensionality, a better understanding of both the semantic taxonomy and the I/P dichotomy was achieved. Suggestions are given for future research. (Contains 2 tables, 7 figures, and 21 references.) (Author/SLD)
THE DIMENSIONALITY OF RESPONSES TO SAT ANALOGY ITEMS

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The Dimensionality of Responses to SAT Analogy Items

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Abstract

This study continued the research on analogy problem-solving on psychometric tests pursued by Bejar, Chaffin and Embretson (1991). In specific, characteristics of a semantic taxonomy and a cognitively and empirically motivated intensional/pragmatic (I/P) dichotomy were explored. There were two research questions: (1) Could Bejar et al.'s results be replicated with SAT items? and (2) Would factor analyses support the bidimensional processing structure suggested by the I/P distinction? A specially constructed test of disclosed SAT analogies was administered to a group of 189 undergraduate students. Though factor analyses did not support the expected bidimensionality, a better understanding of both the semantic taxonomy and the I/P dichotomy was achieved. Suggestions for future work were given.
The Dimensionality of Responses to SAT Analogy Items

Research on analogical reasoning has at times emphasized processing (e.g. Sternberg, 1977) and knowledge (Whitely, 1976; Pellegrino & Glaser, 1979). Thus on one hand, within the purview of the "normative processor model" proposed by Bejar, Chaffin and Embretson (1991) for analogies of the form A:B:?=?, the individual is assumed to encode A, encode B, and postulate a relationship. Then, each alternative pair, C_i:D_i, is encoded, its relationship induced, and then judged in terms of "most similar to" A:B's relationship. On the other hand, Bejar et al. (1991), and Chaffin and Pierce (1987) proposed a semantic taxonomy to describe types of knowledge or schema that may be required to solve GRE items. They additionally presented a pragmatic-intensional dichotomy to explain empirical aspects of their data. Hence, the purpose of this study was twofold: 1) to replicate Bejar et al.'s 1991 findings with SAT items and 2) to check if the dimensionality of responses to SAT items was consistent with the pragmatic-intensional dichotomy.

To provide some theoretical background, semantic taxonomies have been developed to portray "a limited number of relations ... which can function as explanatory primitives (the most 'primitive' or basic set of relations, to which all others can be reduced; Sowa, 1984, p. 13) in associationist and network theories of mental function (Chaffin & Herrmann, 1988)". Table 1 shows a
taxonomy describing 179 GRE analogies (Bejar et al., 1991). At the very least, these relation categories draw from an examinee's semantic and syntactic knowledge bases.

However, when the difficulties of the semantic classes were examined (Bejar et al., 1991), it was found that the classes fell into two cluster types, called intensional and pragmatic. Analogies involving intensional relations were more difficult (class inclusion, similar, contrast, attribute, nonattribute), while those involving pragmatic relations were easier (cause-purpose, space-time, part-whole, representation). Given this hierarchy, illustrated in Table 2, relations were assigned to a semantic class and thereby simultaneously placed into one of the two type categories.

After an overall examination of the two clusters of relations, certain characteristics were observed. Intensional relations were based on a comparison of the attributes or properties of two concepts while pragmatic relations were based on
the co-occurrence of two things in the world. An example of this distinction (Bejar et al., 1991 p. 68) can be found in Figure 1.

A farmer is by definition a person and a tractor is a vehicle. The relation between these word pairs is intensional because it rests purely on the comparison of two concepts. In contrast, the relation between a farmer and a tractor is pragmatic. It rests on the particular circumstances found in most technological societies.

Since inducing relations between words is a creative, productive ability/skill, solving intensional or pragmatic analogies may impose different process requirements (Klix & van der Meer, 1980; Klix, van der Meer & Preuß, 1985; Rumelhart, 1989). Therefore, the way intensional and pragmatic items covary should confirm an intensional-pragmatic processing distinction. In specific, factor analyses of analogy item responses should confirm a very specific dimensionality -- bidimensionality. Further, since different levels of analysis can highlight different structural features, factor analysis models reflecting this dimensionality constraint were run on a cluster level defined by the semantic classes, as well as an item level (Dorans & Lawrence, 1992; Wainer & Lewis, 1990).
To this end, the SAT instrument and sample are described and outcomes are compared with previous GRE results. Since Bejar et al.'s results were replicated, confirmatory and exploratory factor analysis models were run to support the substantive hypothesis of an intensional-pragmatic bidimensionality.

Method

Instrument

An analogy test of 40 items was created from a pool of 399 disclosed SAT analogy items. Two undergraduates, chosen for their high verbal ability, classified all items into the mutually exclusive classes of the taxonomy. Disagreements were resolved through discussion. Then, four items from each semantic class were selected such that, of the four items, one was high on the difficulty continuum, one low, and two fell at the middle difficulty quartiles, where difficulty was measured by the ETS delta statistic. Care was also taken that no words would appear more than once.

Subjects

The subjects consisted of 189 Trenton State undergraduate volunteers. There were 118 females and 70 males and most students (94%) were 22 years old or younger. All students, except four, spoke English as a first language.
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The instrument was group administered by one of the authors as part of a class exercise and subjects were allowed as much time as they needed for the task.

Results

Descriptive Results

In calculating scores, omitted items were scored as incorrect, for the following reasons: (1) response patterns showed items missing throughout the test, i.e. the patterns did not fulfill a typical expectation of many omits occurring at the end of the test. (2) students often got correct answers at the end of the test, even with many omits earlier on (This was not unexpected as the items were ordered by accession number instead of, as is usual, presenting them in order of difficulty.), (3) the two parameter IRT model, with no guessing parameter, fit best and (4) there was a strong difficulty-number missing relationship (r=.71, p<.01), demonstrating that more difficult items, as independently measured by ETS's item delta statistic, tended to be those with larger numbers of omits. It is unlikely that rights scoring would have much impact on these results.

Hence a frequency distribution of total scores is shown in Figure 2. The distribution had a negative skew (Sk=−.603) and a mean of 25.7 out of a possible 40, indicating that the items may have been on the easy side for this sample. Note that the SAT
items are actually designed for a college bound, high school students population.

In addition, analyses were carried out to see if some previous findings from Bejar et al.'s (1991) work were replicated with SAT items. For example, Bejar et al. found, as others have (Lord, 1975), a substantial and persistent negative correlation ($r=-.51$) between item difficulty (delta) and item discrimination ($r$-biserial) for analogy items. However, since typically the criterion used to calculate the biserial correlation has been the SAT verbal score, some alternative approaches were considered here. First, using biserial correlations and delta statistics from TESTFACT, where the criterion was the analogy instrument administered, a negative correlation was still evident ($r=-.55$, $p<.01$). Figure 3 shows the biserial by delta scatterplot.

Second, the relationship of other estimates of discrimination and difficulty, $a$, a measure proportional to the item characteristic curve (ICC), and $b$, the ICC's point of inflection, was examined. The correlation between the $a$ and $b$ parameters from the best
fitting IRT model, a two parameter model, was still negative but not significantly different than zero ($r=-.35$), even though $b$ and delta correlated .96 and $a$ and $r_{b}$ correlated .95.

Bejar et al. (1991) also found that items classified as intensional were more difficult and less discriminating than items classified as pragmatic, as indicated by mean ETS delta and $r_{b}$-bserial statistics. The data for this sample replicated this finding: (1) the mean number correct for pragmatic items was 13.87 (sd=2.97, out of 20 items), as compared to intensional items' mean of 11.82 (sd=3.67), (2) the distribution of pragmatic items were more negatively skewed (sk=-.78) than intensional's (sk=-.36), but, (3) while intensional items remained more difficult than pragmatic items, the class' rank ordering of the difficulty and discrimination changed somewhat, perhaps because when this instrument was constructed, an effort was made to include an entire spectrum of item difficulties.

As analogy items are intended to measure analogical reasoning ability, another important concern is how much variance in item difficulty is due to vocabulary knowledge. Certainly, an examinee cannot hope to reason analogically if she does not know item word meanings; once the hurdle of vocabulary knowledge is surpassed, then analogical reasoning can begin. Therefore, it is expected that some part of the variance in item difficulty is due to vocabulary. This was the case for Bejar et al.'s 1991 study.
where a significant 10% of the variance in delta was explained by stem and key word frequency (Four word frequencies, two for the stem and two for the key pairs, were collated from Kucera and Francis' (1967) text; the natural log of the minimum frequency of a pair's two words was then chosen to represent each pair.). In the present sample 12% of the variance was explained by stem and key word frequency (Kucera & Francis, 1967) but the relation was not significant, probably due to less power (N=40 items here versus N=179 in Bejar et al.'s study.). So SAT items do not seem to draw on vocabulary knowledge more or less than GRE items. Interestingly, if a variable taking into account the average word frequency of the alternatives is added to the regression equation, 24% of the variance in delta was explained and the relation was significant (p<.05).

These same regressions were also run for intensional (N=20 items) and pragmatic items (N=20) separately, as Bejar et al. (1991) did. Here, only the average minimum frequency of the alternatives for intensional items explained difficulty; word frequency was not related to difficulty when looking at pragmatic items. However, these regression analyses had even less power.

A priori Confirmatory Factor Analysis

There were several structural models suggested by the above mentioned intensional/pragmatic distinction\(^2\). Model I was a direct and uncomplicated picture of two correlated factors:
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intensional and pragmatic clusters/items loaded on their corresponding factors. Model II had a hierarchical form with an overall exogenous Proficiency factor explaining the correlation between two endogenous factors (intensional and pragmatic). While theoretically relevant, this model could not be identified with only two endogenous factors (Rindskopf & Rose, 1988). Model III sought to portray the idea that every concept is at its center intensional, or definitional, but some also have pragmatic characteristics. Note that Model III was a bi-factor analysis (Gibbons & Hedeker, in press). Please see Figures 4, 5 and 6.

Insert Figures 4, 5 and 6 about here

These models were tested at both the cluster and item level. Unfortunately, the limitations of the available software and the assumptions of factor analysis dictated possible approaches. A cluster level analysis did not violate factor analysis assumptions; four items per cluster formed continuous variables and traditional LISREL analyses could be run. However, factor analyses on item level data are notoriously problematic (For a review, see Dorans & Lawrence, 1992). Hence, TESTFACT was used to calculate smoothed tetrachoric matrices, with a statistical guarantee of being positive definite (Bock, Gibbons & Muraki, 1988). Unfortunately, TESTFACT had limited capabilities for
confirmatory models. For example, to identify the models, TESTFACT assumed that all factors were uncorrelated. Additionally, it was only possible to allow loadings to be free or fix them equal to zero. Therefore, LISREL also was used to examine item level confirmatory analyses by importing the smoothed tetrachoric matrices.

Moreover, problems due to a large "number of items to total number of students" (40 items to 179 students) ratio necessitated subset analyses for item level confirmatory models in LISREL. Consequently, for Model I and III, specific subsets of the data were further probed. Since the class inclusion and case relation classes were the theoretically most pristine class exemplars of intensional and pragmatic properties (Chaffin, 1992, personal communication), the models were rerun with just these eight items. Also, since item difficulty has historically provided a recurrent methodological theme (Dorans & Lawrence, 1992), additional subsets of items were surveyed. A subset of extreme items, hard and easy, called hilo, were scrutinized. Easy and hard items may draw on different skills for people with varying ability levels, or, methodological factors may be revealed. In addition, the items lying at the inner quartiles of difficulty, called middle, were considered more stable class representatives.
Cluster Level Results

As a first step, a one factor model was checked out: it fit. Since the hypothesized models were postulated a priori, the outcomes were looked at, even though it would be hard to justify accepting a less parsimonious solution. Model I fit, but the factors correlated $r=0.97$. Models II and III were rejected due to estimation and identification problems (e.g., negative variances). Some of these difficulties were undoubtedly caused by the high intercorrelation between factors. Unidimensionality was again verified using Bejar's approach (1980). That is, $b$ estimates for the whole test were plotted with $b$ estimates for the intensional and pragmatic subtests; this scatterplot is shown in Figure 7. If the test data were unidimensional, points should lie primarily on a straight line going through 0 and with a slope of 1, which was the case.

Insert Figure 7 about here

In addition, the two parameter model fit best for both subtests as well as the entire test.

For this sample and set of analogy items, a one factor solution was deemed most descriptive, presenting a rare and informative case of unidimensionality at the cluster level.
Perhaps different kinds of relations reflect a coherence in mental structure leading a unidimensional test.

**Item Level Results**

Model I was run on the case relation/class-inclusion and middle subsets of data at the item level. Following the same reasoning described above, a one factor solution fit best for middle items. Further, despite high expectations, Model I was rejected for case relation/class-inclusion and middle items. For the hilo data, the hypothesis of methodological or difficulty factors was rejected.

**Exploratory Factor Analyses**

**Cluster Level Results**

To check out if a two dimensional model could possibly fit, clusters of four items were randomly grouped together. Indeed, though meaningless theoretically, a bidimensional model could be supported. This finding strengthens the impact of finding a unidimensional data set when the items were clustered by semantic classes.

**Item Level Results**

In contrast, exploratory analyses of all 40 items with TESTFACT indicated that an item level, one factor model did not fit; the one factor solution was rejected for all sets of data. In fact, except for the case relations/class inclusion data subset, the two factor solution significantly improved fit over
the one factor model, evaluated using change in $\chi^2$ statistics. Clearly, at the item level, something more was going on. Only one interpretation seemed sensible. In the unrotated two factor solution for the entire test, a main overall factor dominated, perhaps like $g$, while the loadings for the second factor, with negative and positive values, correlated significantly with delta ($r=.79$, $p<.01$). This provided weak support for a method factor (Dorans & Lawrence, 1992). This was the case despite rejection of the confirmatory models using the hilo subset of data.

Discussion

A review of cognitive research and recent empirical results using GRE items led to the hypothesis that analogy items falling into an intensional and pragmatic dichotomy should reflect, through a bi-dimensional factor structure, their respective cognitive processing requirements. This study examined this issue. Further, some of Bejar et al.'s 1991 results were checked for replication on SAT items.

Initial descriptive results showed that SAT items functioned similarly to GRE. A negative correlation between delta and $r_{biserial}$, appearing consistently over many analogy item sets, persisted here as well. Further, intensional items were more difficult than pragmatic. This occurred despite an effort in instrument development to keep a consistent spectrum of difficulty over classes. These results suggested that certain
characteristics of intensional items created more difficult items. In addition, vocabulary levels for stem and key pairs did not seem to contribute more or less to item difficulty for these SAT items as compared to the GRE's.

However, confirmatory and exploratory analyses lead to unexpected outcomes. By clustering items according to class, the test remained unremittingly unidimensional. Yet, using another tactic to cluster items, though the clusters were not theoretically relevant, did allow for two factors. Further, two factors fit better than one in item level exploratory analyses. So the test was not in all forms unidimensional. The cohesiveness of the clusters may be a way for test developers to design theoretically, rather than statistically, unidimensional analogy tests. These results also lend credence to this taxonomy being appropriate for SAT items too.

While these results were not supportive of the expected bi-dimensionality, there were several differences between the two studies that might account for this outcome. The sample tested was not drawn from a traditional SAT population. These students had emerged as undergraduates from a much larger group of 'possibly college bound students'. They had presumably further matured and learned. Hence, the items seemed to be easier for this sample. Also, no time constraints were imposed. In addition, these particular analogies were drawn from an older set
of items with some historic problems. Since that time, distractors have been written differently. Perhaps the noted importance of the alternatives in predicting item difficulty was related to past problems.

It may also be that factor analytic tools are not sensitive enough to pick up fine processing differences on analogy item types of the form A:B::?::?. Whitely and Schneider (1981) mentioned three possible reasons for results of this nature: (1) there may not be individual differences in processing abilities on these two dimensions, (2) these abilities may not have been reliably measured, and (3) the two distinct processing abilities may be highly correlated. So theory need not be rejected on these grounds alone.

Nonetheless, theoretical issues should still be questioned. It may be that the procedure of categorizing items into classes, which automatically placed them into the type dichotomy, was not appropriate. Just because an item's relation is part-whole, for example, does not mean it was treated by the processor as pragmatic. In fact, when the items were post hoc reclassified by one of the authors as intensional or pragmatic, independently of the taxonomy, item difficulty was significantly explained ($R^2=.25$, $p<.001$). Further, certain stem relations, when considered within the context of the alternatives, may be treated by the processor as primarily intensional or pragmatic. Researchers (Bejar et al.,
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1991; Barnes, 1980) have long noted that multiple choice alternatives affect the way an item rationale is formulated. Future work with this dichotomy must involve a separate categorization of the taxonomy and dichotomy and a classification methodology for the dichotomy that takes into account alternative choice context effects.
References


Chaffin, R. & Pierce, L. (draft). *A taxonomy of semantic relations for the classification of GRE analogy items and an algorithm for the generation of GRE-type analogies*.


Footnotes

1 The effort to control for difficulty met with moderate success. The midpoints of delta at the 25\textsuperscript{th} and 75\textsuperscript{th} quartiles ranged from 8.14 to 13.48; if difficulty had been tightly controlled, this range would have been smaller.

2 Bejar et al. (1991) could not check dimensionality as only item statistics were available.
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Captions for Figures and Tables

Figure 1  Intensional/Pragmatic Example
Figure 2  Total Score Histogram
Figure 3  Delta by Biserial Scatterplot
Figure 4  Model I
Figure 5  Model II
Figure 6  Model III
Figure 7  Subtest by Total Test Scatterplot of b Estimates

Table 1  Semantic Taxonomy of Relations
Table 2  The Intensional/Pragmatic and Semantic Class Hierarchy
Table 1
Semantic Taxonomy of Relations

<table>
<thead>
<tr>
<th>Relation</th>
<th>Example</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Inclusion</td>
<td>robin:bird</td>
<td>A is a member of class B</td>
</tr>
<tr>
<td>Part-Whole</td>
<td>engine:car</td>
<td>A is a part of B</td>
</tr>
<tr>
<td>Similar</td>
<td>breeze:gale</td>
<td>B is a more intense A</td>
</tr>
<tr>
<td>Contrast</td>
<td>default:pay</td>
<td>A is the opposite of B</td>
</tr>
<tr>
<td>Attribute</td>
<td>beggar:poor</td>
<td>B is an attribute of A</td>
</tr>
<tr>
<td>Non-Attribute</td>
<td>harmony:discord</td>
<td>B is not an attribute of A</td>
</tr>
<tr>
<td>Case Relation</td>
<td>tailor:suit</td>
<td>A works on B</td>
</tr>
<tr>
<td>Cause/Purpose</td>
<td>hunger:eat</td>
<td>A is the cause of B</td>
</tr>
<tr>
<td>Space/Time</td>
<td>judge:court</td>
<td>A can be found in B</td>
</tr>
<tr>
<td></td>
<td>summer:harvest</td>
<td>B occurs during A</td>
</tr>
<tr>
<td>Representation</td>
<td>building:print</td>
<td>B is a representation of A</td>
</tr>
</tbody>
</table>
Table 2
The Intensional/Pragmatic and Semantic Class Hierarchy

<table>
<thead>
<tr>
<th>Relation Dichotomy</th>
<th>Intensional</th>
<th>Pragmatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class Inclusion</td>
<td>Case Relation</td>
<td></td>
</tr>
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<td>Similar</td>
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<td></td>
</tr>
<tr>
<td>Attribute</td>
<td>Part-Whole</td>
<td></td>
</tr>
<tr>
<td>Non-Attribute</td>
<td>Representation</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1
Intensional/Pragmatic Example

Pragmatic
PERSON --------------- VEHICLE
<table>
<thead>
<tr>
<th>Intensional</th>
<th>Intensional</th>
</tr>
</thead>
</table>
FARMER ----------- TRACTOR
Pragmatic

Figure 2

Dimensionality and SAT Analogies
Figure 3

Dimensionality and SAT Analogies
Figure 4
Model I

Dimensionality and SAT Analogies
Figure 5
Model II

Dimensionality and SAT Analogies

Cluster Level

Item Level

Class
C1
Inclusion

Similar
C2

Contrast
C3

Attribute
C4

Nonattribute
C5

Part - Whole
C6

Case Relations
C7

Cause - Purpose
C8

Space - Time
C9

Representation
C10

η1
Intensional

η2
Pragmatic

ξ1
Proficiency

λ21

λ31

λ41

λ51

λ72

λ82

λ92

λ102

γ11 = 1

φ11 = 1

γ21

ψ21 = 0

η1

ξ1

η2

χ1

1

λ21

γ11 = 1

φ11 = 1

γ21

ψ21 = 0

χ1

1

λ72

λ82

λ92

λ102
Figure 6
Model III

Dimensionality and SAT Analogies

W21 = 0

\[ \eta_1 \]

Intensional

\[ \eta_2 \]

Pragmatic

\[ \zeta_1 \]

\[ \zeta_2 \]

\[ \psi_21 = 0 \]

\[ \beta_{21} \]

\[ \lambda_{21} \]

\[ \lambda_{31} \]

\[ \lambda_{41} \]

\[ \lambda_{51} \]

\[ \lambda_{61} \]

\[ \lambda_{71} \]

\[ \lambda_{81} \]

\[ \lambda_{91} \]

\[ \lambda_{101} \]

\[ \lambda_{62} = 1 \]

\[ \lambda_{72} \]

\[ \lambda_{82} \]

\[ \lambda_{92} \]

\[ \lambda_{102} \]

\[ \varepsilon_{11} \]

\[ \varepsilon_{12} \]

\[ \varepsilon_{13} \]

\[ \varepsilon_{14} \]

Cluster Level

Item Level

Class

I1

I2

I3

I4

Inclusion

C1

Similar

C2

Contrast

C3

Attribute

C4

Nonattribute

C5

Part-Whole

C6

Case Relations

C7

Cause-Purpose

C8

Space-Time

C9

Representation

C10

\[ \varepsilon_{10} \]
Figure 7

Total Test Parameters

Subtest Parameters

Type of Item
○ Pragmatic
× Intensional
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