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ABSTRACT

Mathematical meanings can be developed when individuals construct translations between algebra symbol systems and physical systems that represent one another. Previous research studies indicated (1) few high school students connect whole number manipulations to algebraic manipulations and (2) students who encounter algebraic ideas through manipulating physical models gain conceptual knowledge of algebra. Five high school algebra classes used algebra tiles to study operations with algebraic expressions. Results suggested no differences between groups who used or did not use manipulatives when tested with traditional chapter tests. Results of diary narrative data indicated that the majority of students stated that the tiles added a mental imagery that made learning "easier." They indicated that they found it easy to think about algebraic manipulations when they visualized the tiles. (Author)

Results of using Algebra Tiles as meaningful representations of algebra concepts

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Abstract

Mathematical meanings can be developed when individuals construct translations between algebra symbol systems and physical systems that represent one another. Previous research studies indicated (a) few high school students connect whole number manipulations to algebraic manipulations and (b) students who encounter algebraic ideas through manipulating physical models gain conceptual knowledge of algebra. Five high school algebra classes used algebra tiles to study operations with algebraic expressions. Results suggested no differences between groups who used or did not use manipulatives when tested with traditional chapter tests. Results of diary narrative data indicated that the majority of students stated that the tiles added a mental imagery that made learning "easier". They indicated that they found it easy to think about algebraic manipulations when they visualized the tiles.

Algebra tiles as meaningful representations of Algebra concepts

Objectives

Because most high school algebra curriculums require students to demonstrate rote memorization of procedures and isolated knowledge of basic skills, many algebra classes immediately begin the year with isolated disjoint manipulation of abstract symbols, such as x, rather than with a concrete conceptual basis, such as graphic or physical models of x. Few students are encouraged to make generalizations or connections between manipulations with whole numbers which may have been learned in elementary school and manipulations with algebraic symbols. The objective of this research was to study the use of algebra tiles as a forum to provide opportunities for high school students in Iowa to acquire traditional algebraic concepts.

Perspectives

Theoretical. Kaput[1] established that "meanings are developed within or relative to particular representations or ensembles of [particular representations]" (p. 168). So, students who successfully make connections between physical representations and mathematical representations have created meaning of mathematical ideas. Such extensions into physical systems or natural language may enrich or create the basis of meaning. "They can create and test conjectures within the symbolic system using a symbol manipulator or can use different combinations of representations from concrete examples to graphs to representations perhaps not yet thought of"[2]. So, meaning might be achieved or at least enhanced when individuals construct translations between algebra symbol systems and physical systems that represent one another.

Concrete representations of abstract mathematical ideas, such as algebra tiles, offer opportunities for creation of situations to facilitate translations between manipulation of algebraic expressions and manipulation of concrete examples. Clearly, students will not merely absorb mathematical ideas from sheer contact with manipulatives even if those blocks are richly designed to transfer mathematical wisdom to the user. Wodd, Cobb, and Yackel[3] articulated such concerns about the use of a variety of instructional representations, including manipulatives. However, they stated that mathematical learning through use of manipulatives can be successfully orchestrated by the teacher who creates the appropriate environment and who coordinates these activities with similarly appropriate linguistic acquisition.

In terms of curriculum, in 1989, The National Council of Teachers of Mathematics (NCTM) stated specific components that should be present in the school mathematics experiences of all students. For example, these standards[4] stated that "in grades 9-12, the mathematics curriculum should include the continued study of algebraic concepts and methods so that all students can appreciate the power of mathematical abstraction and symbolism" (p.150).

Applications. Lee and Wheeler[5] studied high school sophomores' responses (n=268) to interview questions about algebraic equivalencies. For the interview, they confronted students with incorrect algebraic statements such as: $(a^2 + b^2)^3 = (a^6 + b^6)$. When they were prompted to check with numbers and different results occurred some students "indicated that they did not really expect the same result with arithmetic" (p.43). Lee and Wheeler concluded that the

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[1] Kaput, J (1989) *Linking representations in the symbols system of algebra.* In Wagener & Kieran (Eds.) The Learning and Teaching of Algebra. (pp. 167-194). NCTM: Reston VA.

[2] Kaput, J. (1992) Jim Kaput's summarized presentation (pp.31-33). In Algebra for the Twenty-first Century Proceedings of the August 1992 Conference. NCTM: Reston VA.

[3] Wood, T.. Cobb P., & Yackel, E. (1995). *From alternative epitemologies to practice in education: rethinking what it means to teach and learn.* In Steffe, L. & Gale, J. (Eds.) Constructivism in Education (pp.401-422). LEA: London.

[4] NCTM (1989). Curriculum and Evaluation Standards for American Schools. NCTM: Reston VA.

[5] Lee, L, & Wheller, D. (1989). *The arithmetic connection.* Educational Studeis in Mathematics, 20, p. 41-54.

students did not even consider an attempt to forge a relationship between algebra and arithmetic on any level, abstract or concrete. In contrast, Peck and Jencks[6] studied fifth grade children who were taught to understand parts of arithmetic in depth and then were asked to repeat these ideas using algebraic language. To this end, they taught the children to use physical materials and rectangular regions to learn about multiplication of whole numbers. When the students were asked to make extension to algebraic expressions, the children used their materials to justify their solutions. The fact that the connection was made when attempted with the younger students indicates that the connections can be made. Peck and Jencks felt that these situations were made possible because of the children's appropriate use of manipulatives.

Methods

A total of 5 high school algebra classes were involved in this study. Two rural high-school algebra classes (100% white) and three suburban high-school algebra classes (85% white, 10% African-American, 5% Hispanic) were involved in the experiment. The students in the treatment groups used algebra tiles or algebra tiles together with other manipulatives to study adding, subtracting, multiplying, and factoring algebraic expressions. The students in the control groups either did not use other manipulatives or did not use any manipulatives at all. The teachers used the curriculum from their textbooks and followed assignments from the text. Although the classes from the two districts possessed basic differences, these general procedures of the teachers were similar.

Experiment #1: Factoring-only. The suburban students involved in the factoring experiment during the second year of the study ranged in ages from 15 to 18. The treatment group (n=11) used algebra tiles only during the unit on factoring. The first control group (n=13), taught by the same teacher, was not involved with the algebra tiles or manipulatives in any way. It served as a control for the manipulative usage. The second control group (n=13) was taught by a different teacher and served as a control class for both teacher and manipulative.

The students of the treatment group encountered the algebra tiles from the first day of their spring semester, when they studied factoring. An attempt was made to try and re-organize the students' knowledge of multiplication of binomials by using tiles and then to connect this conceptual information to factoring expressions. The students in the two control groups encountered the same examples and the same quizzes and the same pages from the textbook. However, neither group encountered the algebra tiles. At the end of the third week, all three classes took the same departmental test over factoring.

Experiment #2: Year-of-manipulatives, all-operations. The two groups of students from the rural school ranged in ages from 13 to 16 along with one 9-year-old boy who had been classified as *gifted* by the district. This research was designed to compare students who have prior experiences with manipulatives to students who have not encountered manipulatives. The treatment group (n=10) experienced one manipulative-relevant problem each week prior to the study. The control group (n=10) was not involved with manipulatives prior to the experiment. Both groups of students involved in the study encountered the algebra tiles for the entire three weeks of the study. For the entire year, both of these groups of students had kept journals as part of their daily routine in algebra class.

For each of the 22 weeks prior to the three weeks of the experiment, the treatment group solved problems using manipulatives. A sample problem is shown:

Using the pattern blocks, create the following shapes:
- *a triangle that is 1/3 green and 2/3 yellow*
- *a parallelogram that is 1/2 red and 1/2 green*
- *a trapezoid that is 1/16 green, 9/16 red and 3/8 blue*

Draw the solutions.

[6] Peck, D. & Jencks, S. (1988). *Reality, arithmetic, algebra.* Journal of Mathematical Behavior, 7, 85-91.

Results

Because the sample sizes were relatively small, the data from all groups were first analyzed via student t-distributions. These analyses were based upon the assumptions that the scores within any given group were normal, that the standard deviations were the same, and that individual tests scores were independent. According to the student t-distribution, no significant differences (a = .025 *two-tailed*) between group means in either of the experiments were found.

Table 1
Mean scores and standard deviations of students in the factoring-only study

	First semester		Chapter Test	
Group	Mean	St. d.	Mean	St.d.
Treatment	68.3	12.4	66.7	19.0
Control (same teacher)	75.0	9.8	77.2	17.0
Control (different teacher)	66.9	19.0	74.8	23.3

Table 2
Mean scores and standard deviations of students in the year-long manipulatives study

	First Semester Grades		Chapter Test Scores	
Group	Mean	St. d.	Mean	St. d.
Treatment *(Year of manipulatives)*	85.5	4.6	83.6	8.3
Control *(Algebra tiles only)*	90.4	6.9	91.7	7.7

Next, confidence intervals for the estimate of the mean of the chapter tests scores were calculated for each group of students who encountered the tiles. Then, confidence intervals for the estimate of the mean of the previous semester's grades were calculated for each of these groups. These calculations were computed at the 95% confidence level.

Table 3
95 % Confidence intervals for students who encountered the Algebra Tiles

	Confidence Intervals	
Group	First Semester Grade	Chapter Test Score
Treatment *(Factoring only)*	60 - 76	54 - 79
Treatment *(Year of manipulatives)*	82 - 88	77 - 89
Control *(Algebra Tiles only)*	85 - 95	86 - 97

Individual student scores were then compared to the two confidence intervals for their group. Those students for whom test scores fell in regions other than where their semester score was located were determined to have scored in an *unusual* manner and were selected for further analysis. For example, if a student's test score was within the chapter test score confidence interval, but their previous semester's grade had been below that confidence interval, then he or she was selected for further study. It might be the case that the experiences of using the tiles in this study could explain some of the reasons for this change in scoring habits.

These *unusual* students' writing and their interview responses were analyzed. According to Glesne and Peshkin[7], narrative data can be analyzed by categorizing and looking for patterns within the statements. The interpretation of those statements will result from recognition of relevant categories that naturally emerge from the data. Reflection about this natural sorting will result in transformation of the data from raw information into communication about the meaning of the study's findings. Such methods were used to analyze the qualitative data of the interviews and the journal writing.

[7] Glesne, C. & Peshkin, A. (1992). Becoming qualitative researchers: An introduction. NY: Longman.

Factoring only. Five of the eleven students in the treatment group of the factoring-only experiment scored in *unusual* ways. Three students improved upon their usual scoring habits while two students, scored below typical expectations. Of the students who showed improvement, the teacher said, "Of all of my students, I believe that they would benefit the most from what the tiles have to offer. They often try to apply procedures to everything whether they are the appropriate rules or not. The tiles gave enough conceptual understanding for them to experience some success. Also, the tiles gave them some of the motivation that they needed in order to succeed."

For the two students who scored below their *unusual* habits, the teacher stated that the use of the tiles was "upsetting" to them because they had never seen manipulatives before. They didn't like the change in the routine business of learning mathematics. In fact, these two students ended the semester with some of the highest scores in the class. The teacher said, "She didn't need the tiles. The other student saw the tiles as play time and didn't really take them seriously."

In light of Kaput's comments about students' assignment of meaning on a variety of levels, the students who improved upon their usual scoring habits seem to have been interested and willing to think about algebra in new ways.

Year-of-manipulatives, all-operations. Because all of these students experienced the tile manipulatives and no significant differences were found between groups, the journal entries of the students who did not use manipulatives for the 22 preceding weeks were as interesting to analyze as the students in the treatment group. For the treatment group, the chapter test confidence interval was between 77 and 89. The semester grade confidence interval was between 82 and 88. For the control group, the chapter test confidence interval was between 86 and 97. The semester grade confidence interval was between 85 and 95. Four of the ten students in the treatment group and three of the ten control students demonstrated *unusual* scoring habits.

The journal entries of one of the treatment group students who improved, mirrored his companions' journal comments. "I am glad we are doing this, it is nice to get out of the daily routene." This was followed by, "I think the idea is good, but I don't like the tiles. It's kind of confusing to learn the math and the tiles and drawing is kind of a pain." Two days passed and he changed his mind, "I see what is going on with the tiles and it is getting easier." His final journal entry foretold his future, "I think I will have phenomenol performance on the test." He seems to have been able to successfully accommodate several representations: tiles, drawings, and algebraic symbols and hence to have created meaning.

The low scores of three of the four students who scored below typical expectations could be explained. The third-grade student's journal indicated his independence from the manipulatives. He simply did not want to use them, it reminded him of his regular 3rd grade classroom. Two of the four students who scored below typical expectations, experienced deaths in their families shortly before the chapter test. The fourth student's score is a bit of an anomaly. We have yet to find a reason for his poor performance. He enjoyed the tiles, his homework scores were acceptable, and his attitude was good.

When the journal entries of the control group was compared to those of the treatment group, one of the glaring differences was that the students in the treatment group were at ease with the general uses of manipulatives in the early phases of the experiment. For instance, on the first day of the experiment one control group students' journal entry reflected his lack of knowledge: "I like working with those little square and rectangular shaped things." During the second week, he said, "I worked with the tiles again. I really enjoy it, I hope we keep doing it ... helps me understand problems better." This indicated some maturity in the uses of the tiles. Finally, during the last week he understood that he was to use the tiles whenever he needed and that he should use algebra symbols whenever possible. "I worked with tiles for a little bit, but then I did the self-test. It's pretty easy." By the end of the experiment, all students made statements that indicated a fair comfort level with using tiles.

The most common transition for all students in both groups was to connect the journal entry to the mathematics being learned rather than to a description of the tiles. Students' early journals entries spoke of "learning about how to use the tiles" and "learning about math" as two separate ideas. As the days continued, these two ideas became merged and eventually all students reported only the "math" that they were learning. The tiles became a tool and the students saw them as such as evidenced by one student's late journal entry. "Today I learned about factoring. I like it. It's a lot of fun." Although the class had used algebra tiles the entire day, her writing reflected a learning activity that focused on math, not tiles. She saw the lesson as being *about* factoring. As Kaput hypothesized, as the students' representation systems included more connected items, they assigned more meaning to the ideas. These students have internalized algebra tiles as another form of the basic algebraic ideas of 1, x, and x^2. Such connections demonstrate student possession of mathematical meaning.

Conclusions

Although differences did not appear in the analyses of the chapter test data from the treatment and control groups of the factoring-only experiment, the treatment teacher from the suburban school stated that on the final exam (4 months after the experiment), some of the students from the treatment group drew pictures of algebra tiles to help them answer some of the questions. Similarly, the teacher from the small rural school indicated that 3 months after the experiment ended, he encountered a student who was unsuccessfully pondering a question. After prompting that student to "think of the tiles", the answer seemed clear and the student immediately went on to solve the problem. These examples would support Kaput's suggestion that students learn algebra through several modes of representation. These students relied on their knowledge about pictures and tiles to help answer questions during a time when the tiles were no longer part of their daily routine. It may be the case that they were trying to understand and answer the question by using a representation system that made sense to them.

As the narrative evidence suggests, the greatest power of the tiles is not in increasing test scores. Rather, it is in providing alternative representation systems internalized within individual students in ways that memorized facts and rote manipulations do not. Students spoke and wrote of being able to visualize the problems. A typical student who did not score in an *unusual* manner, said, "I'd never have been able to figure out this problem if it hadn't been for the tiles." When asked, he responded, "I can visualize the problem with them." He attached meaning to the algebraic topics that he studied.