The use of the cohort-survival method for projecting student enrollments is widely known in educational finance literature; however, the limited information provided by the model impedes planners in making future operational decisions. The cohort-survival method employs historical rates of usage to predict future patterns of usage and produces a grade-by-grade forecast for each student cohort. This paper presents a model that develops multipliers for several student-support programs to create a broad base of information. The model accommodates the number of regular public school students by grade level as well as an estimate of certain student-support programs--compensatory education, bilingual education, and 11 categories of special education. The model incorporates student-support-program requirements and adds the numbers of retained members to the next cohort, thus presenting a more accurate view of the enrollment picture. Two figures are included. (Contains nine references.) (LMI)
A Broad Base National Enrollment Model

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Introduction

The use of the cohort survival method for projecting student enrollments is well known in educational finance literature, but provides only limited information for planners to use for future operational decisions. Shaw (1984), in reviewing which enrollment forecasting methods work best, reported that for forecasts requiring grade-by-grade information that percentage survival and cohort survival are the most viable methods of forecasting enrollments.

The cohort survival method is a method that uses historical rates of usage to predict the patterns of usage. The prediction is based on the number of students who remain with the cohort through the series of school grades by applying a retention factor to each grade. The retention factor is a historical usage factor developed from several generations of cohorts passing through each grade. A retention factor is generated for each grade by averaging the retained students to passing students ratio over several generations of cohorts. Enrollments for future cohorts are obtained by multiplying birth data for a future cohort by a historical usage factor for kindergarten.

The kindergarten population is then multiplied by the retention factor for grade 1 through high school graduation. In each of the events there will be a portion of the cohort which does not progress to the next grade. The
assumption is that some of the members of the cohort move through each grade without repeating any one grade level. The result of this method of enrollment prediction produces a grade-by-grade forecast for each cohort. A broader base of information is required, such as student support programs, to formulate future operational planning.

Englehart (1974), alluded to the use of multipliers to provide additional information in conjunction with the grade-by-grade data supplied by the cohort survival method, but does not develop them. The model presented in this paper develops multipliers for several student support programs. The student support programs that are provided in this model include: compensatory education, bilingual education, and eleven categories of special education. Although these are not all the categories of student support programs that need to be considered, they will provide enough categories to illustrate the development of educational service need requirements projections when combined with the grade-by-grade information provided by the cohort survival method.

**Model Conception and Development**

In conceptualizing a model it is necessary to formulate a question for the model to answer. The question must be formulated in a manner that will allow the model to produce sufficient data to answer the question. For example, in forecasting enrollment it is desirable to know more than just the total number of students who will use the system. It is also helpful to know...
how many students will be in each grade and how many students will require student support programs.

It is also necessary to look at various possible methods to answer the question. An enrollment model that provides grade-by-grade enrollment information, such as the cohort survival method, is very helpful and desirable. The grade-by-grade information derived by the cohort survival method also provides a basis to answer questions about educational program requirements. The base population for each grade can be multiplied by a usage factor for each category of student support programs to produce a forecasted enrollment for each category. A usage factor is calculated by dividing the number of students who use these programs by the total membership in their cohort over several generations. Model conception allows the modeler to develop a foundation and a possible method for obtaining results.

System

In developing the model it is necessary to define the system under consideration. Systems are classified by type such as static, dynamic, homeostatic, or cybernetic. The educational system can be thought of as a cybernetic system. Cybernetic systems are those systems which are affected by environmental shifts, but have a means through feedback control to continue to meet system objectives (Athey, 1982). System objectives are not
rigid, but are adaptable to changing conditions and responsive to new understandings (Athey, 1982).

The educational system is capable of modifying its goals in response to external and internal pressures. These pressures can be thought of as the school systems' environment and control system. Its control and feedback component are through the various policies of the government bodies that regulate schools. The public school system also adapts itself to a variety of students, with differing program needs by modifying its process methods to meet those special needs such as special education, bilingual education, or compensatory education. In this study these special needs programs are referred to as student support programs to avoid confusion with special education programs.

The public education system's students are drawn from a subset of the general population restricted in eligibility only by age. The students progress through a sequence of process steps known as grades, taking a prescribed curriculum, and most exit the system as graduates.

The educational system typically recognizes that not all students are the same. As a result, differing process techniques are used with students requiring special education, compensatory, or bilingual education to produce the required output. The system also realizes that some students will undergo a rework process, i.e. grade retention, the student who is retained in the same grade must meet certain criteria for a given process step or grade
before moving to the next process step or grade. All of these considerations must be taken into account when modeling the system.

**Model Components**

There are four basic components in a cybernetic system that require attention in model formation. These include: (1) inputs, (2) process function, (3) control, and (4) output blocks. Each of these functional blocks plays a role in the development of the model. The input component, includes students and their program needs. Other public education inputs such as facilities, staff, and funds are tied to student counts that are a reflection of enrollment and are not taken into consideration in this model.

The process and control functions must be included as they directly affect the output. The outputs must be considered as the final step in the system. A schematic diagram is needed to represent the system so that the appropriate measure can be applied to the model.

**Schematic**

The schematic provides a pictorial view of how the system components interact to meet the goals of the system. By examining the interactions of system components, those variables found not to play a part in the interaction of the system are considered extraneous and are dropped from consideration in the model. The variables incorporated into the equation are
only those variables that truly affect the operation of the model with regard to the question being asked of the model.

The schematic must identify the elements of the system that directly impact the question being asked of the system. It is not necessary to model the entire process. Only the number of students entering the system, those leaving the system as graduates and those process considerations which affect the number of students in the cohort are placed in the model. In an enrollment model these process considerations are known as the retention multipliers.

As students complete their prescribed course of study, they are transformed into graduates that have greater economic value than when they arrived as inputs. Graduates then can be viewed as the output of the system. Thus, we are not directly concerned with the entire process of the system, but instead with the relation of the inputs to the outputs as controlled by the retention ratio for each grade.

**Input Considerations**

The input consideration for an enrollment model is limited to simply the general population which is qualified to participate in the public school system. For the purposes of the model we are interested in the historic births, generally five years prior to entry into public school system. Not all of the persons born in a given year will enter public school system six years
later for various reasons such as death, migration, or by choosing private education. To correct for these occurrences, a factor will be developed.

The other source of students is the persons of school age who immigrate to this country from other countries. It is reasonable to expect a portion of this population will not speak the English language and will require additional educational programs to pursue an education in this country. Immigrating students are not the only source of Non-English speaking students. There are also native born Non-English speaking students, such as Native Americans, found in various segments of the general population. As a result, it is necessary to provide an estimate of these populations so that resources can be found to implement suitable programs for this population segment. It is also reasonable to assume from experience that not all students will arrive at the school without disabilities. Disabilities have cost implications to the school systems of this country. A portion of the population also will not possess sufficient income to prepare their children for school. These children may require additional educational services to assist them in getting through the educational system.

**Historic Usage Multiplier for Kindergarten and First Grade**

The historic usage multiplier is developed by taking separate averages of the past usage rate of the kindergarten and first grade. The multiplier derived for kindergarten is known as the kindergarten ratio. It is calculated
by dividing the number of incoming kindergarten students by the number of children born in their birth year. The same calculation is performed for students entering the first grade. It is assumed that the rate of usage will remain constant over time. The number obtained by dividing kindergarten students by births is then used as a multiplier for future enrollment into kindergarten. This technique may be applied to other program needs such as compensatory, bilingual and special education as well.

Cohort Considerations

A cohort is the entire eligible number of persons in a given birth year who will use the public system at one time minus those individuals who either choose private education or do not survive to reach mandatory attendance age plus any changes due to immigration and students who were retained in any grade. A cohort can be expressed as the following equation:

\[ C_i = \sum P - \sum A - \sum t \]

Where:

- \( C_i \) = Cohort Birth year population, i.e. student cohort
- \( \sum P \) = the sum of all persons born in the cohort year, native and immigrant
- \( \sum A \) = the sum of those persons choosing private education
- \( \sum t \) = the sum of all persons born in the cohort year who die
Educational Process Considerations

Each cohort undergoes an iterative process in progressing through the public system, i.e. grade progression from the first to twelfth grade. Each grade has its own cost implications due to class size and specialization of instruction (Johns, Alexander, and Rossimiller, 1969). Moving through each of the grade levels is an interactive process that has cost implications.

The input is the number of eligible native persons in a given birth year plus immigrating persons of the same year minus those who will choose private education and those persons who do not survive to reach school age.

In addition to these basic considerations, the identification of the number of students who qualify for student support programs is required. These students pose additional cost impacts. Each of the student support programs must be considered and expressed as a set of multipliers. The multipliers are then used to determine the number of special education service requirements that exist for a given cohort. The above considerations then comprise the model and is schematically represented as shown in figure 1.

Grade Retention Multiplier

Experience has shown that not all students will progress from one grade to another grade in the normally expected period of one school year.
Experience has also shown that there is not a uniform multiplier that can be applied to all grade levels. In reviewing the National Center for Education Statistics published Public school grade retention rates, the rates vary from 91.1 for the grade 10 to grade 11 progression to 111.0 for the eighth to ninth grade progression using 1992 rates (Gerald and Hussar, 1995). The reason for this variance is that some grade levels will have more students retained than others and in some grades more private students will transfer from private to public school. Some students will terminate their education for a time and return to the school system later. Others will not return to school at all. These factors are addressed by a retention multiplier.

Students who have failed, $R_i$, any one grade translates to an increase in the Cohort currently in grade $i$ which is represented by $C_{t+1}$. Conversely, these same students will represent a decrease to the cohort which has just completed grade $i$ and is represented by $C_t$. Moreover, those members of the cohort who die are treated as a subtraction against the cohort and are represented by $F_i$. Those students transferring from private schools and immigrants who provide an increase to the cohort in grade $i$ are represented by $P_i$. The retention multiplier becomes the following equation:

$$C_t = (C_i + P_i + R_{t+1}) - (R_t + F_t)$$

Where:

- $C_t$ = Total cohort population in current year
- $C_i$ = Cohort (birth) population in current year
R_{(i+1)} = Retained members of previous cohort

F_i = Cohort members in grade_i who die grade_i

P_i = Incoming population transfers from private school and immigrant influx.

In practice the multiplier is derived in a more direct manner. To obtain the multiplier, divide the enrollment of each cohort in each grade by those students leaving grade_i by those who entered grade_i, repeat this process for several generations. Sum the retention ratio for each grade and then average the results to obtain the retention multiplier for each grade transition. This calculation accounts for all the above concerns.

Dropouts Considered as a Part of the Retention Multiplier

Some students make the decision to dropout. Their membership in the cohort is terminated. Dropout rates may be overestimated and subject to some undefined level of error as defined by Crouch (1991). This is because some dropouts will eventually return to school and become members of a different or the same cohort when they return depending on the length of time they are gone. Other dropouts will not return and still other students will repeatedly dropout and return. Dropouts are considered in the retention multiplier as a subtraction against the birth year cohort in grade_i and can be represented as D_i.

The remaining cohort is expressed as:
\[ C_t = \{(C_i + I_i + R_{(i-1)}) - (R_i + D_i + F_i)\} \]

Where:

- \( C_t \) = Total Cohort population in Current year
- \( C_i \) = Cohort (birth) population in current year
- \( R_{(i-1)} \) = Retained members of previous cohort
- \( R_i \) = Cohort members in grade\(_x\) who will be retained in grade\(_x\)
- \( D_i \) = Population in grade\(_x\) that drops its membership by dropping out.
- \( F_i \) = Deaths in the cohort in grade\(_x\)
- \( P_i \) = Incoming students transferring to public school and immigrant influx

The number of students retained from a cohort is then expressed as the following equation:

\[ RC_i = \text{Cohort in Grade}_x - (\text{Cohort in Grade}_x \times R_i) \]

This number of students will remain in the same grade for the next year.

The use of the National Center for Educational Statistics (NCES) grade retention factors is adequate for most projections. The retention factor, which is a multiplier in this study, is a value calculated for each grade level by dividing those leaving a grade level by those who entered that grade level. The individual retention factors are summed and averaged over a five year
period. An average is taken of each grade, kindergarten to twelve grades, for the five year base period.

The retention multiplier equation is:

\[
R_i = \frac{\text{Enrollment in Grade}_{i}}{\text{Enrollment in Gradex}_{i+1}}
\]

According to Simpson (1987), the cohort survival model assumes that children move on one year at each school year, without repeating years, as they grow older. Simpson’s assumption is not representative of the actual process. This assumption does not account for dropouts, immigration, or emigration. In a systems approach, failures are treated as a rework of an input. Thus the students who have failed the grade need to be added to the cohort entering that grade. This changes the direct retention calculation for each grade in the following manner:

\[
R_i = \left\{ \frac{\text{Enrollment in Grade}_{i} + \text{Members of the previous cohorts retained in Grade}_{i}}{(\text{Enrollment in Grade}_{i}) + (\text{Enrollment in Grade}_{i+1})} \right\}
\]

The adding of nonsurviving cohort members of each preceding cohort in Grade to the cohort currently in Grade has been incorporated into this study’s model to reflect this reality. The value of \(R_i\) is then increased as the cohorts are mixed.
Retention Multiplier Values

Retention multipliers can take any value from zero to some value greater than 1.0, but the values generally range from .90 to 1.11. Retention multipliers reflect the ability or the efficiency of the cohort to reach some point in the educational process. A hundred percent movement or a one hundred percent efficiency would be represented by no loss in membership within the birth cohort's initial population and is indicated by the value 1.0. Since, the cohort can increase by transfers from private to public schools, immigration, and so forth it possible that the size of the cohort may exceed the value of 1.0. Any multiplier having a value greater than 1, therefore, would represent an increase in the size of the cohort.

Kindergarten Enrollment Processes

Kindergarten enrollment numbers do not provide stable data for future enrollment projections (Pettibone & Bushan, 1990) The reason is that kindergarten enrollment is not as firm as first grade enrollment. This is because some states permit alternatives to kindergarten enrollment and have different reporting criteria for kindergarten and preschool. As a consequence the total number of children in kindergarten will underestimate total enrollment in future years. The NCES data, for example, in addition to
reporting kindergarten also contains some preschool information (NCES, 1994). Since kindergarten is provided at most public schools a projected enrollment must be provided in any enrollment model. However, the best estimator of current and future needs of kindergarten are factors developed from past usage rates for kindergarten. Retention information is not available for kindergarten and was not included in the kindergarten equation. As a result, kindergarten is treated as a stand-alone category of public school grade enrollments.

The equation for Kindergarten enrollment becomes:

\[ K_t = C_i \times (K_{ratio}) \]

Where:

- \( K_t \) = Cohort total enrolled in kindergarten
- \( C_i \) = Cohort population
- \( K_{ratio} \) = Derived kindergarten enrollment coefficient

First Grade Enrollment Processes

First grade enrollment is fairly consistent in terms of percentage of students enrolled. The age standard is typically six years old at the time of entrance. Since all the states require enrollment in first grade as a result of compulsory attendance laws, the numbers reported to NCES are more accurate than kindergarten enrollment. Since first grade is a stable figure and does not contain any additional enrollments, it will be used as the base
grade in the enrollment projection model. Retention figures are reported for the first grade and each grade beyond to graduation. Since retention is an important factor in the size of the cohort it is placed in the first grade equation.

The first grade equation becomes:

\[ G_{t1} = C_t \left( F_{\text{ratio}} \right) + R_{t+1} \]

- \( G_{t1} \) = Cohort total students enrolled in first grade
- \( C_t \) = Cohort entering grade
- \( F_{\text{ratio}} \) = First grade historic enrollment ratio
- \( R_{t+1} \) = Retained students from previous year

**Second through the Tenth Grade Enrollment Process**

In the second grade through the tenth grade, the enrollment process is basically the same as the first grade process except that each grade level is dependent upon the number of surviving students from the previous grade. The accuracy of the calculated retention multiplier then plays the prime role in influencing the change in the number of students in the next grade level.

The equation for each grade's enrollment is:

\[ C_{tx} = ( G_t + RC_i ) \]

The second to tenth grade enrollment into the next grade level equation after considering retention becomes:

\[ C_{tx+1} = ( G_t + RC_{t+1} ) \times R_i \]
This equation is then used to calculate enrollment into the next grade for every grade from grade 2 to grade 10.

Grade 10 to Twelfth Grade Enrollment Process

After a cohort has progressed into high school, a new factor must be considered which is the ability of students to voluntarily drop their enrollment in school. Information regarding dropouts at lower grades is not readily available as most states require enrollment until a student has reached sixteen years of age. This study will regard dropping out as generally coinciding with attaining the age of sixteen and is typically reflected in enrollments at the tenth grade. With the addition of this factor, represented by \( d_x \), the equation for enrollment changes to:

\[
C_t = ((G_t + RC_i) - d_x) \times R_i
\]

Twelfth Grade to Graduation Enrollment Process

The completion of the twelfth grade represents the final process step in the public education process. As a result the enrollment equation for the twelfth grade becomes:

\[
C_{t12} = (G_{12} + RC_{12}) - D_{12}) \times R_{12}
\]

\( C_{t12} \) is defined as the number of students who will potentially graduate, i.e. become an output of the educational system. Graduating students have
completed all the process steps and are the output of the system described by this model.

**Student Support Program Equations**

This model is limited to three broad categories of student support programs. These categories are special education, compensatory education, and bilingual education. Each of these special groups is drawn from the population.

Since these special populations are drawn from the general population it is assumed that the general population mix for student support programs is somewhat constant over time. As a result, it is reasonable to apply a multiplier for each program and to each grade level to obtain the demand for student support programs.

**Special Education**

Special education is broken into twelve sub-categories of disability in the NCES literature. Although, special educators have tried to avoid the use of labeling categories, these categorizations have been used to obtain the funds to provide an education to students who are assigned to those categories. The special education categories in this model are the following: deaf, deaf and blind, hard of hearing, mentally retarded, multiple disabled, orthopedic, serious emotional disturbance (emotional), specific learning
disabled, speech, visually impaired, autistic and other health impaired. The special education equation in this study is:

\[
\text{SED} = \frac{\text{Sum of all Special Education Categories}}{\text{Total Student Population}}
\]

This is the general equation for all special education services. The individual categories in this model use a separate multiplier. The multiplier for each special education category is developed in the same manner as the kindergarten ratio. Each multiplier is applied across the grade levels to produce an equivalent grade-by-grade enrollment.

**Compensatory Education**

Compensatory education is considered in this model as a student support program. The central idea behind compensatory is provide support to lower socio-economic students who need supporting programs to maintain normal progress in their education. The defining indicator, typically, is a family income level in the lower twentieth percentile.

Census data provides an estimate of family income that complies with this definition. The number of persons receiving state aid or falling into the lower twentieth percentile is divided by the total number of students to obtain a multiplier. The compensatory education equation is:

\[
\text{CED} = \frac{\text{Number of Low SES Students}}{\text{Total Student Population}}
\]
Bilingual

Bilingual education is treated similarly to compensatory education by this model. The multiplier is calculated by dividing the total students needing bilingual education by the total number of students in the school system. The bilingual education service need equation is:

\[ \text{BED} = \frac{\text{Number of Non-native English Speaking Students}}{\text{Total Student Population}} \]

The Model Algorithm

The model algorithm is developed from the above system components and factors to provide the desired projection of enrollments by grade and student support program requirements. The flow diagram for the algorithm used in this model is presented as figure 2. The algorithm reflects the concerns delineated in this model such as adding retained members into the next cohort and the developing student support program requirements for specific educational services.

The model's output is presented immediately after the algorithm flow chart. All enrollment figures are presented in terms of thousands enrolled. Unclassified enrollments are presented to provide a comparison with NCES enrollment. Unclassified enrollments are those students in schools that do not use the customary grade levels to denote student progress through the
system. Student support program enrollments are presented as separate tables below the grade-by-grade enrollment data.

**Summary**

The basic components of the model in this study have been described. The number of regular public school students by grade level is accommodated in this model as well as an estimate of certain student support programs. Model components are brought together as a set of equations that will form the operating equation for the developed model. The addition of student support program requirements and adding retained members to the next cohort presents a more accurate view of the enrollment picture within the public school system. Incorporating that available information provides a wealth of information that cannot be overlooked in the budget planning process.
Figure 1
School System Enrollment Model:
Students as Inputs and Graduation as Outputs

- Issue In: Immigrant population
- Issue In: Native Population
- > Req'd Entry Age
- Delay: State Required Age for Entry
- Decision
- Private
  - Procedure: Private Education
  - Decision: In process
    - Yes
    - Issue Out: Graduation
    - No
    - Issue Out: Terminated
- Public
  - Decision: Terminated
    - Yes
    - Issue Out: Terminated
    - No
    - External Process
      - Yes
      - Issue Out: Terminated
      - No
      - Terminated Fail to Reach Req'd Age
      - Issue Out: Terminated
Figure 2
Model Algorithm

Birth Data

Kindergarten Process

Apply Factors

\[ K_t = C_i \left( \frac{K_{\text{ratio}}}{R_i} \right) \]

First Grade Process

Apply Factors

\[ G_{t1} = C_i (\text{Fratio}) + R \]

Second to Grade 10 Process

Apply Factors

\[ C_{tx} = (G_{t1} + R C_{i1}) \cdot \frac{D_x}{R_i} \]

Grade 10 to Grade 12 Process

Apply Factors

\[ C_{tx} = (G_{t1} + R C_{i1} + 1) \cdot \frac{R_x}{R_i} \]

Grade 12 Graduation

Apply Factors

\[ C_{tx} = (G_{t1} + R C_{12}) \cdot \frac{D_x}{R_{12}} \]
References


