The choice of a cutting score for criterion-related tests influences decisions related to classifying people into dichotomous categories. This paper proposes an empirical methodology for determining the best cutting score when there is information about the test score frequency distribution of test-takers defined as actually successful and actually unsuccessful on some criterion. The method is based on two statistics calculated for each possible cutting score. The first is a pure hit rate, representing the proportion of correct classifications above those expected by chance. Second is a chi-square statistic for testing the significance of the difference between the population frequencies of the two types of misclassification errors. A cutting score summary table is developed based on the information about the test score frequency distributions of two validation samples based on actually successful and actually unsuccessful samples. Cutting scores are divided into those that yield equal frequencies of the two types of misclassification errors and those in which the frequency of one type of error is higher than that of the other. The cutting score summary table facilitates the determination of the best cutting score in each category. (Contains 2 tables and 19 references.) (SLD)
On the Cutting Score Determination in Dichotomous Classifications

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ON THE CUTTING SCORE DETERMINATION
IN DICHTOMOUS CLASSIFICATIONS

INTRODUCTION

The choice of a cutting score for criterion-related tests influences decisions related to classifying people into dichotomous categories - for example, decisions based on tests for admitting students to a college, hiring job applicants, prescription of preventive psychopathological therapy, etc. In general, choosing an appropriate cutting score is an essential issue in setting standards on educational, psychological, and occupational tests which explains the considerable amount of publications related to this topic. All major works on determining optimal cutting scores focus on the estimation of two possible types of misclassifications errors by using different models for the true score distribution - mainly Bayesian models and binomial models (e.g. Hambleton & Novick, 1973; Klein & Cleary, 1967; Huynh, 1976; Lord & Stocking, 1976; Wilcox, 1977). The sum of the estimated misclassification error probabilities, multiplied by judgmentally specified misclassification "losses", define the expected loss and, most frequently, the test score that minimizes the expected loss is taken as the best cutting score. However, factors like need for testing model assumptions, judgmental nature of the misclassification losses, and relatively difficult calculation of the expected losses still keep the door open for a search of technically simple procedures which do not include assumptions about the true score distribution.

In an attempt to make a step in this direction, the present paper proposes an empirical methodology for determining the best cutting score when there is an information about the test score frequency distribution of test-takers defined as actually successful and actually unsuccessful on some criterion (educational, clinical, professional, etc.).

METHOD

The approach proposed here is methodologically based on the following two statistics calculated for each possible cutting score:

1) A "pure hit rate", PHR, representing the proportion of correct classifications above the expected by chance.

2) A $\chi^2$ - statistic for testing the significance of the difference between the population frequencies of the two types of misclassifications errors.

Table 1 represents the general form for the two-way classification frequency distribution yielded by a given cutting score. Cell A is the frequency of correct classifications of the type "predicted successful - actually successful" (PS-AS), and cell D is the frequency of correct classifications of the type "predicted unsuccessful - actually unsuccessful" (PU-AU). Cell B is the frequency of misclassifications of the type "predicted unsuccessful - actually successful" (PU-AS), and cell C is the frequency of the other type misclassifications: "predicted successful - actually unsuccessful" (PS-AU). The proportion of the correct classification is called hit rate:

$$HR = \frac{A + D}{N}, \text{ where } N = A + B + C + D.$$
Table 1

<table>
<thead>
<tr>
<th>Actual classification</th>
<th>Predicted classification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Successful</td>
</tr>
<tr>
<td>Successful</td>
<td>A</td>
</tr>
<tr>
<td>Unsuccessful</td>
<td>C</td>
</tr>
<tr>
<td>Total</td>
<td>A + C</td>
</tr>
</tbody>
</table>

The hit rate, as calculated by (1), is taken into account in many empirical approaches for determining optimal cutting scores (e.g. Berk, 1976; Allen & Yen, 1979, p. 104). However, its value includes a proportion of correct classifications that may occur by chance. In order to avoid this problem and increase the reliability of the cutting score determination, we propose the use of the Cohen's kappa, \( \kappa \), (see Cohen, 1960). In the context of the present study, \( \kappa \) will represent the proportion of correct classifications, PS-AS and PU-AU, above that expected by chance, i.e. the "pure hit rate", PHR, and is calculated by the respective formula:

\[
PHR = \frac{HR - P_c}{1 - P_c}
\]

where: HR is the hit rate calculated by (1); \( P_c \) is the proportion of correct classification expected to occur by chance and, in terms of the cell frequencies in Table 1, is: \( P_c = \frac{(A + B)N + (C + D)N}{A + B + C + D} \).

The question about the equality of the misclassifications in both "directions" PS-AU and PU-AS, is related to testing the following null hypothesis: For a given sample of test-takers, the entries B and C in Table 2 differ only as a result of chance sampling. If this is true, the expected number of PS-AU misclassifications equals the expected number of PU-AS misclassifications and is given by the average of C and B, i.e. \((B + C)/2\). Hence, the null hypothesis can be tested by the use of a \( \chi^2 \)-statistic, which is the sum of the squared differences between the observed and expected frequencies, each divided by the expected frequency:

\[
\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(C - B)^2}{E} + \frac{(B - C)^2}{E} = \frac{(B - C)^2}{B + C}.
\]

Hence, the calculation of the \( \chi^2 \)-statistic for testing the significance of the difference between the population frequencies of the two types of misclassification errors is given by the formula:
This is, in fact, an application of the McNemar test for significance of changes for the situation represented by Table 1. In this case, 2 x 2 tables, the degrees of freedom are df = 1 which makes the use of $\chi^2$ suspicious when the expected frequencies $\frac{B+C}{2}$ are less than 5. The Yates' correction for continuity leads to the following corrected form:

$$\chi^2 = \frac{(B-C-1)^2}{B+C};$$  \hspace{1cm} (see McNemar, 1969, pp. 260-263).

For example, if a given cutting score leads to the following cell frequencies in Table 1: A=45, B=15, C=10, and D=30, by using formula (3), we calculate: $\chi^2 = \frac{(B-C)^2}{B+C} = \frac{(15-10)^2}{15+10} = 1.00$. This number is less than the critical value, $\chi^2 = 3.841$, at level of significance $\alpha = .05$ and degrees of freedom df = 1. Hence, in this case, the cutting score yields equal population frequencies of the two types of misclassifications errors, PS-AU and PU-AS. On the other hand, the pure hit rate yielded by this hypothetical cutting score will be PHR = .49, after applying formulas (1) and (2) for the calculation of the hit rate, HR, and the pure hit rate, PHR, respectively.

Thus, the $\chi^2$ statistic and the PHR index answer two very important questions related to each possible cutting score:

1) Does the cutting score yield equally serious misclassification errors, PS-AU and PU-AS?
2) What is the proportion of correct classifications above that expected by chance?

Cutting score Summary Table (CTS)

Proposed here is a table that summarizes the $\chi^2$-values, the PHR values, and the cell frequencies A, B, C, and D from Table 1 yielded by each possible cutting score. This table, called "Cutting score Summary Table" (CST), is based on the information about the test score frequency distributions of two validation samples of people defined as actually successful and actually unsuccessful. Table 2 represents a CST for hypothetical data including a test scale, given in column C1, and the frequencies over this scale of actually successful (AS) and actually unsuccessful (AU) test-takers, given in columns C2 and C3, respectively. The calculation of the numbers in columns C4, C5, ..., C10 is straightforward:

- C4 = the number of correct PS-AS classifications (cell A in Table 1), obtained as cumulative frequencies from column C2;
- C5 = the number of PS-AU misclassifications (cell C in Table 1), obtained as cumulative frequencies from column C3;
- C6 = the number of PU-AS misclassifications (cell B in Table 1), obtained as $N_s - A$, i.e. by subtracting the column C4 numbers from the total number of successful people, $N_s$;
C7 = the number of correct PU-AU classifications (cell D in Table 1), obtained as N_u - C, i.e. by subtracting the column C5 numbers from the total number of unsuccessful people, N_u;
C8 = the hit rate, HR, calculated by formula (1);
C9 = the pure hit rate, PHR, calculated by formula (2);
C10 = the \( \chi^2 \) statistic, calculated by formula (3).

<table>
<thead>
<tr>
<th>Test score</th>
<th>Actually Success</th>
<th>Actually Unsuccess</th>
<th>PS-AS (A)</th>
<th>PS-AU (C)</th>
<th>PU-AS (B)</th>
<th>PU-AU (D)</th>
<th>HR</th>
<th>PHR</th>
<th>Chi-sq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>75</td>
<td>46</td>
<td>.423</td>
<td>.078</td>
<td>75.00</td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>2</td>
<td>27</td>
<td>2</td>
<td>57</td>
<td>44</td>
<td>.546</td>
<td>.218</td>
<td>51.27</td>
</tr>
<tr>
<td>7</td>
<td>26</td>
<td>7</td>
<td>53</td>
<td>9</td>
<td>31</td>
<td>37</td>
<td>.692</td>
<td>.393</td>
<td>12.10</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>10</td>
<td>64</td>
<td>19</td>
<td>20</td>
<td>19</td>
<td>.700</td>
<td>.347</td>
<td>0.02 **</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>8</td>
<td>74</td>
<td>27</td>
<td>10</td>
<td>19</td>
<td>.715</td>
<td>.321</td>
<td>7.81</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>78</td>
<td>31</td>
<td>6</td>
<td>15</td>
<td>.715</td>
<td>.290</td>
<td>16.89</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>7</td>
<td>80</td>
<td>38</td>
<td>4</td>
<td>8</td>
<td>.677</td>
<td>.152</td>
<td>27.52</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>84</td>
<td>44</td>
<td>0</td>
<td>2</td>
<td>.661</td>
<td>.055</td>
<td>44.00</td>
</tr>
<tr>
<td>0 or 1</td>
<td>0</td>
<td>2</td>
<td>84</td>
<td>46</td>
<td>0</td>
<td>0</td>
<td>.646</td>
<td>.000</td>
<td>46.00</td>
</tr>
</tbody>
</table>

The ** in column C10 indicates a \( \chi^2 \)-statistic which is less than the critical \( \chi^2 = 3.841 \), at the \( \alpha = .05 \) level of significance. The respective cutting score yields equally serious misclassification errors, PS-AU and PU-AS, in the sense that it yields equal frequencies of the two types of errors over the entire population of test-takers.

As one can see from Table 2, the test score of 6, if taken as a cutting score, is the only one that yields equally serious misclassification errors, PS-AU and PU-AS, because its \( \chi^2 \)-statistic (=.02) is the only one which is less than the critical \( \chi^2 = 3.841 \) (with \( \alpha = .05 \) and df = 1). Hence, under the assumption of equally serious misclassification errors, we can choose the cutting score of 6. One can also see that all cutting score above the cutting score of 6 yield higher frequency of the PU-AS error compared to the frequency of the PS-AU error. Hence, if we prefer more PU-AU errors over the entire population of test-takers, we can choose the cutting score of 7 as the best cutting score because it yields the highest pure hit rate (PHR=.393) among all cutting scores above the cutting score of 6. Finally, if we prefer more PS-AU errors over the entire population of test-takers, we can choose the cutting score of 5 as the best one because it yields the highest pure hit rate (PHR=.321) among all cutting scores below the cutting score of 6.
CONCLUDING REMARKS

The method proposed here for determining the best cutting score is based on the idea of the pure hit rate (PHR = proportion of correct classifications above that expected by chance) and on the fact that the McNemar $\chi^2$ test in the context of dichotomous classification tables (see Table 1) divides the set of all possible cutting scores into three categories:

A) Cutting scores that yield equal frequencies of the two types of misclassification errors, PS-AU and PU-AS, over the entire population of test-takers. These cutting scores yield $\chi^2$-statistics which are less than the critical $\chi^2$ (e.g. $\chi^2 = 3.841$ at the level $\alpha = .05$). The best cutting score is the one that yields the highest pure hit rate among all cutting score in this category, assuming that the two types of misclassification errors are equally serious.

B) Cutting scores for which the frequency of the PU-AS error is higher than this of the PS-AU error over the entire population of test-takers. These cutting scores yield $\chi^2$-statistics which are greater than the critical $\chi^2$ (e.g. $\chi^2 = 3.841$ at the level $\alpha = .05$) and they are greater then the cutting scores from the above category, A). The best cutting scores yields the highest pure hit rate among all cutting scores in this category, B), assuming that the PU-AS errors are less serious than the PS-AU errors.

C) Cutting scores for which the frequency of the PS-AU error is higher than this of the PU-AS error over the entire population of test-takers. Like the cutting scores in category B), the cutting scores in this category also yield $\chi^2$-statistics greater than the critical $\chi^2$ (e.g. $\chi^2 = 3.841$ at the level $\alpha = .05$), but they are less than the cutting scores in category A). The best cutting score is the one that yields the highest pure hit rate among all cutting scores in this category, C), if it is assumed that the PS-AU errors are less serious than the PU-AS errors.

The Cutting score Summary Table (CST), illustrated by Table 2, facilitates the determination of the best cutting score in dependence of the category, A), B), or C), reflecting the assumption about the seriousness of the misclassification errors. The development of the CTS is straightforward for a simple use of a calculator or some statistical software. For example, the description of columns C4, C5, ..., C10, given in relation to Table 2, is directly interpretable in MINITAB commands. This is an important advantage of the method for either real data manipulations or computer simulations in the process of determining the best cutting score for the purposes of dichotomous classifications.
REFERENCES


