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ABSTRACT

A software tool called 3D-LAB has been developed for learning and teaching three-dimensional geometry. With this microworld, educators and students can display three dimensional solid objects, rotate them, modify them, open them up, draw points and segments, and measure lengths, areas, volumes, and angles. The major characteristics of this tool are the interactivity of the manipulations and the variability of the objects. Two conflicting aims--ease of manipulation and variety of manipulable solids--are realized by a combined method of loading solids first and modifying them next. Both quantitative and qualitative observations can be made using 3D-LAB. Reproductions of computer screens illustrate the discussion. (Author/BEW)

Paper (M4-202B)

A Laboratory for Learning and Teaching 3D Geometry

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Abstract

We have developed a software tool named 3D-LAB for learning and teaching three-dimensional geometry. By using 3D-LAB, students and teachers can display a basic three-dimensional solid object (such as a cone, pyramid, cylinder, or polyhedron) in either wire-frame or surface mode, rotate it interactively, modify it, find its segments that intersect with a cutting plane, truncate it by means of a cutting plane, open the basic or truncated solid, draw points or segments in three-dimensional space, and measure lengths, areas, volumes, and angles. The major characteristics of this tool are interactivity of manipulation and variability of the manipulated objects. Two conflicting aims—ease of manipulation and variety of manipulable solids—are realized by a combined method of loading solids first and modifying them next. A three dimensional microworld for learning geometry is realized through the ability to make qualitative and quantitative observations while using 3D-LAB on a personal computer.

Overview

We previously developed two tools in the Mathematics Laboratory Series on personal computers as domestic products of IBM Japan: Calculus Lab (for calculus in high-school mathematics (Akaishi, 1992)) and GeoBlock (for two-dimensional geometry in junior high-school mathematics (Hidaka, 1990; Hidaka, 1992)). Our basic principle was to provide tools for thinking, learning, and teaching, with simple user interfaces. Following the same principle, we have developed another software tool, named 3D-LAB, which allows students to manipulate solids and observe them quantitatively and qualitatively.

By using 3D-LAB, students and teachers can display a basic three-dimensional solid object (such as a cone, pyramid, cylinder, or polyhedron) in either wire-frame or surface mode, rotate it interactively, modify it, find its segments that intersect with a cutting plane, truncate it by means of a cutting plane, open the basic or truncated solid, draw points or segments in three-dimensional space, and measure lengths, areas, volumes, and angles.

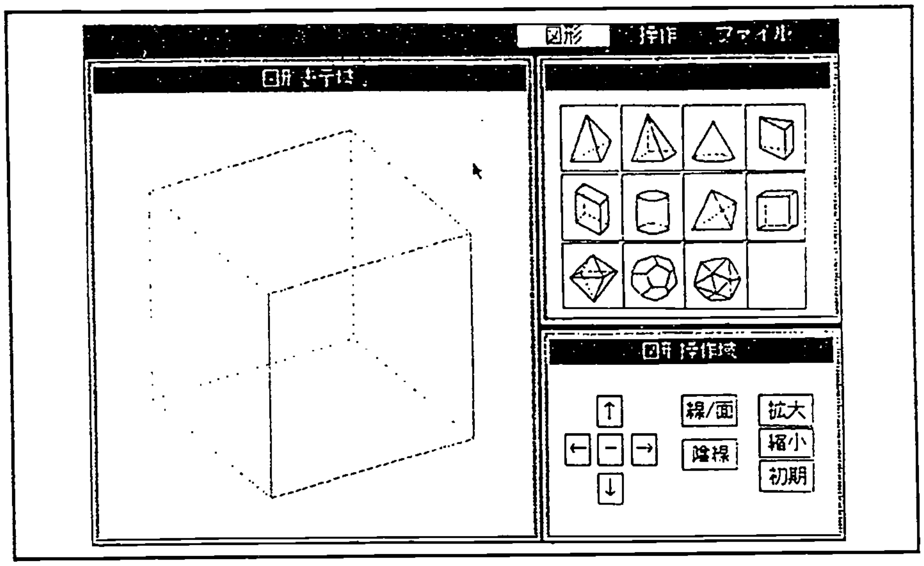

Figure 1. shows the initial display of 3D-LAB. Almost all operations can be performed by selecting from menus, pushing buttons, and dragging points directly with a mouse.

Let us take an example of learning with 3D-LAB. Students can load the pyramid shown in Fig. 2(a) by pushing the pyramid icon. To this "3D-drawings", they can add a segment, the perpendicular line from the top vertex, A, to the base, BCDE (Fig. 2(b)). They can measure the length of AH, the area of BCDE, and the volume of ABCDE, and find experimentally that the volume is one third of the product of AH and BCDE. During this operation, they can rotate the solid, and thus assimilate its three-dimensional structure (Fig. 2(c)).

They can display a cutting plane (Fig. 2(d)) and move it by dragging three control points. The intersecting segments can be calculated and displayed automatically, when the cutting plane is relocated (Fig. 2(e)). Students can also find the area bounded by these intersecting segments, and investigate the relations between the height and volume of the pyramid.

Using modifying functions, they can modify the pyramid so that the top vertex is in a different position, and again measure the height, area, and volume (Fig.2.(f)). This manipulation will help them understand the mathematical principle of the height and volume of a pyramid. Furthermore, they can divide the above pyramid by applying an arbitrary cutting plane repeatedly (Fig. 2(g)), and can open and close the original pyramid or truncated pyramid.

It should be clear now that the major characteristics of this software are interactivity of manipulation and variability of the manipulated objects. In this sense, 3D-LAB is a three-dimensional microworld that allows students to learn the principles of geometry for themselves.

Research was done by Professor Kenjiro Suzuki (University of Tokyo) and Sakiko Wakita (IBM Japan), on how to use the rotating and cutting functions in geometry classrooms. They reported such tool software was effective for improving students' understanding on three-dimensional objects (Suzuki, 1991; Wakita, 19919). We are confident that free manipulation of solids, as described above, will interest students and promote their understanding on 3D geometry.

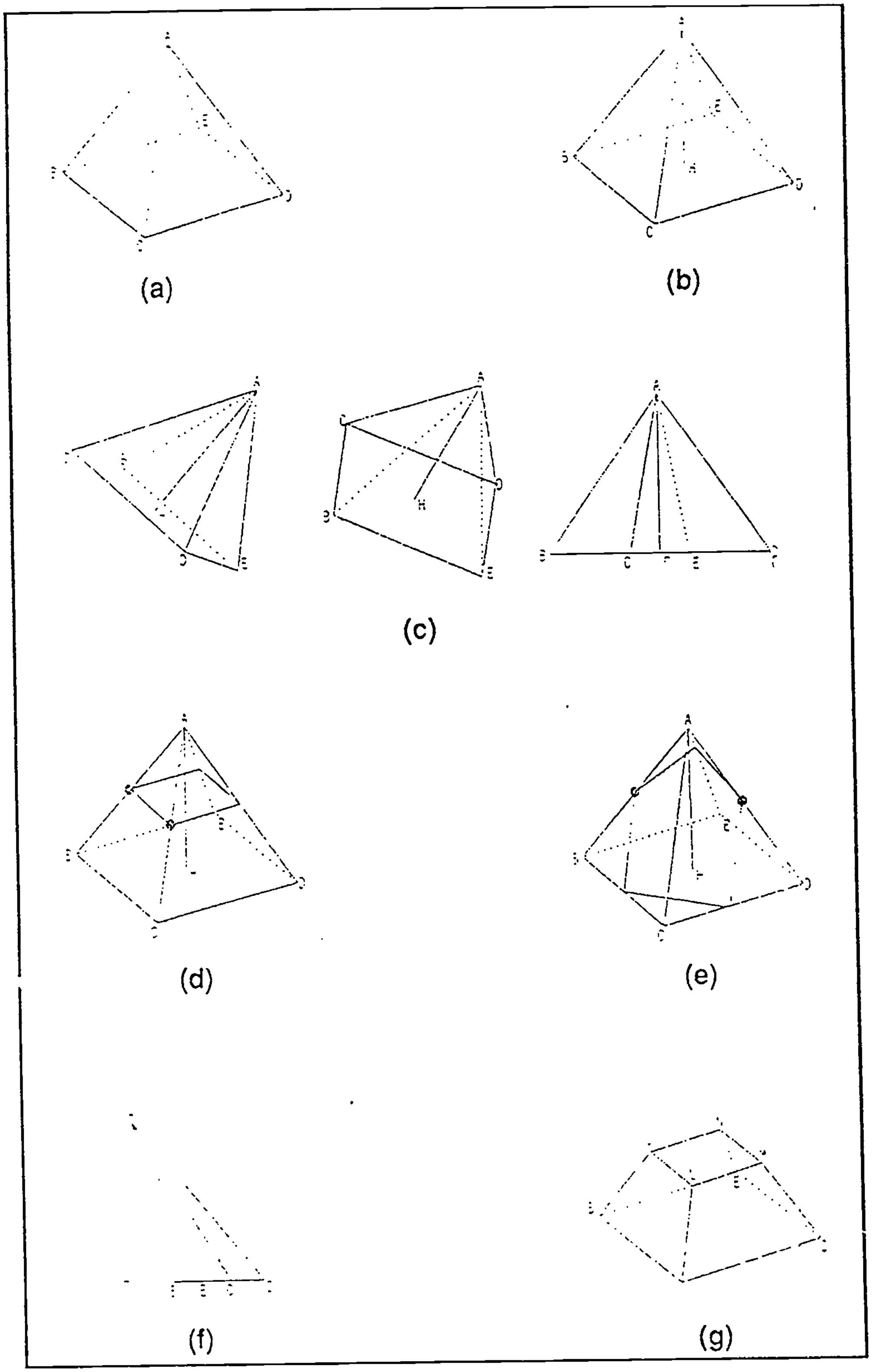

Figure 2. Manipulating a solid.

Implementations

Generally speaking, it is difficult to define and draw a three-dimensional object on a personal computer's display, even if a three-dimensional CAD system is used. Students and teachers are often novice users of computers, and therefore have special difficulties in manipulating three-dimensional objects. To ensure both ease of manipulation and a variety of solids, we designed this software so that a solid can be defined by (1) loading it, and then (2) modifying or truncating it.

Basic solids

Students start learning by loading solids. 3D-LAB handles 11 basic solids: triangular and square pyramids, cone, cylinder, cuboid, triangular prism, regular tetrahedron, cube, octahedron, dodecahedron, and icosahedron (see Fig. 1). These objects can be displayed in either wire-frame mode or surface mode. The hidden edges are represented by dotted lines, and can be turned on or off. We think that providing functions for changing mode (wire-frame, surface, or hidden-edge on/off) is an efficient way of helping students to understand the meaning of hidden edges, hidden planes, and the depth of three-dimensional space. Users can label vertices with alphabetic characters in order to designate vertices, edges, and faces, and the colors of edges and planes can be changed to highlight specific parts.

Rotating

A solid on the display can be rotated in two directions (vertically and horizontally) by pushing rotation buttons. We designed a rotation matrix for calculating a new two-dimensional image of a solid, so that the rotation operation matches the user's mental model. That is, a solid rotates in three-dimensional space, in the direction indicated by the arrow of the pushed button. It can be rotated at any time while the object is being manipulated. We believe that observation of a solid from an arbitrary direction will familiarize students with the structure of the solid.

Modifying

Pyramids, cones, cuboids, and cylinders can be modified by relocating the position of the top vertex, changing the shape of the base, or changing the height. Modification is executed on two planes: the vertical plane and the horizontal one (Fig. 3). During the modification of the solid, the lengths of edges and the angle values can be displayed. The modifying function enables a user to define solids with various shapes from a single basic solid. It also provides teachers with a simple way of creating teaching materials, including various figures.

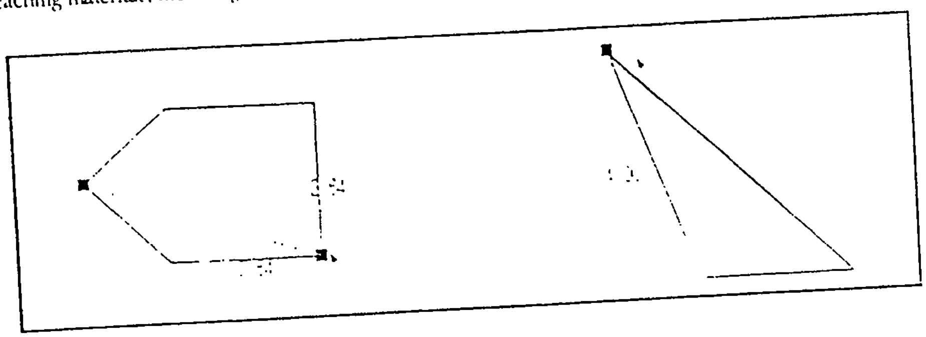

Figure 3. Modifying a solid.

Drawing

Segments and points can be drawn on the edges of the original or truncated solid. In analog mode, points and the starting and ending points of segments are located along all the edges or pre-drawn segments. In digital mode, these points are located only at the dividing points (1/3, 1/4, 1/5) of edges or pre-drawn segments. The digital mode enables a user to draw a specific segment, such as one connecting the midpoints of two opposite edges of a cube. By pushing the perpendicular segment button, students can draw a perpendicular segment from any vertex to (1) an edge or pre-drawn segment, (2) a face, or (3) a cutting plane. Figure 2(b) is constructed by using this function. Students could never draw such complicated figures in three-dimensional space without a computer software tool such as 3D-LAB.

Cutting

A cutting plane is defined by three control points on three edges, and can be modified by (1) changing the locations of these control points, (2) pushing the parallel transformation button, or (3) pushing the rotating transformation button. If the parallel transformation button is pushed, the cutting plane can be moved in the direction normal to this plane. If the rotating transformation button is pushed, the cutting plane can be rotated around the segment connecting two control points. By using these operations, students can interactively observe how the lines of intersection change (Fig. 4). This function gives students a qualitative understanding of solids and cutting planes.

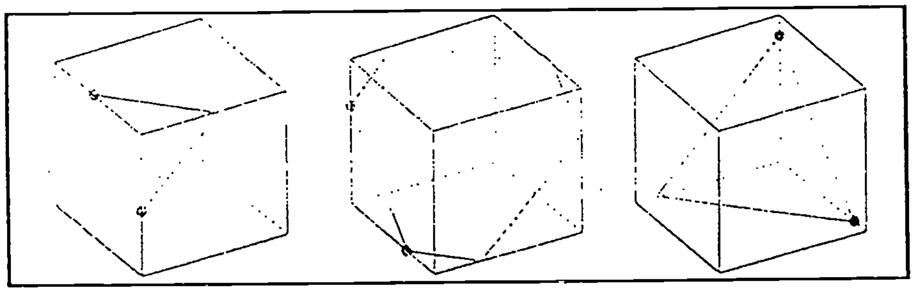

Figure 4. Moving a cutting-plane.

Projecting

There are three projected images corresponding to the xy, yz, zx orthogonal planes (Fig. 5). During projecting, rotation buttons can be used to rotate the object image on each of the three planes. If a user pushes the x-increment button, the image on the yz plane rotates counterclockwise, while if he/she pushes the z-decrement button, the image on the xy-plane rotates clockwise. The original image and the three projected images can be turned on and off on the display. There are two ways of using this manipulation. One is to conjecture each projected image from a solid; the other is to conjecture the original solid from the three projected images. The latter is a more advanced way of conjecturing, and cultivates the students' spatial ability.

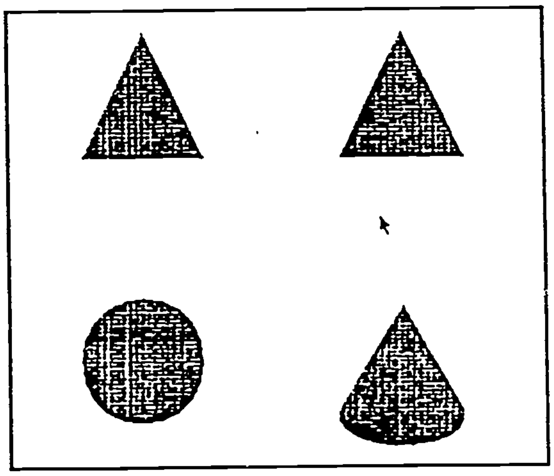

Figure 5. Projecting a cone to three orthogonal planes.

Opening and Closing

There are four types of manipulation in opening and closing a solid. These are opening as specified by the user, automatic opening, closing as specified by the user, and automatic closing. The details of these functions have been described by Y. Hase (Hase, 1993). Figure 6 shows the opening of a cube.

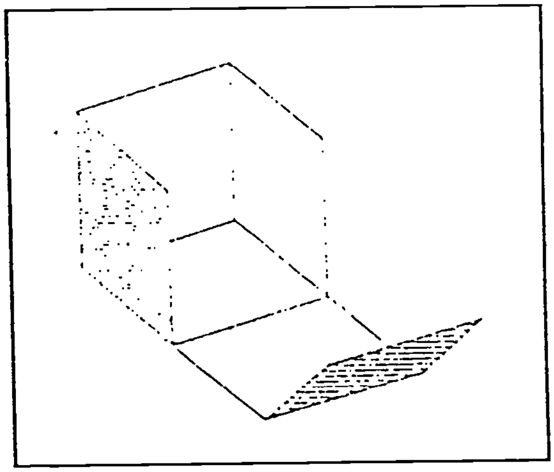

Figure 6. Opening a cube.

Measuring

The user can measure lengths, areas, volumes, and angles in three- dimensional space. Angles are defined by (1) two segments, (2) a segment and a plane, and (3) two planes. On the display, there are two measuring boxes showing measured values, and a calculation box showing the result of calculation based on the two values in the measuring boxes. These three boxes provide a good user interface for quantitative observation of solids.

Conclusions

We developed a three-dimensional "laboratory" named 3D-LAB on a personal computer. In this laboratory, students can manipulate solids interactively and observe them qualitatively and quantitatively. The possible manipulations include rotating, modifying, drawing, truncating, opening and closing, projecting, and measuring. Ease of manipulation is realized by the combination of a mouse and buttons, and by the method of first defining three-dimensional solid objects in the process of loading them, and then modifying them. We believe that there is no similar environment for learning three-dimensional geometry on the basis of experimentation. In the previous research phase, the basic functions of rotating and cutting solids were evaluated in several classrooms by Professor K. Suzuki and S. Wakita. We are planning to evaluate how the complete set of 3D-LAB's functions improves three-dimensional geometry classes.

Acknowledgments

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