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ABSTRACT

A simple technique, developed by A. Phillips (1987) is used to approximate the covariance between the Mantel-Haenszel log-odds-ratio estimator for a 2 x 2 x k table and the sample marginal proportions. These results are then applied to obtain an approximate variance estimate of an adjusted risk difference based on the Mantel-Haenszel odds-ratio estimator. The adjusted risk difference is of potential value in those applications where at least one of the sample rates is descriptive of a relevant population rate. The example applies to the use of the Mantel Haenszel estimator to study the differential difficulty of test questions across groups of examinees. (Contains 11 references.) (Author/SLD)

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A NOTE ON THE COVARIANCE OF THE MANTEL-HAENSZEL LOG-ODDS-RATIO ESTIMATOR AND THE SAMPLE MARGINAL RATES

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ABSTRACT

A simple technique, developed in Phillips (1987), is used to approximate $Cov(\hat{\theta}_{MH}, \hat{p}_i)$ $i = 1,2$ where $\hat{\theta}_{MH}$ is the Mantel-Haenszel log-odds-ratio estimator for a 2x2xK table and the $\hat{p}_i$ are the sample marginal proportions. These results are then applied to obtain an approximate variance estimate of an adjusted risk difference based on the Mantel-Haenszel odds-ratio estimator.

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1. INTRODUCTION

Consider the standard 2x2xK table whose k^{th} 2x2 layer is specified below.

	1	0	Total
Group 1	A_k	B_k	n_{1k}
Group 2	C_k	D_k	n_{2k}
Total	m_{1k}	m_{0k}	t_k

Probability models for the 2x2xK table include the two Binomial model (2B), Hauck, Anderson & Leahy, (1982) and the non-central (or "extended") hypergeometric model (NCH), Breslow, (1981). In both models the different 2x2 layers of the 2x2xK table are statistically independent. In the 2B model, A_k and C_k are independent binomial variates, with $A_k \sim B(p_{1k}, n_{1k})$ and $C_k \sim B(p_{2k}, n_{2k})$. In the NCH model, A_k has the non-central hypergeometric distribution given in (1.1) in which the margins n_{1k}, m_{1k} and t_k are garded as fixed, and ψ_k is the non-centrality parameter.

$$\text{Prob}(A_k = a|\psi_k, n_{1k}, m_{1k}, t_k) = (\psi_k)^a \binom{n_{1k}}{a}\binom{t_k-n_{1k}}{m_{1k}-a}/D, \tag{1.1}$$

where

$$D = \sum_u (\psi_k)^u \binom{n_{1k}}{u}\binom{t_k-n_{1k}}{m_{1k}-u} . \tag{1.2}$$

In (1.2), the range of summation is given by

$$\max(0,m_{1k} + n_{1k} - t_k) \le u \le \min(m_{1k}, n_{1k}). \tag{1.3}$$

Note that in (1.1) the integer, a, is also subject to the inequalities in (1.3). When $\psi_k = 1$, (1.1) reduces to the usual hypergeometric distribution. The NCH model may be viewed as the conditional distribution of A_k in the 2B model given the total $A_k + C_k = m_{1k}$.

The common odds-ratio assumption may be expressed in both the 2B and the NCH models. In the 2B model it is expressed by assuming that the odds-ratio, i.e.,

$$\psi_k = (p_{1k}/(1-p_{1k}))/(p_{2k}/(1-p_{2k})),$$

does not depend on k, i.e., $\psi_k = \psi$.

The common-odds-ratio assumption may be expressed in the NCH model by the assumption that ψ_k in (1.1) does not depend on k.

The Mantel-Haenszel (1959) estimator, $\hat{\psi}_{MH}$, for ψ under the common odds-ratio assumption is defined by

$$\hat{\psi}_{MH} = (\sum_k A_k D_k/t_k)/(\sum_k B_k C_k/t_k) \; . \tag{1.4}$$

It is often convenient to work with the natural log of ψ, i.e.,

$$\theta = \ln(\psi)$$

and the corresponding estimator

$$\hat{\theta}_{MH} = \ln(\hat{\psi}_{MH}). \tag{1.5}$$

The two marginal rates (or risks) are the sample proportions:

$$\hat{p}_1 = (\sum_k A_k)/n_1,$$

and

$$\hat{p}_2 = (\sum_k C_k)/n_2, \tag{1.6}$$

where

$$n_1 = \sum_k n_{1k} \text{ and } n_2 = \sum_k n_{2k}.$$

The main results of this note are simple asymptotic expressions for the covariance between $\hat{\theta}_{MH}$ and $\hat{p}_1$ and $\hat{p}_2$ that are valid for both the 2B and the NCH models. My approach is to exploit two useful formulas ((1.8) and (1.9) below) that were developed in Phillips (1987) and given in Phillips and Holland (1987). These are summarized in the following two lemmas.

Lemma 1: Let $R = \sum_k A_k D_k / t_k$ and $S = \sum_k B_k C_k / t_k$ and let $r = E(R)$ and $s = E(S)$ where these expectations are taken with respect to either the 2B or the NCH models. Define $\tilde{\theta}_{MH}$ by

$$\tilde{\theta}_{MH} = \theta + \frac{R-r}{r} - \frac{S-s}{s}. \tag{1.7}$$

Then

$$\hat{\theta}_{MH} = \tilde{\theta}_{MH} + o_p\left(\frac{Var(R)}{r^2} + \frac{Var(S)}{s^2}\right). \tag{1.8}$$

The virtue of (1.8) is that it allows the non-linear $\hat{\theta}_{MH}$ to be approximated by $\tilde{\theta}_{MH}$ which is linear in R and S. In addition, (1.8) shows the way in which the distribution of $\tilde{\theta}_{MH}$ approximates $\hat{\theta}_{MH}$ --namely, that the quantity $\frac{Var(R)}{r^2} + \frac{Var(S)}{s^2}$ must be small. Phillips and Holland (1987) show how this condition is satisfied in both the "large stratum" and "sparse data" situations.

Lemma 2: Let $x^{(k)} = x(x-1)\ldots(x-k+1)$ denote the descending factorial. If α, β, γ, δ are non-negative integers and ϵ is any non-negative integer not exceeding $\min(\alpha,\delta)$ then under either the 2B or the NCH model we have

$$E(A_k^{(\alpha)} B_k^{(\beta)} C_k^{(\gamma)} D_k^{(\delta)}) = \psi_k^{\epsilon} E(A_k^{(\alpha-\epsilon)} B_k^{(\beta+\epsilon)} C_k^{(\gamma+\epsilon)} D_k^{(\delta-\epsilon)}). \tag{1.9}$$

Equation (1.9) may be used to establish useful relationships between various covariances that involve A_k, B_k, C_k and D_k. This is illustrated in the proof of Theorem 1.

Theorem 1: Under the common odds-ratio assumption and either the 2B or the NCH model

$$\text{(a) } Cov(\tilde{\theta}_{MH}, \hat{p}_1) = 1/n_1,$$

and

$$\text{(b) } \mathrm{Cov}(\tilde{\theta}_{MH},\ \hat{p}_2) = -\ 1/n_2.$$

From Lemma 1 we may use $\tilde{\theta}_{MH}$ as an approximation to $\hat{\theta}_{MH}$ and thereby use Theorem 1 to obtain asymptotic approximations to $\mathrm{Cov}(\hat{\theta}_{MH},\ \hat{p}_1)$ and $\mathrm{Cov}(\hat{\theta}_{MH},\ \hat{p}_2)$ under the 2B and NCH models. In section 2, I discuss an application of these covariance calculations.

<u>Proof of Theorem</u> 1. From the definitions made in Lemma 1,

$$\mathrm{Cov}(\tilde{\theta}_{MH},\ \hat{p}_i) = \mathrm{Cov}(\theta + \frac{R-r}{r} - \frac{S-s}{s},\ \hat{p}_i) = \mathrm{Cov}(\frac{R}{r} - \frac{S}{s},\ \hat{p}_i)$$

$$= \frac{1}{r}\left\{\mathrm{Cov}(R,\hat{p}_i) - \frac{r}{s}\,\mathrm{Cov}(S,\hat{p}_i)\right\}. \tag{1.10}$$

Hence we need expressions for $\frac{r}{s}$, $\mathrm{Cov}(R,\ \hat{p}_i)$ and $\mathrm{Cov}(S,\ \hat{p}_i)$.

It is well-known (and an easy application of Lemma 2 with $\alpha = \delta = 1$, $\beta = \gamma = 0$, and $\epsilon = 1$) that under either the 2B or the NCH models we have

$$E(A_kD_k) = \psi_k\ E(B_kC_k). \tag{1.11}$$

Then (1.11) may be used to show that under the common odds-ratio assumption

$$r = E(R) = \psi\ E(S) = \psi\ s,$$

or

$$\frac{r}{s} = \psi. \tag{1.12}$$

Parts (a) and (b) of Theorem 1 are proved in a similar manner so I will consider only (a).

We need expressions for $\mathrm{Cov}(R,\ \hat{p}_1)$ and $\psi\mathrm{Cov}(S,\ \hat{p}_1)$. But

$$\mathrm{Cov}(R,\ \hat{p}_1) = \sum_k \frac{1}{t_k}\,\mathrm{Cov}(A_kD_k,\ \hat{p}_1)$$

$$= \sum_{k,j} \frac{1}{t_k}\,\frac{1}{n_1}\,\mathrm{Cov}(A_kD_k,\ A_j). \tag{1.13}$$

But, for $k \neq j$, the variates are independent so that (1.13) reduces to

$$\mathrm{Cov}(R, \hat{p}_1) = \frac{1}{n_1} \sum_k \frac{1}{t_k} \{E(A_k^2 D_k) - E(A_k D_k)\ E(A_k)\} . \tag{1.14}$$

Similarly we have

$$\psi \mathrm{Cov}(S, \hat{p}_1) = \frac{1}{n_1} \sum_k \frac{1}{t_k} \{\psi E(A_k B_k C_k) - \psi E(B_k C_k)\ E(A_k)\}. \tag{1.15}$$

But

$$\begin{aligned} E(A_k^2 D_k) &= E(A_k(A_k - 1)D_k) + E(A_k D_k) \\ &= E(A_k^{(2)} D_k) + E(A_k D_k) . \end{aligned} \tag{1.16}$$

Now apply Lemma 2 to (1.16) with $\alpha = 2$, $\beta = \gamma = 0$, $\delta = 1$ and $\epsilon = 1$ and obtain

$$E(A_k^2 D_k) = \psi E(A_k B_k C_k) + E(A_k D_k). \tag{1.17}$$

Hence, from (1.17) and (1.11), (1.14) becomes

$$\mathrm{Cov}(R, \hat{p}_1) = \frac{1}{n_1} \sum_k \frac{1}{t_k} \{\psi E(A_k B_k C_k) - \psi E(B_k C_k)\ E(A_k) + E(A_k D_k)\} \tag{1.18}$$

$$= \psi \mathrm{Cov}(S, \hat{p}_1) + \frac{1}{n_1} \sum_k \frac{1}{t_k} E(A_k D_k). \tag{1.19}$$

Thus, combining (1.19) with (1.10) and (1.12) we have

$$\mathrm{Cov}(\tilde{\theta}_{MH}, \hat{p}_1) = \frac{1}{r} \{\frac{1}{n_1} \sum_k \frac{1}{t_k} E(A_k D_k)\} = \frac{1}{r} \frac{1}{n_1} r = \frac{1}{n_1}. \quad \underline{\text{QED}}$$

2. A THEORETICAL APPLICATION AND EXAMPLE

In this section we develop the necessary formulas for applying the Taylor series- or δ-method to obtain standard errors for functions of $\hat{\theta}_{MH}$, $\hat{p}_1$ and $\hat{p}_2$ and then apply these results and those of section 1 to obtain an approximate standard error for a particular function of $\hat{\theta}_{MH}$ and $\hat{p}_2$ that arises in the use of the Mantel-Haenszel estimator to study differential difficulty of test questions across groups of examinees (Holland, 1985; Holland and Thayer, 1988).

2.1 A general formula

Let $\hat{f} = f(\hat{\theta}_{MH}, \hat{p}_1, \hat{p}_2)$ be a differentable function of $\hat{\theta}_{MH}$, $\hat{p}_1$ and $\hat{p}_2$. The δ-method (Bishop, Fienberg and Holland, 1975) may be used to derive the large-sample variance of $\hat{f}$. It is summarized below.

Theorem 2: *As the variances of $\hat{\theta}_{MH}$, $\hat{p}_1$ and $\hat{p}_2$ go to zero, the variance of $\hat{f}$ is approximated by:*

$$\begin{aligned}\mathrm{Var}(\hat{f}) = {} & \left(\frac{\partial f}{\partial \theta}\right)^2 \mathrm{Var}(\hat{\theta}_{MH}) + \left(\frac{\partial f}{\partial p_1}\right)^2 \mathrm{Var}(\hat{p}_1) \\ & + \left(\frac{\partial f}{\partial p_2}\right)^2 \mathrm{Var}(\hat{p}_2) + 2\,\frac{\partial f}{\partial \theta}\frac{\partial f}{\partial p_1}\mathrm{Cov}(\hat{\theta}_{MH}, \hat{p}_1) \qquad (2.1) \\ & + 2\,\frac{\partial f}{\partial \theta}\frac{\partial f}{\partial p_2}\mathrm{Cov}(\hat{\theta}_{MH}, \hat{p}_2) + 2\,\frac{\partial f}{\partial p_1}\frac{\partial f}{\partial p_2}\mathrm{Cov}(\hat{p}_1, \hat{p}_2).\end{aligned}$$

From Theorem 2 the covariances derived in section 1 may be combined with variance estimates of $\hat{\theta}_{MH}$, $\hat{p}_1$ and $\hat{p}_2$ and the covariance of $\hat{p}_1$ and $\hat{p}_2$ to yield an approximate standard error for $\hat{f}$. Robins, Breslow, and Greenland (1986) give an estimator, $\hat{\sigma}^2(\hat{\theta}_{MH})$, of the variance of $\hat{\theta}_{MH}$ that is valid in a variety of asymptotic situations. This variance estimator may be expressed in the notation of Lemma 1 as

$$\hat{\sigma}^2(\hat{\theta}_{MH}) = \frac{2}{R}\sum_k t_k^{-2}[A_kD_k + \hat{\psi}_{MH}B_kC_k][A_k + D_k + \hat{\psi}_{MH}(B_k + C_k)]\ . \qquad (2.2)$$

This variance estimator is also discussed in Phillips and Holland (1987).

The following lemma summarized the variances and covariance of $\hat{p}_1$ and $\hat{p}_2$ in the 2B and NCH cases.

Lemma 3: (a) In the 2B case:

$$\mathrm{Var}(\hat{p}_i) = \frac{1}{n_i} \sum_k w_{ik}\, p_{ik}\,(1-p_{ik}) \tag{2.3}$$

where $w_{ik} = n_{ik}/n_i$, and

$$\mathrm{Cov}(\hat{p}_1, \hat{p}_2) = 0. \tag{2.4}$$

(b) In the NCH case:

$$\mathrm{Var}(\hat{p}_i) = \frac{1}{n_i^2} \sum_k \mathrm{Var}(A_k) \tag{2.5}$$

and

$$\mathrm{Cov}(\hat{p}_1,\hat{p}_2) = -\frac{1}{n_1 n_2} \sum_k \mathrm{Var}(A_k). \tag{2.6}$$

In Theorem 3, part (a), it is clear that estimates of the variances and covariances of $\hat{p}_1$ and $\hat{p}_2$ under the 2B model are straightforward. For example, A_k/n_{1k} and C_k/n_{2k} can be used as estimates of p_{1k} and p_{2k}, respectively, in (2.3). On the other hand, by Jensen's inequality we have

$$\sum_k w_{ik}\, p_{ik}\,(1-p_{ik}) \le \overline{p}_i(1-\overline{p}_i) \ , \tag{2.7}$$

where $\overline{p}_i = \sum_k w_{ik}\, p_{ik}$. Hence from (2.3) and (2.7) we see that the simple "binomial variance" estimate,

$$\frac{1}{n_i}\, \hat{p}_i(1 - \hat{p}_i), \tag{2.8}$$

provides an estimate of $\mathrm{Var}(\hat{p}_i)$ that is, at worst, an over-estimate. When K is large and some of the t_k are small (2.8) is often a better estimate of the variance of $\mathrm{Var}(\hat{p}_i)$ than the one obtained by substituting the sample proportions, $\hat{p}_{ik}$, for p_{ik} in (2.3).

From part (b) of Lemma 3 it is evident that estimates of the variances and covariances of $\hat{p}_1$ and $\hat{p}_2$ under the NCH model all require estimates of $\sum_k \mathrm{Var}(A_k)$, which, in turn, involves the estimates of the variance of the NCH variate, A_k. Harkness (1965) discusses the moments of A_k in the NCH case. I do not know whether or not those results can be used to give valid estimates of $\sum_k \mathrm{Var}(A_k)$ that are needed to use the NCH part of Lemma 3.

2.2 An application to a "Mantel-Haenszel adjusted risk difference".

In biomedical applications, $\hat{\Psi}_{MH}$ provides an adjusted estimate of the relative odds of getting a disease in an exposed group of individuals compared to an unexposed group. The adjustment is for differences in the distribution of potential confounding variables that may exist between the two groups.

Holland (1985) discussed the use of $\hat{\theta}_{MH}$ as an adjusted measure of "bias" in test questions. In this use of the $\hat{\theta}_{MH}$, "getting the disease" is replaced by "getting the test item right" and the "exposed" and "unexposed" groups are replaced by a reference and a focal group of examinees (i.e. White and Blacks or Males and Females). The adjustment is for overall test performance. Since that suggestion, the use of the Mantel-Haenszel procedure to measure "item bias" has become wide-spread at testing organizations such as Educational Testing Service. In these testing applications, there is an interest in expressing the estimated logit differences, $\hat{\theta}_{MH}$, in terms of the probability scale as an adjusted difference in proportions (eg. Dorans and Kulick, 1986). One way of expressing $\hat{\theta}_{MH}$ in the "p-scale" is the following statistic that in biometric terms might be called the "Mantel-Haenszel adjusted risk difference",

$$RD_{MH} = \hat{p}_2 - \frac{\hat{p}_2 \exp\{\hat{\theta}_{MH}\}}{(1-\hat{p}_2) + \hat{p}_2 \exp\{\hat{\theta}_{MH}\}} . \tag{2.9}$$

The second term in the right-hand-side of (2.9) is the value of p_1 that would be obtained if $\hat{p}_2$ were "adjusted" by $\hat{\theta}_{MH}$, i.e. if p_1 were chosen to solve for p_1 in the equation,

$$\hat{\theta}_{MH} = \log\left(\frac{p_1}{1-p_1} \Big/ \frac{\hat{p}_2}{1-\hat{p}_2}\right) . \tag{2.10}$$

The raw risk difference, $\hat{p}_2 - \hat{p}_1$, makes no use of the matching or stratification that is available and, for this reason, is not of much practical value except in special circumstances. The Mantel Haenszel adjusted risk difference _is_ based on the stratification and is therefore a type of "standardized" risk difference.

If RD_{MH} from (2.9) is used, the need for its standard error arises and the results of sections 1 and 2.1 may be used to obtain an estimate of the variance of RD_{MH} under the 2B model. This is given in Theorem 3, below.

Theorem 3: _Under the 2B model, the common odds-ratio assumption and any conditions that insure the approximation of $\hat{\theta}_{MH}$ by $\tilde{\theta}_{MH}$, the variance of RD_{MH} in (2.9) is estimated by_

$$\frac{1}{n_2}(1-G)^2\, \hat{p}_2\,(1-\hat{p}_2) + G^2\,[\hat{p}_2(1-\hat{p}_2)]^2\, \hat{\sigma}^2(\hat{\theta}_{MH}) + \frac{2}{n_2}\,G(1-G)\,\hat{p}_2(1-\hat{p}_2) \tag{2.13}$$

where

$$G = \hat{\psi}_{MH}/(1-\hat{p}_2 + \hat{\psi}_{MH}\,\hat{p}_2)^2$$

and $\hat{\sigma}^2(\hat{\theta}_{MH})$ is the Robins-Breslow-Greenland variance estimate of $\hat{\theta}_{MH}$ given in (2.2).

Proof: Let $f(\hat{\theta}_{MH}, \hat{p}_1, \hat{p}_2)$ be defined by RD_{MH} in (2.9). Then the relevant derivatives from Theorem 2 are easily shown to be

$$\frac{\partial f}{\partial \theta} = -\,p_2(1-p_2)G,\quad \frac{\partial f}{\partial p_1} = 0,\ \text{and}\ \frac{\partial f}{\partial p_2} = 1 - G. \tag{2.14}$$

These derivatives when combined, via Theorem 2, with the covariance in Theorem 1b, and using (2.8) instead of (2.3) yields the result. _QED_.

3. DISCUSSION

The technique used to prove Theorem 1 (i.e. Lemma 1 and 2) is useful in its own right since it is simple and yet widely applicable to computations involving the Mantel-Haenszel estimator. In addition, I think it is rather remarkable that the asymptotic covariances of $\log(\hat{\psi}_{MH})$ and the $\hat{p}_i$ are as simple as they appear in Theorem 1.

The adjusted risk difference, RD_{MH}, is of potential value in those applications where at least one of the sample rates, say $\hat{p}_2$, is descriptive of a relevant population rate. This occurs in the testing applications referred to earlier but may also arise in prospective epidemiological studies as well. The variance estimate in Theorem 3 is asymptotically valid whenever the Robins-Breslow-Greenland estimate of the variance of $\hat{\theta}_{MH}$ is valid with the added proviso that the "binomial variance estimate", (2.8), be an appropriate estimate of the variance of $\hat{p}_2$. Thus the variance estimate in Theorem 3 will be most useful in the so-called "sparse-data" case where K is large and the t_k are not. In the large stratum case, i.e. when K is small and the t_k are large, it may be better to use formula (2.3) to estimate the variance of $\hat{p}_2$. This substitution would only change the first term of formula (2.13).

I have not performed a small sample study of the behavior of the variance estimate in Theorem 3, but because of its close connection to the Robins-Breslow-Greenland variance estimate for $\hat{\theta}_{MH}$ I would expect it to perform quite well in both the sparse data and the large stratum cases.

To extend Theorem 3 to the NCH model it would be necessary to have a useful estimate of $\sum_k \mathrm{Var}(A_k)$ for the NCH case in order to apply Lemma 2(b). I do not know of any results in this area. However, in the NCH model, while the sample marginal rates $\hat{p}_1$ and $\hat{p}_2$ are still <u>defined</u> by (1.6), it is not clear what meaning to attach to them as <u>estimates</u> of population rates. Thus, RD_{MH} may not be a useful parameter in the NCH case.

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