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ABSTRACT

In the Hypothesis-Experiment-Instruction (HEI) method students are presented with a problem with three or four answer alternatives, which leads them later to recognize the distribution of choices, discuss a variety of ideas among them, and integrate distributed pieces of information. This study examined the effects of presenting a problem with or without answer alternatives on the deployment and products of group discussion in the case of adding fractions with different denominators. Fourth- and fifth-grade students (n=289) were assigned class by class to either Group With Alternatives or Group Without Alternatives. Results showed no significant difference between the two groups on a post test. Researchers surmised that group discussion beginning with presenting a problem with answer alternatives may be effective for discussion among a few competing but equally plausible ideas, but not effective for discussion in the situation where the correct answer can easily be recognized by excluding other implausible alternatives. (MKR)

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Construction of Mathematical Knowledge through the Whole Class Discussion : Effects of Presenting a Problem with Answer Alternatives

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Considering that a group as a whole has a richer data base than any of its members, it is likely that more varied interpretations can be offered and more convincing and less biased pieces of evidence can be presented jointly than individually. It is thus important to develop instructional procedures inducing lively group discussion through which students collect and coordinate relevant pieces of information distributed among them. The Hypothesis-Experiment-Instruction method (HEI method, hereafter), originally devised as a method for science education (Itakura, 1971), seems to represent a prototype of such procedures. In this method students are presented with a problem with three or four answer alternatives, which leads them later to recognize the distribution of choices, to discuss a variety of ideas among them, and to integrate distributed pieces of information, by engaging in spontaneous and flexible division of labor based on partisan as well as epistemic motivation (Hatano and Inagaki, 1991).

Does a similar procedure, that is, presenting a problem with answer alternatives at the beginning, work in mathematics learning? Whereas in science alternative ideas can be quite many in number because students are likely to propose their ideas based on any direct or indirect experience, in mathematics the number of alternative ideas would be limited because they are derived primarily through inference from a few given premises. Thus in the present study we examined effects of presenting a problem with or without answer alternatives on the deployment and products of group discussion in the case of adding fractions with different denominators.

METHODS

Subjects

Subjects were 289 of 4th- and 5th-graders from two classes each of six elementary schools. Subjects from three schools (A, B, and C) were 4th-graders and those from the other three (D, E, and F) were 5th-graders. They were assigned class by class to either Group With Alternatives (A1 - F1) or Group Without Alternatives (A2 - F2) in each grade and in each school. They had learned how to add fractions with common denominators and how to add decimal fractions, but had not yet learned how to add fractions with different denominators.

Target problem

A target sentence problem was: "Taro drinks milk 1/2 liter at breakfast and 1/5 liter at supper. How many liters of m[illegible] does he drink a day?" Group With Alternatives was given the following three answer alternatives, i.e., (a) 1/2+ 1/5=2/7, answer is 2/7 liter; (b) 0.5+0.2=0.7, answer is 0.7 liter; or (c) 1/2+1/5=7/10, answer is 7/10 liter. Group Without Alternatives was not given any answer alternative.

Procedure

Students in Group With Alternatives were given the following six steps as the HEI method: (1) they were presented the target problem, and asked to choose one of the alternatives and to give a reason for choice individually; (2) they were shown the tabulated distribution of their choices on the blackboard by show of hands; (3) they were invited to state their reasons to the whole class and discuss them; (4) they were required to solve the target problem once again: they could change their solutions; (5) they were informed the correct (or most appropriate) answer, i.e., the alternative (c), without further explanation; (6) they were asked to solve the initial problem again without answer alternatives as a post test and two additional problems (i.e., 1/2+1/3 and 1/4+2/5) as a transfer test.

The procedures given to Group Without Alternatives were the same as those given to Group With Alternatives, except that the target problem required students to construct their answers by themselves at step (1) because it had no answer alternatives, and that step (2) was omitted.

The average time spent for the discussion was 10 1/2 min. (range, 6 min. to 18 min.).

RESULTS

The students' Responses to the Target Problem Before and After the Discussion

In Group Without Alternatives, 63 students generated by themselves the solution corresponding to the alternative (a), 42 gave the solution equivalent to the alternative (c), and two, the alternative (b). That is, a majority (74.8%) of the students constructed one of the same solutions as the answer alternatives presented to Group With Alternatives.

Table 1 shows the tabulated distributions of students' responses before and after the discussion. Though the alternative (b) was seldom constructed spontaneously by students themselves, it had some appeal to the students as a plausible solution when it was presented in Group With Alternatives; 27.4% of them chose it before the discussion. A majority (87-88%) of the students in both groups came to give the correct answer (solution (c)) after the discussion but before the feedback; the number of choices of the alternative (c) significantly increased after the discussion in all the classes of Group With Alternatives and all but one of Group Without Alternatives by McNemar's test (range, $\chi 2(1) = 4.2$, $p < .05$ to $\chi 2(1) = 24.0$). This indicates that the whole class discussion, irrespective of presenting a problem with or without alternatives, helped a considerable number of the students to figure out the correct solution.

The Deployment of the Discussion

Though a great majority of the students came to adopt the correct solution after the discussion, as described above, processes leading to it seemed to be different between the two groups, because the deployment of the discussion was different, especially in the examination of the solution (c)--whether it is really the correct one. First, when we counted students' remarks consisting of explanations, questions and counter arguments during the discussion, the students in Group Without Alternatives produced more remarks than those in Group With Alternatives; the total number of such remarks for each solution was 67 in Group Without Alternatives [20 for solution (a), 0 for (b), 28 for (c) and 19 for "others"], whereas only 31 in Group With Alternatives [13 for (a), 8 for (b), and 10 for (c)], and the difference between the two groups was significant by U-test when we treat class as a unit ($U=3$, $p< .05$). In addition, separate comparisons concerning the numbers of remarks for (a) or (c) between Groups With and Without Alternatives revealed that Group Without Alternatives tended to make more remarks only for the solution (c), $U=6$, $p < .10$.

Second, the students in Group Without Alternatives gave more varied explanations about the solution (c) than those in Group With Alternatives. That is, either the explanation about the necessity of making different denominators equal, or the proposal of the detailed procedure for equalizing denominators was observed once or more in all the six classes of Group Without Alternatives, whereas such an explanation was found in only one class of Group With Alternatives ($p < .05$, by the Fisher probability test).

Third, some generalized explanations, such as demonstrating how the idea would work on other problem like 1/6+1/3, were observed in four classes of Group Without Alternatives, while such explanations were never found in the Group With Alternatives ($p < .05$, by the Fisher probability test).

Why did the discussion of Group Without Alternatives elicit the varied types of explanations described above? This was probably because the students' proposed explanations about the solution (c) were critically examined by others who supported (c) in Group Without Alternatives. That is, the supporters of (c) in Group Without Alternatives asked questions to one another, or challenged another explanation, which elicited further explanations. The discussions among supporters of (c) were observed in five classes of Group Without Alternatives, while in one class of Group With Alternatives; the difference was significant by Fisher's probability test ($p < .05$).

Supporters of (a) explained their solution, saying, "This is an addition, so numbers have to be added after they are grouped into denominators and numerators," in all the classes of Group With and Without Alternatives. Counterarguments relying on either "semantics" (e.g., "This cannot be correct because its answer, 2/7, is smaller than 1/2, though 1/5 is added to it") or on "syntax" (e.g., "It is impossible to add the denominator to the denominator") were also observed in all the classes. Supporters of (a) could not refute the students giving these counterarguments in both groups; it seemed easy for the students to recognize something wrong in solution (a).

Results of the Post Test and the Transfer Test

At the post test given after the correct answer had been taught without further explanation, 96% (range, 85-100) of the students in Group With Alternatives and 98% (95-100) of those in Group Without Alternatives solved the target problem correctly and there was no significant difference between them.

Students' responses at the transfer test consisting of two items were classified into one of the following five categories: strategy A, i.e., giving the correct solutions for both items; strategy B, i.e., obtaining a common denominator correctly for both items but not getting numerators properly; strategy C, approximating fractions to be added always by fractions with 10 as the denominator; strategy D, adding denominators as well as numerators as in solution (a); and Others, making responses not classified into any of the above categories including inconsistent solutions for the two items, or no answer. Data from the school B (classes B1 and B2) were excluded from the analysis, because the subjects from this school did not have enough time to solve the transfer task.

Though there was no marked difference in the use of strategy A, most elaborabed, correct solution, between both groups, Group Without Alternatives adopted strategy B more often and strategies C and D less often than Group With Alternatives (See Table 2). When we take the fact into account that the discussion time was fairly short, about 10 minutes, it seems reasonable that we regard the users of strategy B as students who, though imperfectly, learned through the discussion how to obtain common denominators. Thus we used the combined percentage of strategies A and B as an indicator of learning for a statistical analysis. A two-way ANOVA with conditions (2) and schools (5) as between-subject factors indicated a marginally significant main effect of conditions, $\chi2(1)= 3.01$, $p < .10$, confirming the above picture. (Though a main effect of school was significant ($\chi2(4)=6.09$, $p < .05$), an interaction effect was not significant).

DISCUSSION

Why the students in Group With Alternatives tended to learn less about the necessity of and the procedure for making denominators equal than the students in Group Without Alternatives? This was, we interpret, due to the insufficient examination of the alternative (c) during the discussion in Group With Alternatives. The experimental procedure for Group With Alternatives, presenting a problem with three alternatives and then a tabulated distribution of the students' responses, may have led them to find the correct answer by excluding other "weak" alternatives rather than by figuring out why the alternative (c) is right. In contrast, the students in Group

Without Alternatives needed to convince themselves and others why the "alternative (c)" is correct by giving varied explanations. This interpretation is supported by the fact that a majority of the students from Group With Alternatives chose the correct answer after the discussion (before feedback) even in the classes where reasonable explanations for the alternative (c) had not been proposed.

Partisan motivation might also explain in part the above results. Partisan motivation was induced in Group With Alternatives, as indicated by the fact that the explanations given by the supporters of (c) were seldom critically examined by students in the same camp. If partisan motivation had worked effectively, as observed in science lessons (Hatano & Inagaki, 1991), the supporters of (a) and/or (b) would have challenged the explanations by the supporters of (c), which would have led the students to examine more in detail the solution (c). However, these scenes were seldom observed in Group With Alternatives. On the other hand, the supporters of (c) in Group Without Alternatives, where distributions of students' responses were not presented and thus partisan motivation would not be induced or would be weak if induced, might be motivated to resolve apparent incongruity among the explanations given by the supporters of (c), which would lead the students to examine the solution (c) from varied points of view.

In sum, group discussion beginning with presenting a problem with answer alternatives may be effective for discussion among a few competing but equally plausible ideas, as often seen in science, but not effective for discussion in the situation where the correct answer can easily be recognized by excluding other implausible alternatives. It is noteworthy to specify when presenting answer alternatives tends to be beneficial and when no effect or even detrimental. Although the present findings suggest educators that answer alternatives be constructed by students themselves in the case of addition of fractions with different denominators, further studies are needed for more varied units across different subjects.

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TABLE 1. Distributions of students' responses before and after the discussion

	classes	A1		B1		C1		D1		E1		F1		Total	
Group With Alternatives		before	after	before	after	before	after	before	after	before	after	before	after	before	after
	alternative (a)	3		12		9	3	7	2	11		17	1	59	6
	alternative (b)	7		6	1	3	3	8		9	3	7	4	40	11
	alternative (c)	8	18	15	32	2	8	5	18	13	30	4	23	47	129
	Total	18	18	33	33	14	14	20	20	33	33	28	28	146	146
	classes	A2		B2		C2		D2		E2		F2		Total	
Group Without Alternatives		before	after	before	after	before	after	before	after	before	after	before	after	before	after
	(a)	3		23		8		7	1	8	3	14	1	63	5
	(b)									2	3			2	3
	(c)	7	18	6	32	4	15	8	19	11	17	6	24	42	125
	others	8		4	1	3		5		11	9	5		36	10
	Total	18	18	33	33	15	15	20	20	32	32	25	25	143	143

Note 1. The responses of Group With Alternatives were classified as (a), (b), (c) and others, corresponding to the three answer alternatives (a)-(c) presented to the Group With Alternatives.

TABLE 2. Percentages of students adopting each strategy at the Transfer Test

	strategies\classes	A1	(18)	C1	(14)	D1	(20)	E1	(33)	F1	(28)	Total	(113)
Group With Alternatives	strategy.A	16.7	(3)	28.6	(4)	25.0	(5)	42.4	(14)	39.3	(11)	32.7	(37)
	strategy.B	16.7	(3)	35.7	(5)	25.0	(5)	15.2	(5)	10.7	(3)	18.6	(21)
	strategy.C	11.1	(2)	7.1	(1)	10.0	(2)	15.2	(5)	10.7	(3)	11.5	(13)
	strategy.D	11.1	(2)	7.1	(1)	10.0	(2)	0.0	(0)	7.1	(2)	6.2	(7)
	Others	44.4	(8)	21.4	(3)	30.0	(6)	27.3	(9)	32.1	(9)	31.0	(35)
	strategies\classes	A2	(18)	C2	(15)	D2	(20)	E2	(32)	F2	(25)	Total	(110)
Group Without Alternatives	strategy.A	27.8	(5)	33.3	(5)	20.0	(4)	28.1	(9)	28.0	(7)	27.3	(30)
	strategy.B	11.1	(2)	46.7	(7)	25.0	(5)	40.6	(13)	28.0	(7)	30.9	(34)
	strategy.C	22.2	(4)	6.7	(1)	5.0	(1)	6.3	(2)	4.0	(1)	8.2	(9)
	strategy.D	0.0	(0)	0.0	(0)	0.0	(0)	3.1	(1)	4.0	(1)	1.8	(2)
	Others	38.9	(7)	13.3	(2)	50.0	(10)	21.9	(7)	36.0	(9)	31.8	(35)

Note 1. School B (classes B1 and B2) were excluded, because the subjects from this school did not have enough time to solve the transfer test.
2. Figures in parentheses show the number of subjects.