ABSTRACT

When using item response theory (IRT) models in educational and psychological measurement, it is standard practice to estimate the operating characteristics of test items from examinees' item responses alone. This is the final report of a project that employed Bayesian and empirical Bayesian methods to exploit additional information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Although collateral information about examinees and items is rarely employed in IRT, it is straightforward to incorporate it using Bayesian and empirical Bayesian methods. Employing collateral information is mandatory to obtain correct Bayesian and empirical Bayesian inferences if it was used to assign items to examinees. Practical and theoretical results obtained in a series of six research reports are summarized. (Contains 3 tables and 12 references.) (Author/SLD)
FINAL REPORT

EXPLOITING COLLATERAL INFORMATION IN THE ESTIMATION OF ITEM PARAMETERS

Robert J. Mislevy

This research was sponsored in part by the Cognitive Science Program Cognitive and Neural Sciences Division Office of Naval Research, under Contract No. N00014-85-K-0683

Contract Authority Identification No. NR 150-539

Robert J. Mislevy, Principal Investigator

Educational Testing Service
Princeton, New Jersey

September 1988

Reproduction in whole or in part is permitted for any purpose of the United States Government.

Approved for public release; distribution unlimited.

BEST COPY AVAILABLE
Exploiting Collateral Information in the Estimation of Item Parameters (Unclassified)

When using item response theory (IRT) models in educational and psychological measurement, it is standard practice to estimate the operating characteristics of test items from examinees' item responses alone. This is the final report of a project that employed Bayesian and empirical Bayesian methods to exploit additional information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Practical and theoretical results obtained in a series of research reports are summarized.
Exploiting Collateral Information in the
Estimation of Item Parameters

FINAL REPORT

Robert J. Mislevy
Educational Testing Service

September 1988

This work was supported by Contract No. N00014-85-K-0683, project designation NR 150-539, from the Cognitive Science Program, Cognitive and Neural Sciences Division, Office of Naval Research. Reproduction in whole or in part is permitted for any purpose of the United States Government. The author thanks Murray Aitkin and Peter Pashley for their comments on an earlier version of this report.
Abstract

When using item response theory (IRT) models in educational and psychological measurement, it is standard practice to estimate the operating characteristics of test items from examinees' item responses alone. This is the final report of a project that employed Bayesian and empirical Bayesian methods to exploit additional information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Practical and theoretical results obtained in a series of research reports are summarized.

Key words: Bayesian Estimation, Collateral Information, Differential Strategies, Empirical Bayes Estimation, Information Matrices, Item Response Theory, Missing Data
Introduction

Item response theory (IRT) models in psychometrics give the probability that an examinee will respond correctly to a given test item in terms of parameters for just that examinee and that item. This formulation makes it possible to solve many practical measurement problems that are difficult or intractable under classical test theory, including adaptive ability testing, large population equating studies, and test construction to targeted operating specifications.

It is standard practice to estimate IRT item parameters solely from the observed responses of a sample of examinees. This project was motivated by a desire to improve estimation by exploiting collateral information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Table 1 lists the reports from the project exploring both practical and theoretical aspects of the problem. The present report summarizes the main results. The interested reader is referred to the individual papers for details, derivations, and examples.

Table 1 about here
Incorporating Collateral Information into IRT

The initial thrusts of the project were to determine how to incorporate collateral information into estimation procedures when the IRT model is correct, and to gauge its impact on estimation precision. Bayesian and empirical Bayesian methods were employed to this end. This section describes the basic model (Mislevy, 1987; in press).

Under an IRT model, the probability of response $x_j$ to Item $j$ with a possibly vector-valued item parameter $\beta_j$ from an examinee with proficiency parameter $\theta$ is given as

$$P(x_j | \theta, \beta_j) = f(x_j | \theta, \beta_j) ,$$

(1)

where the form of the item response function $f$ is known up to the item parameters. Under the usual assumption of local independence, the conditional probability of the response pattern $\mathbf{x} = (x_1, \ldots, x_n)$ to $n$ test items is simply the product of expressions like (1):

$$P(\mathbf{x} | \theta, \beta) = \prod_j P(x_j | \theta, \beta_j) ,$$

(2)

where $\beta = (\beta_1, \ldots, \beta_n)$. Let the data matrix $\mathbf{X} = (x_1, \ldots, x_N)$ represent response vectors observed from a sample of $N$ examinees from a population in which $\theta$ follows the density $p(\theta)$. The likelihood for $\beta$ induced by $\mathbf{X}$ is obtained as
Marginal maximum likelihood (MML) estimates of item parameters (e.g., Bock and Aitkin, 1981) are obtained by maximizing (3) with respect to $\theta$.

Suppose that in addition to item responses, values of collateral variables $y$ are also available from examinees. The appropriate marginal likelihood is now

\[ L_{xy}(\theta|X,Y) = \prod \int f(x_i|\theta, \beta) p(\theta|y_i) d\theta. \]  

(4)

MML estimates of item parameters that exploit collateral information about examinees are obtained by maximizing (4) with respect to $\theta$ (Mislevy, 1987).

Bayesian item parameter estimates are obtained from posterior distributions for $\theta$, which arise as the normalized product of a likelihood function such as (3) or (4) and a prior distribution for $\theta$, say $g(\theta)$. If, before observing data, one possesses no information to differentiate expectations about the parameters of different items, an exchangeable prior for $\theta$ is appropriate; that is, the items are modeled as if they were $n$ random draws from the same distribution. In this case the posterior distribution is given by
depending on whether collateral information is available about examinees. If values on the collateral variable z are additionally available about items, they are incorporated as

\[ p_{xz}(\beta|X,Z) \propto L_{x}(\beta|X) \prod_{j} g(\beta_j|z_j) \quad (7) \]

or

\[ p_{xyz}(\beta|X,Y,Z) \propto L_{xy}(\beta|X,Y) \prod_{j} g(\beta_j|z_j) \quad (8) \]

(Mislevy, in press). Standard Bayesian procedures for estimating item and population parameters that do not employ collateral information extend to (7) and (8) in a straightforward manner (Mislevy, 1987, in press).

Increase in Information: Theoretical Results

Using general results about missing data problems, such as Orchard and Woodbury's (1972) "missing information principle" it is possible to derive upper and lower bounds for the expected precision of item parameter estimates with and without collateral

\[ \text{ problematic section due to overlap or truncation} \]

\[ p(X|\beta) \propto L_{x}(\beta|X) \prod_{j} g(\beta_j) \quad (5) \]
or

\[ p_{xy}(\beta|X,Y) \propto L_{xy}(\beta|X,Y) \prod_{j} g(\beta_j) \quad (6) \]
information (Mislevy and Sheehan, 1988, in press). The results are expressed most easily in Bayesian terms.

Consider first the impact of collateral information about examinees. Let $V(\beta|\theta, X, Y)$ represent the posterior variance of $\beta$ that would be obtained after observing values of not only item responses $x$ and collateral variables $y$ from a sample of $N$ examinees, but values of their latent proficiencies $\theta$ as well. Let analogous expressions represent posterior variance of $\beta$ when values of one or more types of variables are not observed; for example, $V(\beta|X)$ when only item responses are observed. The following relationships may be derived:

\[
E[V(\beta|\theta, X, Y)] - E[V(\beta|\theta, X)] 
\leq E[V(\beta|X, Y)] 
\leq E[V(\beta|X)] ,
\]

where $A \preceq B$ means that the matrix difference $B - A$ is at least positive semidefinite. Thus the precision of item parameter estimation when using collateral information about examinees along with item responses is at least as great as that expected when using item responses alone, but cannot exceed the precision that would be expected with the same sample size if values of the latent variable $\theta$ could be observed as well.

An obvious lower bound holds the impact of collateral information about items:
that is, expected precision when using collateral information about items in addition to item responses, equals or exceeds precision expected when not using it. No ordering holds between $E[V(\theta|X,Z)]$ and $E[V(\theta|X)]$ in general. In particular, when $Z$ is employed along with $X$, it is possible to exceed the precision obtainable with $\theta$ and $X$.

Increase in Information: Practical Results

By examining the structure of information matrices with and without collateral information, and by applying the methods to data from the National Assessment of Educational Progress (NAEP) and the Profile of American Youth surveys, it was found that modest increases in the precision of item parameter estimates can be achieved by using collateral information (Mislevy, 1987, in press; Mislevy and Sheehan, 1988, in press).

From collateral information about examinees, increases in information depend on the strength of the relationship of the collateral variables with $\theta$. In typical educational and psychological settings where collateral information can often account for about a third of the population variance, and with item reliabilities typical of those settings, gains equivalent to 2 to 6 additional test items can be expected. This gain is substantial when few responses are available from each examinee, as in educational assessments, and may be useful in adaptive testing where tests are short but well-targeted. It is
unimpressive in individual achievement testing, where tests of sixty items or more are common.

From collateral information about items, increases equivalent to hundred and fifty additional examinees were found for Rasch item difficulty parameters in a junior high fractions test (Mislevy, in press). While a gain of this magnitude would be unimpressive in applications where data from thousands of examinees is already at hand, it is meaningful in situations when either (1) few examinees have been tested, as in the fractions example or in local testing problems, or (2) no examinees have been tested, as when approximating item statistics for newly-written test items.

In addition to small-sample applications, collateral information about items can play an important role in both item construction and diagnosis regardless of sample size. The conditional distributions of item parameters, \( p(\beta | z) \), express item operating characteristics such as difficulty in terms of salient features of the items. To the degree that these distributions succeed in explaining item operating characteristics, the test constructor can manipulate the features to modify items in intended ways or to create new items that tap the same essential skills. To the degree that items depart from the centers of these predictive distributions, they are hard or easy for reasons other than those held most important in describing the domain. Outliers are suspect as flawed or irrelevant. The approach implied by (5) and (6) is a step in the direction of integrating educational and
psychological theory into the measurement process. (Its application to the items in the Document Utilization scale of the NAEP Survey on Adult Literacy is currently in progress.)

When Collateral Information Must Be Used

The preceding sections discuss how, when all examinees are presented all items, collateral information about examinees and items may be exploited to obtain more precise item parameter estimates. Consistent estimates are still obtained in this case if the collateral information is not used (Mislevy and Sheehan, in press). The same results apply when each examinee receives only a random subset of items.

This is not the case that obtains in many practical applications of IRT, however. In order to obtain more information about item or examinee parameters per observed response, items are often administered to examinees as a function of item and examinee collateral variables. Fourth grade students may be presented an easier test form than the overlapping form fifth graders receive, for example; and a high school graduate may be presented a harder item first in an adaptive test than a nongraduate. In order to obtain consistent MML item parameter estimates, it is mandatory to employ collateral information about examinees--i.e., to use (4) rather than (3) (Mislevy and Sheehan, in press). In order to obtain the correct Bayesian inferences, it is mandatory to use collateral information about items as well--i.e., to base inferences on (8) rather than (4) (Mislevy and Wu, 1988). Mislevy
and Sheehan (in press) give a simple counterexample with the Rasch model to demonstrate an asymptotic bias in item parameter estimation in such a case if collateral information is ignored.

Modeling Item Responses when Different Examinees Follow Different Solution Strategies

Initial work on using collateral information about items assumed that the IRT model was strictly correct. Thinking about the features of items that made them easy or hard, however, made it clear that difficulty depends on the way that the examinees are attempting to arrive at their answers. In particular, different features of items can make them differentially difficult for examinees who follow different solution strategies. This insight led to the formulation of a mixture of IRT models (Mislevy and Verhelst, in press). Resolving the mixture demands a type of collateral information that plays no role whatsoever in traditional psychometrics, including standard IRT: psychological theory about the different strategies that examinees might follow.

The key idea is to model item difficulty in terms of salient item features—features that tend to make an item easy or difficult under various strategies. The Mislevy-Verhelst model makes the following assumptions:

1. A finite number of known solution strategies apply.
2. Each examinee is applying the only one of these strategies for all the items in the set.
3. The responses of an examinee are observed but the strategy he or she has employed is not.

4. The responses of examinees following Strategy k conform to an item response model of a known form.

5. Substantive theory posits relationships between observable features of items and the probabilities of success enjoyed by members of each strategy class. The relationships may be known either fully or only partially—e.g., known as to parametric form but not parameter values.

Let \( \theta = (\theta_1, \ldots, \theta_K) \) be an examinee proficiency parameter, with the element \( \theta_k \) corresponding to proficiency if Strategy k is employed. Let \( \phi = (\phi_1, \ldots, \phi_K) \) be an examinee strategy parameter, with all elements zero except for the single element \( k \) corresponding to the strategy that is employed; this element takes the value 1. Let the operating characteristics of Item j under Strategy k be given as follows:

\[
P(x_j|\theta_k, \beta_k(z_{jk}|\alpha), \phi_k = 1) = f_k(x_j|\theta_k, \beta_k(z_{jk}|\alpha)),
\]  

(9)

where \( \beta_k(z_{jk}|\alpha) \), the item parameter for Item j that applies when examinees follow Strategy k, depends on its salient features \( z_{jk} \) under that strategy and a relatively small number of basic strategy parameters \( \alpha \). The MML function for estimating \( \alpha \) induced by the data matrix \( X \) from a sample of N examinees and the item/strategy collateral variables \( Z \) is obtained as
\[ L(\alpha|X,Z) = \prod_{i=1}^{N} \sum_{k=1}^{K} \prod_{j=1}^{n} f_{ij}^{k}(x_{ij} | \theta, \beta_k(z_{jk} | \alpha)) \ g_k(\theta) \ d\theta , \quad (10) \]

where \( g_k \) is the density of \( \theta_k \) among those examinees following Strategy \( k \), and \( \pi_k \) is the proportion of the population who do so.

If the \( g_k \)s and the \( \pi \)s are not known, they too can be estimated via MML by maximizing (10) with respect to them as well.

If the \( \alpha \)s, \( g_k \)s, and \( \pi \)s are known or well estimated, it is possible to calculate for a given examinee the probability that his response vector was produced under a given strategy and to estimate his ability under each possibility. By Bayes theorem, the posterior probability of Strategy \( k \) and proficiency \( \theta \) under that strategy is obtained as

\[ P(\theta, \phi_k = 1|x) = \frac{C}{\pi_k} f_k(x | \theta, \beta_k(z_{jk})) \ g_k(\theta) \ \pi_k , \]

where \( C \) is the normalizing constant obtained as

\[ C^{-1} = \sum_{k} \int f_k(x | \theta, \beta_k(z_{jk})) \ g_k(\theta) \ d\theta \ \pi_k . \]

The posterior probability that Strategy \( k \) was employed is

\[ P(\phi_k = 1|x) = \int P(\theta, \phi_k = 1|x) \ d\theta \]

and the posterior mean proficiency conditional on \( \phi_k = 1 \) (i.e., supposing that Strategy \( k \) was used) is
The significance of this model lies in its ability to express how examinees solve items rather than just how many they solve. The latter is all that the standard models of test theory can do. Areas of potential benefit include psychological investigations of alternative processing models, educational decisions involving level of understanding, and determinations of alternative mental models in problem solving. The approach opens the door to such applications as (1) adaptive testing schemes designed to infer how examinees solve problems as well as how well they solve them, and (2) studies of changes in the structure as well as the level of intelligence in the course of human development.

Inferring Examinee Ability When Some Item Responses Are Missing

In practical applications of item response theory (IRT), there are several reasons that item responses may not be observed from all examinees to all test items. The reason most germane to the collateral information problem is the intentional administration of only subsets of items to examinees, with the subset depending on collateral information. It was mentioned above that collateral information must be taken into account in these cases. In addition to this type of missingness, Mislevy and Wu (1988) studied problems of inference that arise with several other types of missingness that arise frequently in IRT.

To preface the results of their study, we review Rubin's (1976) notions about "ignorability" of missing data. Ignoring the
missingness process under direct likelihood inference means using a pseudo-likelihood that includes terms for only the responses that were observed, without regard for the processes by which they came to be observed. The resulting inferences are appropriate if the pseudo-likelihood is proportional to the correct likelihood that does account for the missingness process. In this case the correct point estimate of the maximum likelihood estimate (MLE) is obtained. Sampling-distribution inferences based on the MLE are appropriate only if the missingness pattern does not depend on the values of the observed data. When this condition holds, sampling-distribution inferences can be drawn with regard to repeated samples of responses to only those items whose responses were observed. The missingness process is ignorable with respect to Bayesian inference if the correct Bayesian posterior is proportional to the product of the pseudo-likelihood and an appropriate prior distribution.

For five common types of missingness in IRT, Mislevy and Wu first used Rubin's (1976) theorems to determine whether ignorability holds under direct likelihood and Bayesian inference about examinee parameters $\theta$ when item parameters $\beta$ are known. In those cases in which the correct value of the MLE is obtained under direct likelihood inference, they asked whether sampling distribution inferences based on the MLE were appropriate. They then considered the analogous questions for inferences about $\beta$ when the examinee parameters are eliminated by marginalization, as
in (3)-(8). The findings are summarized below. Tables 2 and 3 highlight the results on ignorability.

-------------
Tables 2 and 3 about here

-------------

Case 1: Alternate Test Forms. When an examinee is assigned one of several alternative test forms by a random process such as a coin flip or a spiralling scheme, the process that renders missing the responses to items on the forms not presented is ignorable for all three types of inference, both for estimating $\beta$ and for estimating $\theta$ when $\beta$ is known.

Case 2: Targeted Testing. When collateral variables such as educational or demographic status are used to assign an examinee one of several test forms that differ in their measurement properties, the resulting missingness on forms not given is ignorable under direct likelihood inference for $\theta$ given $\beta$, but not under Bayesian inference unless the prior information about examinees that led to differential assignments is conditioned on. This information must be taken into account for both likelihood and Bayesian inferences about $\beta$; for Bayesian inference, prior information about $\beta$ used to select items must additionally be taken into account. Sampling distribution inferences may be based on MLEs for $\beta$ and for $\theta$ given $\beta$, conditional on the observed patterns of form administration within values of the examinee variables used for targeting.
It should be emphasized that these conclusions depend on the veracity of the IRT model. In particular, it is necessary that the regression of a correct response on ability be invariant with respect to collateral information. This assumption may well fail in a situation of currently increasing interest: An item pool is calibrated using an IRT model, and a school is allowed to measure students using only those items it deems relevant to its curriculum. If students from different schools have had different opportunities to learn the skills tapped by different items, then tailoring tests to their strengths leads almost certainly to item by school by ability interactions—a violation of the IRT model. Estimates for schools and individuals within schools tend to overestimate the scores they would have received had they been given all items, or randomly selected subsets of items. This use of IRT may hold practical value nonetheless, provided that such scores are viewed not as consistent estimates of performance in the total pool but as indicators of a kind of maximal performance.

Case 3: Adaptive Testing. In adaptive testing, item assignment proceeds item by item for each examinee according to the values of his responses to preceding items. The same conclusions as for Case 2 hold for direct likelihood and Bayesian inference. Ignorability under direct likelihood inference means that the correct points are identified as MLEs of \( \theta \) given \( \beta \) and of \( \beta \). The usual MLE properties under sampling-distribution inference need not hold, however, because the probabilities of missingness patterns depend on the values of observed responses.
Case 4: Not-reached Items. When some examinees run out of time before they see the last items on a nearly nonspeeded test, the not-reached process is ignorable with respect to direct likelihood inference about $\theta$ given $\hat{\theta}$, and the MLE supports sampling distribution inferences that pertain to repeated administrations of the items that were actually reached. This missingness process is not ignorable under Bayesian inference unless speed and ability are independent. An only then can direct likelihood inferences about $\theta$ ignore the missingness. Furthermore, Bayesian inferences about $\theta$ require that collateral variables for items be employed if they played a role in determining which items would not be reached, as when items are ordered from easy to hard.

Case 5: Intentional Omission. When examinees are presented items, have a chance to appraise their content, and decide for their own reasons not to respond, the missingness is not ignorable. Inferences must be drawn from a full model for the joint distribution of missingness and item response.

Not surprisingly, modeling this nonignorable nonresponse is difficult. Neither of the two most ambitious approaches proposed to date, namely Lord's (1983) model for omits and the use of multiple-category IRT models (e.g., Bock, 1972), handles the issue of local independence in a fully satisfactory manner. Under Lord's (1983) model, the marginal model for item responses is not a standard IRT model depending on $\theta$ alone and exhibiting local independence. Under the multiple-category model approach, local
independence fails unless all examinees at any given ability level have the same propensity to omit items they are unsure of, rather than guess at random.

If one assumes that examinees are perfect judges of their chances of responding correctly, and omit only if it is in accordance with the strategy that maximizes their expected score, Lord's (1974) treatment of omits as fractionally correct can be justified as providing the expectation of a conditional term in the full likelihood for omission probabilities and correct-response probabilities. This procedure is readily incorporated into standard complete-data IRT algorithms and avoids having to specify the full likelihood, but sacrifices information about examinee and item parameters conveyed by the observed pattern of missingness. Given the complexity of models for the full likelihood, however, this expedient seems to be a good practical choice—provided that, as Lord urges, examinees are clearly informed about how omits will be scored and which omitting strategy maximizes their chances of scoring well.

Conclusion

Although collateral information about examinees and items is rarely employed in item response theory (IRT), it is straightforward to incorporate it using Bayesian and empirical Bayesian methods. If the IRT model is correct and examinees are assigned items independently of values on collateral variables, then collateral information can be used to improve item parameter estimation modestly. Employing collateral information is
mandatory to obtain correct Bayesian and empirical Bayesian inferences if it was used to assign items to examinees.

Aside from considerations of efficiency, employing collateral information about items is a step toward integrating educational and psychological theory into the measurement process. Two aspects of this idea were developed in the course of the project.

The first, which takes a more traditional measurement perspective, assumes that a single IRT model provides an acceptable fit to the data of interest. Modeling items' operating characteristics in terms of salient features can make estimation more precise, but more importantly it elucidates the reasons that items are hard or easy, and why some are more discriminating than others. A formal framework is thus available for item construction and diagnosis, expressing relationships among substantive theory, item features, and measurement properties.

The second is a response to a growing awareness of the fact that traditional psychometric models (IRT as well as classical test theory) measure what is essentially an overall level of proficiency—losing in the process qualitative differences among examinees that arise from different cognitive solution strategies. In order to extend psychometric analysis to these problems, and to bring to bear the findings of recent research upon applied measurement problems, it is mandatory to employ collateral information about examinees and items that bears upon the ways that people solve problems. A mixture of IRT models that applies to some problems of this type was introduced in the project.
References

Bock, R.D. (1972). Estimating item parameters and latent ability when responses are scored in two or more nominal categories. Psychometrika, 37, 29-51.


BEST COPY AVAILABLE

19
Table 1
Research Reports

-----------------------------------------------------


-----------------------------------------------------
Table 2
Ignorability Results for Estimating $\theta$ Given $\beta$

<table>
<thead>
<tr>
<th>Type of Missingness</th>
<th>Direct Likelihood</th>
<th>Bayesian</th>
<th>Sampling Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternate Forms</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Targeted Forms</td>
<td>Yes</td>
<td>Yes, given examinee variables</td>
<td>Yes</td>
</tr>
<tr>
<td>Adaptive Testing</td>
<td>Yes</td>
<td>Yes, given examinee variables if they are used</td>
<td>No</td>
</tr>
<tr>
<td>Not-Reached</td>
<td>Yes</td>
<td>No, unless speed and ability are independent</td>
<td>Yes</td>
</tr>
<tr>
<td>Intentional Omissions</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

* Conditional on the observed pattern of missingness.
Table 3

Ignorability Results for Estimating $\theta$ After Marginalizing over $\theta$

<table>
<thead>
<tr>
<th>Type of Missingness</th>
<th>Type of Inference</th>
<th>Direct Likelihood</th>
<th>Bayesian Sampling Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternate Forms</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Targeted Forms</td>
<td>Yes, given examinee variables</td>
<td>Yes, given examinee and item variables</td>
<td>Yes, given examinee variables</td>
</tr>
<tr>
<td>Adaptive Testing</td>
<td>Yes, given examinee variables if they are used</td>
<td>Yes, given item variables and examinee variables if they are used</td>
<td>No</td>
</tr>
<tr>
<td>Not-Reached</td>
<td>No, unless speed and ability are independent</td>
<td>No, unless speed and ability are independent</td>
<td>No, unless speed and ability are independent</td>
</tr>
<tr>
<td>Intentional Omissions</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

* Conditional on the observed pattern of missingness.
Dr. P-A. Federico  
Code 51  
NPRDC  
San Diego, CA 92152-6800

Dr. Leonard Feldt  
Lindquist Center  
for Measurement  
University of Iowa  
Iowa City, IA 52242

Dr. Richard L. Ferguson  
American College Testing  
P.O. Box 168  
Iowa City, IA 52243

Dr. Gerhard Fischer  
Liebiggasse 5/3  
A 1010 Vienna  
AUSTRIA

Dr. Myron Fischl  
U.S. Army Headquarters  
DAPE-MRR  
The Pentagon  
Washington, DC 20310-0300

Prof. Donald Fitzgerald  
University of New England  
Department of Psychology  
Armidale, New South Wales 2351  
AUSTRALIA

Mr. Paul Foley  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Alfred R. Fregly  
AFOSR/NL, Bldg. 410  
Bolling AFB, DC 20332-6448

Dr. Robert Glaser  
Learning Research  
& Development Center  
University of Pittsburgh  
3939 O'Hara Street  
Pittsburgh, PA 15260

Dr. Bert Green  
Johns Hopkins University  
Department of Psychology  
Charles & 34th Street  
Baltimore, MD 21218

DORNIER GMBH  
P.O. Box 1420  
D-7990 Friedrichshafen 1  
WEST GERMANY

Dr. Ronald K. Hambleton  
University of Massachusetts  
Laboratory of Psychometric  
and Evaluative Research  
Hills South, Room 152  
Amherst, MA 01003

Dr. Delwyn Harnisch  
University of Illinois  
51 Gerty Drive  
Champaign, IL 61820

Dr. Grant Henning  
Senior Research Scientist  
Division of Measurement  
Research and Services  
Educational Testing Service  
Princeton, NJ 08541

Ms. Rebecca Hetter  
Navy Personnel R&D Center  
Code 63  
San Diego, CA 92152-6800

Dr. Paul W. Holland  
Educational Testing Service, 21-T  
Rosedale Road  
Princeton, NJ 08541

Prof. Lutz F. Hornke  
Institut fur Psychologie  
RWTH Aachen  
Jaegerstrasse 17/19  
D-5100 Aachen  
WEST GERMANY

Dr. Robert Glaser  
Learning Research  
& Development Center  
University of Pittsburgh  
3939 O'Hara Street  
Pittsburgh, PA 15260
Dr. Paul Horst  
677 G Street, #184  
Chula Vista, CA 92010

Mr. Dick Hoshaw  
OP-135  
Arlington Annex  
Room 2834  
Washington, DC 20350

Dr. Lloyd Humphreys  
University of Illinois  
Department of Psychology  
603 East Daniel Street  
Champaign, IL 61820

Dr. Steven Hunka  
3-104 Educ. N.  
University of Alberta  
Edmonton, Alberta  
CANADA T6G 2G5

Dr. Huynh Huynh  
College of Education  
Univ. of South Carolina  
Columbia, SC 29208

Dr. Robert Jannarone  
University of South Carolina  
Columbia, SC 29208

Dr. Douglas H. Jones  
Thatcher Jones Associates  
P.O. Box 6640  
10 Trafalgar Court  
Lawrenceville, NJ 08648

Dr. Milton S. Katz  
European Science Coordination Office  
U.S. Army Research Institute  
Box 65  
FPO New York 09510-1500

Prof. John A. Keats  
Department of Psychology  
University of Newcastle  
N.S.W. 2308  
AUSTRALIA

Dr. G. Gage Kingsbury  
Portland Public Schools  
Research and Evaluation Department  
501 North Dixon Street  
P. O. Box 3107  
Portland, OR 97209-3107

Dr. William Koch  
Box 7246, Meas. and Eval. Ctr.  
University of Texas-Austin  
Austin, TX 78703

Dr. James Kraatz  
Computer-based Education Research Laboratory  
University of Illinois  
Urbana, IL 61801

Dr. Leonard Kroeker  
Navy Personnel R&D Center  
Code 62  
San Diego, CA 92152-6800

Dr. Jerry Lehnus  
Defense Manpower Data Center  
Suite 400  
1600 Wilson Blvd  
Rosslyn, VA 22209

Dr. Thomas Leonard  
University of Wisconsin  
Department of Statistics  
1210 West Dayton Street  
Madison, WI 53705

Dr. Michael Levine  
Educational Psychology  
210 Education Bldg.  
University of Illinois  
Champaign, IL 61801

Dr. Charles Lewis  
Educational Testing Service  
Princeton, NJ 08541-0001

Dr. Robert L. Linn  
Campus Box 249  
University of Colorado  
Boulder, CO 80309-0249
Dr. James B. Olsen  
WICAT Systems  
1875 South State Street  
Orem, UT 84058

Office of Naval Research,  
Code 1142CS  
800 N. Quincy Street  
Arlington, VA 22217-5000  
(6 Copies)

Office of Naval Research,  
Code 125  
800 N. Quincy Street  
Arlington, VA 22217-5000

Assistant for MPT Research,  
Development and Studies  
OP 0187  
Washington, DC 20370

Dr. Judith Orasanu  
Basic Research Office  
Army Research Institute  
5001 Eisenhower Avenue  
Alexandria, VA 22333

Dr. Jesse Orlansky  
Institute for Defense Analyses  
1801 N. Beauregard St.  
Alexandria, VA 22311

Dr. Randolph Park  
Army Research Institute  
5001 Eisenhower Blvd.  
Alexandria, VA 22333

Wayne M. Patience  
American Council on Education  
GED Testing Service, Suite 20  
One Dupont Circle, NW  
Washington, DC 20036

Dr. James Paulson  
Department of Psychology  
Portland State University  
P.O. Box 751  
Portland, OR 97207

Dept. of Administrative Sciences  
Code 54  
Naval Postgraduate School  
Monterey, CA 93943-5026

Department of Operations Research,  
Naval Postgraduate School  
Monterey, CA 93940

Dr. Mark D. Reckase  
ACT  
P. O. Box 168  
Iowa City, IA 52243

Dr. Malcolm Ree  
AFHRL/MAA  
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt  
HumRRO  
1100 South Washington Street  
Alexandria, VA 22314

Dr. Carl Ross  
CNET-PDCD  
Building 90  
Great Lakes NTC, IL 60088

Dr. J. Ryan  
Department of Education  
University of South Carolina  
Columbia, SC 29208

Dr. Fumiko Samejima  
Department of Psychology  
University of Tennessee  
310B Austin Peay Bldg.  
Knoxville, TN 37916-0900

Mr. Drew Sands  
NPRDC Code 62  
San Diego, CA 92152-6800

Lowell Schoer  
Psychological & Quantitative Foundations  
College of Education  
University of Iowa  
Iowa City, IA 52242

Dr. Mary Schratz  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Dan Segall  
Navy Personnel R&D Center  
San Diego, CA 92152
Educational Testing Service/Mislevy

Dr. W. Steve Sellman
OASD (MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujisawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street, Suite 120
Alexandria, VA 22314-1713

Dr. Richard E. Snow
School of Education
Stanford University
Stanford, CA 94305

Dr. Richard C. Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Symson
Navy Personnel R&D Center
Code-62
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research Laboratory
103 S. Mathews Avenue
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820
Dr. Vern W. Urry  
Personnel R&D Center  
Office of Personnel Management  
1900 E. Street, NW  
Washington, DC 20415

Dr. David Vale  
Assessment Systems Corp.  
2233 University Avenue  
Suite 440  
St. Paul, MN 55114

Dr. Frank L. Vicino  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Howard Wainer  
Educational Testing Service  
Princeton, NJ 08541

Dr. Ming-Mei Wang  
Lindquist Center  
for Measurement  
University of Iowa  
Iowa City, IA 52242

Dr. Thomas A. Warm  
Coast Guard Institute  
P. O. Substation 18  
Oklahoma City, OK 73169

Dr. Brian Waters  
HumRRO  
12908 Argyle Circle  
Alexandria, VA 22314

Dr. David J. Weiss  
N660 Elliott Hall  
University of Minnesota  
75 E. River Road  
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman  
Box 146  
Carmel, CA 93921

Dr. Douglas Wetzel  
Code 51  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. Rand R. Wilcox  
University of Southern California  
Department of Psychology  
Los Angeles, CA 90089-1061

German Military Representative  
ATTN: Wolfgang Wildgrube  
Streitkraefteamt  
D-5300 Bonn 2  
4000 Brandywine Street, NW  
Washington, DC 20016

Dr. Bruce Williams  
Department of Educational Psychology  
University of Illinois  
Urbana, IL 61801

Dr. Hilda Wing  
NRC MH-176  
2101 Constitution Ave.  
Washington, DC 20418

Dr. Martin F. Wiskoff  
Defense Manpower Data Center  
550 Camino El Estero  
Suite 200  
Monterey, CA 93943-3231

Mr. John H. Wolfe  
Navy Personnel R&D Center  
San Diego, CA 92152-6800

Dr. George Wong  
Biostatistics Laboratory  
Memorial Sloan-Kettering Cancer Center  
1275 York Avenue  
New York, NY 10021

Dr. Wallace Wulfeck, III  
Navy Personnel R&D Center  
Code 51  
San Diego, CA 92152-6800
Dr. Kentaro Yamamoto  
03-T  
Educational Testing Service  
Rosedale Road  
Princeton, NJ 08541  

Dr. Wendy Yen  
CTB/McGraw Hill  
Del Monte Research Park  
Monterey, CA 93940  

Dr. Joseph L. Young  
National Science Foundation  
Room 320  
1800 G Street, N.W.  
Washington, DC 20550  

Mr. Anthony R. Zara  
National Council of State  
Boards of Nursing, Inc.  
625 North Michigan Avenue  
Suite 1544  
Chicago, IL 60611  

Dr. Peter Stoloff  
Center for Naval Analysis  
4401 Ford Avenue  
P.O. Box 16268  
Alexandria, VA 22302-0268