This is a one-semester case study of the perceptions of five preservice teachers and their mathematics professor as participants in a reform-style mathematics classroom under the Maryland Collaborative for Teacher Preparation (MCTP) program. Researchers interviewed the professor and the teacher candidates throughout the semester, gathering data to address this question: "Do the instructor and the pre-service teachers perceive the instruction in their mathematics course as exemplifying the kind of teaching and learning they would like to promote as upper elementary/middle level mathematics and science teachers, and if so, how?" Data were collected and analyzed through analytic induction, constant comparison, and discourse analysis for patterns of similarities and differences between the professor and the preservice teachers' perceptions. The data indicate that the professor and the students perceived vast differences between traditional instruction and the teaching and learning they experienced in this class. All expressed a clear image of what they thought teaching in grades four through eight should be. This image was quite consistent with the teaching and learning that the students experienced in the class. Explanations are offered for why issues of pedagogy were not discussed in the class. (Contains 44 references.) (Author/NAV)
Modeling Reform-Style Teaching in a College Mathematics Class from the Perspectives of Professor and Students

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Abstract

Fundamental changes in teaching and learning have been proposed for mathematics education in the United States. As part of the reform effort, several publications directed at college mathematics teachers stress the importance of modeling reform-style teaching to undergraduate students (e.g., Mathematics Association of America, 1988; Mathematical Sciences Education Board, 1995; Tucker & Leitzel, 1995). This study presents the perceptions of five pre-service teachers and their mathematics professor as participants in a reform-style mathematics classroom. The following a priori research question is addressed: Do the instructor and the pre-service teachers perceive the instruction in their mathematics course as exemplifying the type of teaching and learning they would like to promote as upper elementary/middle level teachers of mathematics and science? And if so, how?

An analysis of the data indicated that the professor and the teacher candidates perceived vast differences between traditional instruction and the teaching and learning they experienced in this class. Moreover, both the professor and the teacher candidates expressed a clear image of what they thought teaching in grades 4 through 8 should be. Their image of ideal teaching was quite consistent with the teaching and learning that they experienced in this class. The experiences of these teacher candidates and this professor has implications for teacher education programs interested in preparing pre-service teachers to achieve the standards for teaching and learning set forth in the reform documents.
Introduction

Fundamental changes in teaching and learning have been proposed for mathematics education in the United States. The National Council of Teachers of Mathematics [NCTM] (1989, 1991, 1995), the Mathematical Sciences Education Board [MSEB] (1990, 1991, 1995), the Mathematical Association of America [MAA] (Tucker & Leitzel, 1995) and the National Research Council [NRC] (1991) have issued documents proposing a framework for change in mathematics education at all levels, elementary through college. The framework is based on the philosophy that students are active learners who construct knowledge through their interpretations of the world around them. The above reform documents present goals for mathematics education which state that all students should: learn to value mathematics, become confident in their ability to do mathematics, become mathematical problem solvers, learn to communicate mathematically, and learn to reason mathematically.

Several publications directed at college mathematics teachers stress the importance of modeling reform-style teaching to undergraduate students, especially pre-service teachers (MAA, 1988; MSEB, 1995; NRC, 1991; National Science Foundation [NSF], 1993; Tucker & Leitzel, 1995). Modeling reform-style teaching at the college level is important for the following reasons. First, the result of modeling good teaching is a better education for all students, not just future teachers, in that all students benefit from good teaching (NRC, 1991; NSF, 1993). Also, since the literature on teacher education posits that teachers tend to teach as they have been taught when they were students (Brown & Borko, 1992; Kennedy, 1991), teachers (including college level teachers) should model the type of teaching that is consistent with the reform documents (MSEB, 1995). Moreover, as a consequence of this finding, there are implications specific to college teaching. While all teachers serve as role models for students who want to become teachers, college faculty are the people teaching pre-service teachers as they train for their careers; thus, college faculty should be especially concerned about modeling good teaching. "Unless college and university mathematicians
model through their own teaching effective strategies that engage students in their own learning, school teachers will continue to present mathematics as a dry subject to be learned by imitation and memorization" (NRC, 1991, p. 29).

However, in looking at the literature on reform-style teaching in mathematics available to college faculty, Brown and Borko (1992) state that existing research, provides limited evidence about the design and implementation of good mathematics teacher education programs. Careful documentation of the experiences of teachers in such programs and the resulting changes in their knowledge, beliefs, dispositions, thinking, and actions will provide further insight into the process of becoming a mathematics teacher (p. 235 - 236).

The Maryland Collaborative for Teacher Preparation [MCTP] is addressing this need for the design, implementation, and documentation or a reform-based teacher education program at ten colleges and universities in Maryland. The MCTP is a National Science Foundation funded project with the mission to develop, implement, and evaluate an interdisciplinary mathematics and science, upper elementary/middle level teacher preparation program consistent with the goals for reform in mathematics and science education as described above. MCTP involves college faculty from mathematics, science, and education departments who are collaborating to develop and implement the program. In designing the courses and field experiences, the following basic principles guide the faculty participating in the MCTP program. These principles are outlined in an MCTP abstract developed by the principle investigators of the project.

1. Preservice teachers should be actively involved in the learning of mathematics and science through instruction that models practices that they will be expected to employ in their teaching careers.

2. Courses and field experiences should reflect the integrated nature of mathematics and science so that prospective teachers can develop an understanding of the connections between mathematics and science.
3. The programs of all preservice teachers should include field internships that involve them in genuine research activities of business, industrial, or scientific research institutions and informal teaching activities of educational institutions such as science centers, zoos, or museums.

4. The courses and experiences of all preservice teachers should focus on developing their ability to use modern technologies as standard tools for problem solving.

5. The courses and experience of all preservice teachers should prepare them to deal effectively with the broad range of students who are in public schools today.

6. The teacher graduates should be given assistance and continued support during the critical first years in the teaching profession.

These principles are consistent with the recommendations of the reform documents in that they emphasize active learning, mathematics and science connections, real-world experiences, the utilization of technology, teaching to diverse student populations, and ongoing professional support.

In addition to developing a teacher education program, the MCTP has dedicated significant efforts to teacher education research. The primary purpose of the research is gaining knowledge and understanding about the experiences of the pre-service teachers and the college faculty in the process of implementing a mathematics and science education program which is based on reform-style teaching and learning. More specifically, this study presents the perceptions of five pre-service teachers and their mathematics professor as participants in a reform-style mathematics classroom. The goal is to promote understanding which can inform future research on the teaching and learning practices of college level mathematics instructors from a constructivist perspective and thus contribute to the preparation of pre-service mathematics teachers.
Objectives

The purpose of this study is to provide a description and an interpretation of an MCTP professor and five MCTP teacher candidates who are attending to teach and learn in a class consistent with the goals set forth by the reform documents. This study addresses the following a priori research question: Do the instructor and the pre-service teachers perceive the instruction in their mathematics course as exemplifying the kind of teaching and learning they would like to promote as upper elementary/middle level teachers of mathematics and science? And if so, how?

Theoretical Perspective and Methodology

Theoretical Perspective

The research was conducted from a perspective which combines ideas of interactionism and constructivism. This perspective is consistent with the philosophy toward teaching and learning that underlies the framework for reform in mathematics education and with the philosophy of the Maryland Collaborative for Teacher Preparation [MCTP].

First, according to the perspective of interactionism, people invent symbols to communicate meaning and interpret experiences (Alasuutari, 1995; Blumer, 1986; Romberg, 1992); moreover, people create and sustain social life through interactions and patterns of conduct including discourse (Alasuutari, 1995; Gee, 1990; Hicks, 1995; Lave & Wenger, 1991). Furthermore, this position is in accordance with the constructivist perspective of learning in that individuals develop understandings based on their experiences and knowledge as it is socially constructed (Bruffee, 1986; Ernest, 1991; Gergen, 1985; Romberg, 1992).

Simon and Schifter (1991) adopted the following view of constructivism which combines aspects of radical (e.g., von Glasersfeld, 1990) and social (e.g., Ernest, 1991) constructivism:
1. Constructivism is a belief that conceptual understanding in mathematics must be constructed by the learner. Teachers' conceptualizations cannot be given directly to students.

2. Teachers strive to maximize opportunities for students to construct concepts. Teachers give fewer explanations and expect less memorization and imitation. This suggests not only a perspective on how concepts are learned, but also a valuing of conceptual understanding. (p. 325)

Cobb and Bauersfeld (1995) discuss the social aspects of learning and knowledge by advocating viewing mathematics education through the perspectives of interactionism and constructivism. Incorporating the interactionist perspective with constructivism, Cobb and Bauersfeld (1995) state that,

[The authors of the book] draw on von Glasersfeld's (1987) characterization of students as active creators of their ways of mathematical knowing, and on the interactionist view that learning involves the interactive constitution of mathematical meanings in a (classroom) culture. Further, the authors assume that this culture is brought forth jointly (by teachers and students), and the process of negotiating meanings mediates between cognition and culture (p. 1).

In regard to research based on a constructivist view, Noddings (1990) states, "We have to investigate our subjects' perceptions, purposes, premises, and ways of working things out if we are to understand their behavior . . . We have to look at their purposive interactions with those environments" (p. 15). Through such methods as participant observation, the ideas of interactionism and constructivism provide a strong framework within which the researcher constructs meanings to interpret and explain the observed and inferred perceptions, actions, and interactions of the study participants (Bogdan & Biklen, 1992, Cobb & Bauersfeld, 1995). Simon and Schifter's (1991) definition of constructivism along with the perspective presented by Cobb and Bauersfeld (1995) reflect what is both implied and stated in the reform documents and in the MCTP philosophy, and they reflect the researchers' perspective on teaching and learning; and thus, they represent the perspective from which the research was conducted.
Methodology

Since this study involves an in-depth examination of a phenomenon, the research strategy best suited to helping researcher understand the perceptions, actions and interactions of faculty and students is the case study with a qualitative methodology (Goetz & LeCompte, 1984; LeCompte, Millroy, & Preissle, 1992; Merriam, 1988; Romberg, 1992; Stake, 1995). While a case study in and of itself is not a methodology and has been applied to both quantitative and qualitative research methods, a "qualitative case study is characterized by the main researcher spending substantial time, on site, personally in contact with activities and operations of the case, reflecting, revising meanings of what is going on" (Stake, 1994, p. 242).

In this research project, the case study methodology enables the researcher to develop an in-depth story about the selected professor and teacher candidates which might serve to provide a framework from which other educators can reflect on their experiences and to inform future research (Merriam, 1988; Romberg, 1992; Stake, 1995). It is a study of the participants' and the researchers' perceptions of their experiences teaching and learning in an MCTP course throughout the semester. For this study, the case is bounded in time by the academic semester (Fall, 1994).

As part of this case study, the professor and the teacher candidates engaged in ongoing interviews and observations throughout the semester to obtain data regarding their perceptions and actions toward teaching and learning and the extent to which the instruction modeled the kind of teaching and learning appropriate for grades 4 through 8, the focus of the MCTP program. The data were collected and analyzed through the use of the qualitative techniques of analytic induction, constant comparison, and discourse analysis for patterns of similarities and differences between the professor's and teacher candidates' perceptions (Bogdan & Biklen, 1992; Gee, 1990; Goetz & LeCompte, 1984; LeCompte, Millroy, & Preissle, 1992).
Data Sources and Collection Methods

Setting

The research setting was an undergraduate mathematics classroom at a large state university. The mathematics course was developed and taught by a university professor (pseudonymous Dr. Taylor) as part of the Maryland Collaborative for Teacher Preparation. The mathematics course was open to both MCTP teacher candidates (intending teachers who have been accepted into the MCTP program and plan to enroll in MCTP courses throughout their undergraduate program) and non-MCTP undergraduates. In addition to education majors, the course served departments such as English, business, theater, and journalism.

Participants

Participants in this study were the course instructor, Dr. Taylor, and five MCTP teacher candidates in his mathematics class. Dr. Taylor was an experienced university professor with a joint appointment to the mathematics and education departments. The teacher candidates were first year undergraduates, and ranged in age from 17-19 years old. Because they were in their first semester, none of the teacher candidates previously had taken an MCTP course or an education course; however, they were all concurrently enrolled in an MCTP science course (either physics or chemistry), and a one-credit MCTP Seminar Course. (The purpose of the Seminar Course was to make connections between the mathematics and science courses that the MCTP teacher candidates were taking and to discuss issues related to their future teaching of these subjects.) Four teacher candidates were women, and one student was a man.

Data Tools

Research tools used included interviews with individual participants, group interviews, participant observation, and artifact collection. All participants were interviewed individually at the beginning and end of the semester, and the interviews were audio taped and
transcribed (see Appendix for interview protocols). The interviews were semi-structured in that they contained a set of standard questions; however, additional questions were posed based on the participants' responses. In addition, two group interviews were conducted with only the teacher candidates and a researcher present.

Also, throughout the semester, data for Dr. Taylor's and the teacher candidates' actions in the process of teaching and learning were obtained through class observations and field notes. To further inform the researchers, informal interviews with the instructor and the teacher candidates were conducted prior to and following the class observations. Finally, in the process of analyzing data and writing the research report, selected participants were consulted as a means of member checking and establishing validity (Stake, 1995).

Findings

An analysis of the data indicated that Dr. Taylor and the teacher candidates perceived vast differences between traditional instruction and teaching and learning "this way" (Julie, interview, 12/8/94) as modeled by Dr. Taylor. Moreover, both Dr. Taylor and the teacher candidates expressed a clear image of what they thought teaching in grades 4 through 8 should be. This image of ideal teaching was quite consistent with the teaching and learning that they experienced in Dr. Taylor's class. Five categories emerged from the data in regard to the participants' perceptions of traditional teaching and learning, teaching and learning in Dr. Taylor's class, and the participants' image of what teaching and learning should be for grades 4 through 8. These categories are presented below.

1. Doing Mathematics in Typical (Traditional) Courses Means Mimicking the Teacher and Following Prescribed Steps Without Understanding

   Teacher Candidates' Perceptions of Traditional. All five teacher candidates expressed the same view of how mathematics teaching and learning typically takes place. Their usual experience with mathematics is that it is dry, rule-based, and consists of a set of procedures, each which leads to a single correct (or incorrect) answer. The teacher
candidates were accustomed to doing large sets of similar mathematics problems without understanding the meaning or purpose of the problems.

Julie relates her prior experiences with mathematics as consisting entirely of procedures without understanding when she says,

*Before when I would have math classes, . . . it's just that I had to be able to mimic what the teacher did; I just had to be able to follow the steps and just do it without understanding what I was actually doing. So later on, it would be . . . so much easier for me to forget the things because I hadn't really understood it, I was just following what the professor had done* (Interview, 10/5/94).

Also, Kevin discusses the lack of interest he felt and observed from other classmates:

*[Typically in mathematics classes] they stress memorizing formulas and things like that, or they'd give you the formula and then you'd have to go home and do 20 like that for homework. . . I've had classes where you sit down and people will fall asleep, and the teacher was goin' on talking* (Interview, 12/8/94).

In addition, Heidi relates the lack of active participation found in most mathematics classes:

*Math my classes were always, you sat at a desk with your book, and you had examples to do, and the teacher would write on the board, and. . . and I mean, that was math, and that's what you expected from math. You sit and listen to the teacher* (Interview, 12/8/94).

**Dr. Taylor's Perception of Traditional.** While Dr. Taylor did not focus his discussions on his perceptions about traditional mathematics teaching to the extent that the teacher candidates did, his view of what happens in traditional mathematics classrooms was consistent with what the teacher candidates shared. He states,

*In a traditional class, they learn "how to" problems, they go home and they do their problems, and the other kind of stuff is just immaterial* (Interview, 12/6/94).

**II. Doing Mathematics in this Course Means Emphasizing Concepts and Understanding, Not Just Memorizing or Doing Procedural Routines**

**Teacher Candidates' Perceptions of Class.** All of the teacher candidates perceived Dr. Taylor's class as different from what they were used to in mathematics class. They recognized that the course was focused on concepts and understanding and learning meaningful mathematics.
Julie explains how the course emphasized concepts over memorization and understanding the significance of mathematics.

[In this course the emphasis was on] concepts. It was a lot of understanding just in general, like knowing how things work - more than just a memorization of facts - just understanding what we were doing and not just kind of following what he said to do, and what the book said to do. . . . You have to do a lot more thinking about the bigger picture; that's always what [he] stresses, is looking for the bigger picture and finding the great significance in it, and not. . . . the knit-picky things, but understanding the overall process (Interview, 12/8/94).

Beth compares the focus on understanding in Dr. Taylor's course to an emphasis on memorizing empty facts that she experienced in previous mathematics courses.

[Dr. Taylor's course] has definitely been more of understanding of how to solve the problems as opposed to the memorization of facts and stuff (Interview, 12/8/94).

Dr. Taylor's Perception of Class. The teacher candidates' perceptions of the course emphasizing concepts and understanding are consistent with what Dr. Taylor envisioned in planning the course. When he discussed his intentions for teaching and learning early in the semester he emphasized that the course would not focus on procedures without understanding:

I think that one thing that we [do not do] is a lot of procedural routines. . . . that stuff on the board (Interview, 9/16/95).

Dr. Taylor later describes what he considers to be important learning for the students in his class: learning based on reasoning, connections, and meaningful problems. He states,

[The students should] be able to explain [methods of problem solving] . . . [it's] not going to be just memory of a fact, it's going to be understanding of a whole way of reasoning about a problem. . . . We're trying to help students. . . . make the connection between the real object and the mathematical representation or the mathematical model of it. . . . We're trying to have the course problem-based in a sense that the mathematical ideas will be encountered first in looking at the context of working on a problem of some kind rather than "here's how we're gonna do today's problems". It's trying to embed the mathematics in problem-solving activity. . . . It's more an applied problem. . . . more making sense of a real situation and patterns in data (Interview, 9/16/94).
Dr. Taylor’s description of what he considered to be important in mathematics teaching and learning is consistent with what the researcher observed as the focus of activities and discussions during class and in the course materials.

III. Doing Mathematics in this Course Involves Communication and Collaboration

Teacher Candidates’ Perceptions of Class. An important component of conceptual learning of mathematics based on understanding is perceived to be discussing ideas and working together to gain an understanding of mathematics. Four of the five teacher candidates made specific references to the importance of communication and collaboration in the process of learning mathematics.

Kevin discusses how working with others helps in generating ideas and strategies for problem solving:

[Dr. Taylor] gives you a problem that you have to solve, and you get together with other students and you all try to solve the problem together, so you’re coming up with all these different ideas of ways to conquer this problem (Interview, 10/5/94).

In addition, Julie states that the process of explaining her reasoning to others is a necessary part of understanding and being able to do mathematics:

[In this class] it’s like I have to do this [mathematics] here, I have to understand it right now, and I have to be able to explain it to someone else, and I have to be able to move with this (Interview, 10/5/94).

Dr. Taylor’s Perception of Class. Dr. Taylor stresses the importance of communicating and collaborating to learn mathematics. At the beginning of the term, Dr. Taylor expressed his interest in incorporating these things in the teaching and learning process:

[I am] asking students to collaborate with each other and to work cooperatively. Quite often asking students to present...to communicate their ideas in writing, submitting write-ups about their solutions to a problem or talking, sharing what their group has come up with orally in class (Interview, 9/16/94).

Dr. Taylor’s commitment to communicating and collaborating throughout the semester is evidenced by classroom observations which reflected regular use of group work and oral
and written reports from students. In addition, toward the end of the semester, Dr. Taylor discussed the notion that explanation of ideas and reasoning played an important role in students demonstrating what they knew on exams:

[On the exams],...there was a lot of problem solving in the sense of using techniques that they'd learned to analyze a situation. . ., and they were asked to explain. . .why they did what they did  (Interview, 12/8/94).

IV. Teaching Mathematics in this Course Means Facilitating and Guiding Understanding

Teacher Candidates' Perceptions of Teaching. In several instances, the teacher candidates discussed the actions of Dr. Taylor: what he did as a teacher to create the learning environment described above. All of the teacher candidates, in one way or another, mentioned that Dr. Taylor acted as a facilitator or guide to learning as opposed to a lecturer who delivers information and facts to students.

Kevin explains how Dr. Taylor would ask questions in an effort to engage students in thinking about a problem:

The teacher will come around and sort of direct you in a certain direction, or ask you more questions, get you thinking more. It seems, that you're sort of widening your focus on math instead of running a single process, and you will learn that process, but you also, along the way, you know, sort of pick up this other stuff. And you're not just copying things copying things off the board (Interview, 10/5/94)

Also, Julie states that Dr. Taylor's questions would help to re-direct their thinking if they were having difficulties approaching a problem:

[Dr. Taylor] would step in and kind of guide us the right way, maybe asking us questions in different ways so that we can see in a different way what he's trying to get across, and that way remember it because we understand it (Interview, 12/8/94).

The notion that Dr. Taylor was always "walking around" and "asking questions" to guide learning was prevalent in the teacher candidates' comments and in the researcher's observations of the class. The teacher candidates quickly became accustomed to this approach to teaching and seemed to welcome his involvement in their learning.
Dr. Taylor's Perception of Teaching. Dr. Taylor explains that his intention in teaching was not to tell students information and what to do to solve a problem, but instead, it was to let the students attempt solving the problem. According to Dr. Taylor, what was important for him to do was to "get them thinking" not necessarily to arrive at a specific answer. In describing an example of how he employed this method of teaching, Dr. Taylor mentions a probability problem he presented in class...

The context was in a store and the average salesperson is successful on two out of five customers on average, and two different people were working in that store, and one of them has a day when they only sell to four out of 15 customers, another one has 8 out of 15 customers. Does it seem fair for the person who only sold the four out of 15 person to be fired as incompetent or substandard? And so I let them discuss what their reaction was. And to some extent what it gave me [was information about which students] had any inkling that...there could be a chance phenomena operating. ...I was using [the problem] to get them thinking about what might be involved, and also, I guess that rather than me saying, "Here is a problem that you can study with probability, and here is how you can do it,"...I use it more [as] a way of getting 'hem to think about what the issues are in a situation" (Interview, 9/16/95).

Dr. Taylor's description of the probability activity is typical of what the researcher observed in his class and in the course materials. Usually, students were presented with a problem that would stimulate discussion and some form of data collection as a basis for reasoning through a problem. Rarely, were the students given problems that had a single, correct numerical answer.

V. Image of What Mathematics Teaching and Learning Should Be for Grades 4 through 8.

Teacher Candidates' Image. After experiencing mathematics in a reform-style classroom, the teacher candidates perceived Dr. Taylor's teaching as modeling the type of teaching and learning that they would like to promote when they begin teaching in the elementary/middle grades. A...e of the teacher candidates described an image of what mathematics teaching and learning should be for grades 4 through 8 in a manner consistent with the type of teaching and learning they experienced in Dr. Taylor's class. They stressed the importance of meaningful mathematics, an emphasis
on conceptual understanding, students' active involvement in learning activities, students working collaboratively in groups to solve problems, and teachers acting as facilitators and guides in the learning process. Moreover, the teacher candidates believed that this type of teaching and learning promotes better understanding in that the mathematics they have learned is more meaningful to them in life.

For example, Beth describes her image of good mathematics teaching and learning as the teacher serving as a facilitator and promoting collaboration:

[Good mathematics teaching and learning involves] more interaction with the students instead of just, like, standing up there and saying, "Okay. This, this, this." Because lecturing doesn't really work and, at least for me, it doesn't really work. . . So, like, more like letting the kids work together, or working with students, asking them questions and having them say what they think (Interview, 10/5/94).

Also, Paula states that a good mathematics teacher motivates students to be interested in mathematics through the use of meaningful mathematics that applies to real-world situations:

[A good mathematics teacher is] someone who gets you interested in what you're doing, who doesn't just give you problems, and tell you to answer them, and show you how to do it; somebody who maybe applies it...applies math,...shows how math is used in the real world, other than just giving you random problems and just having you solve them--showing students that you can use this. This is something that can be helpful to you in life, it's not just something you're doing in school (Interview, 10/5/94).

Dr. Taylor's Image. Dr. Taylor also believed that the type of teaching and learning that took place in his undergraduate mathematics course modeled what should be happening in grades 4 though 8. He states,

The [NCTM] Standards' model of the instruction and curriculum are problem oriented learning, contextualized learning, learning in true collaboration with other people, learning through active investigation of things, and so we try to do all those things. And those things seem to be appropriate, at least as far as we know, appropriate guidelines for intermediate school instruction (Interview, 12/6/94).
Implications and Educational Significance

The experiences of these teacher candidates and this professor have implications for teacher education programs interested in preparing pre-service teachers to achieve the standards for teaching and learning set forth in the reform documents.

A First Step

First, a major implication gained from this qualitative study is that the college students who experienced a reform-style mathematics classroom completed a first step in achieving the vision for reform of mathematics education: constructing an initial model of mathematics teaching and learning which embraces the ideals of the reform movement.

Although not at the undergraduate level, research shows that this type of construction has occurred for other students who experienced learning in reform-style classrooms. In a study of two elementary school classrooms, Cobb, Wood, Yackel, and McNeal (1992) discuss this notion of students constructing a new idea of what it means to do mathematics. Cobb, et al. (1992) investigate and contrast instructional situations in mathematics which promote teaching and learning for understanding and instructional situations that do not promote understanding. The researchers view the classroom interactions in terms of five distinct types of classroom social norms (regulations, conventions, morals, truths, and instructions) and focus on the mathematical explanations and justifications that occurred during the lessons. Mathematical explanations and justifications are considered to be essential components of teaching and learning for understanding as is recommended by the goals of the reform movement in mathematics education (Cobb, et al., 1992), and these components were also important in Dr. Taylor's class.

Cobb, et al. (1992) characterize two distinct classroom mathematics traditions in their descriptions of the classrooms studied. In the first classroom, doing mathematics means following procedural instructions, and thus mathematical explanations and justifications are not valued or expected. In the second classroom, doing mathematics means co-constructing
a mathematical reality based on the students' and teacher's experiences with created and manipulated abstract mathematical objects. Correspondingly, in the second classroom, mathematical explanations and justifications are expected and valued. Thus, when a teacher uses a more traditional style of mathematics teaching, the students continue to view and act on mathematics as strictly procedural and rule-based; however, when a teacher believes and behaves in a way that models and supports the ideals of reform-based teaching and learning the students respond by changing their views of mathematics.

Based on the findings, Dr. Taylor's students' experiences were similar to that of the second classroom in Cobb, et al.'s (1992) study. In order to justify this claim, Cobb, et al.'s (1992) study is examined more closely. Cobb, et al. (1992) describe the first teacher's actions as facilitating "her students' enculturation into what Lave (1988) called the folk beliefs about mathematics" (p. 589.). (Folk beliefs about mathematics include the idea that mathematics consists of standard procedures only appropriate for "school-like" tasks (p. 589).) In contrast, the second teacher facilitated the students' enculturation into mathematical ways of knowing which consisted of "taken-as-shared mathematical meanings and practice" (Cobb, et al., 1992, p. 595). A similar process of enculturation seemed to occur for Dr. Taylor's students. Being in a classroom where reform-style teaching was modeled and where students were engaged in active learning through meaningful problem solving and collaboration enabled the students to construct a new model of mathematics teaching and learning.

Exploring this notion of enculturation further, consider Gee's (1990) ideas on enculturation. He makes a distinction between acquisition and learning. Gee (1990) defines these terms as follows:

**Acquisition** is a process of acquiring something subconsciously by exposure to models, a process of trial and error, and practice within social groups, without formal teaching. It happens in natural settings which are meaningful and functional in the sense that acquirers know that they need to acquire the thing
they are exposed to in order to function and they in fact want to so function. This is how most people come to control their first language.

Learning is a process that involves conscious knowledge gained through teaching (though not necessarily from someone officially designated a teacher) or through certain life-experiences that trigger conscious reflection. This teaching or reflection involves explanation and analysis, that is breaking down the thing to be learned into its analytic parts. It inherently involves attaining, along with the matter being taught, some degree of meta-knowledge about the matter (p. 146).

Based on these definitions, it seems that while Dr. Taylor’s students may have been learning mathematics, they were acquiring ideas about the teaching and learning of mathematics. The students were being exposed to Dr. Taylor’s model of teaching and learning, and it was in the natural setting of teaching and learning: a classroom. Formal teaching about mathematics occurred; however, formal teaching about the teaching and learning process was not present. (This lack of formal teaching about the teaching and learning process is discussed further in the next section.)

Gee (1990) goes on to say that, “Acquisition must (at least, partially) precede learning; apprenticeship must precede ‘teaching’ (in the normal sense of the word ‘teaching’)” (p. 147). Here, Gee (1990) links acquisition to apprenticeship. This notion of apprenticeship is also discussed by Lave and Wenger (1991); however, they prefer to use the term “situated learning” (p. 31). Lave and Wenger (1991) stress the importance of situated learning as “learning by doing” (p. 31). These ideas apply to the teacher candidates in Dr. Taylor’s class in that they were encultured into the ideas of reform-style teaching and learning by experiencing it as a student. They were “learning by doing” from the perspective of students. What has not yet taken place is the “teaching” of how to become a reform-style teacher. However, it seems that the phase of enculturation into the social practices associated with reform-style teaching is a necessary first step.

The idea of needing to experience mathematics as a student in a reform-style classroom before being able to create a reform-style teaching and learning environment as a
teacher are evident in the experiences related by Schifter and Fosnot (1993). They studied practicing teachers who participated in SummerMath, a summer workshop for teachers interested in implementing reform goals in their elementary mathematics teaching. One of the key premises of the SummerMath program is that, "If teachers are expected to teach mathematics for understanding [as defined in the reform documents] they must themselves become mathematics learners" (Schifter & Fosnot, 1993, p. 16). Moreover, the Professional Teaching Standards (NCTM, 1991) calls for such experience when they state, "If teachers are to change the way they teach, they need to learn significant mathematics in situations where good teaching is modeled" (p. 191). In other words, while all teachers do not necessarily need a full college-level, reform-style course in mathematics, they do need experiences as learners (or students) in a reform-style environment before they can be expected to emulate it as teachers.

However, this initial experience as a student in a reform-style mathematics classroom is not enough for preparing pre-service teachers. In accordance with the findings of Borko, Eisenhart, and colleagues (Borko, et al., 1992; Eisenhart, et al., 1993), the teacher candidates in Dr. Taylor's class believed that further educational coursework and field experiences would be necessary before they would be prepared to "do the things that [Dr. Taylor is] doing now" (Beth, Interview, 12/8/94) in their own teaching. This finding suggests that while one content course taught from a constructivist perspective is not sufficient in preparing pre-service teachers to meet the goals for reform, it is an important step beginning the process of preparing pre-service teachers to incorporate reform-based practices into their future mathematics teaching.

What Was Not Said

Another implication for the preparation of pre-service teachers rests in what was not discussed or taught in Dr. Taylor's class. Earlier, the claim was made that formal teaching about the teaching and learning process (pedagogical issues) did not take place in Dr.
Taylor's class. In observing the classes and talking to the participants, the researcher never heard overt talk about how the teacher candidates' experiences in Dr. Taylor's class might translate to their future practice as elementary/middle school teachers unless they were specifically asked to discuss this by the researcher. It seems that discussions of pedagogical issues relevant to pre-service teachers were considered to be inappropriate discourse.

In an effort to validate this finding and to understand why issues of pedagogy were not discussed, the researcher asked Dr. Taylor and Julie (the key informant among the teacher candidates) for their views on this matter. Dr. Taylor said that he did not "recall talking explicitly about [his] teaching as a model of how one would teach middle school kids" (electronic communication, 2/9/96). However, he did address his general rationale behind approaching teaching and learning in a way that was different from what teacher candidates were used to experiencing in a mathematics class. He says, "We did fairly often talk about why the innovative features of the course were being used - my rationale for doing things in different ways (in part this was a periodic pep-talk to encourage them that things were going reasonably well, even if different)" (electronic communication, 2/9/96).

Julie's recollection about talking about pedagogical issues was similar to Dr. Taylor's in that she states that Dr. Taylor "alluded" to reasons why he was approaching topics at times, but never directly discussed how teaching and learning in his class related to their future teaching in the elementary and middle-level schools. Julie continued by saying that this type of conversation did not seem appropriate for a mathematics course since they were there to learn math. These comments from both Dr. Taylor and Julie are consistent with what the researcher observed.

However, both Dr. Taylor and Julie revealed that pedagogical issues were discussed in the MCTP Seminar Course which was taught by Dr. Taylor and an MCTP science professor. (This course is beyond the bounds of this study.) As mentioned earlier, the purpose of the Seminar Course was to make connections between the mathematics and
science courses that the MCTP teacher candidates were taking and to discuss issues related to their future teaching of these subjects. In addition to the seminar, Julie said that outside of class (in the hallway to and from class) the five MCTP teacher candidates occasionally discussed how their experiences in Dr. Taylor’s class might relate to their future teaching. Thus, pedagogical issues were appropriate for discussion outside of mathematics classes.

Next, the question to Dr. Taylor was, “What were his reasons (if any) behind not discussing pedagogical issues pertinent to future elementary/middle school teachers?” Dr. Taylor said, “In part, this was because of the low density of MCTP students [in the class]” (electronic communication, 2/9/96). (There were five MCTP teacher candidates in the class, and approximately 8 out of 20 students who intended to teach - including the MCTP students.) In pursuing whether more MCTP teacher candidates or other education students would have affected his decision to include discussions about pedagogy, Dr. Taylor stated that even if the class were entirely composed of education students, he does not believe he would have included pedagogical discussion. In fact, he preferred that the course not be offered exclusively to education majors. He wanted to concentrate on the mathematics and not turn it into a pedagogy course. Also, Dr. Taylor was sensitive to the perception that a mathematics course designed exclusively for pre-service teachers might be viewed by other mathematics department faculty as a course that was made easier even though that would not be true.

Dr. Taylor’s concern about the perception that college faculty might have (regarding content courses designed specifically for education majors as being less rigorous) appears to be supported given the recommendations by mathematics and science faculty from colleges and universities throughout the United States published in an NSF document (NSF, 1993). In this document there is concern expressed that “watered down” versions of content courses for pre-service teachers be avoided (NSF, 1993) with the implication that this watering down is a perceived risk of specialized content courses for future teachers.
The question that remains is, "Why is it significant that pedagogy was not discussed in a mathematics course?" Shulman (1986) brought the notion of pedagogical content knowledge to the forefront of teacher education. He defines pedagogical content knowledge as going "beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (Shulman, 1986, p. 9). Included in the category of pedagogical content knowledge are: "the ways for representing and formulating the subject that make it comprehensible to others, [and] . . . an understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p. 9). Shulman (1986) calls for teacher education programs which offer instruction focusing on content that includes "knowledge of the structures of one's subject, pedagogical knowledge of the general and specific topics of the domain, and specialized curricular knowledge" (p. 13). In other words, pre-service teachers need to learn about the pedagogical issues in the context of subject matter knowledge. This need is also stated in the reform documents (e.g., NCTM, 1991).

Furthermore, much has been said about the value of metacognition in learning (e.g., Flavell, 1979, 1981; Schoenfeld, 1992). Flavell (1981) defined metacognition as "knowledge or cognition that takes as its object or regulates any aspect of any cognitive endeavor" (p. 37). There seems to be a metacognitive component to the notion of pedagogical content knowledge as it relates to learning in Dr. Taylor's class. Referring back to Flavell's (1981) definition, the object of the learning is the mathematics content; however, for the teacher candidates, an important metacognitive aspect of learning is relating the mathematical content to ideas regarding their future teaching of mathematics. While this metacognitive aspect of connecting the teacher candidates' experiences learning mathematics with pedagogical issues related to their future teaching of mathematics did not take place in Dr. Taylor's class, it did seem to occur outside of the class in the seminar course. (It should be noted that while metacognition in relationship to pedagogical issues was not a part of the class, Dr. Taylor did incorporate metacognition in the students' reflection on their own
mathematical learning and problem solving. He states, "Students really are asked, and encouraged, to think a lot more about their own thinking" (Interview, 12/6/94). Regardless of whether the metacognitive learning that facilitates the development of pedagogical content knowledge occurs within or outside of the mathematics classroom, this learning is important for the development of future teachers.

The need for pedagogical content knowledge has implications for classes like Dr. Taylor's. However, a paradox exists concerning what is needed for the preparation of pre-service teachers in regard to pedagogical content knowledge and what content professors like Dr. Taylor are willing to include (or not include) as a part of their courses. Dr. Taylor seems to have sound reasons in his context for focusing on content at the near exclusion of pedagogical discussions, and many other mathematics and mathematics education faculty probably agree with his reasons. However, does this mean that pedagogical discussions must be delayed until pre-professional education courses? It seems that to delay would be missing a significant opportunity for the development of pedagogical content knowledge. So, how is this paradox resolved? If professors are unwilling or unable to include pedagogical discussions in mathematics content courses, then perhaps providing opportunities such as the MCTP seminar is important complementary environment for pre-service teachers. In other words, if conversations which promote reflecting on and making connections between the pre-service teachers' learning experiences in a mathematics course and their future teaching are not taking place in mathematics classrooms, then teacher education programs should consider initiating forums where this type of conversation can concurrently take place to foster pedagogical content knowledge. One additional note: In the case of Dr. Taylor, he was in the position of teaching both the content course and the seminar course that dealt with pedagogical issues. In situations where one person is not able to serve in both roles, further efforts may need to be made to bridge the content course and the pedagogical discussions and to emphasize the notion that neither area is valued more.
Reactions of Key Informants

In an effort to validate these findings and implications, member checking (Stake, 1995) was used with two key informants, Dr. Taylor and Julie. Dr. Taylor and Julie were provided with a draft of this manuscript and asked to react to the interpretations of the researchers. Dr. Taylor indicated that the only thing that the paper did not capture was his feelings of the difficulty and the struggles involved with instructional decision making in this type of course. However, these struggles were not apparent in either the his interviews or in the teacher candidates’ perceptions. Perhaps this suggests that creating this kind of teaching and learning environment is far more complex than it may seem as Simon (1995) has indicated.

Julie said that she agreed with the interpretations and added, “I found it fascinating how we (students and professor) were so much on the same wave length” (Written communication, 3/27/96). Also, she wanted to be sure it was understood that she believed that “the lack of addressing [pedagogical issues] was not necessarily inappropriate because we were in a math class” (Written communication, 3/27/96). This statement confirms earlier findings that both the teacher candidates and Dr. Taylor do not see the inclusion of pedagogy as important in a content course, and again, this indicates that other venues for the discussion of the connections between pedagogy and content are necessary.

Remaining Questions

Some of the many research questions that remain are: How will these pre-service teachers continue to develop and learn about reform-style teaching? Will experiences such as what Dr. Taylor’s students’ had combined with further educational coursework and field experiences enable these pre-service teachers to meet the goals for reform in their teaching? What components of the MCTP program (such as the Seminar course or field experiences) are most significant in ensuring the pre-service teachers development and what implications does this have for other programs? Furthermore, how many and what types of content and education courses are necessary? As we continue to follow MCTP teacher candidates
throughout their undergraduate preparation for teaching and in their first years of teaching, we hope to gain a better understanding of answers to these questions.
References


Appendix

Student Interview Protocols

Interview #1

1. What does it take for a student to be successful in mathematics?
2. What do you expect of a good math teacher?
3. What does it take for a student to be successful in science?
4. What do you expect of a good science teacher?
5. Can a student do well in both mathematics and science?

Interview #2

1. Has the instruction in [Dr. Taylor's] class helped you make connections between mathematics and science?
2. To what extent has this class involved the application of technologies (e-mail, cd's, computers, calculators, etc.)?
3. Has the instructor made significant attempts to understand your understanding of a topic before instruction? Did the tests reflect this emphasis?
4. To what extent has this course stressed reasoning, logic, and understanding over memorization of facts and procedures?
5. Do you think the teaching you experienced in this course models the type of teaching that you believe should be done in grades 4 - 8? How? Why?
6. Did your instructor explicitly encourage you to reflect on what you learned in this class?
7. After participating in this content class, what are your expectations regarding your mathematics and science methods classes? How should they each be taught? What should be in the curriculum?

Faculty Interview Protocol

(Used for both interviews - with verb tense changed for second interview.)

1. To what extent is the instruction in this class planned to highlight connections between mathematics and the sciences?
2. To what extent will this class involve the application of technologies (e-mail, cd's, computers, calculators, etc.)?
3. To what extent will you make significant attempts to access you students' prior knowledge of a topic before instruction? What techniques will you use?
4. To what extent do the tests and exams of this course stress reasoning, logic and understanding over memorization of facts and procedures? Would you provide copies of these materials?

5. In what ways do you think your teaching in this course models the type of teaching that you believe should be done in grades 4 - 8?

6. To what extent will you explicitly encourage your students to reflect on changes in their ideas about topics in your course? Can you give an example? What techniques do you anticipate using?