Students may be able to perform strictly manipulative mathematics and not be able to solve word problems or understand more complex mathematical concepts due to weak reading skills. This paper discusses the interrelationships between mathematics and language. First, highlights from research are given. Following is a discussion of characteristics of urban college students. Finally, suggestions to minimize the effects that reading has on students' mathematics performance are given, including text selection, textbook use, lectures, and assignments. The appendix contains a homework assignment to help students become more familiar with their textbook. (MKR)
Are You Assessing Language or Mathematics?

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Introduction

In mathematics and statistics classes, the interrelationships between mathematics and language are of increasing importance. There are several reasons. The number of students from other countries attending college is growing, at least in the metropolitan areas. For example, in the statistics classes at Hunter College, approximately 46% of the students speak a language other than English at home. In addition, even the students who speak only English often have difficulties reading and comprehending technical materials. Students may be able to perform strictly manipulative mathematics, but may not be able to solve word problems or understand more complex mathematical concepts due to weak reading skills. Furthermore, this problem is not necessarily confined to students who are clearly reading-deficient, as defined by the CUNY Assessment Tests. This problem may also exist for students who have achieved an "acceptable" level of reading expertise. After all, there are varying levels of complexity in English.
Some Important Ideas from Research on Reading and Mathematics

The interrelationships between mathematics and reading have been identified in research efforts aimed at pre-college students. Some highlights of the research follow:

- The students' reading ability and computation proficiency are factors important to success in solving word problems (Balow 1964; Barney 1972; Cohen and Stover 1981; Glennon and Callahan 1968; West 1977). Also, when students are taught to read mathematics, their problem solving performance improves (Aiken 1972).
- Even when students improve their vocabulary in mathematics their scores are raised on verbal problem solving tasks (Henney 1971; VanderLinde 1974).

How the mathematics is written also affects the students' mathematics performance. In word problems, both mathematical vocabulary—words like mean, sum, and equals—and non-mathematical vocabulary can influence problem difficulty. The number of words in the problem, as well as the length of individual sentences, affect the difficulty level of problems (Jerman 1973; Jerman and Rees 1972; Jerman and Rees 1974). Difficulty is also affected by grammar. Placing the question first appears to focus the student on what is desired and appears to make problems easier (Williams and McCright 1965). The position of certain punctuation can also influence problem difficulty (Carpenter, Moser, and Romberg 1982; Nesher 1982; Riley, Greeno, and Heller 1983). It is also true that the (measured) "readability level" of the material appears to influence problem difficulty (Thompson 1967; Linville 1969). This readability level is influenced by the number of passive sentences, as well as the number of conditional sentences and the lengths of sentences.
Characteristics of Urban College Students

Urban college students are older than the elementary school students involved in much of the cited problem solving research; however, they do have many of the same learning difficulties. The error patterns in computational tasks that a student exhibits as a child will persist into adulthood without specific intervention to change it (Ashlock, 1976, Clarkson 1981). Many students come to college with learning disabilities, reading deficiencies, and an inability to do long division or to solve simple algebraic equations. It seems likely that the elementary school students’ reading problems are also present for adult students.

Studies conducted at Hunter College to determine whether students' reading proficiency level affects their performance on mathematics "word" problems on exams have given some insight into the reading problems of these college level students. Based on these studies, the conclusions are as follows:

1. Reading ability is a separate, quantifiable factor which impacts the performance of all students on mathematics word problems.
2. Less complex writing leads to better results on word problems for all students.
3. Less complex writing leads to even more improvement in test results for “weaker” readers [those needing reading remediation] than for “average” readers [those exempting reading remediation]. The effects are non-linear (Clarkson and Williams, 1994).

Impact of Research on Learning Materials for College Students

Existing research has had little impact on text materials. After seeing how reading affects students’ performance in statistics, we conducted a review of available texts. Many texts use complex sentence structures and conditional phrases and passive voice.
Analysis of text writing, using a common grammar checker, yielded text passages that measured at the Grade 22 level in reading difficulty!!

Minor changes teachers can make in their teaching to insure maximum success

To minimize the effects that reading has on a student's mathematics performance, one must adjust the writing style--or provide instructor guidance that minimizes its effect--throughout the course. For surely, the assessment of a student's mathematics performance includes the assessment of how that student has learned--using the text, attending lectures, doing the assignments and taking the exams. The teacher is in a unique position to influence the students' learning in all these areas. Following is a list of suggestions gathered over three decades of teaching.

1. **Text Selection:**

   Before selecting a text, one should read the Table of Contents and determine if the content is appropriate and sufficient for the course. Are topics presented in the desired order? The order of presentation unlikely to be random: so do not assume that the order of presentation can be easily altered. The presentation that an author uses for a particular topic determines the prerequisite topics. If the text and syllabus differ, consider rethinking the syllabus or the textbook.

   Students need user-friendly textbooks. Before selecting a text, one should read selected portions to determine whether the chapter material is easy to read and whether examples and mathematical procedures are clearly delineated and easy to follow. The teacher must take responsibility for choosing a text that is well-written and on a level appropriate for the students. Otherwise, students will miss out on a valuable learning resource. And, require that students buy a text only if they will be expected to use it.

   Before selecting a text, one should determine whether the variety and difficulty level of the problems is suitable for the students. Are there enough easy problems so that
borderline students can use these to work their way into the more difficult problems? Are some answers given in the back of the book? Are the problems interesting and relevant to the students?

II. Textbook Use:

When the textbook is first introduced, offer some incentive for the students to become familiar with the text. An introductory exercise has been included in the appendix to guide instructors in making up one of their own. How a teacher presents the text influences how a student accepts it. One should not make negative comments about the text to students. If the text is not appropriate, it is the responsibility of the teacher to keep that text out of the classroom. After a text is selected, it is the teacher’s responsibility to introduce it in such a way to make it a meaningful part of the students’ learning experience.

Many students must be taught to use the text. A teacher can facilitate this learning by doing some of the following. Consider giving open book texts. This is a guaranteed (almost) way to get students to use the book. It also allows the teacher to test a student’s ability to apply the mathematics and not simply to test the student’s memory. If one refers to the textbook when lecturing, giving the page number that examples appear on, students will be able to refer to the book when reviewing their notes. Students who learn to use the text well are on their way to becoming more independent learners.

III. Lectures:

A well-organized lecture is crucial for student learning. Students who are weak readers or with little facility with English can benefit tremendously by a clear, logical presentation. And all teachers need to organize. “Off the cuff” comments may be entertaining but students need a structured lesson that highlights the important aspects of the topic. When a problem is presented, the teacher should work it out, from beginning to
end, in the format desired from the student. Then, the teacher should return to this solved problem to highlight possible misunderstandings, explain mathematical procedures and emphasize the steps necessary in the solution.

A teacher should give students structure. To help students identify the important ideas, one should highlight definitions and "rules" on overheads, on handouts and in writing assignments. The teacher should use lectures to teach students how to take notes, solve problems and present those solutions on an exam.

The teacher should choose the words used in a lecture very carefully. Mathematical procedures should be referred to as they are in the text and in other courses. Where it may be cute and entertaining to call something the Tuesday Rule or Clarkson's Clue, the teacher should resist that. A student who lacks sophistication in a subject needs help in learning the vocabulary and pronunciation and essential structure. Many of these students will take additional mathematics courses and must be prepared. The material and the students should be treated with respect.

The teacher should teach without using idioms and colloquial expressions. Many students are very capable in proper English but have much less facility with slang and jargon. Language can be used to deny equal learning access to many students. A teacher who is using language "new" to the class should resay things in a different way to make sure that students understand the meaning. If a definition or procedure is new to the students, and particularly if it requires mathematical computation that may be difficult, one should consider giving an additional example or restating a definition in simpler words.

IV. Assignments:

Before assigning homework, the teacher should read the assignment. If the procedures required are new to the students, the teacher should make sure the directions are clear. Sometimes it is helpful to work one of the problems, or assign it as classwork to
make sure that it is clear. The teacher should tell students whether the answers are in the back of the book and explain how to use those answers. One should also define words and terms that are unclear. The teacher should also provide students with a procedure for evaluation.

It is often helpful to assign short writing assignments requiring students to give answers in their own words to questions that are related to statistics but not necessarily related to solving statistics problems. Some suggestions appear in the Appendix.

**Minor changes teachers can make in testing to insure maximum success**

One of the most important uses of language is in the assessment process. Students often know how to do procedures, but cannot determine when to do the procedure because of how a question is posed. Teachers are often at fault when a student fails to use the correct procedures. The way one words a task may affect the results. In fact, many college, pre-calculus mathematics courses test much more than mathematics alone. Solving "word" problems in the language of mathematics first requires a clear understanding of the language (English) in which the problems are written, followed by the related, but different, ability to translate the verbal language into symbolic mathematical expressions. Only then does the specific ability to manipulate the symbolic "words" of mathematics come into play. The overall result is that if a "word" problem is done incorrectly, there is no unambiguous way to conclude that the fault lies in any well-defined area of mathematics rather than in reading proficiency.

Before giving a test, the teacher should preview test items and work out the problems. Words can often be the "problem" in solving problems. In rereading old exams, one may be surprised to find that many of the questions that seemed so clear last year may be a bit confusing now. One often reads meaning into something that, by itself and out of context, has no clear meaning. Teachers should be encouraged to write their
exams ahead of time, put them aside for a few days, and then reread them to see if the questions are clear and are testing the desired skill.

Additionally, the teacher should discard culturally dependent items. Using baseball as an example may be reasonable for native-born students, but for immigrants, the choice may result in making a mathematics problem impossible. Other situations to look out for: cards, dice or any items depending on previous knowledge not universally available. The teacher should think of what information a student has to bring to the class in order to solve the original problem. When one assumes that students know about cards or baseball teams or other "culturally specific topics," one automatically exclude some students from learning. And, unless one has unusual rapport with the students (or unusual students), one may not know the results of including such items until after the exam or the assignment.

Additional elements of a problem may seriously affect its difficulty level. Test items should be written eliminating "if-then" constructions. One should also shorten sentences and eliminate passive voice. Put the question first if possible.

EXAMPLE:

Original Problem--The weights of several children are given as 40 pounds, 41.5 pounds, 56 pounds, 33.4 pounds, 42.1 pounds. If each of the measures is increased by 2.3 pounds, by how much does the mean increase?

Possible Rewrite--We weigh five children. The weights are 40 pounds, 41.5 pounds, 56 pounds, 33.4 pounds, 42.1 pounds. Each child gains exactly 2.3 pounds. What is the mean of the five new weights? How much greater is the new mean than the original mean?
The rewritten version of the problem is more specific about the desired outcome. The original problem most often yields the new mean as the final answer. The revised problem indicates that there are two steps required. If a student misses the original problem, one will not know whether he knows how to solve a two-step problem or whether he was confused by the problem statement. The second problem will sort out the errors.

Conclusions

Teachers can make a huge difference in how students learn. There is information available from research that is very important. However, a teacher cannot expect that the results of the research has impacted commercially available materials. It is still the responsibility of the teacher to select a student-friendly text with appropriate material, sufficient problems and with a reading level appropriate to the students’ ability levels.

Teachers should organize their order of presentation, structure their lessons and choose appropriate material from the text to maximize the students’ ability to learn. One of the most crucial areas of teaching is assessment. The wording of test items is extremely important. The teacher must pay attention to the meaning of the problem, the clarity of the question, the words used in the problem, the sentence structure and the sentence length. The students of today come from more varied backgrounds than ever before. To maximize their chances of learning, we as teachers should be very careful to make sure that we are assessing mathematics and not reading.

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STATISTICS 113: HOMEWORK ASSIGNMENT [This assignment is given so that students will become familiar with their textbook]

First read Chapter 1. Next complete each of the following questions by referring to Chapter 1, the Table of Contents, Index and indicated pages. This assignment is due on September 7 and will not be accepted later than that.

1. "The word statistics has two meanings." (Section 1.1) What are they?

2. Much of statistics is based on the distinction between populations and samples. Describe what is meant by a sample? How do you distinguish between a population and a sample? Why don't we always use populations when we are gathering statistics?

3. Discuss the differences between the following terms in your own words:
   - descriptive statistics and inferential statistics
   - simple random sample and representative sample
   - sampling with replacement and sampling without replacement
   - element or member of a sample
   - observation or measurement
   - quantitative variable and qualitative variable
discrete variable and continuous variable.

\[ \sum (x^2) \text{ and } (\sum x)^2 \]

4. Give the page numbers for the following references. Use the Index.
   - Paired differences
   - Rectangular histogram
   - Rejection region

5. What is the title of Chapter 7? What page is Section 7.2 on?

6. Where would I find problems 5.6? 7.18? and 9.40?

7. What answers are located in the text? Where are they?

8. Who is the author of your text? Where does he teach?

9. What are our names? Where are our offices? When are the office hours for the course?
Sample Writing Assignments [These assignments are given so that students will become familiar with writing about statistics in their own words.]

⇒ In your own words, 25 or less, define "variability" so that a group member missing class can use this definition to understand the concept.

⇒ Give three uses for the INDEX in your text.

⇒ What kind of students do you think your text author is writing for?

⇒ Take the third problem you did for homework and write an explanation of that problem to a group member so they can do the problem without seeing your work.

⇒ How do you intend to use statistics in your field of study? If you do not know, tell how you wish you could use it. If you do not intend to use it, tell why you think your field cannot benefit from it. Or, write your own question about using statistics and answer it. You may use one page to answer this question, no more.

⇒ How did you use arithmetic last week? List as many ways as you can.

⇒ Write three questions that you had as you listened to the lecture today.