The issue of how a child constructs higher order, logical mathematical intelligence within the context of a multiple-intelligences classroom is explored in this story. Teacher journal observations of one 5th-grade child are woven with selected literature on multiple intelligences and research trends in elementary mathematics education. The story presents an intuitive set of conclusions about how children construct higher order, logical mathematical intelligence. Contains 23 references. (Author/MKR)
Wanderer, the road is your footsteps, nothing else; wanderer, there is no pain, you lay down a path in walking. In walking you lay down a path and when turning around you see the road you'll never step on again. Wanderer, path there is none, only tracks on ocean foam.

Antonio Machado
(translated from Spanish by Francisco Varela)

FOOTSTEPS

(A story of one child's construction of higher order, logical mathematical intelligence)

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Abstract

How does a child construct higher order, logical mathematical intelligence within the context of a multiple intelligences classroom? That issue is explored in this story. Teacher journal observations of one 5th grade child are woven with selected literature on multiple intelligences and research trends in elementary mathematics education. The story presents an intuitive set of conclusions about how children construct higher order, logical mathematical intelligence.

major descriptors: multiple intelligences, logical mathematical intelligence, Howard Gardner

minor descriptors: teacher research, elementary school mathematics education, constructivism
Introduction

Forrest Gump's IQ was 85. Yet he became a star football, player, war hero, faithful husband, millionaire, and finally, a model single parent. If Gump had followed the script, then his schooling would have prepared him to run errands for the corner grocer. He would have been a lifelong errand boy.

A fictional character from a best picture of the year film? Yes. But seen as a metaphor, Gump stands for the millions of children that schools have tracked by IQ scores, "the ability to answer items on tests of intelligence." This operational definition of intelligence, according to Howard Gardner, had often carved a child's school script in stone. (Gardner, 1983, 1993)

83 years into the 20th century, Howard Gardner's Frames of Mind was born. His book debunked the old definition of intelligence, replacing the outdated idea the way Stephen Gould's biological possibilities had replaced biological determinism. Ten years later, Gardner's multiple Intelligences (MI) theory had contrasted Benet's "...ability to answer items on tests of intelligence " with the following definition: "an intelligence entails the ability to solve problems or fashion products that are of consequence in a particular cultural setting or community." (Gardner, 1983; 1993)

The old view claimed that intelligence was singular and fixed. Gump's IQ was 85. It would never be higher. But Gardner says people have at least seven intelligences (verbal linguistic, logical mathematical, musical, visual spatial, bodily kinesthetic, interpersonal, and intrapersonal) that can be improved with teaching, coaching and experience over a lifespan. (Gardner, 1993)

Gump may not have scored well on a measurement of intelligence, but he is relatively gifted in bodily kinesthetic, interpersonal, and intrapersonal intelligences. Likewise, Ayanna, the 5th grader I have been observing, has an IQ that is greater than 130, but she may be relatively gifted in six of the seven intelligences. In neither case does the IQ test tell the story.
The old fashioned, single definition that squeezes intelligence into a single score misses all the other ways people can be smart. It also fails to account for the effect good teaching and parenting has on a child. As Vygotsky had discovered, skilled adults can raise the level of a child's performance. This idea can be generalized to say that each of the seven intelligences can be improved with teaching, coaching, and experiences. That is one of Gardner's key arguments: intelligences can grow. (Gardner, 1993; 1994)

By implication, this generalization means restructuring the living creature we call schooling to create a system based on the idea that every child (every Forrest Gump and Ayanna Smith) can achieve—i.e. improve his or her intelligences. That is the heart of Superintendent Hornbeck's plan for the Philadelphia Public School System.

Says Gardner "...the purpose of school should be to develop intelligences and to help people reach vocational and avocational goals that are appropriate to their particular spectrum of intelligences."

"People who are helped to do so, I believe, feel more engaged and competent, and therefore more inclined to serve the society in a constructive way," he adds.

To achieve this purpose, a school system would design activities and assessments that facilitated the growth of each of the seven intelligences for each child.

"Footsteps," is a story that explores a hologram, a piece, of the implications MI theory has for schooling. This narrative describes how Gardner's purpose of education and new definition of intelligence translates into a 5th grade, urban classroom at a School Wide Project setting with no prior commitment to the development of multiple intelligences. Then it shows how multiple intelligences (MI) theory provides a context for describing one 5th grade child's construction of higher order, logical mathematical intelligence. Next, it tells Ayanna's story, highlighting her core capacities in logical mathematical intelligence. Afterwards, it provides a context for teaching her to construct higher order, mathematical intelligence by describing the constructivism trend in elementary mathematics education. Finally, the narrative offers a set of intuitive ideas about how a child constructs higher order, logical mathematical intelligence.

...
We join spokes together in a wheel, but it is the center hole that makes the wagon move.

We shape clay into a pot, but it is the emptiness inside that holds whatever we want.

We hammer wood for a house, but it is the inner space that makes it livable.

We work with being, but non-being is what we use.

passage #11
Tao Te Ching
(Stephen Mitchell’s translation)

Conception of Ayanna’s MI-theory classroom at Pennell

When the final decade of the 20th century began, Joseph Pennell Academics Plus Elementary School had become a tired, gray building on a hill in an aging, urban neighborhood. However, blessed with a cadre of veteran teachers endowed with youthful energy, a principal with a drive for excellence, and 11 new kids on the block (a group of teachers who transferred in the Fall of 91 to usher its inaugural year as a School Wide Project), Pennell became peppy and green. By 1994, Pennell had been recognized as a top School Wide Project.

Joseph Pennell's evolution is an enactment of Gardner's definition of intelligence. On the one hand, Carolyn Garvin, Ruth Anderson, Cathy Hebert, and Karen Zucker, the first core members of the leadership team who wrote and implemented the plan between 1991-1993, had sought to solve the problem of low student achievement.
Specifically, they hoped to raise achievement levels of the number of Chapter One students. Based on City Wide Test scores, number of students achieving a year of growth in reading, reduction of D and F grades and improved attendance, they succeeded. By its third year, Pennell's School Wide Project had been renewed for another three year funding period. Around the Philadelphia public school circuit, Pennell's reputation for excellence was growing.

On the other hand, Pennell's core leadership team had fashioned a product, namely the School Wide Project Plan. This plan defined activities and achievement over a three year span. However, a set of problems had been introduced by the very plan aimed at solutions. These problems included lack of a clear theoretical base such as whole language, critical thinking, instrumental enrichment, cognitive apprenticeship, Neo-Piagetian theory, or multiple intelligences theory. A multiple intelligences (MI) theory framework, for instance, would have enabled teachers to select the Houghton Mifflin Reading, Math, and Social Studies activities that developed one or more of Gardner's seven intelligences. MI-theory might have encouraged an individual centered learning approach complete with performance based assessments. (Gardner, 1993)

Lack of a coherent, theoretical base presented contradictions. For example, the Pennell staff replaced the Philadelphia Mathematics Evaluation Test (PMET) with Houghton Mifflin Cumulative Reviews (CR) While the CR assessments are based on sequential instruction and, therefore, are better than the non sequential PMETs, they are still decontextualized, multiple choice presentations of calculation problems--void of any real measurement of higher order, logical mathematical understanding.

The cumulative reviews are several steps down from NCTM standards, and the pacing schedule that drives teachers to finish enough chapters to take these multiple choice assessments encourages superficial coverage rather than in depth understanding of mathematics. Yet the Houghton Mifflin program offers Problem Solving Reviews, Cooperative Problem Solving Tasks, critical thinking activities that do, in fact, offer contextualized assessments of mathematical understanding as defined in the 13 points making up the NCTM standard.
While the program may fall short of the eleven of the problem solving strategies that Reys, Suydam, and Lindquist describe in their text, *Helping Children Learn Mathematics*, it does point out areas that may be supplemented with a literature and mathematics connection, NCTM addenda, and critical thinking in mathematics programs such as Anita Harnadek's. (Reys, Suydam, and Lindquist, 1992)

The multiple choice, cumulative reviews measure, primarily, the "computational skills Arithmetic--addition, subtraction, multiplication, and division with whole numbers, fractions and decimals." But as Reys, Suydam, and Lindquist explain so much more is involved in teaching mathematics. (Reys, Suydam, and Lindquist, 1992)

They say the following.

Mathematics is

- a study of patterns and relationships
- a way of thinking
- an art, characterized by order and internal consistency
- a language, using carefully defined terms and symbols
- a tool

(Reys, Suydam, and Lindquist, 1992)

The Pennell's staff's commitment to cumulative reviews and a rigid pacing schedule has doomed it to stand on the edge of the diving board of national mathematical excellence instead of taking the plunge. If the staff had used MI-theory to select assessments, they might have selected the richer Houghton Mifflin assessments, depth of coverage, restructured time, math portfolios or processfolios, math/science journals, NCTM standards, and a range of activities using each of the multiple intelligences in the mathematics domain in order to reach across all learning styles to develop depth of mathematical understanding from K-5.

In addition, even a small push (what a Tai Chi master calls "4 ounces of force") in the direction of emphasizing student construction of higher order, logical mathematical intelligence might have touched recent thinking about what it means to learn mathematics.

According to the *Handbook of research on mathematics teaching and learning*, the "shift in learning psychology trend" from cognitive science describes how children learn mathematics. It gives a picture of understanding not painted in quick coverage and decontextualized tests.
In summary, cognitive psychologists have provided the concept of ‘well-organized schemata’ to explain how people impose order on experiential information. Assimilation, accommodation, and mode of functioning in response to new information are important in the enterprise of schooling. Without schemata into which new information can be assimilated, experience is incomprehensible, and little can therefore be learned from it. But the scheme by which a student assimilates a lesson may not be that assumed by teachers or mathematicians. This mismatch can easily escape detection because the student will often be able to repeat segments of the text and lecture even though he or she understands them in terms of an incorrect, incomplete, or inconsistent framework. Indeed, students may develop specialized frameworks for maintaining the particular identity of lesson material in order to cope with the demand for veridical reproduction. Scheme use must be a dynamic, constructive process, for people do not have a schema stored to fit every conceivable situation. In this view, acquisition of knowledge implies changes in schemata, not just the aggregation of information. (my italics) (NCTM Handbook, 1992)

Far south of the border this trend has been described in the classic volume of Paulo Freire. The old practice saw teachers as depositing information in the minds of students. Freire called this the banking model. In contrast, he says “knowledge emerges only through invention and re-invention, through the restless, impatient, continuing, hopeful inquiry men pursue in the world, with the world, and with each other.” (Freire, 1990)

Unfortunately, Pennell’s superficial assessment of mathematics in multiple choice tests and its rigid pacing schedule adhere more to the banking model and less to the problem posing model. Also, these practices miss the mark set by the shift in research on teaching and learning: the practices ignore the need to have students construct mathematical schemata one well planned year after another.
Hope has been kept alive, though. Thanks to the efforts of Melodie Hayes, Pennell's new Principal, the core leadership team, and several teachers, the faculty has been learning about learning styles during in-depth staff development sessions with Barbara Moore Williams, a top school district staff developer. A foundation for using multiple intelligences in the mathematics program as well as across the curriculum has been laid. In addition, local initiatives such as Superintendent Hornbeck's systems plan for student achievement and national reform mandates such as Goals 2000 demand that teachers at Pennell take math and science learning more seriously.

But the most significant recent development has been the staff's March 1995 revision of its School Wide Project Plan. Using a "collapsing consensus design," Melodie Hayes facilitated a workshop that involved the entire instructional staff in a revision of Pennell's plan as outlined in Superintendent Hornbeck's ten point framework for systemic change.

The mission statement that emerged from that session put critical thinking and problem solving in the forefront of the school's vision. It gave each teacher permission to use the numerous critical thinking and problem solving activities offered in the three Houghton Mifflin programs (Mathematics, Reading, and Science) at Pennell. It gave permission to use the National Intellectual Standard to assess intellectual products. It gave permission to use methods such as the National Council for Excellence in Critical Thinking Instruction (NCECT) framework, David Perkins' knowledge as design for critical and creative thinking or the Sandra Parks-Black and Robert Schwartz approach to infusing critical thinking across the curriculum. It gave permission to use content area methods such as Anita Hardanek's critical thinking in Mathematics and Eggin and Main's critical thinking in Science. Finally, it gave all the stakeholders at Pennell permission to become a community of strong sense critical thinkers. (Paul, 1990; Fluellen, 1993)
Note. Richard Paul, NECT president and inventor of the construct, says the strong sense critical thinker can be defined as this:

One who is predominantly characterized by the following traits:
1) an ability to question deeply one's own framework of thought;
2) an ability to reconstruct sympathetically and imaginatively the strongest versions of points of view and frameworks of thought opposed to one's own; and
3) an ability to reason dialectically (multilogically) in such a way as to determine when one's own point of view is at its weakest and when an opposing point of view is at its strongest. (Paul, 1990)

Staff has answered a key strategic planning question: What business are you in? But a companion question is What business should you be in? The business Pennell should be in is intelligence, namely, Gardner's theory of multiple intelligences. This business involves development of the whole person and encompasses critical and creative thinking as well. So even as the laser printer ink dries on the final copy of the 1995-1996 School Improvement Plan, the strategic business of Pennell may not be the best it could be.

Yet this plan does place the school squarely in the critical thinking movement sweeping North America, if not the world. For the first time in the 1990s, the plan driving activities and assessments at the school might be rooted in a clear theoretical base. Also, the critical thinking movement and the multiple intelligences movement seem to sprout from the same rich soil: constructivism.

In summary, because Gardner's framework includes critical thinking, creative thinking, and problem solving across the curriculum, a commitment to developing multiple intelligences would have gone beyond Pennell's new mission statement. Similar to New City School in St. Louis, MI-theory might have required a new set of goals and assessments in the two main areas highlighted in Pennell's School Wide Project Plan, namely mathematics and reading. From the point of view of MI-theory, this focus would have become higher order, logical mathematical and verbal linguistic intelligences.
Problems aside, the first core leadership team's pioneering effort to solve a complex problem and fashion an intellectual product of value in one or more cultural settings may have prepared Joseph Pennell to rank 53rd among several hundred public schools in Philadelphia according to a *Philadelphia Inquirer* article.

The first core leadership team also provided a context for creating multiple intelligences classrooms at Pennell before the turn of the millennium, and they created a climate for strong sense critical thinkers to grow and teacher research projects such as this one to flourish. They are to be congratulated for a job well done. If Pennell ultimately becomes a smart school, in the David Perkins sense, it will be because of the team's pioneering efforts.

Add a literature base to this story.

Multiple intelligences literature ranges from Project Zero research reports at Harvard University to an expanding data base reported in ERIC ≥1983. Though not limited to Gardner's multiple intelligences view, the base includes the increasing number of how to books for his theory. David Lazear's four volumes and Thomas Armstrong's recent book represent this genre.

Taken as a whole, the literature explores the theory and practice of Gardner's multiple intelligences from 1983-1995 but provides little hard data to support the MI-theory framework. The 32 works listed in the ERIC data base, for example, seem to be primarily descriptive. However, the theory is just ten years old and several studies are in progress across the nation. Eventually, a balance between descriptive studies and empirical studies, university studies and teacher research projects such as this one will be achieved.

The youth of MI theory does not subtract from its ability to connect subject matter, method, teaching, learning, and assessing--Silva's variables of schooling. Nor does it discount still unanswered questions such as the one guiding this teacher research project: **How does a 5th grade child construct higher order, logical mathematical intelligence within the context of a multiple intelligences classroom?** (Silva, 1994; Fluellen, 1994)

...
True perfection seems imperfect,
yet it is perfectly itself.
True fullness seems empty,
yet it is fully present.

True straightness seems crooked.
True wisdom seems foolish.
True art seems artless.

The Master allows things to happen.
She shapes events as they come.
She steps out of the way
and lets the Tao speak for itself.

passage #45
Tao Te Ching
(Stephen Mitchell's translation)

Story Two: Ayanna's growth

Ayanna transferred into my class on 6 October 1994. That was two
months after I had become part of Susan Lytle's teacher research seminar
at University of Pennsylvania. On fellowship in the Philadelphia Writing
Project, I designed an inquiry around one child's logical mathematical
intelligence during the Summer Institute I with two facilitators (Michele
Sims and Margo Ackerman). I knew one child would be the target
population, but I didn't know whom.

Ayanna had been in Ms. Brooks' room before the school reorganized.
When her 5th grade class transferred in and my 4th graders transferred
out, I was without the child previously selected for the inquiry. The first
thing I noticed about Ayanna was her ability to concentrate during lessons
on a range of subjects. My intuition said she might be worth studying
rather than choosing another child by random selection.
During the first report card period, I made several journal entries about her multiple intelligences. This period was also the first few months of developing the MI-theory classroom that would become a context for describing Ayanna’s construction of logical mathematical intelligence. I was exploring both a theoretical framework for teaching and learning and constructing a teacher research project. Ayanna was constructing logical mathematical intelligence while learning about all seven of her God given intelligences. We were both true to the African proverb “one who learns, teaches” and its corollary one who teaches, learns.

10-21-94

Ayanna seems to have a number of well developed intelligences.

Logical mathematical intelligence

She scored 100 on the first Cumulative Review Math examination. This multiple choice test measured properties of addition, and addition and subtraction computation. More importantly, her oral recitations express clear, accurate understanding of mathematical ideas. Her metacomprehension is high; i.e., she understands when she understands a mathematical idea such as zero property of addition; she understands when she does not understand such as the differences between associative and cumulative properties of addition. She is able to use the problem, plan, solve, check model to work through math story problems. And in Science, she is able to write an essay that clearly and accurately compares definitions of observation, inference and opinion—three science process skills. Also, she can classify a 20 item list of mammals according to clearly demarked attributes.

Verbal-linguistic intelligence

Ayanna’s reading level is 5/2, one half year above grade level. During the first report period, she wrote an original science fiction short story for a city wide contest and wrote an essay about Frederick Douglass. The essay developed a main idea with relevant details. She gave the second best dramatic recitation of William Henly’s “Invictus.” Her recitation had expression and showed an understanding of the shifts in meaning and feeling throughout the poem. She used KWL accurately to study a chapter about slavery in the Houghton Mifflin Social Studies text and delivered a concise oral report on “The Plantation Society.”
Musical Intelligence
I have not observed her use of musical intelligence as yet.

Visual spatial Intelligence
I have not observed her use of visual spatial intelligence as yet.

Bodily-kinesthetic Intelligence
Her performance of the Tai Chi exercises has been noteworthy because Tai Chi addresses all the core abilities of bodily kinesthetic intelligence. She does the pre patterned movements in the strength, balance, and flexibility exercises well. She should be able to learn the form.

Interpersonal Intelligence
She seems to be a natural leader and gets along well with most of her fellow students. She organizes other students in her cooperative learning group. However, of the dozen or so names nominated for student of the month during the class meeting, she was not on the list. The qualifications for nomination were excellent work and acceptable behavior. In my view she qualified. In her fellow students' views she did not.

Intrapersonal Intelligence
Ayanna may be highest in intrapersonal intelligence though this is not as easy to observe as the other six intelligences. David Lazear, facilitator of the Phi Delta Kappa Seminar on teaching for seven ways of knowing, said use of metacognitive strategies is part of intrapersonal intelligence. In that case Ayanna learned David Perkins' knowledge as design method for critical thinking. She accurately used it to make a cognitive map of the math concept addition. Her math journal reveals an understanding of the purpose, structure, model cases, and arguments of addition.

Also, she made clear, relevant responses using de Bono's six thinking hats to understand the poem "Invictus."

In addition, when given the choice of working in teams or working alone, she has chosen to work alone.

She seems able to concentrate throughout a range of activities.
Ayanna has been in the class now about two weeks. Mr. Manning, her 4th grade teacher said she was his only straight A student. He had her tested. She placed in the Mentally Gifted program.

Note. LM-I

She was the only one of 23 math students to earn 100 on the second cumulative review math test. The test had 10 multiple choice items, 3 of which asked students to recognize pre algebra properties of addition. But the test was not as noteworthy as her insightful responses to a knowledge as design discussion of math concepts and story problems. She seems able to explain the problem, plan, solve, check model clearly, and her homework story problems have been consistently right.

I need to find her threshold. What are her limits? Can she write story problems? Can she teach the problem solving model to a weaker student? Could she use the full knowledge as design method of critical thinking to discuss a big idea in math such as divisibility?

Note. VL-I

Ms. Smith has been working with kids for a sci-fi short story writing contest. She selected Bjrian and Dontel from my class. I sent Ayanna with them. A week later, Ayanna told me that Ms. Smith had rewritten her short story.

"She made it boring," said Ayanna.

Note. M-I

She seemed to enjoy the song version of Dunbar's "When Malindy Sings." As part of a year long poetry and performance class to develop verbal linguistic intelligence, we have been reciting Dunbar's poem and singing the Jazz version that Abby Lincoln wrote based on three verses. Ayanna was the only student who could recite the poem from memory as well as sing the song.

Note. VS-I

I still don't know much about her art smarts yet. Her cognitive maps in math are functional but not as artistic as James's, Kenny's or Ryan's, three boys with high visual spatial intelligence.

Note. BK-I

Ayanna may have a long attention span in this intelligence. If the class were smaller, I could begin teaching her a Tai Chi form right now.
Note. Inter-I
While she sometimes has chosen to work alone, she continues to also work well in groups. Today she was a good follower in her group, making suggestions and contributing information while another child served as leader.

Note. Intra-I
She seems to enjoy the 7 minute silent period after lunch, and she seems to like listening to Bach Cello concertos playing softly in the background. I imagine her brain waves during this activity have reached the desired alpha state.

10-29-94
Ayanna sings well.
Yesterday, during organized free time, several children broke into song. Ayanna and three other girls were playing Medical Monopoly in the portable learning center at the time.
"You should hear Ayanna sing Toni Braxton, Mr. Fluellen!" That is what one girl exclaimed when the group had finished their song.
On cue, Ayanna left the board game and walked over to the desk where the group of singers had clustered. She closed her eyes, held back her head and let out a husky, 10 year old voice—clear, uncracked, melodic. She sang a Toni Braxton love ballad as if in a recording studio.
That was my first observation of her musical intelligence.

11-4-94
Ayanna surprised me at the class Halloween party. She said I'm dressed as a model. She looked more like a hooker: skin tight dress, traffic light red lipstick, high heel shoes. But her costume wasn't the surprise.
The girl can dance!
She does all the isolations my daughter can do. That doesn't seem like much at first glance. But consider this: my daughter has been studying ballet and jazz at Phildanco since she was five years old. Now 14, she is currently in the advanced classes at Phildanco and a freshmen dance major at the High School for Creative and Performing arts. My daughter is a superb dancer. Ayanna, on the other hand, never had a dance lesson in her life, yet she can present each isolation with the same clarity as my daughter.
While there is so much more to dance than isolations, Ayanna's performance does indicate some bodily-kinesthetic intelligence. Coupled with the intensity and ease with which she performs the Tai Chi exercises, she has the core abilities of bodily kinesthetic intelligence.

11-19-94

Ayanna earned all A grades in achievement for major subjects on the first report card. She and one other student, a boy, were the only A students in mathematics.

12-12-94

Ayanna's mother is not coming for her report card. She sent a note asking me to give the report to her daughter.

12-20-94

I named her student of the month for January. No one objected even thought the class nominated and voted on Students of the Month for October, November, and December. In each case I had selected one of the top three nominees. Ayanna had not been nominated for any of the class elections even though her academic performance and school citizenship indicate she is the best qualified student in the class. When I announced that Ayanna would represent Room 303 for the January Student of the Month because she had the best report card and behavior ratings of 2s and 1s, not a single student objected out loud.

12-30-94

Let's summarize. Ayanna's class transferred into mine when the school was reorganized. That was 6 October 1994. She has been with me for three months. In this time, I have made observations about each of her seven intelligences.

It would be useful to organize these observations along the lines of core abilities for each intelligence. Such a schema would not only summarize the year, but set up a way of seeing what she does in the next months of instruction.

In David Lazear's Seven ways of knowing, a fact sheet is offered for each intelligence in addition to exercises that awaken each intelligence and research summaries. Each fact sheet presents the core capacities involved for that intelligence, the neurological process involved, ways to strengthen the target intelligence, and the developmental journey from basic to higher order. David includes projected careers that highlight the intelligence.

For the purpose of organizing new observations of Ayanna's logical mathematical intelligence just the core capacities are used.
**Logical/Mathematical Intelligence** is activated in situations requiring problem solving or meeting a new challenge, as well as situations requiring pattern discernment and recognition," says David Lazear in his factsheet on logical/mathematical intelligence. Core capacities of this intelligence are as follows:

- abstract pattern recognition
- inductive reasoning
- deductive reasoning
- discerning relationships and connection
- performing complex calculations
- scientific reasoning

Based on observations up to 1-16-95, Ayanna's logical mathematical intelligence can be described along the lines of the core capacities.

"I don't understand this!" That's what Ayanna exclaimed when faced with making a factor tree to find the Greatest Common Factors of 18 and 27. The steps I had given them broke the problem into three parts which I then modeled in a think aloud at the blackboard.

1. Find the factors first.
2. Circle all the common factors.
3. Select the greatest common factor.

After her initial reaction, Ayanna was able to solve similar problems that involved abstract reasoning. As a rule she has been able to solve both pre algebra problems involving an unknown or properties of addition and multiplication. Abstract pattern reasoning may not be her weakness. Even her exclamation about what she did not understand shows high metacomprehension. She understands when she understands as well as when she does not understand.

One clear cut example of inductive reasoning in a scientific activity demonstrated Ayanna's use of inductive reasoning. From scratch, she was able to develop a set of properties to classify a list of 15 mammals, a list including dolphins, rats, zebras, wolves, and elephants.

Also, the Science class has begun a unit on force. This unit featured a number of assessments that required inductive reasoning so I will make more observations about Ayanna's capacity.
It seems to me the steps for finding CiCF in math is an example of inductive reasoning. I'll see how she does with this on the cumulative reviews and the open ended problems.

Ayanna has shown numerous performances of deductive reasoning in mathematics. All of the story problems begin with the Problem, Plan, Solve, Check model. She has been able to use the model to work out solutions more successfully than any other student in her math class. Of course these same story problems demonstrate inductive reasoning as well since they lead up to a solution.

I will look for specific examples of both reasoning patterns during the final report card period.

The classification of mammals exercise involved discerning relationships and making connections. Specifically, the assessment required using all of the science process skills learned to date: observation, inference, and classification.

Ayanna had to decide which property or properties grouped several of the mammals together. For example, the dolphins, sea lions, harbor seals and Humpback whales she classified as one group. She put the wolf, Gingham dog, Bassett hound, and fox in another group. While she realized the common connection between all the mammals was the set of characteristics for all mammals, the relationships she made within the classification groups involved inferences she made about a subordinate set of properties: mammals in the sea, mammals on land that seem to be in the dog family, horse family, and rat family.

To date, long division has given Ayanna the most experience in performing complex calculations. She can do one and two digit quotient calculations flawlessly. But her richest experience came from doing a real life problem deriving from the poster contest. She was on a team responsible for finding the averages of one judge's rating of saving wildlife posters. This activity was more in line with Gardner's "distributed intelligence" including use of a calculator, however. It was not conducive to individual observations of Ayanna's performance of complex calculations because of its cooperative learning structure.
The upcoming series of Science Fair Project experiments ("Sound of Music: exploring the question what is sound?") should yield observations about her ability to use scientific reasoning in its full context. To date I only have observations of her use of process skills. She knows the differences among opinion, observation, inference, and classification and can write a clear, relevant, accurate essay to explain the differences. She can use each thinking skill in Science, Mathematics, and other subjects as well.

During "Sound of Music," 303's entry in the school wide Science Fair Project, she will use these process skills plus generalization in solving problems with the scientific method over the next several months.

In sum, Ayanna shows logical mathematical intelligence in each of the core capacities. Now the challenge is to develop these core capacities along the lines of higher order, logical mathematical intelligence.

David Lazear says logical mathematical intelligence can be strengthened by "learning and using various 'patterns for thinking' (e.g. prediction, intuitive and deductive thinking, etc.); working with symbolic language; using complex problem solving; and learning and using number patterns." (Lazear, 1991)

For the third report card period, Lazear's set of awakening activities will guide the instructional program for mathematics and science in Ayanna's 5th grade class at Joseph Pennell Elementary. Future observations of children will comment on their construction of higher order, mathematical intelligence earlier in the teacher journal observation period. Thus more time will be available to observe higher order, logical mathematical intelligence.

The model for observation I created in this project, however, might be repeated: observe for all seven intelligences; observe for components of logical mathematical intelligence; observe for components of higher order logical mathematical intelligence including processfolio and portfolio.

If this three part approach became a Carini-like descriptive review, then a context for examining a child's higher order logical mathematical intelligence would be provided by data about all seven intelligences and the more general components of logical mathematical intelligence.
Note. David Lazear defines higher order thinking in LM-I as follows:

- finding unknown qualities
- linking operations for complex problem solving
- inductive/deductive reasoning processes
- metacognitive processing skills
- logic

When this project is done in 1995-1996, his definition will be kept in mind right from the start.

It was Jane Drucker, an elementary Mathematics education professor at Temple University, who ignited my search for trends in the research about mathematics teaching and learning. Such a literature review provides a final context for the insights about a child's construction of higher order, logical mathematical intelligence.

This brief review begins with a summary of research trends in mathematics as cited in NCTM's 1992 research handbook. Then, it examines selected items related to constructivism in mathematics learning including a summary of how children learn mathematics as presented in chapter 4 of the Reys, Suydam, and Linguist text, and selected items from the ERIC data base, ≥1990. Finally, the review includes an environmental scan of recent mathematics education journal issues. The review, itself, provides a context for the closing remarks I make about Ayanna.

In NCTM's 1992 research handbook, five broad trends are cited:

- growth of research
- diversity in research methods
- shift in epistemology
- shift in learning psychology
- growth of political awareness
Trend one

Growth of research in mathematics teaching and learning has been exponential in the last 30 years and is likely to continue multiplying because of widespread research communities and current reform movements.

Trend two

Research methods have diversified from relying, primarily, on empirical studies to inclusion of descriptive research and acceptance of research perspectives from different nations.

Trend three

In the last three decades what it means to do math has been changing because of technology and new understandings of how mathematics is defined.

Trend four

Research from the cognitive science community has significantly shaped views about learning mathematics.

Trend five

And finally, in mathematics research teaching and learning communities awareness of political relationships has increased. (Handbook, 1992)

Of these five broad, interrelated trends (i.e. megatrends), the shift in learning psychology has the most bearing on this teacher research project because it describes new views of how humans learn mathematics or any subject. My concern has been how does a child construct higher order, logical mathematical intelligence—the very constructivism embedded in the new views.

Says the Handbook, "a new view of learning—cognitive science—is an outgrowth of the revolution in psychology that has become dominant during the past decade." Seven notions of how the mind works characterize this new perspective. (Handbook, 1992)

Reys, Suydam, and Lindquist echo the above constructivist view from cognitive science. "Understanding how children learn mathematics," the fourth chapter in their text, outlines a research based perspective on the question how is mathematics learned?

Two key ideas emerge in their chapter: students learn best with bridges from the concrete to the abstract; and, while behaviorism has a place in the teaching learning process, students learn best when they construct mathematical understanding. (Reys, Suydam, and Lindquist, 1992)
Finally, abstracts of articles and dissertations in research summary issues of NCTM's Journal for research in Mathematics Education (July issues) and research summaries in ERIC's Investigations in Mathematics Education tell the story of constructivism within the context of the megatrend “shift in learning psychology.”

This story continues with a search of the literature on constructivism. To cover the last 25 years in depth, three rules can limit both computer and manual searches:

1. Constructivism (or a derivation of the noun form) must appear in the title and/or abstract or research summary.
2. The literature has been referenced in the library of congress data base, ERIC data base, Journal of Research in Mathematics Education, or Mathematics Education Information Report (The ERIC Science, Math, and Environmental Education Clearinghouse).
3. The literature of most interest will be 1983, the year A nation at risk was released and reform movements intensified.

For now, such a comprehensive review lies beyond the time constraints of the deadline given this draft of “Footsteps.” A more exhaustive literature review is to be included when this paper is revised as an ERIC monograph.

If “trend” may be defined as a sustained pattern of growth or change over a significant time, then constructivism in the 20th century has caught fire. Sparks had been set in Vygotsky’s early formulations in Russia and Piaget’s 50 year research program at the Center for Genetic Epistemology at University of Geneva. Before 1966, though, these sparks had been unnoticed by most educators in the United States.

In the ERIC data base period from 1966-1981, three (3) items are identified when constructivism is in the title or abstract. From 1982-1991, thirty-nine (39) items appear. From 1992-1994, seventy-six (76) items are offered. Of the 118 citations about 72 percent were entered in the last two years of the data base. Just 42 citations appear in the 24 year period from 1966-1991. 76 citations appear in the two year period from 1992-1994.
Beyond the data base, the four most recent issues of NCTM's Teaching Children Mathematics (volume 1, numbers 5-8, January to April 1995) published over three dozen articles that seem to be rooted in constructivism. That is about 36 articles in just four months in just one publication—a rate of 144 articles for 1995 alone.

None of these articles used constructivism in the title. Nor would the word constructivism be likely to appear in an abstract. Yet, using an environmental scanning technique to ferret out main ideas, each of these articles deals with constructivism in mathematics. Because the articles dealing with this trend may not be found through a computer search using constructivism in titles and abstracts as its rule, it is feasible that many more articles about constructivism in mathematics education could be found through an environmental scan.

Conservatively, my guess is that constructivism may be a trend in mathematics education. Put in terms of John Naisbett’s framework, in the 20th century, the psychology of learning megatrend has shifted from behaviorism to constructivism. Such a shift parallels the late 20th century world view shift from the Newtonian paradigm to the Fri Capra’s ecological paradigm.

My selected review of the literature ≥1990 (an environmental scan) brings this wide angle shot of constructivism close up. First, Michael Connell’s Internet article defined constructivism and offered several themes. Second a set of debate articles in a special issue of the Journal of Research in Mathematics Education sharpened my definition of constructivism and gave its historical context. Third, an environmental scan of articles in recent issues of Teaching Children Mathematics round out the review.

Dr. Connell’s point of entry on the construct "constructivism" was to propose themes. According to him, three themes emerge:
- mathematical constructivism as practiced by teachers
- technology usage in constructivist classrooms
- mathematics reform in light of NCTM standards
Also, he feels that because constructivist classrooms tend to be personed by constructivist teachers, each class appears idiosyncratic when compared with other constructivist classrooms. Yet a "deep family resemblance which underlies mathematical constructivism, due in large part to the influence Piaget's thinking has had upon the field" unifies these efforts. Essentially, these teachers put into the practice the idea that children's acquisition of mathematical knowledge originates in action. (Connell, 1995)

Martin Simon's article in the Journal for research in Mathematics articulates the constructivist perspective, reviews the literature, and describes a model of a mathematics classroom that explicitly applies a constructivist framework in its pedagogy. Then Steffe and D'Ambrosio critique Simon's article. Following that, Simon responds to their critique. (Simon, 1995; Steffe and D'Ambrosio, 1995)

Finally, the most recent four issues of Teaching Children Mathematics balance this review. Based on an environmental scan, about 36 articles in these issues discuss how knowledge originates in action. They are constructivist perspectives.

The scan used Dr. Silva's key variables of schooling to find differences among the otherwise unified articles. Thus, unlike a tradition environmental scan in the strategic planning field which would classify literature according to categories such as economic, legislative, scientific, social and technological, this mathematics education literature was classified in terms of subject matter, method, teaching, learning, or assessing.

In the heat of the scan, it became clear that some articles touched more than one topic area, so each was classified according to its primary fit. Also, all of the articles dealt with the subject matter mathematics so that topic area became more specific when joined with another area.

Additionally, the definition of each topic borrowed from Dr. Silva's descriptions of the key variables of schooling. Thus, subject matter became content area; method became approach to teaching; teaching became organizing instruction to encourage the construction of schema; learning became student construction of schemata; and assessing became measurement of understanding. The articles were grouped according to two attributes: subject matter and method or teaching or learning or assessing depending on the main thrust of the article. Subject matter was a constant while the other four topics were variables.
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Subject matter and learning early childhood activities April 1995
Subject matter and learning k-6 activities April 1995
Subject matter and learning linear measurement March 1995
Subject matter and learning k-6 activities March 1995
Subject matter and learning k-6 activities February 1995
Subject matter and learning k-6 activities January 1995

Assessing
Subject matter and assessing understanding of math March 1995
Subject matter and assessing vignette as alternative February 1995 assessment tool
Subject matter and assessing children's understanding of math January 1995

What the scan reveals are trends within the megatrend constructivism. For example, the "investigations" feature in each issue makes teachers and students partners in researching mathematical ideas. Each issue presents sets of activities that can be done at home with parents. Each issue publishes at least one novel method for teaching mathematics within the framework of NCTM standards. And each issue connects mathematics with literature in another standard feature.

At a deeper level, all the articles in Teaching Children Mathematics make use of two or more of Gardner's multiple intelligences with an eye on "solving problems or fashioning intellectual products." It could be said that multiple intelligences is the implicit megatrend while constructivism is the explicit megatrend.

...
Conclusions

On a larger scale, it remains to be seen if the magnitude of the correlation between systematic instruction in higher order, logical mathematical intelligence within an MI-theory framework and achievement in Mathematics is significant for a whole elementary school class. No studies to date have addressed that problem. But for the question how does one child construct higher order, logical mathematical intelligence (Ayanna’s case), a few tentative conclusions might be offered in this teacher research project.

Since birth, Ayanna already had the core capacities of logical mathematical intelligence itself. Gardner’s work on MI-theory implies that human beings are born with the core capacities of each intelligence hard wired, though the spectrum of intelligence, relative strengths and weaknesses, varies from individual to individual. Ayanna began school with core capacities in logical mathematical intelligence as well as in the other intelligences.

Also, because each individual’s core capacities can be improved with teaching, coaching, and experience including parenting, Ayanna may have experiences that awakened her logical mathematical intelligence. She didn’t show up in 5th grade with sleeping capacities.

For example, her 4th grade City Wide Test scores were two standard deviations above the norm and she earned an A in 4th grade mathematics. Both achievements would be rooted in the overall development of her number sense up to 4th grade according to Reys, Suydam, and Lindquist and schemata for solving mathematical problems according to recent cognitive science research.
Also, while there is no evidence yet that she grew up in what Seymour Papert called a mathematical environment (in Papert’s terms this is a “mathetic” environment, a home in which mathematical ideas naturally develop through day to day interaction with math literate adults), her K-5 records indicate high grades in Math and Science and CWT scores ranging from one to two standard deviations above the norm. Her logical mathematical intelligence had certainly been awakened. (Papert, 1980; Reys, Suydam, and Lindquist, 1992; NCTM handbook, 1992)

There is also evidence that Mr. Manning, her 4th grade teacher at Joseph Pennell, gave Ayanna the necessary background for this exploratory 5th year. She often referred to an idea that he taught her. That may indicate a schema was already in place for selected math ideas such as long division.

New for her may have been the deliberate attempt to teach higher order, logical mathematical intelligence within the MI theory framework. New may have been the year long metacognitive training in David Perkins’ knowledge as design for critical thinking and KWL for learning mathematical ideas. New may have been the explicit use of literature, music, and art as bridges to mathematical knowledge and math as a bridge to knowledge in Social Studies, Literature, and life. New may have been the use of a National Intellectual Standard to examine intellectual products across the curriculum. New may have been involvement in theme projects such as the Analogy Project to reason analogically in Mathematics, Science, Geography, and Literature.

She is still an A student in Mathematics (she has >90 average for 13 cumulative review tests and over a dozen alternative open ended problem solving assessments), but the grade is now based on activities rooted in verbal linguistic, logical mathematical, visual spatial, musical, bodily kinesthetic, interpersonal, and intrapersonal intelligences.
She can stand before a class and use unifix cubes to explain, clearly, how to make an equivalent fraction for 3/21 that is also stated in its lowest terms 1/7. She can solve magic square puzzles faster than any other student in her class and write her own magic square puzzles. She can observe a pattern of powers of ten (from 10 to the zero power to 10 to the sixth power) and make inferences and generalizations about exponential growth from 10 to a million. She can see an analogy between fraction, decimal, percentage, and ratio. She can use David Perkins' knowledge as design metacognitive method of critical thinking to analyze a mathematical concept in terms of its purpose, structure, model case and set of arguments.

Finally, she can give an oral report of the adult version of Michael Creighton's *Jurassic Park* and discuss many of the novel's chaos theory ideas such as "sensitive dependence on initial conditions." She can design a Venn diagram to compare Carl Sanburg's "The Fog" with T. S. Elliot's verse about yellow fog from "Love Song for J. Alfred Prufrock.

Also, she can use inductive and deductive reasoning to solve a teacher authored open ended problem such as the following two.

If Michael Jordan were to score 60 of the Bulls' 188 points in a game against the 76ers, then he would have scored 60/188 of the Bulls; total. Using base ten blocks and a chart in your science/math journal, how might Jordan's contribution be described in terms of fraction, decimal, percentage, and ratio? What pattern do you see between these different forms of divisibility? Is there a generalization or rule you can make about a pattern your see? What would you infer about the game?

or

How old will you be in the year 2050 A. D.? Make a chart of mathematical analogies that tell your age in 2050 in terms of months, weeks, days, and minutes. Figure out the analogous ages for several friends and family members. Include them in your chart. What patterns do you discover from the data?

Ayanna is showing signs of higher order, logical mathematical thinking, i.e. finding unknown quantities, linking complex operations, deductive/inductive reasoning, use of metacognitive strategies, and use of logic. But her higher order, logical mathematical intelligence is only now a bud.

Footsteps
Implications

Because number sense develops slowly, it may be years before her higher order, logical mathematical operations flower. Meanwhile, intuitively, I can make a few tentative statements about the opening question (how does one child construct logical mathematical intelligence within a multiple intelligences classroom?).

• First, the teacher must create what Seymour Papert coined a “mathetic” environment. In such a classroom the language of mathematics is spoken and math centered thematic units, alternative assessments, use of appropriate technology, math/science journals, and processfolios are the order of the day.
• Second, the child must be given chances to use all of his or her intelligences as bridges to logical mathematical intelligence. Use of multiple intelligences assumes that mathematical ideas are presented in activities that match each child’s learning style. Also, these bridges from other intelligences may help each child to construct schemata for higher order, logical mathematical intelligence.
• Third, each assessment of logical mathematical intelligence must be intelligence fair, contextualized and consistent with a national standard of excellence. Though other intelligences may be used as bridges to the land of mathematical understanding, the assessment of that understanding must be in the mathematics domain. More so, the problems solved or the intellectual products fashioned must aim at NCTM’s national standard of excellence. A rubric based on the standard can make this kind of assessment reliable and at least ecologically valid.
• Finally, beyond the shores of this teacher research project, what I now believe is this: If each student is to construct schemata for higher order, logical mathematical intelligence, then my entire approach to teaching elementary school Mathematics will need to be restructured.

Multiple intelligences theory seems to provide the explicit theoretical framework needed to guide this restructuring because it connects subject matter, methods, teaching, learning, and assessing while at the same time allowing for individualized instructional programs. The megatrend toward constructivism in the cognitive science research on teaching and learning elementary school mathematics also supports this restructuring. The ecological paradigm for the 21st century demands it.
Future world citizens must understand deep ecology, the complex problems it tackles, and the systems level, sustainable solutions it will offer. Thus, increasing higher order, logical mathematical intelligence is a first giant step for humankind to provide a critical mass of future, ecocentric citizens. Restructuring my classroom is one small step. Adding systematic development of creative intelligence some time in the near future would be another step. (Capra, 1991; Sessions, 1995)

With an eye on Gregory Bateson's presupposition that science probes but does not prove, each of the four conclusions raises several new questions for new teacher research projects, though the present question remains rather virginal. Indeed this whole project reminds me of a Zen story.

Doken had been ordered to spend a few years outside of the monastery. Afraid his studies would be disrupted, he asked his friend Sogen, an advanced student, to join him in the country side. That way his studies would continue.

"Yes," Sogen replied. "I will join you. But first answer these questions."

Doken perked up. He knew from past encounters with his friend that questions would provide both newly lit knowledge and expose vast new areas of ignorance.

"If you are hungry, what relief to your belly if I eat rice? If you are lame, what relief is there for you in my racing along merrily? If your bladder is full, what relief to you if I piss?"

Doken understood that his search for knowledge would yield more questions. Each new question would be mother to still another family of questions. His search for knowledge (and expanding ignorance) would spiral into infinity.

Likewise, Ayanna's story doesn't have an ending so I'll let one of my favorite characters from the novelization of Star Trek: The Next Generation's final episode sum things up.

"Q went on, his voice a distant drone. 'The real voyage of exploration has yet to begin, Jean-Luc...a voyage vastly unlike any other in your experience. And it has nothing to do with mapping star systems and charting nebulae. It's a voyage of perceptions...of thoughts...of moments and possibilities...'"

Footsteps

30
Works Cited


____(1994, July 17-22) Seven myths about multiple intelligences. Keynote address at the Sixth International Conference on Thinking, Massachusetts Institute of Technology.


