Some Consequences of the Uncertainty in IRT Linking Procedures.

In many practical applications of item response theory, the parameters of overlapping subsets of test items are estimated from different samples of examinees. A linking procedure is then employed to place the resulting item parameter estimates onto a common scale. It is standard practice to ignore the uncertainty associated with the linking step when drawing inferences that involve items from different subsets, a situation that arises, for example, in the measurement of change. This paper outlines how the uncertainty can be accounted for and exemplifies the ideas with a jackknife approximation for the Stocking-Lord linking procedure. Examples from the National Assessment of Educational Progress suggest that the resulting uncertainty will usually be negligible for inferences about individuals, but can constitute a major source of estimation error in aggregate statistics such as changes in group means. (Contains 2 figures, 9 tables, and 13 references.) (Author)
SOME CONSEQUENCES OF THE UNCERTAINTY IN IRT LINKING PROCEDURES

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and

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Some Consequences of the Uncertainty in IRT Linking Procedures (Unclassified)

Kathleen M. Sheehan and Robert J. Mislevy

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Some Consequences of the Uncertainty in IRT Linking Procedures

Abstract

In many practical applications of item response theory, the parameters of overlapping subsets of test items are estimated from different samples of examinees. A linking procedure is then employed to place the resulting item parameter estimates onto a common scale. It is standard practice to ignore the uncertainty associated with the linking step when drawing inferences that involve items from different subsets, a situation that arises, for example, in the measurement of change. This paper outlines how the uncertainty can be accounted for, and exemplifies the ideas with a jackknife approximation for the Stocking-Lord linking procedure. Examples from the National Assessment of Educational Progress suggest that the resulting uncertainty will usually be negligible for inferences about individuals, but can constitute a major source of estimation error in aggregate statistics such as changes in group means.

Keywords: Item Response Theory Linking Transformations The Stocking-Lord Linking Procedure
1.0 Introduction

A widely cited advantage of item response theory (IRT) in educational measurement is its capability to provide proficiency estimates on a common scale when different examinees are administered different items, or when examinees are administered different items at different points in time. A common practice is to estimate the parameters of a large number of test items, treat the estimates as known true parameters, and calculate proficiency estimates for individuals or groups based on responses to selected subsets of items. Practical considerations often preclude administering all items to a single sample of examinees in order to obtain the initial item parameter estimates; rather, estimates for overlapping sets of items are obtained from separate samples of examinees, then linked to a common scale. While it is generally recognized that the parameters of the required linking functions used in practice are estimates rather than known constants, the effects of the uncertainty associated with them upon subsequent analyses are rarely taken into account.

This paper lays out a framework for incorporating the uncertainty associated with IRT linking procedures in subsequent estimates of individual or group change. The ideas are implemented for the linking procedure given by Stocking and Lord (1983), and illustrated with data from the 1984 and 1986 reading surveys of the National Assessment of Educational Progress.
2.0 The 3-Parameter Logistic Item Response Model

The 3PL model expresses the probability of a correct response to an item as a function of (i) the examinee's proficiency level \( \theta_i \), and (ii) three parameters characterizing the item, \( \beta_j = (a_j, b_j, c_j) \) for \( j = 1, \ldots, n \). The parameter \( a_j \), called the discrimination or slope parameter, characterizes the item's sensitivity to proficiency. The parameter \( b_j \), called the threshold parameter, is a measure of item difficulty. The parameter \( c_j \) is the probability that an individual with very low proficiency will respond correctly to the item. The conditional probability of a correct response to any single item, denoted \( P_j(\theta_i) \), is obtained as

\[
P(x_{ij} = 1|\theta_i, \beta_j) = \frac{c_j + (1-c_j)/(1+\exp[-1.7a_j(\theta_i-b_j)])}{1+\exp[-1.7a_j(\theta_i-b_j)]},
\]

where the item response \( x_{ij} = 1 \) if correct and 0 if not. Under the usual assumption of local or conditional independence, the probability of a vector of observed item responses, \( x_i = (x_{i1}, \ldots, x_{in}) \), given a known proficiency value \( \theta_i \), can be expressed as a product over items as follows

\[
P(x_i|\theta_i, \beta) = \prod_{j=1}^{n} P(x_{ij} = 1|\theta_i, \beta_j)^{x_{ij}} (1-P(x_{ij} = 1|\theta_i, \beta_j))^{1-x_{ij}}
\]
Because \( P_j(\theta_i) \) is defined as a function of \( a_j(\theta_i - b_j) \), the origin and unit of measurement of the proficiency metric are undetermined. That is, for any rescaling constants \( A \) and \( B \), if 
\[
\theta_i^* = A \theta_i + B, \quad b_j^* = A b_j + B \quad \text{and} \quad a_j^* = A^{-1} a_j,
\]
then \( a_j^*(\theta_i^* - b_j^*) = a_j(\theta_i - b_j) \) and \( P_j(\theta_i) \) is unchanged. Since any such linear transformation of the scale retains the meaning and the implications of all parameter values, the unit-size and origin of the \( \theta \) scale must be determined arbitrarily by the researcher.

Two widely used procedures for estimating the item parameters \( \beta = (\beta_1, \ldots, \beta_n) \) of \( n \) items under the 3PL model are: joint maximum likelihood, the approach incorporated in the LOGIST program (Wingersky, Barton, and Lord, 1982); and marginal maximum likelihood, the approach incorporated in the BILOG program (Mislevy and Bock, 1982). In both of these programs, the aforementioned linear indeterminacy is resolved by standardizing the distribution of proficiency in the calibration sample in one way or another. The resulting item parameter estimates, and the scale they implicitly define, are then typically taken as fixed when used to estimate individual examinees' proficiencies (as may be required for selection or placement decisions) or population characteristics such as group means (as may be required in educational surveys such as NAEP). In order to focus attention on

\[
\frac{1}{n} \sum_{j=1}^{n} P_j(\theta_i)^x j^2 (1 - P_j(\theta_i))^{1 - x j^2}. \quad (2)
\]
the impact of the uncertainty in the linking functions, we shall not deal with the uncertainty in the item parameter estimates themselves. The interested reader is referred to Lewis (1985) and Tsutakawa (1986) for more on this latter topic.

3.0 Linking Transformations

Often, it is not feasible to administer all of the items in a large item pool to a single sample of examinees. Instead, overlapping subsets of items are administered to different samples of examinees. When practical considerations preclude a concurrent calibration of all sample data together, as may be the case when the various samples are collected at different points in time, then independent calibrations must be performed on the data collected from each sample. If the IRT model is true, the parameter estimates obtained for items common to two or more calibrations will differ by (i) estimation error, and (ii) an unknown linear transformation.

In this paper, we address the simple case of two tests that share a subset of common items. Each test is independently calibrated on a different sample of examinees. The two calibration samples could represent the same group of examinees tested at two different points in time, or two different groups of examinees for which comparisons are to be made. We refer to the scale established by the calibration of the first sample as the target scale and the scale established by the calibration of the second sample as the provisional scale. The inferential problems
are, first, to estimate the linear transformation needed to bring the item parameter and proficiency estimates from the provisional scale to the target scale, and second, to account for the uncertainty of the linking procedure when stating the precision of resulting statistics. This simple case can be generalized to the more complex calibration problem which arises when multiple forms of a test are calibrated on several independent samples of examinees.

3.1 The Stocking-Lord Linking Procedure

A number of approaches have been suggested for estimating linking transformations. Several attempt to match characteristics of the distributions of a and b parameter estimates on the target scale and reexpressed scale (e.g., Marco, 1977), possibly with differential weighting of estimates to account for the precision with which they have been estimated (Linn, Levine, Hastings, and Wordrop, 1980) or to discount the influence of outliers (Bejar and Wingersky, 1981). The Stocking-Lord (1983) procedure, which we employ in the sequel, minimizes the average squared difference between test characteristic curves (TCCs) estimated from the two sets of item parameters available for the common items.

The input data to the Stocking-Lord procedure consists of two sets of parameter estimates for the common items, one set expressed on the target scale and one set expressed on the provisional scale. For item j, we denote these estimated parameters as \((\hat{a}_j^{T}, \hat{b}_j^{T}, \hat{c}_j^{T})\) and \((\hat{a}_j^{P}, \hat{b}_j^{P}, \hat{c}_j^{P})\) respectively.
The goal is to estimate the parameters A and B of the linking transformation that can be used to produce rescaled parameter estimates \( (\hat{\theta}_j, \hat{\beta}_j, \hat{\gamma}_j) \), where

\[
\begin{align*}
\hat{\theta}_j &= A^{-1} \hat{\theta}_j, \\
\hat{\beta}_j &= A \hat{\beta}_j + B, \quad \text{and} \\
\hat{\gamma}_j &= \hat{\gamma}_j.
\end{align*}
\]

(Note that the estimate of the lower asymptote parameter \( \hat{\gamma}_j \) is unaffected by the transformation.) After A and B have been estimated from the items common to both calibrations, this same linking transformation is applied to the parameters of the items that appeared in the second calibration only, in order to bring them to the target scale.

Estimation of A and B is accomplished by minimizing the squared difference between estimated true scores (expected numbers correct) on the \( n_c \) common items at N preselected values of \( \theta \). The function to be minimized is

\[
f(A, B, \theta) = \frac{1}{N} \sum_{i=1}^{N} (\zeta_1(1, 0, \theta_i) - \zeta_2(A, B, \theta_i))^2
\]

where \( \zeta_1(1, 0, \theta_i) \) is the true score associated with the proficiency level \( \theta_i \), calculated from the common items using the item parameter estimates expressed on the target scale, and \( \zeta_2(A, B, \theta_i) \) is the true score associated with the proficiency level \( \theta_i \), calculated from the common items using the item parameter estimates which were originally obtained on the provisional scale.
and then reexpressed on the target scale with the rescaling parameters $A$ and $B$. That is,

$$\zeta_1(1,0,\theta_1) = \sum_{j=1}^{n_c} \hat{c}_{j1} + (1-\hat{c}_{j1})/(1+\exp[-1.7\hat{\alpha}_{j1}(\theta_1-\delta_{j1})])$$

and

$$\zeta_2(A,B,\theta_1) = \sum_{j=1}^{n_c} \hat{c}_{j2p} + (1-\hat{c}_{j2p})/(1+\exp[-1.7\hat{\alpha}_{j2p}(\theta_1-(A\delta_{j2p}+B))])$$

$$= \sum_{j=1}^{n_c} \hat{c}_{j2r} + (1-\hat{c}_{j2r})/(1+\exp[-1.7\hat{\alpha}_{j2r}(\theta_1-\delta_{j2r})]) .$$

The values $\theta=(\theta_1,\ldots,\theta_N)$, which are selected rather than estimated, play the role of the independent variables in a regression analysis. They should be selected to ensure that the equation given in (3) is minimized over the entire (expected) range of the target proficiency scale.

We note in passing that under this procedure, the common items end up with three sets of item parameter estimates, one set expressed on the provisional scale, and two sets expressed on the target scale. Alternative procedures for combining the two sets of estimates expressed on the target scale are given in McKinley (1988).
3.2 A Jackknife Approximation for the Uncertainty of the Stocking-Lord Linking Procedure

The uncertainty associated with the estimated rescaling parameters A and B of the Stocking-Lord linking procedure can be approximated using a Jackknife procedure (Mosteller and Tukey, 1977). Although alternative Jackknife implementations may be appropriate for the problem described here, for the purposes of illustration, we present a single variation only. The variation presented is an example of an interpenetrating Jackknife procedure. It consists of three steps. First, the set of $n_c$ common items used to define the transformation are divided into ten equal length subsets with approximately equal average difficulty. Second, the function given in (3) is minimized ten times. Each minimization is accomplished using all but one of the item subsets defined in step 1. Finally, the observed variation among the A and B parameter estimates obtained from the ten minimizations is used to estimate a covariance matrix which quantifies uncertainty due to (i) the imprecision of the estimated item parameters, and (ii) lack of fit from the IRT model. This procedure is illustrated with data from the National Assessment of Educational Progress in Section 5.

The jackknife procedure described above measures variation arising from two sources: estimation error and model misfit. The uncertainty associated with estimation error can often be decreased by increasing the size of the calibration samples. To decrease the uncertainty associated with model misfit, it is also necessary to have a large number of linking items. To see this,
note that, if the IRT model were correct, the differences between sets of \((a,b,c)\) estimates obtained from different increasingly large samples of examinees would be accounted for totally by a linear transformation. In this case, consistent estimates of the linking parameters could be obtained with as few as two linking items. When the IRT model does not fit, however, different sets of linking items will tend to provide different estimates of the linking parameters even as calibration sample sizes increase without bound. In this latter case, it is clear that the model misfit component of uncertainty can only be reduced by increasing the number of linking items. Moreover, the linking items should be chosen so as to be representative of the set of all items which might have been used to estimate the linking function.

4. How the Uncertainty in Linking Procedures Propagates to Subsequent Analyses

In this section, we show how the uncertainty associated with an IRT linking procedure can be accounted for, in the context of measuring change. As before, we consider the simple case of only two tests sharing a single subset of common items. The first test is administered to a group of examinees at time 1. The second test is administered to the same group of examinees at time 2. Our primary interest is to measure the change in proficiency observed over time for individual examinees and for specified population subgroups. We assume that a covariance matrix quantifying the uncertainty associated with the parameters of the
linear transformation used to link the two tests has been estimated (as with a jackknife approximation, for example).

We first consider the problem of estimating the change in proficiency for a single examinee. Let \( \hat{\theta}_{i1} \) denote a proficiency estimate calculated for the \( i \)th examinee at time 1 using the estimated item parameters which were originally obtained on the target scale. Let \( \hat{\theta}_{i2p} \) denote a proficiency estimate calculated for the same examinee at time 2 using the estimated item parameters which were originally expressed on the provisional scale. And finally, let \( \hat{\theta}_{i2r} \) denote a proficiency estimate obtained for the same examinee at time 2 using the item parameters which were originally estimated on the provisional scale and subsequently reexpressed on the target scale; that is, \( \hat{\theta}_{i2r} = A \hat{\theta}_{i2p} + B \). Since \( \hat{\theta}_{i1} \) and \( \hat{\theta}_{i2r} \) are both expressed on the target scale, an estimate of the change in proficiency for this examinee can be obtained from the difference, \( \hat{D}_i = \hat{\theta}_{i2r} - \hat{\theta}_{i1} \). If the parameters of the linking transformation were known without error, then the standard error of this estimated change would be given by

\[
SE(\hat{D}_i) = SE(\hat{\theta}_{i2r} - \hat{\theta}_{i1}) = (\sigma^2_{i2r} + \sigma^2_{i1})^{1/2},
\]

where \( \sigma_{i2r} \) and \( \sigma_{i1} \) are the standard errors of the proficiency estimates \( \hat{\theta}_{i2r} \) and \( \hat{\theta}_{i1} \), respectively. (As is usually the case, we have also assumed independent errors across tests.)

Now \( \sigma_{i1} \) will be a function of the item parameters which were originally estimated on the target scale, whereas \( \sigma_{i2r} \) will be a function of the item parameters which were originally estimated on
the provisional scale and then reexpressed on the target scale. Thus, any procedure which accounts for the uncertainty of the transformation used to link the two tests will affect the calculation of $\sigma_{i2r}$ but not $\sigma_{i1}$. To calculate $\sigma_{i2r}$, note that $\hat{\theta}_{i2r} = \hat{A} \hat{\theta}_{i2p} + \hat{B}$, and that the estimated standard error of $\hat{\theta}_{i2p}$, denoted $\sigma_{i2p}$, can be calculated as a function of item parameters which have not yet been rescaled and are thus unaffected by the uncertainty of the linking procedure.

As a first step, define a covariance matrix for $[\hat{\theta}_{i2p}, \hat{A}, \hat{B}]$ as follows:

$$
\Sigma = \begin{bmatrix}
\sigma_{i2p}^2 & 0 & 0 \\
0 & \sigma_A^2 & \sigma_{AB} \\
0 & \sigma_{AB} & \sigma_B^2
\end{bmatrix}
$$

where $\sigma_A^2$, $\sigma_B^2$, and $\sigma_{AB}$ quantify estimation variation for the parameters $A$ and $B$ of the linking transformation. The quantities $\sigma_A^2$, $\sigma_B^2$, and $\sigma_{AB}$ can be approximated using the jackknife procedure given in the previous section. Second, note that

$$
\text{Var}(\hat{\theta}_{i2r}) = \text{Var}(\hat{A}\hat{\theta}_{i2p} + \hat{B})
$$

$$
= \text{Var}(g(\hat{\theta}_{i2p}, \hat{A}, \hat{B}))
$$

$$
= \left[ \frac{\partial g}{\partial \theta_{i2p}} , \frac{\partial g}{\partial A} , \frac{\partial g}{\partial B} \right] \Sigma \left[ \frac{\partial (g)}{\partial \theta_{i2p}} , \frac{\partial (g)}{\partial A} , \frac{\partial (g)}{\partial B} \right]'^t
$$

$$
= \left[ A , \theta_{i2p} , 1 \right] \Sigma \left[ A , \theta_{i2p} , 1 \right]'
$$
Thus, the uncertainty associated with the linking procedure can be accounted for in the estimated standard error of the difference \( D_1 \), as follows:

\[
\text{SE}(D_1) = \text{SE}(\hat{\theta}_{i2r} - \hat{\theta}_{i1}) \\
= (\text{Var}(\hat{\theta}_{i2r}) + \sigma^2_{i1})^{1/2} \\
= (f(\hat{\theta}_{i2p}, \hat{A}, \hat{\Sigma}) + \sigma^2_{i1})^{1/2}
\]

(6)

where \( f(\hat{\theta}_{i2p}, \hat{A}, \hat{\Sigma}) \) is given as in (5).

The same procedure can also be used to incorporate the uncertainty associated with the linking parameters \( A \) and \( B \) in the estimated standard error of aggregate statistics such as the difference between two subgroup means. In this latter case, the \( \theta \) and \( \sigma \) statistics for individuals will be replaced by corresponding point estimates and standard errors for subgroup means.

5. A Numerical Illustration

In this section, data available from the National Assessment of Educational Progress (NAEP), a congressionally mandated survey of the educational achievement of American students, is used to approximate the uncertainty of the Stocking-Lord linking procedure and to evaluate the consequences of that uncertainty. Data from two NAEP surveys are used: the 1984 Reading Survey and the 1986...
Readir Survey. Both of these surveys were independently scaled using a three parameter logistic IRT model. Item parameters were estimated using BILOG (Mislevy & Bock, 1982) and mean proficiencies for population subgroups were obtained using the plausible values methodology given in Mislevy and Sheehan (1987). These data are used to illustrate the consequences of the uncertainty of the transformation parameter estimates from the Stocking-Lord linking procedure. Because NAEP data support inferences about aggregate statistics such as group means but not about individuals' proficiencies, we use real NAEP data to demonstrate procedures for changes in group means but simulated data for changes in individual proficiencies.

5.1 The NAEP Data

Mean reading proficiencies for the three age groups which were assessed by NAEP in 1984 and 1986 are given in Table 1. The first row of the table provides 1984 age group means expressed on the 1984 calibration scale. For the purpose of this illustration, the 1984 calibration scale is designated as the target scale. The second and third rows of the table provide 1986 age group means expressed on the provisional scale (the 1986 calibration scale) and the target scale (the 1984 calibration scale). The Stocking-Lord linking procedure was used to estimate the linear transformation needed to express the 1986 means on the 1984 calibration scale. The table also provides estimated standard errors for each mean.
5.2 Quantifying the Uncertainty of the NAEP Link

The 1984 NAEP survey contained 128 cognitive reading items. The 1986 NAEP survey contained 107 cognitive reading items, 76 which were common to the 1984 assessment and 31 which were administered for the first time in 1986. The linking transformation needed to express the item parameters obtained from the calibration of the 1986 data on the scale established by the calibration of the 1984 data was estimated using the Stocking-Lord linking procedure, as implemented in the TBLT computer program (Stocking, 1986). The generally satisfactory results can be seen in Figure 1, which shows the TCCs of the first and second calibrations of the common items after reexpression, and in Figure 2, which plots the b-parameter estimates from the first and reexpressed second calibrations. The jackknife procedure described in Section 3 was used to approximate the uncertainty associated with the estimated parameters of the linking transformation. The results are given in Table 2.

Figures 1 and 2 and Table 2 about here
5.3 Inference for a Single Examinee

The artificial data set constructed for this analysis contained simulated responses for five examinees to two tests. The first test consisted of 30 items selected from the 1984 NAEP reading survey. The second test consisted of 30 items selected from the 1986 NAEP reading survey, half of which were common to the 1984 survey. For a given examinee, responses were generated in accordance with the 3PL, with item parameter estimates for the first test taken from the 1984 NAEP calibration run and item parameter estimates for the second test taken from the 1986 NAEP calibration run. So that the proficiency of a given simulee was the same on both tests, a value of $\theta$ was specified for the first test and $(\theta-B)/A$ was used for the second. Simulees' $\theta$ values on the first test were $-1.0$, $-0.5$, $0.0$, $0.5$, and $1.0$. The response vectors generated according to these specifications are given in Table 3.

Table 3 about here

Treating the item parameter estimates as known, maximum likelihood estimates (MLEs) of $\theta$ and associated standard errors were obtained for each response pattern using the BILOG program. They are shown in Table 4, with the values for the second test shown before and after reexpression. Table 5 provides estimated standard errors for the change from the first test to the second using (4), which does not take the uncertainty of $A$ and $B$ into account.
account, and (6), which does. The increase in standard errors is negligible, about 2-percent on the average. An approximate variance components analysis is given in Table 6. For each response pattern considered, the total error variance is estimated using (6) which includes components due to both sampling and linking. The contribution due to sampling alone is estimated using (4) and the contribution due to linking is obtained by subtraction. The table shows that for each response pattern considered, the relative increase in uncertainty is negligible, accounting for about three percent of the total error variance on the average.

Tables 4, 5 and 6 about here

5.4 Inference for Group Means

The changes in the mean reading proficiencies of students aged 9, 13 and 17, over the two year period from 1984 to 1986, as estimated from the NAEP data, are given in Table 7. The table also provides approximate standard errors calculated using (4) and (6). Whereas the size of standard errors increased by only about 2-percent for estimates of change of individuals, the increase in standard errors for groups is about 200-percent! An approximate

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1 These figures are shown for illustrative purposes only, and are not to be taken as estimates of changes in reading proficiency during the period due to certain anomalies in the 1985/86 NAEP data. The interested reader is referred to Beaton (1988) for further information.
variance components analysis is given in Table 8. The table shows that the component due to linking represents approximately 90-percent of the total error variance, on the average. To put these results in another perspective, the change in mean reading proficiency at each age level is expressed in standard error units in Table 9. The table shows, for example, that the decrease in the mean reading proficiency of 9 year olds is approximately three standard errors when the uncertainty of the linking procedure is not accounted for, but only one standard error when it is.

6.0 Summary

A common problem in applied work with item response theory is to express item parameter estimates from separate calibrations on the same scale, based on the multiple estimates for subsets of items common to two or more calibrations. Several methods have been proposed for estimating the optimal linear transformations for this purpose, including the Stocking-Lord (1983) procedure for matching test characteristic curves. After the resulting transformations have been applied, the uncertainty associated with them is rarely taken into account in subsequent analyses of individual or group levels of proficiency.

This uncertainty can be expressed in terms of a covariance matrix of estimation errors, which can be approximated empirically.
through a procedure such as the jackknife. With an approximation of the sampling covariance matrix of estimation errors of the parameters of a linking transformation, one can readily derive standard errors for change scores or comparisons that take this additional uncertainty into account.

Using data from the 1984 and 1986 reading surveys of the National Assessment of Educational Progress, this paper used the jackknife to approximate the uncertainty of the linking transformation between the two assessments. Its effect was found to be negligible in the context of drawing inferences about change of individuals, since its magnitude was much smaller than the uncertainty arising from having only the limited numbers of item responses from individuals that generally characterize individual testing programs. Correct standard errors were only about 2-percent larger than those that ignored linking uncertainty. The effect was substantial in the context of estimating group changes, however, leading to correct standard errors that were 200-percent larger. The differential impact is due to the fact that sampling variances of group means are much smaller than sampling variances of individual scores, while the sampling variance of the linking transformation is the same in both cases.
References


Table 1
Mean Proficiencies
Estimated from the 1984 and 1986 NAEP Reading Surveys
With Standard Errors in Parentheses

<table>
<thead>
<tr>
<th>Year</th>
<th>Scale</th>
<th>Age 9</th>
<th>Age 13</th>
<th>Age 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>84</td>
<td>84 Calib.</td>
<td>-0.752(.020)</td>
<td>0.150(.014)</td>
<td>0.766(.018)</td>
</tr>
<tr>
<td>86</td>
<td>86 Calib.</td>
<td>-0.375(.025)</td>
<td>0.571(.019)</td>
<td>0.874(.018)</td>
</tr>
<tr>
<td>86</td>
<td>84 Calib.</td>
<td>-0.864(.028)</td>
<td>0.198(.022)</td>
<td>0.538(.020)</td>
</tr>
</tbody>
</table>

The 1984 sample included over 22,000 students at each age level. The 1986 sample included approximately 7,000 Age 9 students, 6,000 Age 13 students, and 16,000 Age 17 students.
Table 2

Results of the Jackknife Approximation for the Stocking-Lord Linking Procedure

<table>
<thead>
<tr>
<th>Run</th>
<th>Items</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>76</td>
<td>1.12196</td>
<td>-0.442910</td>
</tr>
<tr>
<td>1</td>
<td>68</td>
<td>1.118018</td>
<td>-0.449670</td>
</tr>
<tr>
<td>2</td>
<td>68</td>
<td>1.126296</td>
<td>-0.447837</td>
</tr>
<tr>
<td>3</td>
<td>68</td>
<td>1.121856</td>
<td>-0.449472</td>
</tr>
<tr>
<td>4</td>
<td>68</td>
<td>1.110982</td>
<td>-0.433893</td>
</tr>
<tr>
<td>5</td>
<td>68</td>
<td>1.114703</td>
<td>-0.426793</td>
</tr>
<tr>
<td>6</td>
<td>68</td>
<td>1.128065</td>
<td>-0.430320</td>
</tr>
<tr>
<td>7</td>
<td>68</td>
<td>1.125834</td>
<td>-0.446748</td>
</tr>
<tr>
<td>8</td>
<td>69</td>
<td>1.128753</td>
<td>-0.440663</td>
</tr>
<tr>
<td>9</td>
<td>69</td>
<td>1.112862</td>
<td>-0.447648</td>
</tr>
<tr>
<td>10</td>
<td>69</td>
<td>1.135424</td>
<td>-0.455858</td>
</tr>
</tbody>
</table>

Parameter estimates, $A$ and $B$, obtained from Run 0 were used to reexpress the 1986 results on the 1984 scale. The parameter estimates obtained from Runs 1 through 10 were used only to estimate the uncertainty of the linking procedure.
Table 3
Simulated Responses To Test 1
Administered at Time 1

<table>
<thead>
<tr>
<th>Generating Value</th>
<th>11000 11000 10011 00101 00000 01010</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>11000 11000 10011 00101 00000 01010</td>
</tr>
<tr>
<td>-0.5</td>
<td>00110 10101 10000 10011 01000 11100</td>
</tr>
<tr>
<td>0.0</td>
<td>00010 11101 11100 00100 01101 11100</td>
</tr>
<tr>
<td>0.5</td>
<td>11111 01111 11111 00111 01101 11111</td>
</tr>
<tr>
<td>1.0</td>
<td>11111 11111 11111 01111 10110 11111</td>
</tr>
</tbody>
</table>

Simulated Responses To Test 2
Administered at Time 2

<table>
<thead>
<tr>
<th>Generating Value</th>
<th>00010 01000 00011 11000 10000 00001</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>00010 01000 00011 11000 10000 00001</td>
</tr>
<tr>
<td>-0.5</td>
<td>11001 01000 01011 11101 01100 11000</td>
</tr>
<tr>
<td>0.39</td>
<td>01100 01101 10011 00111 11111 10100</td>
</tr>
<tr>
<td>0.84</td>
<td>00011 11111 10111 11101 01111 11111</td>
</tr>
<tr>
<td>1.29</td>
<td>11111 11111 11111 10111 10110 01110</td>
</tr>
</tbody>
</table>

Table 4
Maximum Likelihood Estimates of Reading Proficiency
At Time 1 and Time 2
For Five Simulated Subjects
With Estimated Standard Errors in Parentheses

<table>
<thead>
<tr>
<th>Generating Value</th>
<th>Value Estimated at Time 1</th>
<th>Value Estimated at Time 2 Before Reexpression</th>
<th>Value Estimated at Time 2 After Reexpression</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>-1.062 (.625)</td>
<td>-0.375 (.422)</td>
<td>-0.864 (.474)</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.662 (.489)</td>
<td>-0.116 (.534)</td>
<td>-0.574 (.560)</td>
</tr>
<tr>
<td>0.0</td>
<td>-0.502 (.470)</td>
<td>0.249 (.360)</td>
<td>-0.163 (.404)</td>
</tr>
<tr>
<td>0.5</td>
<td>0.748 (.546)</td>
<td>0.824 (.409)</td>
<td>0.482 (.459)</td>
</tr>
<tr>
<td>1.0</td>
<td>1.177 (.662)</td>
<td>1.434 (.512)</td>
<td>1.172 (.574)</td>
</tr>
</tbody>
</table>
Table 5

An Estimate of the Change in Reading Proficiency
From Time 1 to Time 2
For Five Simulated Subjects
With Approximate Standard Errors

<table>
<thead>
<tr>
<th>Change in Generating Values</th>
<th>Estimated Change</th>
<th>S.E. Method 1</th>
<th>S.E. Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.198</td>
<td>0.784</td>
<td>0.790</td>
</tr>
<tr>
<td>0</td>
<td>0.088</td>
<td>0.743</td>
<td>0.779</td>
</tr>
<tr>
<td>0</td>
<td>0.339</td>
<td>0.620</td>
<td>0.625</td>
</tr>
<tr>
<td>0</td>
<td>-0.266</td>
<td>0.713</td>
<td>0.718</td>
</tr>
<tr>
<td>0</td>
<td>-0.005</td>
<td>0.876</td>
<td>0.883</td>
</tr>
</tbody>
</table>

Method 1 refers to the method which assumes that the linking function is known without error, as in equation (4); Method 2 refers to the method which accounts for the uncertainty of the linking procedure as in equation (6).

Table 6

A Comparison of Approximate Variance Components
For Inferences About Change at the Individual Level

<table>
<thead>
<tr>
<th>Generating Value</th>
<th>Total Variance</th>
<th>Component Due to Sampling</th>
<th>Component Due to Linking</th>
<th>Linking Variance as % of Total Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>.6241</td>
<td>.6146</td>
<td>.0094</td>
<td>1.5</td>
</tr>
<tr>
<td>-0.5</td>
<td>.6068</td>
<td>.5520</td>
<td>.0548</td>
<td>9.0</td>
</tr>
<tr>
<td>0.0</td>
<td>.3906</td>
<td>.3844</td>
<td>.0062</td>
<td>1.6</td>
</tr>
<tr>
<td>0.5</td>
<td>.5155</td>
<td>.5084</td>
<td>.0071</td>
<td>1.4</td>
</tr>
<tr>
<td>1.0</td>
<td>.7797</td>
<td>.7674</td>
<td>.0123</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Table 7
An Estimate of the Change in Mean Reading Proficiency
From 1984 to 1986
With Approximate Standard Errors

<table>
<thead>
<tr>
<th>Age</th>
<th>Change</th>
<th>S.E. Method 1</th>
<th>S.E. Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>-0.112</td>
<td>0.034</td>
<td>0.105</td>
</tr>
<tr>
<td>13</td>
<td>0.048</td>
<td>0.026</td>
<td>0.084</td>
</tr>
<tr>
<td>17</td>
<td>-0.228</td>
<td>0.027</td>
<td>0.066</td>
</tr>
</tbody>
</table>

1 Method 1 refers to the method which assumes that the linking function is known without error, as in equation (4); Method 2 refers to the method which accounts for the uncertainty of the linking procedure as in equation (6).

Table 8
A Comparison of Approximate Variance Components
For Inferences About Change at the Group Level

<table>
<thead>
<tr>
<th>Age</th>
<th>Total Variance</th>
<th>Component 1 due to Sampling</th>
<th>Component 1 due to Linking</th>
<th>Linking Variance as % of Total Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.0110</td>
<td>0.0012</td>
<td>0.0098</td>
<td>89.5</td>
</tr>
<tr>
<td>13</td>
<td>0.0071</td>
<td>0.0007</td>
<td>0.0064</td>
<td>90.1</td>
</tr>
<tr>
<td>17</td>
<td>0.0044</td>
<td>0.0007</td>
<td>0.0037</td>
<td>84.1</td>
</tr>
</tbody>
</table>

1 Total Variance refers to the estimated variance of the change in mean reading proficiency from 1984 to 1986.
Table 9
The Estimated Change in Mean Reading Proficiency from 1984 to 1986 Expressed in Standard Error Units

<table>
<thead>
<tr>
<th>Age</th>
<th>Method 1 S.E. Units</th>
<th>Method 2 S.E. Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>-3.29</td>
<td>-1.07</td>
</tr>
<tr>
<td>13</td>
<td>1.85</td>
<td>0.57</td>
</tr>
<tr>
<td>17</td>
<td>-8.44</td>
<td>-3.45</td>
</tr>
</tbody>
</table>
Figure 1
Comparison of Test Characteristic Curves

Solid Line = 1984 Curve
Dashed Line = Reexpressed 1986 Curve
Figure 2
Comparison of Item b Parameter Estimates
Reexpressed 1986 Estimates vs. 1984 Estimates
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