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ABSTRACT

This document provides a laboratory manual for an experiment whose objectives are: (1) measure the constant acceleration of a body; (2) calculate the moment of inertia for various symmetrical shape objects; and (3) use the moment of inertia to solve for the constant acceleration of the body. The paper includes a list of materials needed, theory, procedures, and data sheet. (MKR)

Laboratory Manual

Measurement of Constant Acceleration of a Body: Moment of Inertia

by E.P. Villamoran, Ph.D.

LABORATORY MANUAL

Measurement of Constant Acceleration of a Body:
Moment of Inertia
(by: E. P. Villamoran, Ph. D.)

Objectives:

1. Measure the Constant Acceleration of a body.
2. Calculate the moment of inertia for various symmetrical shape objects.
3. Use the moment of Inertia to solve for the Constant Acceleration of the body.

Materials Needed:

1. Inclined-plane set-up
2. Piece of metal to be made as velocity wheels
3. Calipers
4. Digital timer
5. Protractor

Theory:

The motion of a particle which has constant acceleration is important for several reasons. For one, this type of motion is common in nature. For example near the surface of the earth, all objects fall vertically with the constant acceleration of gravity if air resistance can be neglected and if there are no forces acting on the objects other than the pull of gravity. The acceleration of gravity is designated by g and has the approximate value

$$g = 9.81 \ m/s^2$$

The motion of a body of mass m sliding down a frictionless inclined plane is also that of constant acceleration. Its direction is that of an incline, while its magnitude depends upon the height, h, and the length, x, of the incline.

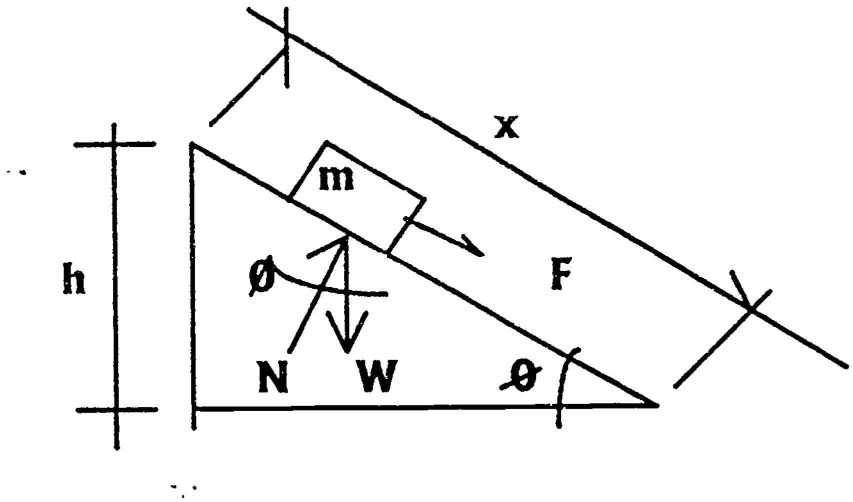


Fig. 1 Body m, moving on an inclined plane.

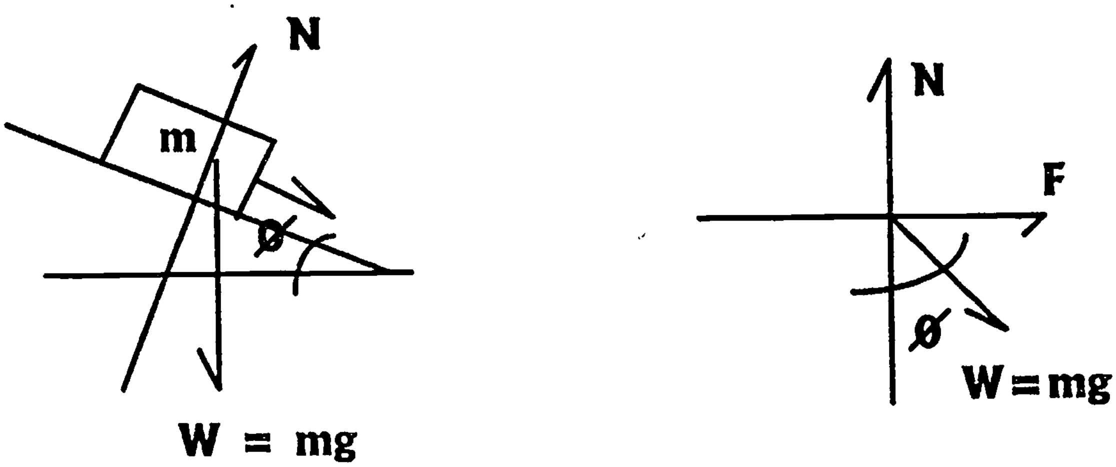


Fig. 2 FBD of mass m, moving down the incline.

$\Sigma F_x = 0;\ N - mg \cos \theta = 0$

$N = mg \cos \theta$

$\Sigma F_g = 0;\ F + mg \sin \theta = 0$

$F = -mg \sin \theta$

The acceleration of the body depends on the acceleration due to gravity, g, and is independent of the mass of the body.

Using the FBD

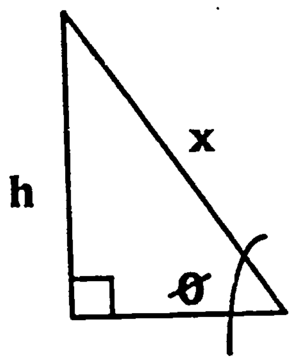
$F = -mg \sin \theta$ (1)

Since F = ma using Newton's second Law of Motion

$ma = -mg \sin \theta$

therefore $a = g \sin \theta$ (2)

From fig. 1



$\sin \theta = h/x$

Substituting in equation (1)

$a = g\,(h/x)$ (3)

If acceleration is constant, the velocity changes linearly with time. If the velocity is V_o at time $t = 0$, its velocity V at a later time is given by

$$V = V_o + at \qquad (4)$$

While the position function is

$$x = x_o + V_o t + 1/2\ at^2$$

at

$$x_o = o\ ;\ x = V_o t + 1/2\ at^2 \qquad (5)$$

$$V_o = o;\ V = at^2 \qquad (6)$$

$$\therefore\ x = 1/2\ at^2 \qquad (7)$$

therefore $x = 1/2\ at^2$

to solve for a: $a = 2x/t^2$

Solving equations (6) and (7)

From (7), $t^2 = 2x/a\ ;\ t = \sqrt{2x/a}$

$$V = a\left(\sqrt{2x/a}\right)$$

$$V^2 = a^2(2x/a)$$

$$V^2 = 2ax \qquad (8)$$

Substitute equation (3) to equation (8)

$$V^2 = 2\ [\ g(h/x)]\ x$$

$$V^2 = 2gh \qquad (9)$$

This is the same equation for a freely falling body dropped from a height h.

This also proves that the sole function of the incline is to change the direction of the velocity. Since friction cannot be totally eliminated in an incline, it can be minimized by using bodies that will roll the incline with less friction.

Shown below is the analysis of the motion of a cylinder or sphere that is made to roll down an incline. (Figure 3).

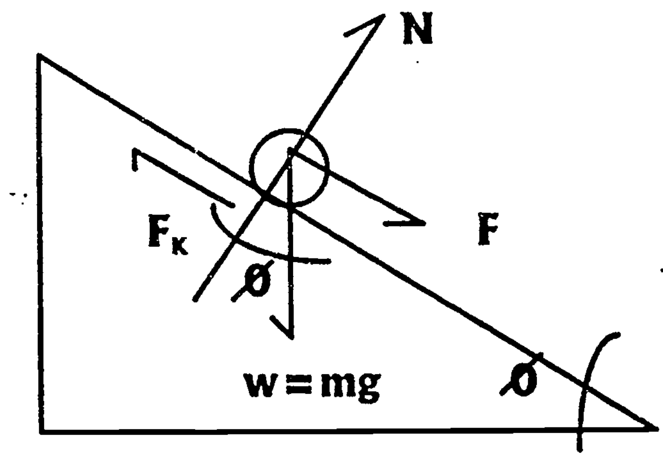
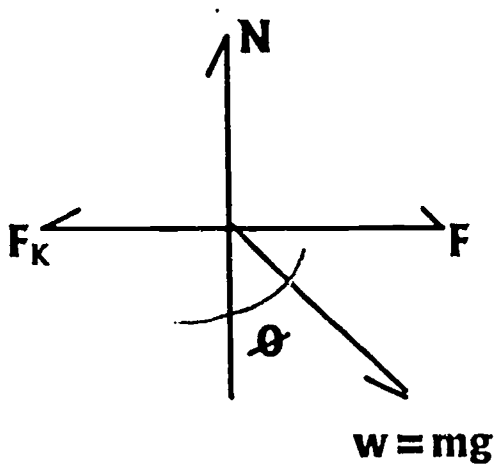


Fig. 3. Forces Acting on a Sphere along on inclined plane.



let m = mass of the sphere or cylinder

N = normal force exerted by the incline on the sphere or cylinder

F_K = force of static friction acting along the incline at the point of contact.

Fig. 4 FBD of all the forces acting on the sphere.

Assume all the external forces act at the center of mass

$\sum Fy = o$; $N - mg \cos \emptyset = o$ (10)

$\sum Fx = ma$; $mg \sin \theta - Fk = ma$ (11)

Where a = acceleration of the sphere in rotting down the incline

Now, the torque due to friction;

$$T = f_k r \quad (12)$$

Where r is the radius of the sphere or cylinder.

but $T = I_o \alpha$ (13)

$F_k r = I_o \alpha$ (14)

where T = torque

α = angular acceleration of the body.

I_o = rotational inertia of of the cylinder

Since $\alpha = a/r$

substituting in equation (14)

$$F_k r = I_o (a/r) ; F_k = I_o (a/r^2) \quad (15)$$

Substituting equation (15) into equation (11)

$$mg \sin \theta - I_o (a/r^2) = ma$$

Solving for a:

$$mg \sin \theta - I_o . a/r^2 = ma$$

$$r^2 mg \sin \theta - I_o a = mar^2$$

$$a (mr^2 + I_o) = r^2 mg \sin \theta$$

$$a = \frac{r^2 mg \sin \theta}{mr^2 + I_o} = \frac{mr^2 (g \sin \theta)}{mr^2 [1 + I_o/mr^2]}$$

therefore $$a = \frac{g \sin \theta}{[1 + I_o/mr^2]}$$

The constant acceleration of the body in an incline can be computed experimentally, by taking the time t, elapsed for the sphere or cylinder to roll down the incline at a distance x. Use equation (7) to solve for a

$$a = 2x / t^2$$

Similarly, the acceleration can also be computed using the moment of inertia of the body used in the experiment. Use equation (16) to solve for this a:

$$a = \frac{g \sin \theta}{1 + I_o/mr^2}$$

For the velocity wheel (Wheel & Axle), since the material is known, then the mass of the cylinders and the, other bodies can be calculated by using $m = vd$, where V is the volume and d is the density.

The wheel and axle prepared to be used in this manual are the cylindrical, conical and spherical shapes with aluminum as the material.

Moment of Inertia Equations:

For the wheel and axle;

$$I_o = I_{axle} + 2 \cdot I_{wheels}$$

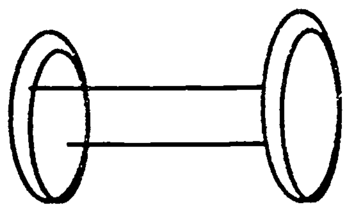
A. Cylindrical wheel and axle

Solid cylinder about an axis

$$I = 1/2\ MR^2$$

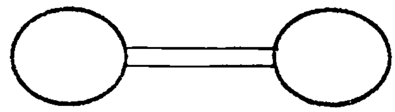
$$I_o = I_{axle} + 2 \cdot I_{wheels}$$

$$I_o = 1/2\ m_{cyl}\, r^2 + 2(1/2 M_{cyl}\, R^2_{cyl})$$



B. Spherical wheel and axle

b. Solid sphere about diameter



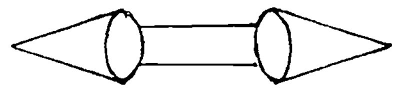
$$I = 2/5\ .MR^2$$

$$I_O = I_{AXLE} + 2\ (I_{WHEEL})$$

$$I_O = 1/2\ M\ cyl\ r^2 + 2\ (2/5)\ M_S R_S^2$$

$$\boxed{I_O = 1/2\ Mr^2 + 4/5\ MR^2} \qquad (18)$$

C. Conical wheel and axle



$$I = (3/10)\ M_C R_C^2$$

$$I_O = I_{AXLE} + 2\ (\ I_{WHEEL})$$

$$I_O = 1/2\ m_{CYL}\ r^2 + 2[(3/10)(M_C R_C^2)]$$

$$\boxed{I_O = 1/2\ m_{CYL} r^2 + 3/5\ M_C R_C^2} \qquad (19)$$

Procedure:

1. Prepare the inclined - plane using smooth edged wood with adjustable screws for variations of height and angle of inclination.
2. Prepare the different shape variations of the wheel and axle.
3. Perform the experiment using the different wheel and axle variations.

a. Take the time elapsed, as the wheel and axle rolls on the incline.

b. Solve for the acceleration of the body using equation (7).

$$a = 2x\ /\ t^2$$

c. Solve for the moment of inertia of the different wheel and axle variations. Use equations 17, 18, & 19.

d. Solve for the acceleration of the body using equation (16),

$$a = \frac{g \sin \theta}{1 + I_o/Mr^2}$$

e. Compare the acceleration computed from (b) & (d).

4. Compute different accelerations by varying the value of θ and then, plot the graph of a versus Sin θ. The slope of the regression line can be computed as

$$= \frac{g}{[\, 1 + I_o/Mr^2 \,]}$$

Note: M = total mass of wheel and axle
r = radius of axle
for which the value of I_o may be calculated.

5. Compute different accelerations by making θ = constant but vary the distance x, then plot x vs t^2. The shape of the regression line equals

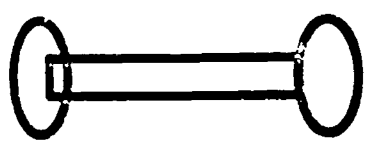
$$= \frac{g}{2\,[\, 1 + I_o/mr^2 \,]}$$

Where : I_o can be computed

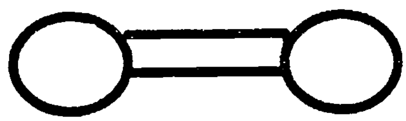
Note: Accuracy of the experiment can be determined by the value of I_o from the dimensions of the sphere or cylinder.

Shaped Variations of the Wheel and Axle

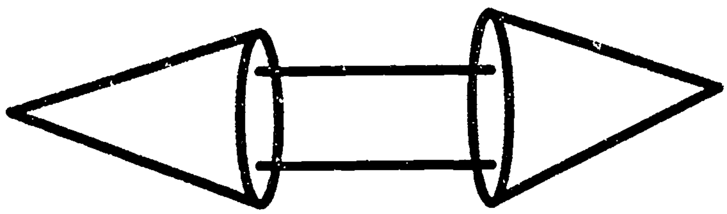
1) Cylindrical - Shaped wheel and axle



2) Spherical - shaped wheel and axle



3) Conical - shaped wheel and axle



Density of Aluminum = 2.7×10^3 kg./m^3

DATA SHEET

TABLE I

Wheel and Axle	Angle of Inclination, °	Distance, cm	Average time, t (s)	Acceleration, a (cm/s^2)
Cylindrical				
Spherical				
Conical				

TABLE II

Wheel and Axle	Diameter, D, cm	Radius,r, cm	Height, h,cm	Volume, V, cm3	Mass, m, g	I_o (total) $g.cm^2$	acceleration cm/s^2
Cylindrical							
Spherical							
Conical							