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ABSTRACT

This paper describes a quantitative assessment project, its findings, and its impact. College mathematics faculty developed an item bank of problems covering material from undergraduate mathematics courses and basic statistics. Instructors use this item bank to develop a questionnaire involving the specific quantitative skills their students need as prerequisites for a particular course and give this test to students within the first 2 weeks of the semester. Thus, early in the semester both students and instructors possess useful information about instructor expectations, student capabilities, and the need for any corrective action. Also discussed are departmental needs and student capabilities revealed by assessment; three levels of quantitative expectations; patterns of student performance on quantitative tasks; and the impact of assessment on participants, the mathematics department, and the entire campus. (MKR)

Assessing the Quantitative Skills of College Juniors

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Two Quantitative Tasks

Suppose we gave the following problem to a group of juniors at your local college or university—what proportion of the students would you expect to give the correct answer?

Problem 1. *A media professor asked the students in his class whether or not they read Time or Newsweek the previous week. The students' responses are summarized in this table:*

		Time Yes	*Time* No
Newsweek	Yes	41	23
Newsweek	No	52	10

How many students said they read Time the previous week?

We have used this and similar problems with many students at the University of Wisconsin-Madison and were surprised to find that only about a third of the students, and rarely more than half, correctly answered this question about data in a cross tabulation table.

Let's consider another, more mathematical, problem. Suppose this time that a group of juniors, who had studied from one to three semesters of calculus, were given Problem 2. How do you think they would do?

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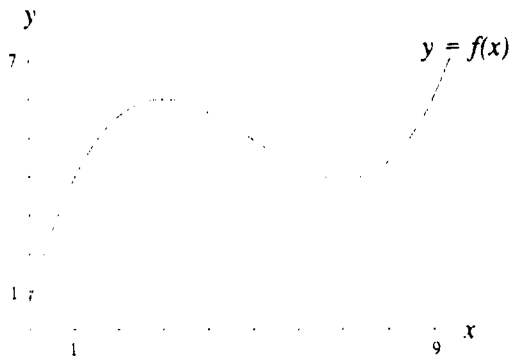

Figure 1.

<u>Problem 2.</u> *Figure 1 is the graph of a function $y = f(x)$. Use the graph to estimate these values as accurately as you can:*

a. $f(3)$ *b.* $f'(7)$ *c.* $f'(1)$

d. For which values of x in the interval $[0,9]$ is $f'(x)$ negative?

The proportion of students in our study who correctly answered a group of questions about a graph, such as this, ranged from 10% (in a course where most students had completed just one semester of calculus) to around 60% (in a course for students who had completed the standard three-semester science and engineering calculus sequence). Again, we were very surprised that so many college juniors had difficulty with such questions.

Our research addresses the extent to which emerging college juniors have the quantitative skills required for success in their chosen upper-division courses. This article describes our quantitative assessment project, its findings, and its impact. Our assessment project may be of special interest to mathematicians because it has been implemented by mathematicians and statisticians, not assessment professionals. The process has several important advantages over more familiar, standardized external assessments: (a) It is faculty driven, promoting faculty involvement and reflecting faculty needs (see [2] and [4]), (b) the assessment *process* and its *outcomes* contribute equally to instructional improvement; (c) it has a specific focus and is tied to particular courses; (d) it reflects the educational choices and needs of students; and (e) it provides useful information both for participants and for departments and campus administrators.

The two problems given above may already have raised some questions (or doubts). For instance:

- ***Are the reported results representative of college juniors generally?*** Although our approach is closely tied to specific courses, repeated assessments reveal similar patterns in a variety of settings. We are confident that our findings are at least representative of undergraduates at this university, and most likely they apply more widely.

- ***Why use these problems—what makes these quantitative skills important?*** All test problems are chosen by upper-division undergraduate course instructors as representing skills needed for success in their course. The items may not always reflect what mathematicians or statisticians believe is important, but they definitely reflect expectations of faculty members in other departments.

- ***Is this another discouraging report of the inadequacies of American college students? If so, do you have specific suggestions to help faculty members deal with the problems?*** Yes, we have found some discouraging information about the capabilities of undergraduates, and the project has identified areas of the undergraduate curriculum that should be improved. But we believe that individual faculty members and departments will respond effectively to this appraisal of their students' capabilities. Precisely because it is so closely tied to the courses they offer, the information we provide should be useful as they plan their responses to problem areas we uncover. A side benefit is that our assessment process encourages faculty members to reflect on and discuss with colleagues the goals of their instruction.

Two Roles for the Math Department

Mathematics has important roles in the undergraduate curriculum that provide the context for our assessment work. On most college and university campuses the subject is considered, to some degree or another, an essential component of the general education of every student; only English has a comparably ubiquitous role in the undergraduate curriculum. Because of this, mathematics faculty members have dual instructional roles in their involvement with introductory undergraduate mathematics. On the one hand, the first two years of the curriculum provide the groundwork for the upper-division and graduate work of those who will specialize in mathematics. Mathematicians, like scholars in other disciplines, are best qualified to decide what preparation their students need to pursue advanced study in mathematics. On the other hand, mathematics also functions in a service role, which often requires much of the department's instructional resources. Typical of this imbalance, our department recently had about 250 declared undergraduate mathematics majors, while more than 6500 students were enrolled in courses up to introductory linear algebra. A significant problem mathematicians face in undergraduate programs is to design instruction that fulfills both instructional roles—preparing majors and serving the quantitative needs of other departments.

This balancing act is a challenge because the roles are not necessarily compatible; tradeoffs must be made. For example, should mathematics departments use specialized, parallel courses, requiring students to make an early (perhaps premature) decision about the direction of their academic career? Or should general introductory courses be used, preserving student options and, perhaps, conserving limited instructional resources—but at the expense of the level of preparation for specific disciplines?

Our Assessment Project

Like many institutions of higher education in this era of *accountability* (see [1]), those in the University of Wisconsin system were directed to implement an assessment plan targeting the quantitative and verbal capabilities of emerging juniors. The faculty committee that worked to meet this Regents' directive on the Madison campus decided to break from existing quantitative and verbal assessments that measure all students on a common scale (e.g., commercial standardized tests). Considering the tremendous range in the quantitative backgrounds of college juniors (from no collegiate mathematics to advanced undergraduate course work), the group instead sought ways to tie assessment to student backgrounds. This was accomplished by linking assessment to specific courses.

Perhaps it was not too surprising, considering prevalent methods of classroom assessment, that the group selected classroom tests to generate the required information. We developed an "itembank" of problems covering material from undergraduate mathematics courses (through introductory differential equations) and from basic statistics. Instructors, selected from a range of departments, use a questionnaire linked to the itembank to identify the specific quantitative skills their students need. The students are then given a test at the start of the semester designed to determine whether they have these skills. We discovered that the careful design of these assessment tests is crucial and have tailored our original plans for an "automated" test-generating procedure accordingly.

About 10 custom-designed free-response (i.e., *not* multiple choice) tests have been given to 300 or more students, mostly juniors, each semester since Fall 1990. The tests, usually given during the second week of classes, assess the extent to which students possess those quantitative skills that their instructors (a) identify as essential for survival in the course, (b) expect students to have from the first day of class, and (c) will not cover during the course. For example, both problems 1 and 2 above met these criteria in several courses. Our role as assessors is to ensure that each test reflects what students in a course need to know. The tests are intended to be neither "wish lists" nor comprehensive examinations of the content of prerequisite mathematics courses.

Corrected test papers are returned to students, along with solutions and specific references for remediation, within one week. Instructors receive information about the students' test performances a few days later. Thus, early in the semester both students and instructors possess useful information about instructor expectations, student capabilities, and the need for any corrective action. We have developed a reliable grading system that allows mathematics graduate students, with limited training, quickly to record information about the students' work and their degree of success on each problem. The coding system provides detailed data for later analysis while allowing the quick return of corrected papers to the students.

Information of two kinds is generated by our assessment process: (a) a detailed picture of those quantitative skills needed for upper-division course work in other departments and (b) an assessment of the quantitative capabilities of emerging juniors outside the context of specific mathematics courses. The first comes from our personal contacts with faculty members as we design the test and interpret the results; the second is provided by analysis of students' performance on the assessment project test and their quantitative backgrounds as shown by university records.

Departmental Needs and Student Capabilities Revealed by Assessment

More than 3000 undergraduates have taken an assessment project test over the first five years of operation. Broadly, we find that instructors do expect their students to have certain skills covered in prerequisite mathematics or statistics courses and that many, even most, students have adequate skills to meet the quantitative demands of their chosen upper-division courses. While the project has not revealed gross mismatches at this institution between instructor expectations, the content of prerequisite courses, or student capabilities, specific areas require some attention. We should caution that the natural inclination to focus on shortcomings, as we did at the start of this article, may convey an exaggerated impression of student unpreparedness.

University transcripts provide supplementary information related to student performance on assessment tests. We discovered a group of students who apparently avoid any college courses with a quantitative component judging by their transcript information, as well as written student and instructor comments on tests and follow-up questionnaires. This behavior may not be inherently undesirable. But taken along with the poor performance on routine, basic statistics and arithmetic tasks needed in their chosen, nonquantitative courses, this finding raises two important questions: Is there a base level of quantitative literacy that should be required of all baccalaureate students? If so, how should these requirements be set and met? A university curriculum committee pursued these questions, ultimately recommending a university-wide quantitative degree requirement. This is probably the clearest example of our assessment work's broad curricular impact.

Three levels of quantitative expectations. There seem to be three levels of quantitative expectations for students in upper-division undergraduate courses. *Level 1* courses, such as *Principles of Advertising* and *Construction of Classroom Tests*, lack any formal quantitative prerequisites. Their instructors expect basic statistical and arithmetic skills along with the ability to read and interpret information presented in tables and graphs. Sometimes Level 1 courses also draw on basic geometric and algebraic capabilities. All these quantitative skills are found in high school curricula. Whether students have these skills is of concern to mathematicians, but they have no place in the college mathematics curriculum.

Student capabilities in courses at the other two quantitative levels depend much more on introductory college mathematics and statistics courses. *Level 2* courses, such as *Finance* and *Quantitative Methods in Agricultural Economics*, require a semester of calculus and perhaps a first course in statistics. *Level 3* courses usually require three semesters of calculus. Examples of Level 3 courses that we have assessed include *Biophysical Chemistry*, *Circuit Analysis*, *Techniques in Ordinary Differential Equations*, and *Mechanics*.

Patterns of student performance on quantitative tasks. Our results in several Level 1 courses revealed gaps in the curricula of the mathematics and statistics departments. That is, students who had difficulties with problems on the assessment tests could not be directed to existing courses that cover the material. For example, an early and surprising discovery was that so many students could not solve problem 1 above. This skill from basic statistics is important to understand experimental results reported in a journal article. Students in Level 1 courses are usually successful with direct computations (e.g., converting temperatures from Fahrenheit to Celsius using a provided formula) and one-step problems (e.g., using the table in problem 1 to say how many students did not read either magazine). Many students at this level run into difficulty when asked to relate information logically, as in problem 1, or to extract information and devise a strategy to use it. Problem 3 is another example of such a task, involving percentages.

Problem 3. *An advertising company is planning the layout for a full page ad in a magazine with 8"x10½" pages. The cost of the ad depends partly on the amount of printed space (excluding margins), so they want to compare the printed area if they use no margins to the printed area using 1" margins. What percentage of the full page will be printed if they use one-inch margins all around as shown in figure 2?*

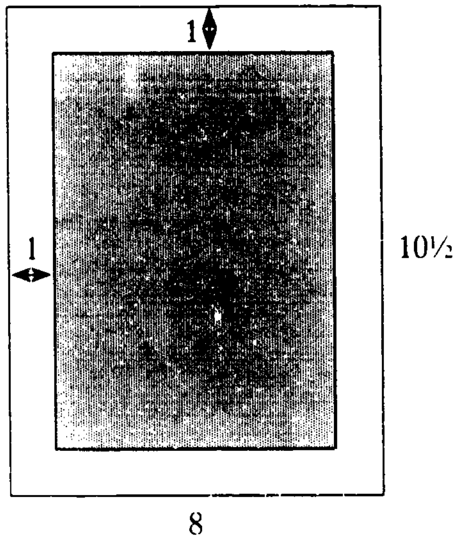

Figure 2.

In one Level 1 course, 20% of the students answered this question correctly; in another about one-third got it. The difficulty is less in computing percentages than in deciding what to do with the information that is provided. When the same classes were told "170 is 85% of a number—find the number," three-quarters of the students gave the correct answer.

Most students in Level 2 courses are successful with tasks from statistics and precalculus; few show proficiency with material from calculus. Conceptual tasks, such as deducing information about a function's derivatives based on a graph of the function (problem 2), are not handled well. In most Level 2 courses many students can handle routine tasks, such as finding a derivative, but have little success with other material from calculus.

Students in Level 3 courses also have difficulty with similar, less routine, more conceptual problems. At this level, a common question involves the evaluation of a definite integral. In one Level 3 course many more students could exactly evaluate a definite trigonometric integral symbolically than could accurately estimate its value from a graph. In other courses many students were unable to make numerical or graphical approximations of integrals. To our surprise, only a quarter of the students in a first differential equations course correctly evaluated a convergent geometric series. Generally, Level 3 students are prepared for the quantitative requirements of their chosen courses. Although students may have difficulties with specific problems, instructors from these technical courses report that students usually regain the necessary skills during the semester.

General Conclusions

It seems that instructors often want students to be able to reason independently, to make interpretations and to draw on basic quantitative concepts in their courses; they are less concerned about whether students remember particular algorithms or procedures. These conceptual expectations were confirmed during our meetings with groups of faculty members from other departments. In contrast, students are most successful with routine, standard computational tasks and often show less ability to use conceptual knowledge or insight to solve less standard quantitative problems. Put another way, in the context of our exams many students can do what they have been shown, successfully handling certain kinds of conventional problems (e.g., using substitution or integration by parts); few students seem able to make connections or to solve more novel problems (e.g., estimating an integral's value from a graph or tabular data). Put another way, many students can do what they have been shown, successfully handling certain kinds of conventional problems (using substitution or integration by parts); few students seem able to make connections or to solve more novel problems on their own in the context of our exams (estimating an integral's value from a graph or tabular data).

Another example may illustrate both the general nature of quantitative skills that faculty in other departments often want and the way in which our process, through ongoing discussions with individual faculty members from across the campus, helps to develop very specific and, we believe, accurate information about the needs of other departments. In 1991 an engineering professor asked us to include a Taylor Series problem on his test. As we reviewed the results of the first semester's tests with him, he observed that he had not really made use of Taylor Series during the semester. Instead, he suggested another problem for the second semester that better represented the quantitative capabilities that students needed for his course. Problem 4, proposed by this engineering professor, illustrates a broader pattern that we have found over the years: Faculty more often want their students to have what could be called *quantitative* or *number sense* than particular, specialized manipulative skills.

<u>*Problem 4.*</u> *Use a [non-graphing, scientific] calculator to help sketch a graph of the function* $f(t) = 4e^{-t}\sin t$ *over the interval* $0 \le t \le 2\pi$. *Choose scales so that the graph makes use of all of the provided grid.*

[A 3-inch square grid for graphing was provided with this problem.]

Use your graph and calculator to estimate the minimum value of $f(t)$ *over this interval.*

The engineering instructor wanted students to use *their own selection of output values* and careful reasoning to understand the behavior of a function over an interval —he was more interested in their problem-solving skills than in their ability to recall and apply algorithms. This problem also illustrates an important attribute of our approach: The tests reflect the instructor's quantitative expectations, not what we as assessors (and teachers of mathematics) think students ought to be able to do. No students in the engineering course where this item was used had access to graphing calculators—only a quarter of the students were successful with this problem. In comparison, about 75% of this same engineering class successfully evaluated $\int_0^2 te^{-t}\,dt$.

Another engineering professor, commenting on his students' performance on the assessment test, observed that he was not surprised that his students did well on our test since it contained fairly standard mathematical problems; he felt that students could "turn the crank," but were unable to connect the mathematics to the physical situation being modeled, such as to use an integral to model hydrostatic force.

Impact of Assessment

Many of the specific results we have obtained are quite interesting in themselves, but an important question is whether this work can have any impact on undergraduate curricula and instruction. The quick answer is "yes," and in a variety of contexts. However, the individualized approach we use has not produced, and is not likely to provide, quick, broadly generalizable results that lead to wide scale, dramatic curricular changes. Instead, the ongoing process promotes faculty reflection on the goals and impact of their instruction; changes have been, and are likely to remain, incremental and gradual.

Impact on participants. The clearest potential for impact is in the particular course being assessed. We have seen a variety of responses from faculty and students, ranging from no action and indifference through extra review and restructuring of courses. It seems we have had the least impact on students. In some technical courses many students report studying both before and after the assessment test, but the most common responses are (a) "I already knew this, so why waste my time" and (b) "If I wanted to be a math major I would have taken a math course—leave me alone." The latter is often heard from students who do not appear to have the tested skills. Most students, when questioned at the end of the semester, recognized that the skills were important in their course, but had not chosen to use to use the assessment information to help with preparation for those requirements.

Instructors, on the other hand, have mostly reacted very favorably to the assessment process. Those who do not report making any changes either found from the tests that students had the prerequisite skills or said that they were already aware of the difficulties and had modified their approach to deal with them—the project simply confirmed what they had suspected. In cases where instructor expectations differed from the results they often reported making changes, either omitting reviews that no longer appeared necessary or including additional work to develop important, missing capabilities.

Our experiences show that the design of the assessment procedure has successfully met the goal of being useful and meaningful for participants, although we would like to find ways to increase student gain from the experience. What about impact at the departmental and university level? To what extent can a highly localized approach provide information at a broader level? It appears that we are having success here, too.

Departmental impact of assessment. The potential value to the Department of Mathematics of the data generated by assessment is quite clear. We report annually to the entire faculty, but we have probably had greater curricular influence by targeting our findings at groups responsible for particular levels or groups of courses, particularly precalculus and calculus. Findings from many assessed courses have shown, for instance, that faculty want students to interpret graphical representations. This had not been highly emphasized in mathematics courses. It was somewhat ironic, but instructive, that in an early meeting to discuss our findings with a curriculum group in mathematics one faculty member remarked about a problem like the second one given in this paper, "I'm not surprised students couldn't do that—I never ask such questions in my class." A colleague responded that he thought such tasks were very important and always emphasized such ideas when he taught calculus. Obviously our assessment work, as in this instance, can stimulate valuable discussions about what is and should be covered in introductory mathematics courses. Our findings about graphical representations have led coordinators at this level to encourage instructors to give increased attention to graphical representations of functions. Perhaps more important, though, is what our work shows about the kind of mathematical skills that are needed in other departments: Instructors seem less concerned about computational, algorithmic knowledge than more conceptual, problem-solving capabilities. This has implications for the way that mathematics is taught, the expectations for what students will do, and not just the content of mathematics courses.

Assessment has also influenced participating departments and we have moved recently to improve communication at this level. When assessment reveals problems, an effective approach has been to send a summary report to faculty members in the affected department. Following the written report, we attend a regular faculty meeting to answer questions and discuss the issues raised by assessment. The information we provide could lead to a variety of departmental changes (so not all problems require resolution by the Department of Mathematics). In one department, a faculty member remarked that students claimed they did not realize they would be expected to know material from a prerequisite calculus course in their later course work! A natural response to this situation involves advising: Faculty in the department should ensure that prospective majors understand that prerequisite courses cover important knowledge that will be used later; that prerequisites are not just some sort of hurdle placed in a student's path.

After finding that many students were unable to handle material from calculus, another department beefed up the prerequisite course from first semester business calculus to two semesters of the regular calculus sequence. They did this not because the students needed the additional content, but to ensure that their students had further developed the necessary fundamental ideas by using and reviewing them in later mathematical work.

Campus-wide impact of assessment. The striking quantitative deficiencies of students who seemed to be actively avoiding any courses with quantitative expectations was mentioned earlier. These findings from some Level 1 courses contributed to a university curriculum committee recommendation that all baccalaureate degree programs include a six-credit quantitative requirement. The recommendation was adopted by the Faculty Senate, indicating that our localized procedure can produce information useful at the broadest institutional levels.

How do faculty respond when significant numbers of students do not have necessary skills, quantitative or otherwise? In some cases, we have found, with resignation: "It would be lovely if we required three or even two semesters of calculus. But one will have to do," was the response from one instructor. Another faculty member said he had chosen to completely leave all quantitative material out of his nontechnical course because (a) students lacked the necessary skills and (b) he had plenty of other material to cover. In other cases, faculty carry on regardless: "The students pick up [the necessary skills] as we go, drop, or perish." It seems that another important function for our group is to bring the range of faculty responses to perceived weaknesses in students' backgrounds to the attention of all faculty members. Individuals, especially junior untenured faculty, may feel pressures that lead them to make choices that are not in the best interests of the institution. An example we have encountered more than once is watering down or eliminating important quantitative material in response to perceived student capabilities, student pressure, or even pressure from colleagues. Our assessment work exposes such patterns of responses in their broader institutional context, and we are looking for appropriate ways to respond to such findings. It does seem that this is an important, campus-wide role that quantitative assessment can play.

Assessment has always had a prominent role in the study of mathematics in colleges and universities. With the exception of graduate qualifying examinations, most of this attention has been at the level of individual courses, with assessment used to monitor student learning during and at the end of a particular class. The natural focus of mathematics faculty members is on their majors and graduate students. We have outlined a locally developed procedure that addresses another important but often neglected dimension of assessment in mathematics: student retention of mathematical knowledge over the longer term and in relation to the quantitative needs or expectations of other departments. A recent article on the nature of research in collegiate mathematics education ([**3**]) listed long term retention of mathematical knowledge as an important issue deserving study. Although this sort of assessment has received little attention in the past, it deserves more prominence because it focuses on the important service role played by most mathematics departments. Quantitative assessment also answers the call for more assessment at the broader institutional level ([**1**], [**2**], and [**4**]). We believe that our approach helps departments and faculty address this important aspect of their educational mission.

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