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ABSTRACT

The purpose of this study was to examine the effects of researcher-developed lessons on students' understanding of two- and three-digit numeration. Digit-correspondence tasks, often used for individual interview assessment of place value understanding, were adapted to be used as problem-solving tasks. The tasks were presented to three classes, grades 3-5. Students were given ample opportunities, in cooperative groups and as a whole class, to discuss and exchange points of view. In the selected classrooms the social norms established by the teacher encouraged such exchanges. A scoring rubric was developed for a whole-class, digit-correspondence task requiring individual written responses. Only 18% of the students were successful on the preassessment. Of the 58 students who were initially unsuccessful, 71% were successful after the instructional intervention as measured by a delayed postassessment. (Author/MKR)

Place-Value: Problem-Solving and Written Assessment Using Digit-Correspondence Tasks

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PLACE-VALUE: PROBLEM-SOLVING AND WRITTEN ASSESSMENT USING DIGIT-CORRESPONDENCE TASKS

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The purpose of this study was to examine the effects of researcher-developed lessons on students' understanding of two- and three-digit numeration. Digit-correspondence tasks, often used for individual interview assessment of place value understanding, were adapted to be used as problem-solving tasks. The tasks were presented to three classes, grades 3-5. Students were given ample opportunities, in cooperative groups and as a whole class, to discuss and exchange points of view. In the selected classrooms the social norms established by the teacher encouraged such exchanges. A scoring rubric was developed for a whole-class, digit-correspondence task requiring individual written responses. Only 18% were successful on the preassessment. Of the 58 students initially unsuccessful, 71% were successful after the instructional intervention as measured by a delayed postassessment.

In digit-correspondence tasks, students are asked to construct meaning for the individual digits in a multidigit numeral by matching the digits to quantities in a collection of objects. As measured by such tasks, even in the fourth and fifth grades no more than half the students demonstrate an understanding that the "5" in "25" represents five of the objects and the "2" the remaining 20 objects (Kamii, 1982; Ross, 1986; Ross, 1990).

Constance Kamii has argued that young students' developing understanding of place value is eroded by traditional algorithmic instruction in addition and subtraction, where individual digits are all treated as "ones" (Kamii & Lewis, 1993). Significant gains in conceptual understanding of place value have been demonstrated among first and second grade children participating in full-year studies where students are encouraged to invent their own methods for multidigit addition and subtraction (cf. Fuson & Smith, 1994; Hiebert & Wearne, 1992; Kamii, 1989).

In this study we examined the learning among older students who in prior grades had experienced traditional algorithmic instruction for multidigit addition and subtraction. In earlier work we had individually interviewed numerous students using digit-correspondence tasks, and had often wondered how children would react if they heard the ideas of other students and had an opportunity to react. We designed this study with two questions in mind. What would students learn from their peers if they share their thinking about the meanings of the digits in digit-correspondence tasks? What could be learned about student thinking by examining their individual responses to a written whole-class assessment instead of by using individual interviews.

Method

We worked in a fifth grade classroom of 22 students, a fourth grade classroom of 20 students, and a combination class that included 19 third and 10 fourth graders. The three heterogeneous classrooms were selected because of the teachers' experience and expertise with problem-based instruction. The teachers had successfully established social norms to encourage students to exchange points of view with respect to their mathematical thinking. All three teachers had worked collaboratively with the university-based research team over a period of three years in a grant-supported teacher leadership program. In the program they studied constructivist theories of learning mathematics and collaboratively designed curriculum and practiced instructional strategies to be consistent with those theories.

The instructional intervention was conducted in February and March, when students were accustomed to the classroom routines and to problem-based instruction. We were in each classroom over a period of five or six consecutive days. Written assessment tasks and four 90-minute lessons were presented.

Written Digit Correspondence Assessment Tasks

We designed tasks that could be administered to the whole class, rather than in individual interviews. Each student received a picture of 35 objects. Aided by an overhead projector transparency of the picture, the researcher elicited a consensus that the number of objects in the picture was 35. The researcher then wrote "35" on the transparency and said "Thirty-five stands for the 35 beans (or squares). She then circled the "5" in one color and asked the students to do the same. She then asked, "What does this part of 35 have to do with how many beans are in the picture? Write down what you think and color the picture to show what you mean. After allowing for response time, she circled the "3" with another color, asked students to do the same and asked "How about this part? What does THIS part have to do with how many beans are in the picture?"

For the preassessment, the picture of 35 objects arranged in a rectangular five-by- seven array. A second version, picturing 35 objects in an ungrouped collection, was administered at the close of the instructional period, and again in June which was three months after the instruction.

Lessons

Each lesson began with a problem-solving task to set the stage for the digit-correspondence (experimental) task, which was to decide what the parts of the number had to do with how many objects are in a collection. The stage-setting tasks were designed to reflect typical intermediate-grades curriculum (topics included area, multiplication, and division), and to provide entry for all students. Manipulative materials and/or drawings were part of all the tasks. A set of detailed lesson descriptions including samples of student work is available from the authors.

144 Squares. Students were asked to decide, in groups, whether or not three gridded paper shapes were the same amount of paper (area). The rectangular shapes were 12cm x 12cm (144), 13cm x 11cm (143), and a shape 6cm x 24 cm with one square centimeter cut off each corner (140). Students reached consensus that the yellow square was the largest, with an area of 144 square centimeters. In the digit-correspondence task, students were asked "what does this part of 144 (circling each individual digit beginning with the 4 in the units place, then the tens digit and finally the "1") have to do with how many square centimeters are in the yellow shape?" We provided each group a transparency picture of the 12 x 12 square for preparing their presentation to the whole class.

124 Cubes. In this lesson we used a "factory" context of filling orders for cubes. Base ten blocks were available as models for cubes "prepackaged" in sets of ten and one hundred. We asked, "How many ways can you fill an order for 124 cubes?" Making a list was modeled as a problem-solving strategy. For the digit-correspondence task, each group was assigned one of the non-standard ways to fill the order (e.g., seven long packages and 54 individual cubes) and asked to "decide which blocks would fill the order for each of the three parts of the number" (digits).

26 Wheels. Students were asked to determine how many toy wheels were contained in a clear plastic bag, based on the following two clues: "There are enough for six cars. There are two left over." After students reached a consensus that there would be 26 wheels we asked what each part of 26 (the "6" and then the "2") had to do with "how many you have." A diagram of the six cars (each with its four wheels) and the remaining two wheels was provided each student as they worked individually and a transparency version was provided to each group.

62 Wheels. "If each car has four wheels, how many cars can be fitted with 62 wheels?" After arriving at a consensus of 15 cars, we asked, What does each part (the "2" and the "6") of 62 had to do with how many wheels you have? Students made their own drawings.

Typically, a member of the research team presented the task, researchers and the classroom teacher circulated among groups during the cooperative-group work time, and the classroom teacher led the whole-class discussion while groups presented their results on overhead transparencies. Teachers used questions and comments to focus attention on differences and similarities among the ideas presented, and often asked students to elaborate by showing with the picture. Special care was taken to provide neither any direct instruction about the "tens and ones" meanings of the digits nor any judgments about the correctness of the ideas presented.

Transcripts of the lessons were based on note-taking by a trained observer and audio recordings. All individual written work and group work, which was usually in the form of overhead projector transparencies, were collected for analysis.

Results and Discussion

We sorted the individual written assessment papers into categories of similar responses and developed a descriptive rubric of nine distinct categories. Reading the 71 preassessment papers was discouraging. Twelve students failed to respond to the questions, and 32 invented meanings that gave no hint of the "3" representing 30. One student gave the response that the "5 meant five squares and the "3" stood for three squares. All the other students attempted to account for the whole collection of 35 squares; "rows of five" and "counting by threes" (even accounting for the remainder) were common responses. Fourteen students used the language of tens and ones in their written responses, but we could not be sure they were talking about the collection of squares in the picture or simply describing the names they had learned for the "coloms" (sic). Eight students wrote responses that strongly suggested that they might understand the meanings of the digits, but included no pictorial evidence of a 30 and 5 partitioning. Only five students gave truly convincing written and pictorial evidence of understanding.

With only one or two students in each classroom demonstrating understanding at the beginning of the instruction, we were concerned that there would be insufficient numbers of knowledgeable peers for the social-interaction design to produce changes in student conceptions. However, students found the lessons engaging and were soon immersed in making sense out of the digits. They examined many ideas, and lively debate often occurred as students exchanged points of view about the meaning of the individual digits. The task that elicited the most heated debate was "26 Wheels." One viewpoint was that the "2" stood for twenty wheels (usually in five cars), and that the "6" stood for the remaining six wheels. Other students were equally adamant that the "2" stood for the two wheels left over and the "6" stood for the six cars or the wheels on the six cars.

The responses on the postassessments were generally both more correct and more expansive than those on the preassessment. In the delayed postassessment, 23 students described the "3" in 35 as representing not simply 30, but also as three sets of ten; 10 of the 23 partitioned the accompanying picture into sets of ten while the remaining 13 partitioned it into 30 and 5. An additional 29 students wrote that the digits represented five squares and 30 squares; 15 of these included pictures. We concluded that there might be three reasons for the improvement. One is that students constructed meanings for the individual digits in a multidigit numeral that were more consistent with our place-value numeration system than those they held before the instructional intervention. Another is that they became better at expressing their mathematical thinking after the experience of talking and writing about their ideas, and hearing and seeing other students' ideas. Finally, because relatively few students used the pictures to show what they meant in the preassessment, we chose to prompt the use of coloring the pictures more assertively in administering the postassessments.

Some small groups seemed to get stymied with a single incorrect interpretation because it was the viewpoint of a student in their group who was a respected leader in the classroom. Students in these groups might have benefited more had we changed the groups so that they could have experienced a more fluid exchange of ideas. Although we were constrained to present the lessons on consecutive days, the lessons might have been more effective if spaced across the school year, because teachers change the composition of the collaborative of groups every few weeks.

To evaluate changes in student thinking about the digits, we compared the preassessments with the postassessments in terms of success. All responses that related the "5" in 35 to a set of five objects and the "3" in 35 to the remaining set of 30 objects or three sets of ten objects were defined as "successful." On the preassessment, the work of only 13 of the 71 students (18%) demonstrated that they knew that the "3" represented thirty of the objects; these 13 were also successful on both the immediate and delayed postassessments. On the immediate postassessment, 45 additional students (63%) were successful, falling to 41 (58%) on the delayed postassessment. Among those who were initially unsuccessful, 65% of the third graders, 76% of the fourth graders, and 63% of the fifth graders were successful on the delayed postassessment. Two were absent and 15 (21%) remained unsuccessful on the postassessments.

Conclusions

The digit-correspondence instructional tasks used in this study are "worthwhile" as defined in the NCTM Professional Teaching Standards (National Council of Teachers of Mathematics, 1991). By presenting a few digit-correspondence tasks in a problem solving mode and allowing students to exchange points of view, teachers may be able to help more students in grades three through five construct an understanding of the meanings of the digits in a multidigit numeral.

In the NCTM's *Mathematics for the Young Child*, Thompson recommends that teachers use digit-correspondence tasks to interview individual students as a way to diagnose place-value understanding (1990, 106-107). Teachers of older students may find the whole-class, written format described in this study to be a useful alternative.

Understanding place value is important to achieving good number sense, estimating and mental math skills, and to an understanding of multidigit operations. The results of this study contribute to a growing body of evidence that students can construct important mathematical concepts and structures through social interaction and communication with their peers about worthwhile mathematical tasks.

References

Fuson, K. C., & Smith, S. (April, 1994,). *Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans.

Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in mathematics Education, 23*(2), 98-122.

Kamii, C. (1989). *Young Children Continue to Reinvent Arithmetic, Second Grade: Implications of Piaget's theory.* New York: Teachers College Press.

Kamii, C., & Lewis, B. (1993). The Harmful Effects of Algorithms in Primary Arithmetic. *Teaching Pre K-8, 23*(4), 36-38.

Kamii, M. (1982). *Children's graphic representation of numerical concepts: a developmental study.* Unpublished doctoral dissertation, Harvard University.

National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Ross, S. H. (April, 1986). *The Development of Children's Place-Value Numeration Concepts in Grades Two through Five.* Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco CA.

Ross, S. H. (1990). Children's acquisition of place-value numeration concepts: The roles of cognitive development and instruction. *Focus on Learning Problems in Mathematics, 12*(1), 1-17.

Thompson, C. S. (1990). Place value and larger numbers. In J. N. Payne (Ed.), *Mathematics for the Young Child*, (pp. 89-108). Reston VA: National Council of Teachers of Mathematics.